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MEDITATIONES ANALYTICÆ,

AB

EDVARDO WARING,

K

REGIÆ SOCIETATIS ET ACADEMIÆ BONONIENSIS INSTITUTI SOCIO,
ET MATHESEOS APUD CANTAB. PROFESSORE LUCASIANO.

MVSEVM BRITANNICVM

PRIMA EDITIO IMPRIMEBATUR ANNIS 1773, 4 & 5: SECUNDA CUM NONNULLIS ADDITIONIBUS ANNIS 1783, 4 & 5, ET EDITA JUN. 4, 1785.

CANTABRIGIÆ,

Typis Academicis excudebat J. Archdeacon;
Veneunt apud J. Nicholson, Cantabrigiæ; J. Rivington & Filios, Londini; et
J. & J. Fletcher, et D. Prince & Co. Oxon.

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TO

SIR JOSEPH BANKS, BART. PRESIDENT OF THE ROYAL SOCIETY;

Who by his Voyages has greatly contributed to the Increaſe of botanical and natural Knowledge; and on all Occaſions has proved himſelf a zealous Patron and active Promoter of every Science.

TO THE

REV. NEVIL MASKELYNE, D.D. F.R.S. AND ASTRONOMER ROYAL;

Diſtinguiſhed for his Knowledge in Optics and phyſical Aſtronomy; who after proving by his Obſervations in two Voyages the Certainty of the lunar Method in finding the Longitude at Sea, recommended it to the Board of Longitude; and ſtrenuouſly aſſiſted them in bringing it into Practice.

AND TO

JOHN WILSON, ESQ. F.R.S. AND KING'S COUNCIL IN THE LAW;

Who was in early Life my Defender, and during a Period of twenty-five Years has been my Friend and Counſellor; from whom in mathematical Enquiries I have received greater Aſſiſtance than from any other Perſon, and than whom I know no one poſſeſſed of more Knowledge or Acuteneſs in theſe Studies.

THE FOLLOWING WORK

IS WITH THE GREATEST ESTEEM INSCRIBED,

BY THEIR VERY HUMBLE SERVANT,

E. WARING.

PRÆFATIO.

CUM langueſcere jam apud noſtrates videantur ſtudia mathematica, & juventutem academicam ab iiſdem dehortari non ceſſent plerique, mirum ſane erit me tempus & pecuniam in conſcribendis operibus mathematicis & imprimendis impendere, nec labori illaudato parcere; argumenta quibus me tuear ſollicite non quæro, hoc conſolans, me officii causâ hæc ſcripſiſſe. Munus quod nactus eſſem profeſſorium erat ornandum, & inſtituti ratio poſtulabat, ut matheſeos fines pro virili latius proferrem, ejuſque partes, quantum in me eſſet, defenderem: rem interea lectori mathematico non ingratam me facturum eſſe arbitror; ſi hìc præfationis loco, diſquiſitionum, quæ in hoc libro contineantur, brevem ſubnectam hiſtoriam ortûs & progreſsûs.

Parabolæ quadraturam & approximationes ad areas curvarum per polygona curvis inſcripta & circumſcripta invenerunt antiqui, ſed notatione algebraicâ egentes haud multa præclara in hâc mathematum parte efficere potuerunt.

Archimedes invenit limites inter quæcunque parallelogramma inſcripta & circumſcripta curvis, etiamque inter inſcriptos & circumſcriptos cylindros ſolidis a rotatione curvarum circa axes ſuos generatis: rationes horum limitum a *Walliſio* & *Neutono* dicuntur ultimæ rationes.

Keplerus fingit curvam ex infinitis punctis conſiſtere, & ejus ordinatam maximam vel minimam evadere, cum per gradus inſenſibiles varietur: *Cavallerius* lineas in indiviſibiles partes diviſit; & exinde deduxit aream curvæ, cujus æquatio eſt $y = ax^n$, ubi n eſt integer numerus.

Carteſius invenit methodum ducendi tangentes ad curvas, circulis ductis, qui curvam tangerent.

Huddenius dedit regulam, ex quâ dici poſſit, utrum duæ radices datæ æquationis ſint inter ſe æquales, necne; & exinde deduxit methodum de maximis & minimis inveſtigandis rationalium quantitatum algebraicarum.

Walliſius, *Fermatius*, *Barrovius*, *Sluſius*, aliique invenere proportionalia incrementa abſciſſæ & ordinatæ ex principiis quæ nunc uſitata ſunt in methodo incrementorum vel fluxionum; hæc vero principia ad irrationales quantitates nunquam ab iis applicata fuere, ſemper enim ita reducebant æquationem, ut in eâ nulli irrationales termini contineantur: *Walliſius* ex interpolatione invenit aream curvæ, cujus æquatio ſit $y = ax^{\frac{n}{m}}$, animadvertit etiam rectangulum ex ordinatâ in fluxionem abſciſſæ fluxioni areæ curvæ eſſe æquale.

Inter *Joannem Bernoulli*, *Keilium*, *Leibnitzium* & *Neutonum* agitata fuit quæſtio, utrum *Neutonus* an *Leibnitzius* primus invenerit fluxionem, vel, quod idem eſt, incrementum irrationalis quantitatis abſque ejus irrationalitatis exterminatione. *Neutonus* in epiſtolâ ad *Collinſium* datâ Dec. 10, 1672, primum quod ſcio, profitetur ſe hujuſce problematis ſolutionem cognoviſſe; in epiſtolâ ad *Oldenburgum* cum *Leibnitzio* communicandâ idem profitetur, ſed haud conſtat illum ejus methodum alicui communicaſſe. Ex epiſtolis *Leibnitzii* dilucide conſtat illum ejus ſolutionem prædictis temporibus haud detexiſſe; in epiſtolâ ad *Oldenburgum* 21 Junii, 1677, ſolutionem prædicti problematis primum dedit *Leibnitzius*. Ex his manifeſto conſtat *Neutonum* primum detexiſſe methodum inveniendi fluxionem irrationalis quantitatis; confitendum eſt tamen nullum teſtimonium extare, ex quo credibile ſit *Leibnitzium* eandem proprio marte haud detexiſſe; nec aliquid in hoc mirabile videtur; res quidem in eo erant, ut ulterior vix ullus in hâc matheſeos parte daretur progreſſus ſine hujuſce problematis reſolutione; nec fuit inveſtigatio difficilis, quæ hujuſmodi eſt: ſit irrationalis quantitas, cujus incrementum requiratur, $(a + bx + cx^2 + \&c.)^m$; ſcribatur v pro $a + bx + cx^2 + \&c.$, & reſultat $(a + bx + cx^2 + \&c.)^m = v^m$; ſed incrementum quantitatis v^m prius inventum fuit mv^{m-1} ductum in incrementum quantitatis v, at $v = a + bx + cx^2 + \&c.$ cujus incrementum prius datum fuit $b + 2cx + \&c.$ in incrementum quantitatis x; in quantitate $mv^{m-1} \times$ incre. quantitatis v pro v & ejus incremento ſcribantur eorundem prædicti valores, & conſit problema: nemo poteſt eſſe teſtis in ſuas partes, is mihi ſemper

per dicendus eſt inventor, qui primus evulgaverit, vel ſaltem cum amicis communicaverit; vix enim inveniatur aliquis dignus mathematici nomine, qui de ſuo ingenio multa a prioribus ſcriptoribus reperta ipſe haud detexerit.

Fluxiones ſecundi, &c. ordinis ex primis per eundem modum ac primas ex earum fluentibus inveniri poſſe docuere *Neutonus*, aliique; ille primus dedit fluentem fluxionis $(a + bx^n)^m x^{\theta-1} \dot{x}$: deinde *Cragius* animadvertendo eandem irrationalitatem in fluente & fluxione contentam eſſe, ex aſſumptis generalibus terminis pro coefficientibus invenit fluentem fluxionis $(a + bx^n + cx^{2n} + \&c.)^m x^{\theta-1} \dot{x}$: fluxionem ad alteram formulam reducendam eſſe, ſi modo plures ejus factores ſint inter ſe æquales primus obſervabat *Joannes Bernoulli*; deinde *Neutonus* inveſtigavit fluentem fluxionis $(a + bx^n + cx^{2n} + \&c.)^l \times (e + fx^n + gx^{2n} + \&c.)^m x^\theta \dot{x}$; & animadvertit, ſi plures factores datæ fluxionis habeant inter ſe communem diviſorem, reducendam eſſe datam fluxionem ad diverſam formulam; quodſi ordinata ſit fractio rationalis irreducibilis cum denominatore ex duobus vel pluribus terminis compoſito, reſolvendum eſſe denominatorem ad diverſos ſuos omnes primos; & ſi diviſor ſit aliquis, cui nullus alius eſt æqualis, tum fluentem exprimi nequire: in hoc libro obſervatur, ſi P ſit algebraica functio quantitatis x, dimenſiones quantitatis x in P majores eſſe per unitatem quam dimenſiones ejuſdem quantitatis in π, ubi $P = \pi \dot{x}$; ni dimenſiones quantitatis x in P nihilo ſint æquales, in quo caſu dimenſiones quantitatis x in P majores erunt quam dimenſiones quantitatis x in π per quantitatem majorem quam unitatem; hinc facile ſequitur, ſi dimenſiones quantitatis x in numeratore π fluxionis $\frac{\pi}{\rho} \dot{x}$ ſint minores per unitatem quam ejus dimenſiones in denominatore, fluentem fluxionis $\frac{\pi}{\rho} \dot{x}$ in finitis terminis non exprimi poſſe. *Neutonus* dicit omnem ordinatam duobus modis in ſeriem reſolvi poſſe, nam index vel affirmativus eſſe poteſt vel negativus, & tentandus eſt uterque caſus, & ſi ſerierum alterutra tandem abrumpitur, habebitur area curvæ in finitis terminis; hic animadvertitur, quod, ſi una ſeries abrumpatur, rebus recte diſpoſitis abrumpetur altera; etiamque docetur methodus inveniendi fluentes fluxionum hujuſce formulæ $(a + bx^n + \&c. + (e + fx^n + \&c.)^m)^r \times x^\theta \times (A + Bx^n + \&c. + (e + fx^n + \&c.)^{m-1} \times (g + \&c.))\dot{x}$ ex principiis, iis a *Cragio* traditis haud multum diſſimilibus; & obſervatur, ſi fluxio quantitatis $a + bx^n + \&c.$ per $(e + fx^n + \&c.)^m$ diviſa æqualis ſit fluxioni quantitatis $(e + fx^n + \&c.)^m$ per $a + bx^n + \&c.$ diviſæ, pro fluente aſſumendam eſſe $(a + bx^n + \&c. + (e + fx^n + \&c.)^m)^r \times \&c.\ \&c.$, & non $(a + bx^n + \&c. + (e + fx^n + \&c.)^m)^{r+1} \times \&c.$; aſſeritur etiam in fluentibus detegendis eaſdem radices omnis irrationalis quantitatis uſurpandas eſſe; & e ſubſtitutione datur methodus inveſtigandi plures fluentes ejuſdem fluxionis inter diverſos variabilis valores contentas, quæ inter ſe ſunt æquales.

Cragius primum fluentes quarundam fluxionum fluentes involventium methodum inveniendi docuit; nempe fluentem fluxionis $\dot{P}\int.\pi\dot{x}$ eſſe $P\int.\pi\dot{x} - \int.P\pi\dot{x}$; eandem perfecit *Joannes Bernoulli* per ſeriem $\int.n\dot{z} = nz - \frac{z^2\dot{n}}{2\dot{z}} + \frac{z^3\ddot{n}}{2.3\dot{z}^2} - \frac{z^4\dddot{n}}{2.3.4\dot{z}^3} + \&c.$ ex hâc ſerie eruit $\int.\frac{\dot{t}}{1+t^2} = t - \frac{1}{3}t^3 + \frac{1}{5}t^5 - \&c. = \frac{t}{1(1+t^2)} + \frac{1.2t^3}{1.3(1+t^2)^2} + \frac{1.2.4t^5}{1.3.5(1+t^2)^3} + \&c.$ & ſic $\int.\frac{\dot{x}}{\sqrt{(1-x^2)}} = \frac{x}{1(1-x^2)^{\frac{1}{2}}} - \frac{1}{3}\frac{x^3}{(1-x^2)^{\frac{3}{2}}} + \frac{1}{5}\frac{x^5}{(1-x^2)^{\frac{5}{2}}} - \&c.$ unde $\int.\frac{\dot{x}}{\sqrt{(1-x^2)}}\int.\frac{\dot{x}}{\sqrt{(1-x^2)}} = \int.\frac{x\dot{x}}{1-x^2} - \frac{1}{3}\int.\frac{x^3\dot{x}}{(1-x^2)^2} + \frac{1}{5}\int.\frac{x^5\dot{x}}{(1-x^2)^3} - \&c.$ poſtea vero *Neutonus* tradidit fluentem fluxionis $\dot{x}\int.\dot{x}\int.\dot{x}\int.\dot{x}\ldots\int.\dot{x}\int.\dot{x}\int.y\dot{x}$, quæ facile e prædictis methodis acquiri poteſt. *Eulerus* dedit fluentem fluxionis $\int.x^\alpha\dot{x}\int.x^\beta\dot{x}\int.x^\gamma\dot{x}\ldots\int.\frac{\dot{x}}{1-x}\ \&c.$; in hoc libro continentur nonnulla hujuſce generis exempla. Deduxit *Neutonus* ex fluentibus $(l-1)$ datarum fluxionum hujuſce formulæ $x^{\theta \pm rn - 1}\dot{x}(e + fx^n + gx^{2n} + \&c.)^{\lambda \pm s - 1}$, ubi θ, λ, e, f, g, &c. maneant ſemper eædem invariabiles quantitates, r vero & s ſint quicunque integri numeri, & $e + fx^n + gx^{2n} + \&c.$

quantitates (l) nominum, fluentem cujuſcunque fluxionis prædictæ formulæ; ſed hìc obſervatur quoſdam caſus, in quibus λ vel θ ſit integer numerus, excipiendos eſſe; etiámque ſi modo ſit exponentialis quantitas $e^{(a+bx^n+cx^{2n}+\&c.)^m}(a+bx^n+cx^{2n}+\&c.)^{m\pm\nu}x^{\theta\pm\pi n}\dot{x}$, ubi ν & π ſint integri numeri & $a+bx^n+cx^{2n}+\&c.$ ſit quantitas l nominum, tum e fluentibus $l-1$ fluxionum independentium prædictæ formulæ, quæ habent $l-1$ diverſos valores quantitatis ν erui poſſe fluentes omnium fluxionum ejuſdem formulæ: & conſimiles propoſitiones de fluentibus hujuſce generis, quæ in diverſas irrationales quantitates ducuntur; de fluentialibus fluxionibus; de fluentibus fluxionum hujuſce formulæ $(a+bx^n+cx^{2n}+\&c.)^{\lambda\pm\pi}\times x^{\theta\pm\alpha n\pm\beta m}\dot{x}$ vel $(a+bx^n+cx^{2n}+\&c.)^{\lambda\pm\pi}\times x^{\theta\pm\alpha n}\dot{x}\int.(e+fx^n+\&c.)^{\pm\rho}x^{\theta'\pm\beta n}\dot{x}$, ubi π, ρ, α & β ſunt integri numeri; in hoc libro traduntur.

In datâ algebraicâ æquatione relationem inter abſciſſam & ejus correſpondentes ordinatas deſignante pro y ſubſtitutit $\dot{v}$ *Neutonus*; & ex hâc ſubſtitutione in quibuſdam caſibus reſultant æquationes ad areas, quarum fluxiones arearum per functionem algebraicam v in $\dot{v}$ denotantur; quod quidem evenit, cum æquatio inter abſciſſam x & ejus correſpondentes ordinatas y ſit $y^m+ax^ny^r=bx^t$. In Actis Philoſoph. Lond. 1764. a me primum docetur $v^n+(a+bx+cx^2)v^{n-1}+(d'+b'x\,..\,x^4)v^{n-2}+\&c.=0$ eſſe æquationem ad aream (v) curvæ, cujus æquatio relationem inter abſciſſam & ejus correſpondentes ordinatas ſit $y^n\pm(\alpha+\beta x)y^{n-3}+\&c.=0$, ſi modo ea in finitis terminis exprimi poſſit.

Jacobus Bernoulli invenit fluentem fluxionalis æquationis $\dot{y}+Py\dot{x}=X\dot{x}$, ubi P & X ſunt fuctiones quantitatis x.

Eulerus idem principium ad fluxionales æquationes ſuperiorum ordinum applicavit, nempe invenit fluentem & ejus (n) multiplicatores æquationis $\overset{n}{\dot{y}}+a\overset{n-1}{\dot{y}}\dot{x}+b\overset{n-2}{\dot{y}}\dot{x}^2+c\overset{n-3}{\dot{y}}\dot{x}^3\ldots+Ay\dot{x}^n=0$, & *D'Alembert* detexit fluentem fluxionalis æquationis $\overset{n}{\dot{y}}+a\overset{n-1}{\dot{y}}\dot{x}+b\overset{n-2}{\dot{y}}\dot{x}^2\ldots Ay\dot{x}^n=X\dot{x}^n$; in hoc libro traditur methodus inveniendi, annon fluxionalis æquatio reddi poteſt integrabilis ex ejus multiplicatione in functionem quantitatum x & $\dot{x}$.

Joannes Bernoulli primum invenit fluxionem exponentialis quantitatis x^x; dedit regulam pro inveniendo fractionis $\frac{P}{Q}$ valore, cum ejus numerator & denominator ſimul evaneſcant, erit enim $\frac{\dot{P}}{\dot{Q}}$ vel $\frac{\ddot{P}}{\ddot{Q}}$, &c.; quæ regula in quibuſdam caſibus fallit; in hoc libro traditur regula pro inveniendo prædicto valore, quæ vix aut ne vix unquam fallit; nempe evaneſcant & numerator & denominator cum $x = a$, pro x ſcribatur $a + v$, & reducantur & numerator & denominator ad ſeriem ſecundum dimenſiones quantitatis v progredientem, & facile conſtabit valor fractionis: eadem principia etiam ad inveniendas differentias, &c. inter quantitates, quæ evadunt infinitæ magnæ, hìc applicantur: invenit etiam curvam, quæ $n - 1$ habet quadrabiles areas; e. g. ſit $y = z + (ax^n + bx^{n-1} + cx^{n-2} + \&c.)^{\frac{r}{s}}$ relationem inter abſciſſam x & ejus correſpondentes ordinatas y exprimens, ubi $\frac{r}{s}$ ſit fractio ad minimos terminos reducta, & $z\dot{x}$ fluxio, cujus fluens inveniri poteſt; & radices vero $\alpha, \beta, \gamma, \delta$, &c. æquationis $ax^n + bx^{n-1} + \&c. = 0$ ſint poſſibiles, tum inveniri poſſunt $n - 1$ quadrabiles areæ curvæ prædictæ inter valores α & β, β & γ, γ & δ, &c. contentæ: inveſtigavit etiam fluentem fluxionis $x^x \dot{x}$ inter valores 0 & 1 quantitatis x contentam $= 1 - \frac{1}{2^2} + \frac{1}{3^3} - \frac{1}{4^4} + \&c.$ docuit methodum detegendi areas curvarum, quarum relationes inter abſciſſas & earum correſpondentes ordinatas per homogeneam vel algebraicam vel fluxionalem æquationem primi ordinis exprimuntur: reduxit fluxionalem æquationem $y^m \ddot{y} = x^n \dot{y}^{2-p} \dot{x}^p$ ad fluxionalem æquationem primi ordinis: invenit etiam curvam omnia puncta contrariæ flexuræ ad fluxionalem æquationem deſignantem in quibuſdam caſibus eſſe algebraicam.

Coteſius invenit radices æquationis $x^n \pm 1 = 0$, ex quibus detexit fluentem fluxionis $\frac{x^{\frac{\lambda}{\mu}n-1}\dot{x}}{x^{2n} \pm ax^n + 1}$.

Manfredi dedit fluentem fluxionalis æquationis primi ordinis, primi vero gradus.

Riccati dedit caſus, in quibus fluens fluxionis $\dot{y} + ay^2\dot{x} = X\dot{x}$ innoteſcit.

Brook Taylor transformavit datam fluxionalem æquationem, in quâ x fluit uniformitur, in alteram, in quâ y fluit uniformiter.

Mac Laurinus invenit quaſdam fluxiones, quarum fluentes inveniri poſſunt ope ellipticorum & hyperbolicorum arcuum, de quâ re poſtea ſcripſere *Eulerus*, *Le Grange*, & alii; at minus utiles ſunt hæ inveſtigationes, quoniam nullæ dantur tabellæ, quæ exhibeant prædictos arcus.

Comes Fagnanus tradidit fluentes quarundam fluxionalium æquationum primi ordinis, in quibus ſimiliter involvuntur variabiles x & y, & exinde detexit ellipticos arcus, quorum differentia eſt finita quantitas, hujus generis plures adjecit *Eulerus*.

Clairaut invenit fluxionem $p\dot{x} + q\dot{y}$ fluentem recipere, ſi modo $\left(\frac{\dot{p}}{\dot{x}}\right) = \left(\frac{\dot{q}}{\dot{x}}\right)$; *Fontaine* idem perfecit pro fluxionibus $p\dot{x} + p\dot{y} + r\dot{z}$, & invenit $A\left(\frac{\dot{B}}{\dot{x}}\right) - B\left(\frac{\dot{A}}{\dot{x}}\right) + \left(\frac{\dot{A}}{\dot{z}}\right) - \left(\frac{\dot{B}}{\dot{y}}\right) = o$, cum æquatio fluxionalis $\dot{x} + A\dot{y} + B\dot{z} = o$ ſit integrabilis; eadem principia diverſis modis ad fluxionales æquationes primi ordinis applicavere *D'Alembert* & *Eulerus*; conſimilia principia a *Marchione Condorcet* & in hoc libro extenduntur ad fluxionales quantitates & æquationes plures variabiles quantitates involventes & majores ordines habentes.

Eulerus invenit fluentes fluxionum $(a + bx^n)^{m \pm r} x^{pn \pm vn - 1}\dot{x}$ inter valores o vel infinitum & $-\frac{a}{b}$ quantitatis x^n, ubi r & v ſunt integri numeri, ex fluente fluxionis $(a + bx^n)^m x^{pn-1}\dot{x}$ inter eoſdem valores variabilis x contentâ, quod quidem e prædictâ *Neutoni* regulâ erui poteſt, nam pro x in fluente per prædictam *Neutoni* regulam deductâ ſcribantur prædicti valores, & quantitatum reſultantium differentia erit fluens quæſita: hinc, cum generalis fluens acquiri poſſit, non difficile erit fluentem prædictam detegere; ex ſubſtitutione enim ſequitur: inveſtigatio fluentis inter o vel infin. & $\frac{a}{b}$ non magis utilis eſt quam inter quoſcunque duos alios variabilis x valores; in nonnullis caſibus conſtat ejus magis facilis reſolutio; e. g. *Eulerus* invenit fluentem fluxionis $\frac{z^{m-1}\dot{z}}{1+z^{n}}$, inter valores o & infinitum quantitatis $z = \frac{\pi}{n \times \text{ſin.} \frac{n}{m}\pi}$, ubi π denotat $\frac{1}{2}$ peripheriam circuli, cujus radius eſt 1, unde facile deduci poteſt fluens fluxionis $\frac{z^{n+s-1}\dot{z}}{z^{2n}+2lk^{n}z^{n}+k^{2n}}$: *Mac Laurinus* detegit, ſi modo $\mathcal{Q}$, G & P denotent fluentes fluxionum $\dot{x}x^{rn-1}(E+Fx^{n})^{l-1} \times \int. \dot{x}x^{sn-1}(e \times fx^{n})^{k}$ & $\dot{x}x^{rn+sn-1} \times (E+Fx^{n})^{l-1}$ & $\dot{x}x^{sn-1}(e+fx^{n})^{k+l}$ inter valores o & $-\frac{E}{F} = d$ quantitatis x^{n} contentas, $e^{l}d^{sn}\mathcal{Q} = G \times P$.

Simpſon dedit legem ſeriei, quæ deducit $\int. x^{m \pm rn-1}\dot{x}(e+fx^{n})^{\lambda \pm s}$ ex datâ $\int. x^{m-1}(e+fx^{n})^{\lambda}\dot{x}$; ubi r & s ſunt integri numeri.

Le Grange invenit variationem incrementi æqualem eſſe incremento variationis, ſi modo infiniteſima ſint; & deduxit *Lexell*, *Eulerus*, *Le Grange* fluxionales æquationes, quarum fluentes inveniri poſſunt: in hoc opere animadvertitur, ſi incrementum fluxionis $\dot{x}$ æqualis ſit fluxioni incrementi, fluxionem incrementi functionis quantitatis x æqualem eſſe incremento fluxionis.

D'Alembert primus dedit exempla de æquationibus relationes exprimentibus inter quantitates hujuſce formulæ $\left(\frac{\dot{P}}{\dot{x}}\right)$, $\left(\frac{\dot{P}}{\dot{y}}\right)$, &c.; ubi hæ quantitates reſpective denotant fluxiones quantitatis P, cum x, y, &c. ſolummodo habeantur variabiles: de hâc re plurima ſcripſere *Eulerus*, *Le Grange*, *Condorcet*, *Le Place*, aliique; at de his perpauca in hoc libro traduntur: ex datâ reſolutione æquationis prædicti generis $\left(\frac{\dot{m}}{\dot{y}}\right)p + \left(\frac{\dot{p}}{\dot{y}}\right)m = \left(\frac{\dot{m}}{\dot{x}}\right)q + \left(\frac{\dot{q}}{\dot{x}}\right)m$ ſequitur multiplicator cujuſcunque fluxionalis æquationis primi ordinis $p\dot{x} + q\dot{y} = o$; & conſimilia principia etiam applicari poſſunt ad fluxionales æquationes ſuperiorum ordinum. Ex quibuſdam datis formulis multiplicatorum dedit *Eulerus* exempla fluxionalium æquationum, quæ redduntur integrabiles per prædictos multiplicatores: ex datâ ſubſtitutione, quæ ita transformat datam fluxionalem æquationem, ut evadat integrabilis, invenit multiplicatorem, qui datam æquationem reddet integrabilem: docuit methodum inveſtigandi in quibuſdam caſibus, annon data æquatio ſit fluens datæ fluxionalis æquationis: ex datis particularibus fluxionalis æquationis $\ddot{y} + P\dot{y}\dot{x}^2 + Qy^2\dot{x}^2 = X\dot{x}^2$ valoribus, ubi P, Q & X ſunt functiones quantitatis x eruit generalem valorem prædictæ æquationis; invenit fluentem fluxionalium æquationum $\frac{n\dot{y}(e + fy + gy^2 + \&c.)}{\surd(ay^2 + by + c)} + \frac{m\dot{x}(e + fx + gx^2 + \&c.)}{\surd(ax^2 + bx + c)} = o$ & $\frac{\dot{x}}{\surd(a + bx^2 + cx^4)} + \frac{\dot{y}}{\surd(a + by^2 + cy^4)} = o$: dedit etiam plura exempla fluxionalium æquationum, quarum fluentes deteguntur: & ſubſequentem propoſitionem; ſi modo $P = a$, ubi a ſit quæcunque invariabilis quantitas, ſit generalis fluens datæ fluxionalis æquationis, tum erit quæcunque functio quantitatis $P = a$ generalis fluens: nonnulla adjecit de ſubtangente curvæ, cujus æquatio $1 . 2 . 3 .. x = y$ ab *Walliſio* primum tradita fuit. Plura exempla fluentium inter valores o & 1 variabilis x contentarum dedit.

In hoc opere 1. obſervatur, quodſi plures dimenſiones fluxionum $\overset{n}{\dot{y}}$, &c. in datâ fluxionali æquatione contineantur, tum inveſtigatio fluentis exigit reſolutionem æquationis algebraicæ, cujus dimenſiones haud ſunt minores quam prædictæ (n). 2. Adjicitur etiam methodus inveniendi fluxionalem æquationem n ordinis, ex quâ methodo conſtat fluxionalem æquationem n ordinis n recipere diverſas generales fluentes primi ordinis, etiamque n diverſos generales multiplicatores; $n . \frac{n-1}{2}$ generales fluentes ſecundi ordinis; & ſic deinceps: exhinc de-

duci

duci poſſunt plurimæ fluxionales æquationes, quæ fluentes recipiunt. 3. Animadvertitur, ſi generales fluentes fluxionalis æquationis n ordinis ſint $P = a$, $P' = d'$, $P'' = d''$, &c., ubi a, d', a'', ſunt invariabiles quantitates, generalem fluentem datæ fluxionalis æquationis eſſe quemcunque functionem quantitatum P, P', P'', &c. 4. Oſtenditur quantitatem $P = o$ non neceſſario eſſe fluentem fluxionalis æquationis $\alpha \dot{P} + \beta P \dot{q} = o$, cum ſit fluens prioris partis æquationis $\alpha \dot{P} = o$, at non fluens poſterioris $\beta P \dot{q} = o$; quæ eſt quantitas $\beta P = o$ ducta in $\dot{q}$: hic animadvertendum eſt fluxionalem æquationem primi ordinis $p\dot{x} + q\dot{y} = o$ ſemper reduci poſſe ad fluxionalem æquationem $a\dot{P} + bP\dot{c} = o$, ubi p, q, a, b & c ſunt functiones quantitatum x & y, & P eſt quæcunque aſſumpta functio quantitatum x & y: etiamque, ſi modo ſit $y = A \times B \times C \times D \times$ &c. $+ a$, ubi a eſt invariabilis quantitas ad libitum aſſumenda, generalis fluens datæ fluxionalis æquationis; $y = A$, $y = B$, $y = C$, &c. eſſe particulares fluentes fluxionalis æquationis: vel magis generaliter, ſi modo $\phi : (x \,\&\, y) \times \phi' (x \,\&\, y) \times \phi'' (x \,\&\, y) \times$ &c. $+ a = o$ ſit generalis fluens datæ fluxionalis æquationis, $\phi (x \,\&\, y) = o$, $\phi' (x \,\&\, y) = o$, $\phi'' (x \,\&\, y) = o$ eſſe particulares fluentes datæ fluxionalis æquationis; hinc pro particulari fluente quærendæ ſunt prædictæ quantitates $\phi (x \,\&\, y) = o$, &c. vel $\phi (x \,\&\, y) \times \phi' : (x \,\&\, y) \times$ &c. $= o$. Et ſic de fluxionalibus æquationibus ſuperiorum ordinum. 5. Aſſeritur fluentes $x^{-\beta} \int . x^{\beta-\alpha-1} \dot{x} \int . x^{\alpha} P \dot{x} = x^{-\alpha} \int . x^{\alpha-\beta-1} \dot{x} \int . x^{\beta} P \dot{x}$, &c. vel generaliter $x^{-\pi} \int . x^{\pi-\beta-1} \dot{x} \int .^{\beta-\epsilon-1} \dot{x} \int . x^{\epsilon-\gamma-1} \dot{x} \,.\,. \int . x^{\tau} P \dot{x} = x^{-\gamma} \int . x^{\gamma-\beta-1} \dot{x} \int . x^{\beta-\pi-1} \dot{x} \int . x^{\pi-\epsilon-1} \dot{x} \int .$ &c. . . . $\int . x^{\alpha} P \dot{x}$; in hiſce duabus iidem continentur indices π, α, β, σ, γ, ϵ, τ, &c.; & ſi præcedens terminus in utriſque ſit $x^{b-\lambda-1} \dot{x}$, tum ducitur in $\int . x^{\lambda - \&c. - 1} \dot{x} \int .$ &c. 6. Animadvertitur, quod, ſi detur algebraica æquatio $ay^{n} + by^{n-1} + cy^{n-2} + dy^{n-3} +$ &c. $= o$, ubi a, b, c, &c. ſint functiones ipſius x, & curva per prædictam æquationem deſignata haud ſit compoſita e duabus vel pluribus curvis, & fluens fluxionis $\frac{b\dot{x}}{a}$ haud inveniri poſſit finitis terminis, tum curva haud quadrari poteſt: conſimilis propoſitio etiam de fluente cujuſcunque algebraicæ functionis quantitatum y & x in $\dot{x}$ ductæ affirmari poteſt; cujus ſolutio petenda eſt e meis Miſcel. Analyt. anno 1762 editis, in quibus primum docetur methodus inveniendi ſummam e ſingulis valoribus cujuſcunque algebraicæ functionis quantitatum x & y & earum fluxionum; deinde Actis Philoſoph. Londin. 1764, ex datâ algebraicâ æquatione relationem inter x & y exprimente a me datur methodus, e quâ inveſtigari poſſit, utrum fluens cujuſcunque fluxionis, quæ ſit algebraica functio literarum in datâ algebraicâ æquatione contentarum & earum fluxionum, inveniri poteſt, necne. 7. Aſſeritur ſummam fluentium cujuſcunque algebraicæ functionis literæ x & y in $\dot{x}$ ductæ ad quemcunque valorem abſciſſæ x pertinentium exprimi poſſe per finitos terminos, circulares arcus & logarithmos radicum datæ algebraicæ æquationis. 8. Docetur etiam methodus, ex datâ algebraicâ, fluentiali, exponentiali, &c. quantitate, quæ exprimit ſummam ſeriei ſecundum dimenſiones literæ x progredientis, inveniendi ſummam alternorum ſeriei terminorum vel denique ſeriei terminorum, quorum diſtantiæ a ſemet ipſis ſit n, ex iiſdem principiis ac iis, quæ cum pluribus aliis ad Regiam Societatem anno 1757 communicata fuerunt. 9. Circa tres axes ſecum quoſcunque angulos facientes gyretur quæcunque curva, cujus abſciſſæ & ordinatæ ſint reſpective x & y; transformetur data curva in alteram ſubſtituendo pro x, $az+bv+c$; & pro y, $pz+qv+r$; gyretur hæc curva circa axem ſuum; & e datis contentis ſolidorum circa tres axes rotatione datæ curvæ generatorum ſequitur ſolidum generatum a rotatione poſterioris curvæ circa axem ſuum.

Vice-comes *Brounker* eruit ſummas ſerierum $\frac{1}{1\cdot 2}+\frac{1}{3\cdot 4}+\frac{1}{5\cdot 6}+$ &c. $\frac{1}{2\cdot 3}+\frac{1}{4\cdot 5}+$ &c. $\frac{1}{2\cdot 3\cdot 4}+\frac{1}{4\cdot 5\cdot 6}+$ &c. *Gregory St. Vincent* primum dedit $\frac{1}{1-x}=1+x+x^2+$ &c. *Mercator* operando in literis ad eundem modum, quo arithmetici in numeris decimalibus dividunt, invenit $\int.\frac{\dot{x}}{1-x}=x-\frac{1}{2}x^2+\frac{1}{3}x^3-$ &c. idem in extractione radicum perfecit *Neutonus:* eorum anteceſſores in re algebraicâ diviſiones & extractiones in infinitum haud promovebant, nullum perſpicientes uſum, cui per applicationem *Walliſiani* theorematis in quadraturis detegendis inſervire potuerit: *Neutonus* ex binomiali theoremate reduxit binomiales algebraicas quantitates ad infinitas ſeries, & exinde deduxit areas curvarum: quantitates multo magis complexæ ad infinitas ſeries adhuc ſolummodo per vulgares methodos diviſionis & radicum extractionis olim traditas reduci poſſunt: in harum regularum exemplis docuit ſeries vulgares, quibus ex arcu correſpondens ſinus vel coſinus detegitur, & e logarithmo numerus: ſeriem $x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 +$ &e. pro logarithmo primus invenit vice-comes *Brounker*; ſeriem vero $t - \frac{1}{3}t^3 + \frac{1}{5}t^5 -$ &c. pro arcu detexit *Jacobus Gregory*; ſeries $A - \frac{c}{2} \times \frac{1 \cdot A - xP}{2} -$ &c. pro arcu ellipſeos, &c. ex novis principiis in hoc opere primum traditur; ſed animadvertendum eſt in quâcunque fluente detegendâ, convergentes eſſe debere ſeries pro duobus valoribus variabilis datæ ſeriei quantitatis.

Leibnitzius a *Neutono* quæſivit caſus, in quibus ſeries exortæ vel ex *Mercatoris* methodo dividendi, vel ex *Neutonianâ* radicum extractione in infinitum convergent: quod *Leibnitzius* quæſivit, in hoc libro primum peragitur; nunquam convergent ſeries e prædictis methodis deductæ ſecundum dimenſiones quantitatis x aſcendentes, ſi x major ſit quam minima radix æquationum reſultantium ex denominatore vel ex radicalibus in infimos terminos depreſſis nihilo æqualibus eſſe aſſumptis; nec deſcendentes, ſi x minor ſit quam maxima radix prædicta. Convergentia pendet ex ratione quam habent radices prædictæ ad x; e. g. ſeries $m + \frac{1}{2}m^2 + \frac{1}{3}m^3 +$ &c. $= \int. \frac{\dot{m}}{1 - m}$ ſemper converget, cum m minor ſit quam 1; ſeries autem $x - \frac{1}{2}x^2 + \frac{1}{3}x^3 -$ &c. $= \int. \frac{\dot{x}}{1 + x}$, cum x non major ſit quam 1: ſi $m = \frac{x}{1 + x}$; tum erit $m + \frac{1}{2}m^2 + \frac{1}{3}m^3 +$ &c. $= x - \frac{1}{2}x^2 + \frac{1}{3}x^3 -$ &c.; hinc per ſeriem $m + \frac{1}{2}m^2 + \frac{1}{3}m^3 +$ &c. ſemper acquiri poteſt log. quantitatis

$1+x$, utcunque magna ſit quantitas x: ſeries autem $x-\frac{1}{2}x^2+$ &c. divergit cum x major ſit quam 1, ergo ex eâ non deduci poteſt log. quantitatis, quæ major eſt quam 2: quantitatum $\frac{a}{10}$, $\frac{a}{100}$, $\frac{a}{1000}$, &c. (dato logar. numeri 10) logarithmi facile detegi poſſunt ex logarithmo quantitatis a, & hinc ex ſerie $x-\frac{1}{2}x^2+$ &c. detegi poteſt log. quantitatis, cum ea fruſtra ſine tale transformatione quæreretur; & magis convergentes evadent utræque ſeries: eadem etiam applicari poſſunt; cum quantitas, cujus logar. innoteſcit, non ſit 10, ſed quicunque alius numerus: in omni caſu ſeries $m+\frac{1}{2}m^2+\frac{1}{3}m^3+$ &c. magis celeriter converget quam ſeries $x-\frac{1}{2}x^2+\frac{1}{3}x^3-$ &c.: in hoc libro ex transformatione fluxionis ſcribendo in eâ $z+a$ & $\dot{z}$ pro x & $\dot{x}$ oſtenditur, quod fluens inter duas proximas radices prædictarum æquationum contenta ſemper acquiri poteſt; etiamque, ſi quæcunque radices prædictæ inter duos valores quantitatis x, inter quos requiritur fluens, conſiſtant, & fluens haud ſit infinita; tum neceſſe eſt plures interpolare diverſos valores quantitatis a, aſſumendos ita quidem ut quantitas z incipiat ad ſingulas prædictas radices, vel in iiſdem terminet; ſin aliter vero omnes ſeries reſultantes haud erunt convergentes: ſi vero plurimi interpolentur valores quantitatis a inter duas proxime ſucceſſivas radices prædictas ita ut maxime celeriter convergant ſeries, tum erunt differentiæ inter valores prædictos in geometricâ progreſſione; unde ex *Mercatoris* & *Neutoni* methodo fluens ſolummodo detegi poteſt cum utrique valores variabilis quantitatis ſint minores quam minima radix vel negativa vel affirmativa; ſed ex hâc ſimplici transformatione ſemper detegi poſſit, cum prædicti valores inter quaſcunque duas proximas radices interponantur, vel magis generaliter cum ea finita ſit: utrum quæcunque duæ datæ quantitates inter eaſdem duas radices interponantur, necne; plerumque ex earum ſubſtitutione in datis æquationibus e mutatione ſignorum datarum & reſultantium æquationum facile conſtabit.

Taylorus dedit ſeriem $S=y\pm\frac{a\dot{y}}{\dot{x}}+\frac{a^2\ddot{y}}{1.2\dot{x}^2}\pm$ &c.

Eulerus, *Mac Laurinus*, &c. cum approximationes inventæ lente convergant, interpolavit alios terminos ad perparvas diſtantias a ſe invicem poſitos. *Eulerus* advertebat, quod, cum ordinatæ fiant infinitæ, tum haud convergent approximationes ſic inventæ, & ad inveſtigandam aream curvæ inter duas ordinatas poſitam interpolavit plures ordinatas, & deinde ex ſerie $y = \frac{\dot{s}}{\dot{n}} - \frac{\ddot{s}}{2\dot{n}^2} + \frac{\dddot{s}}{2.3\dot{n}^3} -$ &c. inveſtigat ordinatam, & ex ordinatâ per ſeriem $s = \int . y\dot{n} + \frac{y}{2} + \frac{\dot{y}}{12n}$ + &c. inveſtigat aream, &c.

Arabes, *Lucas de Burgo*, *Vieta*, *Harriottus*, *Oughtredus*, *Girard*, *De Lagny*, aliique tractarunt de approximationibus ad radices æquationum, docuere methodos approximationum eaſdem, quibus nunc utimur, cum inveniatur quantitas a radice quæſitâ haud longe diſtans; *Neutonus*, *Vietæ* aliorumque veſtigiis inſiſtens, eadem magis elucide perfecit, & animadvertebat, quod ſi e ſit perparva quantitas quæſita, & data æquatio ſit $A + Be + Ce^2 +$ &c. $= o$, tum ex tribus terminis $A + Be + Ce^2 = o$ nihilo æqualibus eſſe ſuppoſitis ſequitur approximatio ad valorem quantitatis e, quæ duplum præbet numerum figurarum approximationis ex æquatione $A + Be = o$ inventæ; ultimo a me primum oſtenditur veram eſſe hanc propoſitionem, & radices æquationis $A + Be + Ce^2 + De^3 = o$ triplum numerum illarum figurarum præbere; & ſic deinceps.

Eulerus in algebraicâ æquatione $A = o$ pro x ſubſtituit ejus valorem (a) prope, & aſſeruit $\frac{A\dot{x}}{\dot{A}}$ eſſe propiorem valorem; ejus demonſtrationem primum dedit *Courtivron*.

Simpſon invenit continuas approximationes ad radices duarum æquationum duas incognitas quantitates habentium.

Hic animadvertendum eſt omnes has methodos ſolummodo particulares præbere caſus regulæ vulgariter dictæ falſi, quæ in hoc libro magis generaliter redditur.

In hoc libro 1. primum oſtenditur approximationem ex hâc methodo inventam ſemper eo magis accuratam fore, quo propior fuit quantitas pro radice ſubſtituta radici quæſitæ quam reliquis. E. g. ſi quantitas data ſit ferè intermedia inter duas proxime ſucceſſivas radices, i. e. ſit radix æquationis ad limites, tum nova approximatio inventa erit infinita quantitas; minime refert, utrum permagnam habeat rationem ad radicem quæſitam, necne. 2. Traditur methodus ex datis approximationibus ad *m* radices æquationis *n* dimenſionum inveniendi approximationes ad ſingulas prædictas radices magis convergentes. 3. Sit æquatio $x^n - px^{n-1} + qx^{n-2} - \&c. = 0$, & ſi una radix multo major ſit quam *m*, multo vero minor quam $n - m - 1$, traditur nova methodus inveniendi ſeriem radicem prædictam exprimentem. 4. Datur etiam theorema quod dico incrementiale theorema, ex quo ſi modo dentur incrementa e ſingulis variabilibus in datâ quantitate contentis, facile erui poſſunt ejus diverſa incrementa; & ex hoc theoremate quamplurimæ æquationes in algebraicas transformari poſſint, ex quibus approximationes ad radices deduci poſſint; etiamque adjicitur nova methodus ex aſſumptis *m* diverſis valoribus pro radice *x*, & ab eâ haud longe diſtantibus, inveniendi æquationem *m* dimenſionum, cujus radix erit approximatio ad radicem *x*: omnia præcedentia principia etiam applicari poſſunt ad plures æquationes plures incognitas quantitates habentes, etiamque ex iis deduci poteſt una in terminis reliquarum.

Neutonus dedit $(\alpha^{2m} + \beta^{2m} + \gamma^{2m} + \&c.)^{\frac{1}{2m}}$, ubi α, β, γ, &c. ſunt radices datæ æquationis, pro maximâ radice.

Daniel Bernoulli invenit maximam radicem datæ æquationis $x^n - px^{n-1} + qx^{n-2} - rx^{n-3} + \&c. = 0$, ſi modo ea ſit poſſibilis; in hoc libro obſervatur idem perfici poſſe pro omnibus æquationibus irrationales vel plures variabiles quantitates involventibus; docetur etiam conſimilis methodus ulterius promota inveniendi radicem, quæ minor ſit quam maxima, major vero quam omnes reliquæ; & eadem principia ad detegendas omnes reliquas radices promovere liceat. Omnia ea, quæ prius protuli de convergentiis ſerierum ex diviſione & extractione radicum ortarum, æque ad haſce approximationes applicari poſſunt; ſed omnes hæ regulæ vel ſupponunt omnes radices eſſe poſſibiles, vel poſſibiles eſſe majores quam impoſſibiles; at in æquationibus ſuperiorum dimenſionum plerumque numerus impoſſibilium radicum multo major erit quam numerus poſſibilium, & conſequenter probabile erit, ut maxima impoſſibilis radix major ſit quam maxima poſſibilis; ergo regulæ non applicari poſſunt, ni inventum ſit poſſibilem radicem eſſe majorem quam ſingulas impoſſibiles. Datur methodus facile deducendi infinitas æquationes, quarum radices ſunt omnes poſſibiles.

In hoc libro docetur etiam methodus reducendi quantitates prædictas ad ſeries ſecundum dimenſiones quantitatis x progredientes, &c. ita ut earum integrales innoteſcant.

Barrovius in quibuſdam caſibus deduxit ex æquatione relationem inter x & y exprimente approximationem ad quantitatem y in terminis quantitatis x; hanc propoſitionem magis generalem reddidit *Neutonus*, cujus particularem caſum dedit in reverſione ſerierum; eadem principia ad fluxionales æquationes applicavit, ſed in hiſce æquationibus, ſi modo reſolutio ſit generalis, tot ſemper occurrent quantitates ad libitum aſſumendæ, quot ſit ordo fluxionalis æquationis, in quibus caſibus plerumque occurrent quædam difficultates, quarum nonnullæ in hoc libro traduntur; e. g. in generali fluente detegendâ per ſeriem fluxionalis æquationis (n) ordinis ſemper occurrent homogeneæ fluxionales æquationes primi ſecundi, &c. uſque ad n ordinem. *Leibnitzius* primum aſſumpſit quantitatem $y = a x^n + b x^{n+m} + c x^{n+2m} +$ &c. generales coefficientes habentem. *Eulerus* recte inſtituit approximationes ad ſingulas quantitates & earum fluxiones, & ex iis per methodos prædictas approximationes deduxit. In hoc libro 1. docentur methodi adjudicandi, utrum hæ ſeries convergent necne, ferè ex iiſdem principiis ac prius tradita fuerunt pro algebraicis æquationibus, &c. 2. Prima approximatio ex quâcunque datâ hypotheſi ita deduci poteſt. Sit æquatio $y^n - a y^{n-1} + b y^{n-2} -$ &c. $= o$, ubi a, b, c, &c. ſint functiones ipſius x; deinde rejectis omnibus terminis, qui haud maximi reſultant ex datâ hypotheſi in prædictis quantitatibus a, b, &c. reſultet æquatio $y^n - \alpha y^{n-1} + \beta y^{n-2} -$ &c. $= o$; quæ erit æquatio, cujus radices erunt primæ approximationes ad ſingulas datæ æquationis radices; ſi modo quædam ex iis longe diſtent a reliquis, tum ex obſervatione adjunctâ facile acquiri poſſunt; aliter exoritur reſolutio quadraticæ, &c. æquationis: idem etiam perfici poſſit per eundem modum, ſi modo quæcunque irrationales quantitates in datâ æquatione contineantur: eadem principia applicari poſſunt ad plures æquationes plures incognitas quantitates habentes: *Le Grange* invenit legem, quam obſervat reverſio ſeriei ex meâ ſerie pro ſummis poteſtatum; eadem ſeries paulo aliter in hoc libro deducitur; etiamque deteguntur approximationes ad maximam vel minimam datæ algebraicæ æquationis radicem; & invenitur y in terminis ſecundum dimenſiones quantitatis x progredientibus, ſi modo prius ita transformetur data æquatio, ut una radix evadat multo major vel minor quam quæcunque alia datæ æquationis radix.

Briggius vel *Vieta* & *Paſcal*, primum quod ſcio invenit binomiale theorema: id ad radices extrahendas primus applicavit *Neutonus*.

Noſtras *Briggius* reddidit calculum pro logarithmis faciliorem, ex eo quod interpolavit plures quantitates ea methodo quam invenit differentiarum ultimo inter ſe æqualium; & prout primæ, ſecundæ, tertiæ, &c. differentiæ exoriantur æquales, diverſas ex methodus habet leges: hanc rem ulterius proſecuti ſunt *Regnaldus* & *Moutonus*, & hi omnes invenere n differentias poteſtatis n quantitatum in arithmeticâ progreſſione eſſe æquales; deinde ex hujuſmodi quantitatibus datis, quarum differentiæ ſint ultimo æquales, inveſtigavere legem, ex quâ interpolari poſſunt prædictæ quantitates, quæ conſtant ex quantitatibus hujuſce formulæ $Ax^n - ax^{n-1} + bx^{n-2} -$ &c. *Neutonus* idem problema magis generale reddidit, aſſumpſit enim quantitatem prædictæ formulæ $Ax^n - ax^{n-1} + bx^{n-2} -$ &c. $= y$, & pro x & y ſcripſit quaſcunque quantitates p, q, r, s, &c. & A, B, C, &c. & ex æquationibus reſultantibus deduxit quantitatem prædictam; ex hâc methodo duxit curvam algebraicam per quotcunque data puncta: ex aſſumptis quantitatibus hujus formulæ $A + B(x-p) + C.(x-p).(x-q) +$ &c. reſolutionem magis facilem reddidere *Nichole*, *Sterling* & *Walzius*: in hoc libro 1. quædam nova traduntur de hâc methodo differentiarum, de ejus convergentiâ, cum termini haud ultimo evadant inter ſe æquales,

i. e. haud terminetur ſeries. 2. Adjungitur etiam problema ex datâ lege, cui m aſſumptæ quantitates conveniant, n vero haud conveniant, ita corrigere legem, ut n poſteriores etiam ei conveniant. 3. Datur methodus correſpondentium valorum, i. e. ſit $y = \frac{x-q\,.\,x-r\,.\,x-s\,.\,\&c.}{p-q\,.\,p-r\,.\,p-s\,.\,\&c.}\times A + \frac{x-p\,.\,x-r\,.\,x-s\,.\,\&c.}{q-p\,.\,q-r\,.\,q-s\,.\,\&c.}\times B + \frac{x-p\,.\,x-q\,.\,x-s\,.\,\&c.}{r-p\,.\,r-q\,.\,r-s\,.\,\&c.}\times C + \&c. = y$. 4. Datur altera nova methodus differentiarum. 5. Methodus correſpondentium valorum applicatur ad plures caſus in transformatione æquationum ex Medit. Algebr. deſumptos. 6. Eadem etiam applicatur ad datos correſpondentes valores trium vel plurium incognitarum quantitatum, &c.

Brook Taylor invenit incrementum cujuſcunque algebraicæ integralis; idem perficitur pro incremento fluentialis integralis ab *Le Grange*, *Eulero*, &c. & in hoc libro; conſimilia principia etiam applicari poſſunt ad incrementa integralium & fluentialium quantitatum detegenda: integralem incrementi $z\,.\,z+\underset{.}{z}\,.\,z+2\underset{.}{z}\,.\,.\,z+(n-1)\underset{.}{z}$, etiamque incrementi $\frac{1}{z\,.\,z+\underset{.}{z}\,.\,.\,z+n\underset{.}{z}}$ dedit *Taylor: Jacobus Stirling* reduxit omnes quantitates hujuſce formulæ $A + Bz + Cz^2 + Dz^3 + \&c.$ ad formulam $a + bz + cz\,.\,(z+1) + dz\,.\,(z+1)\,.\,(z+2) + \&c.$ & quantitates hujuſce formulæ $A + Bz^{-1} + Cz^{-2} + Dz^{-3} + \&c.$ ad formulam $a + \frac{b}{z} + \frac{c}{z\,.\,z+1} + \frac{d}{z\,.\,z+1\,.\,z+2} + \&c.$ ſi vero termini in genere ſint x^{z+n} in $\frac{a}{z} + \frac{b}{z\,.\,z+1} + \&c.$ ubi z ſit diſtantia a primo ſeriei termino, invenit ſummam $\frac{a}{(1-x)\,.\,z} + \frac{b-Ax}{(1-x)\,.\,z\,.\,(z+1)} + \&c.$ & ex quibuſdam æquationibus inter ſucceſſivos terminos & diſtantiam a primo ſeriei termino invenit particulares valores per vulgarem methodum infinitarum ſerierum.

De Moivre & *Daniel Bernoulli* inveſtigavere ſummas recurrentium ſerierum, & e ſummâ datâ deduxere relationem inter ſucceſſivos terminos.

Monmort transformavit ſeriem $A + \frac{B}{r} + \frac{C}{r^2} + \&c.$ ita conſtitutam, ut ultimæ coefficientium differentiæ d, d', d'', &c. evadant tandem inter ſe æquales, in ſeriem finitam $\frac{rA}{r-1} - \frac{rd}{(r-1)^2} + \frac{rd'}{(r-1)^3} + \&c.$ *Nicholas Bernoulli* invenit ſeriem æqualem ſummæ n terminorum hujuſce ſeriei; hujuſce problematis altera datur ſolutio in hoc opere: *Monmort* etiam deduxit ſummam ſeriei, cujus numeratores conſtituant lineam quamlibet erectam in triangulo arithmetico, denominatores vero lineam quamlibet tranſverſam.

Bernoulli, *Mac Laurin*, *Le Grange*, *Eulerus*, &c. multa tradidere de iſoperimetricis problematibus, de quibus haud in hoc opere tractatur.

Walliſius dedit ex interpolatione arearum curvarum inter valores 0 & 1 abſciſſæ x contentarum, quarum ordinatæ ſint reſpective $(1-x^2)^0$, $(1-x^2)^1$, $(1-x^2)^2$, &c. terminum intermedium inter primum & ſecundum, viz. aream curvæ, cujus ordinata ſit $(1-x^2)^{\frac{1}{2}}$, i. e. circuli inter prædictos valores abſciſſæ x contentam $= \frac{2.4.4.6.6.8.8.\&c.}{3\cdot 3\cdot 5\cdot 5\cdot 7\cdot 7\cdot 9\cdot \&c.}$; Vice-comes *Brounker* dedit $\frac{2.4.4.6.6.\&c.}{3.3.5.5.7.\&c.} = \cfrac{1}{1+\cfrac{1}{2+\cfrac{9}{2+\cfrac{25}{2+\&c.}}}}$ de his continuis fractionibus plura & elegantia ſcripſere *Eulerus* & *Le Grange*, &c.; e. g. ſi continuæ fractionis termini ſemper recurrant, tum ejus valor e quadraticâ æquatione facile erui poteſt, & ſic facile transformari poteſt radix quadratica cujuſcunque quantitatis in continuam fractionem: nonnulla de hâc re elegantia dedit *Lambert*: in hoc libro in novâ ſpecie continuarum fractionum, in quibus includuntur radices, exprimitur radix cujuſcunque æquationis hujuſce formulæ $x^m + ax^{m-1} + bx + c = 0$.

Bernoulli & *Eulerus* invenere plura continua contenta inter ſe æqualia; & plura, & multo magis generalia ex novis principiis in hoc libro continentur. E. g. Datur methodus ex ſubſtituendo αx, βx, γx, &c. pro x in datâ quantitate, ubi α, β, &c. ſunt radices æquationis $x^n - 1 = 0$ inveniendi continuum contentum datæ quantitati æquale; ex eo quod inveniuntur quantitates, quæ in ſeriem datæ quantitati vel fluenti æqualem ductæ, eam reddunt magis convergentem. 2. Et ex aſſumptis æquationibus inter terminos facile conſtant continua contenta datis quantitatibus æqualia.

Jacobus Gregory ex datis aliquot ordinatis æquidiſtantibus invenit aſymptoton hyperbolæ generis logarithmici, cujus problematis ope quaſdam ſeries ſummavit.

Jacobus Bernoulli invenit maximum terminum binomii ad integram poteſtatem evecti, punctum vero ejus inflexûs dedit *De Moivre*.

Jacobus Bernoulli aſſumit quantitatem *A*, cujus termini ad infinitam diſtantiam ſunt infinite parvi; ex quantitate *A* ſubtrahit eandem quantitatem per quoſdam terminos diminutam; & reſultat ſeries, cujus ſumma eſt prædictorum terminorum ſumma; ſeriem reſultantem ſemper convergere, primum quod ſcio in hoc libro demonſtratur: hoc problema magis generaliter perfecere *Goldbatch* & *De Moivre* ex multiplicatione in factores nihilo æquales; & in hoc libro idem perficitur per additionem ſine multiplicatione. 2. Etiamque hi duo caſus redduntur magis generales per additionem e ſingulis duobus, tribus, &c. ſucceſſivis terminis. 3. Aſſumuntur duæ vel plures ſeries, & ex iis deducuntur ſeries, quarum ſummæ innoteſcant, e. g. ex ſeriebus $\frac{1}{a} + \frac{1}{a+b}x^b + \frac{1}{a+2b}x^{2b} + \&c.$, $\frac{1}{b} + \frac{1}{b+b}x^b + \frac{1}{b+2b}x^{2b} + \&c.$, $\frac{1}{c} - \frac{1}{c+b}x^b + \frac{1}{c+2b}x^{2b} - \frac{1}{c+3b}x^{3b} + \&c.$ inveniri poteſt ſumma omnis ſeriei, cujus generalis terminus eſt

$$\frac{d'z^m +}{a+bz.a+bz+b.a+bz+2b..a+bz+nb \times b+bz.b+bz+b}$$

$$\frac{b'z^{m-1} + \&c.}{.b+bz+2b..b+bz+nb \times bz+c.bz+\tfrac{1}{2}b+c.bz+b+c.bz+}$$

$$1\tfrac{1}{2}b+$$

$\dfrac{\qquad}{1\frac{1}{2}b+c\,..\,bz+\frac{n''}{2}b+c\times\&c.}$; ubi n, n', n'', &c. & m ſunt integri numeri, & z diſtantia a primo ſeriei termino. 4. Ex datis ſummis ſerierum $\frac{1}{a} \pm \frac{1}{a+b}x^{b} + \frac{1}{a+2b}x^{2b} \pm$ &c., $\frac{1}{a^2} \pm \frac{1}{(a+b)^2}x^{b} +$ &c., $\frac{1}{a^3} \pm \frac{x^{b}}{(a+b)^3} +$ &c., ... $\frac{1}{a^l} \pm \frac{1}{(a+b)^l}x^{b} +$ &c.; $\frac{1}{b} \pm \frac{1}{b+b}x^{b} + \frac{1}{b+2b}x^{2b} \pm$ &c., $\frac{1}{b^2} \pm \frac{1}{(b+b)^2}x^{b} +$ &c., ... $\frac{1}{b^{l'}} \pm \frac{1}{(b+1)^{l'}}x^{b} +$ &c.; acquiri poſſunt per ſimplices æquationes ſummæ omnium ſerierum, quarum generales termini ſunt

$$\frac{a'z^m + b'z^{m-1} + \&c.}{(z+a)^l \times (bz+a+b)^l \times (bz+a+2b)^l\,..\,(bz+a+nb)^l \times (b+bz)^{l'}\,..\,(b+b+bz)^{l'}\,..\,(b+n'b+bz)^{l'}\ \&c.}.$$

5. Datur facilis methodus generaliter inveniendi caſus, in quibus ſumma ſeriei, cujus generalis terminus eſt

$$\frac{1}{a+bz\,.\,a+bz+b\,.\,a+bz+2b\,.\,\&c.\times b+bz\,.\,b+bz+b\,.\,b+bz+2b\,.\,\&c.\times c+bz\,.\,c+bz+b\,.\,\&c.\times\&c.},$$

in finitis terminis exprimi poteſt. 6. Animadvertitur, ſi in denominatore contineatur factor $bz+d$, qui non habet ſucceſſivum $bz+d+nb$, ubi n eſt integer numerus; in finitis terminis non exprimi poſſe ſeriei ſummam: *Nichole* invenit generalem terminum datæ ſeriei

$$\frac{d'z^m + b'z^{m-1} + \&c.}{a+bz\,.\,a+bz+b\,.\,a+bz+2b\,.\,.\,a+bz+nb} = \frac{e}{a+bz+nb} + \frac{f}{a+bz+nb\,.\,a+bz+(n-1)b} + \&c.,$$

in hoc libro prædictus terminus invenitur

$$= \frac{e'}{a+bz\,.\,a+bz+b\,.\,a+bz+2b\,.\,.\,a+zb+nb} + \frac{f'}{a+bz\,.\,a+bz+b\,.\,.\,a+bz+(n-1)b} + \&c.,$$

+ &c., ubi m non major eſt quam n; etiamque terminus $\frac{az^{m} + bz^{m-1} + \&c.}{a+bz\,.\,a+bz + b\,.\,.\,a+bz + nb \times b+bz\,.\,b+bz + b\,.\,b+bz + n'b \times c+bz\,.\,\&c.}$ reſolvitur in plures ſimplices, quarum ſummæ innoteſcunt. *De Moivre* & *Bernoulli* invenit ſummas ſerierum ex inveniendo fluentes utriuſque æquationis partis: *Eulerus* exinde deduxit ſummas ſerierum, quarum generales termini ſunt $\frac{1}{a+bz\,.\,b+bz\,.\,c+bz\,.\,\&c.} \times e^{z}$, ex fluentibus fluxionum $\frac{x^{a-1}\dot{x}}{1 \pm x^{b}}$, $\frac{x^{b-1}\dot{x}}{1 \pm x^{b}}$, &c., cum quantitates a, b, c, &c. ſint inæquales, & exinde facile erui poſſunt ſerierum ſummæ, cum generales termini ſint $\frac{a''z^{m} + b''z^{m-1} + \&c.}{a+bz\,.\,b+bz\,.\,\&c.} e^{z}$: in hoc libro ex datâ ſummâ W ſeriei, cujus generalis terminus eſt V, & cujus fluxio detegi poteſt, deducuntur ſummæ ſerierum, quarum generales termini ſunt $V(a'z^{m} + b'z^{m-1} + \&c.)$; & ex ſummis ſerierum, quarum generales termini ſunt $V \times \frac{1}{\alpha+bz}$, $V \times \frac{1}{\beta+bz}$, $V \times \frac{1}{\gamma+bz}$, &c. deducuntur ſummæ ſerierum, quarum generales termini ſunt $V \times \frac{a'z^{m} + b'z^{m-1} + \&c.}{\alpha+bz\,.\,\beta+bz\,.\,\gamma+bz\,.\,\&c.}$; & ſimiliter de pluribus factoribus inter ſe inæqualibus in denominatoribus contentis. 2. Summæ prædictarum ſerierum per fluentes fluxionum $\dot{W}$, $x^{\frac{\alpha}{b}}\dot{W}$, $x^{\frac{\beta}{b}}\dot{W}$, &c. exprimuntur, &c. 3. Summæ ſerierum, quarum generales termini ſunt $\frac{a'z^{m} + b'z^{m-1} + \&c.}{\alpha+z\,.\,\beta+z\,.\,\gamma+z\,.\,\delta+z\,.\,\&c.} \times l\,.\,\frac{l-1}{2}\,.\,.\,\frac{l-nz}{nz+1} \times e^{nz}$ ſemper exprimi poſſunt per finitos terminos, circulares arcus & logarithmos, ſi modo α, β, γ, &c. vel ſint integri numeri, vel habeant 2 pro ſuo denominatore. 4. Dantur ſummæ ſerierum, quarum generales termini ſunt $\frac{a'z^{m} + b'z^{m-1}}{1\,.\,2\,.\,3\,.\,nz \times \alpha+z\,.\,\beta+z} + \&c.$

$\frac{+ \&c.}{.\gamma + z . \&c.} \times e^{z}$, ex fluentibus fluxionum $z^{\alpha} \times \dot{x}$, ubi $\frac{\dot{x}}{1 \pm x} = \dot{z}$, $z^{\beta}\dot{x}$, $z^{\gamma}\dot{x}$, &c. & ſic de pluribus conſimilibus. 5. Ex dato generali termino invenitur ſeriei ſumma, ſi modo generaliter in finitis terminis exprimi poſſit, rejiciendo omnes ultimos factores & aſſumendo pro ſummâ omnes reliquos in rationalem & integralem functionem quantitatis z diſtantiæ a primo ſeriei termino cum coefficientibus deducendis ductos, & exinde deducendo coefficientes quæ quidem ſemper præbent ſummam, ſi modo in finitis terminis exprimi poſſit; ſin aliter ſeriem in infinitum progredientem. 6. Datur methodus inveniendi ſummas ſerierum, quæ continent irrationales quantitates: noſtras *Landen* poſt primam editionem hujuſce operis editam transformavit ſeriem $x - x^{-1} - \frac{x^{2} - x^{-2}}{2} + \&c. = \log. x$, & quæ ſeries, ut a me primum obſervatur, ſemper divergit, cum x ſit poſſibilis quantitas, in ſeriem ſecundum ſinus & coſinus progredientem; ſeries enim eſt differentia inter duas vulgares ſeries $x - \frac{x^{2}}{2} + \frac{x^{3}}{3} - \&c.$ & $x^{-1} - \frac{x^{-2}}{2} + \frac{x^{-3}}{3} - \&c.$, quarum una divergit, cum altera convergat; hæc ſeries eaſdem propoſitiones præbet, utrum ſit transformatio in ſinus, necne; &c. prior methodus calculum magis facilem plerumque reddit. Hìc datur fluens fluxionis $\overset{(q)}{s^{m}}\,\overset{(q)}{c^{n}} \times \frac{\dot{x}}{\sqrt{(1 - x^{2})}}$.

Joannes Bernoulli invenit ſummas quarundam ſerierum $1 \pm \frac{1}{2^{\lambda}} + \frac{1}{3^{\lambda}} \pm \frac{1}{4^{\lambda}} + \&c.$ ex radicibus æquationis $x - A + \frac{A^{3}}{2 . 3} - \&c. = 0$, vel æquationis $x - \frac{A^{2}}{1 . 2} + \frac{A^{4}}{2 . 3 . 4} - \&c. = 0$; *Eulerus* invenit ſummas plurium ſerierum conſimilis formulæ ex iiſdem principiis & radicibus æquationis $e^{\pm x} \pm e^{\mp x} = \left(1 \pm \frac{x}{i}\right)^{i} \pm \left(1 \mp \frac{x}{i}\right)^{i} = \&c. = v^{i} \pm w^{i} = 0$; a *Coteſio* datis; & exinde ex ſubſtitutione plures deduxit;

duxit; quarundam ſerierum e diviſione rationalium functionum integrorum vel primorum numerorum, & ex comparatione ſerierum exinde ortarum cum ſeriebus e prædictis *Joannis Bernoulli*, &c. deductis ſummas inveſtigavit; invenit etiam ſummam ſeriei $1 + \frac{x}{2^2} + \frac{x^2}{3^2} +$ &c., cum $x = \frac{1}{2}$; *Landen* eandem invenit, cum $x = \frac{1}{2}$ & cum ſint duo alii valores α & β quantitatis x: in hoc libro exinde deteguntur ſummæ ſerierum, quarum generales termini ſunt $\frac{(a'z^m + b'z^{m-1} + \&c.)\, x^z}{(a + nz)^b\, (b + nz)^{b''}\, (c + nz)^{b''} . \&c.}$, ubi m, h, b', b'', &c. ſunt integri numeri, & x vel $= 1$ vel $\frac{1}{2}$ vel α vel β. Cum duo vel plures diviſores denominatoris generalis termini ſint inter ſe æquales, tum non deduci poteſt ſeriei ſumma ex ſummis ſerierum, quæ non habent totidem diviſores inter ſe æquales. *Landen* dedit limites ad quos appropinquant ultimo quædam quantitates; in hoc libro adjiciuntur de his quædam nova principia. *Naudei* problemata ab *Eulero* in reſolutionem aliorum problematum transformantur, at non reſolvuntur; conſimilia quam plurima facile adjici poſſunt, quorum nonnulla in hoc libro traduntur. 2. Datur nova methodus inveniendi approximationes ad fluentes fluxionum $\frac{x^a \dot{x}}{1 \pm x^b}$, ubi $b : \alpha$ non habet rationalem rationem. 3. Cum dentur quantitates infinite magnæ, docetur methodus detegendi earum ſummam, differentiam, &c.

Jacobus Bernoulli primus (quod ſcio) edidit ſummam ſeriei $\sqrt[n]{(a + \sqrt[n]{(a \mp \sqrt{(a + \&c.)})})} = x = \sqrt[n]{(x + a)}$; *Joannes Bernoulli* aſſerit ſe primum inveniſſe; dedit etiam ſeries $x = \sqrt[n]{(a \sqrt[n]{(a\ \&c.)})} = \sqrt[n]{(a x)}$, unde $x^{n-1} = a$; obſervatur in hoc libro, quod in ſingulis hiſce formulis plures dentur radices quam ex hiſce ſolutionibus deducantur. *Joannes Bernoulli* dedit regulam pro inveniendâ ſummâ $1^m + 2^m + 3^m + 4^m +$ &c. ubi m eſt integer numerus; legem, quam obſervat hæc ſeries, debemus *Iſaaco Milner*.

Eulerus invenit proportionem, quam medius terminus ſeriei $1 + nx + \&c. = (1 + x + x^2)^n$ habet ad ſummam ſeriei, &c.

Hoc opus moliebatur mecum, & in partes diſtribuebat eruditiſſimus vir, mihique amiciſſimus *Joannes Wilſon* armiger, & laboris quidem totius particeps fuiſſet; ni ſtudia forenſia, quibus ſe dedit, vetuiſſent; adjutore tali tantoque non ſine magno & meo & rei mathematicæ damno carui; ab illius enim ingenio hæc ſcientia incrementa eſſet captura ab aliis vix aut ne vix quidem ſperanda.

Jam reſtat ea, quæ in hoc libro continentur, breviter recenſere.

In primo capite datur regula, quam obſervat fluxio cujuſcunque exponentialis. 2. Animadvertitur in quadraturis minime ad rem conducere, quomodo partes generantur, e. g. utrum per motum, necne; ſolummodo opus eſt partes correſpondentibus quantitatis aſſumptæ partibus convenire.

In ſecundo capite obſervatur dimenſiones variabilis quantitatis x in fluxione minores eſſe quam ejus dimenſiones in fluente per unitatem; ni dimenſiones fluentis ſint nihilo æquales, in quo caſu dimenſiones in fluxione minores erunt quam ejus dimenſiones in fluente per quantitatem majorem quam unitatem; unde conſtat, ſi dimenſiones quantitatis x in numeratore ſint minores per unitatem quam ejus dimenſiones in denominatore, ejus fluentem in finitis terminis haud exprimi poſſe. 2. Animadvertitur; ſi modo fluxio, quæ ſit functio quantitatis x^n in $\dot{x}$, reducatur ad functionem quantitatis x^{-n} in $\dot{x}$, vel ad functionem cujuſcunque quantitatis, quæ algebraicam habeat relationem ad x^n in $\dot{x}$; & utriuſque fluens per vulgares methodos detegatur, & ſerierum alterutra terminetur, tum (rebus recte diſpoſitis) terminari alteram. 3. Datur methodus inveniendi fluentem fluxionis $(a + bx^n + cx^{2n} + \&c. + (d + ex^n + fx^{2n} + \&c.)(p + qx^n + rx^{2n} + \&c.)^\lambda)^r \times (A + Bx^n + \&c. + \&c.)\dot{x}$, ſi modo in finitis terminis exprimi poſſit. 4. Sit fluxio $(A + B)^m \times (a + bx^n + \&c. + \&c.)\dot{x}$; ubi $\frac{\dot{A}}{B} = \frac{\dot{B}}{A}$ vel magis generaliter ſit $(A + B + C)^m \times \&c.\,\dot{x}$, ubi $\frac{\dot{A}}{B} = \frac{\dot{B}}{C} = \frac{\dot{C}}{A}$; & ſic deinceps; tum non aſſumenda eſt quantitas $(A + B + C\,\&c.)^{m+1} \times$

&c. ſed $(A + B + C \,\&c.)^m \times$ &c. pro fluente. 5. Traditur methodus inveniendi fluentes in finitis terminis (ſi modo per eos exprimi poſſint) per infinitas ſeries. 6. Traduntur fluentes quarundam fluxionalium quantitatum non prius traditarum. 7. Quædam nova adjiciuntur de correctionibus fluentium. 8. Obſervatur in corrigendis fluentibus eaſdem radices ſemper uſurpandas eſſe, i. e. nunquam $-\sqrt{-}$ pro $+\sqrt{-}$ ſubſtituendam eſſe. 9. Ex ſubſtitutione inveniuntur diverſæ fluentes ejuſdem fluxionis inter ſe æquales. 10. Animadvertitur quaſdam eſſe exponentiales quantitates, quæ perpetuo mutantur de poſſibili in impoſſibilem, & vice verſâ de impoſſibili in poſſibilem. 11. Docetur methodus inveniendi, annon datæ fluentes inter datos valores variabilis contentæ ſint finitæ. 12. Ex datâ fluxione, cujus fluens exprimit ſummam ſeriei ſecundum dimenſiones quantitatis x progredientis, deducitur fluxio, cujus fluens exprimit ſummam terminorum prædictæ ſeriei ad (n) diſtantias a ſeſe poſitorum. 13. Nonnullæ fluxiones non prius traditæ, quæ involvunt irrationales functiones variabilis quantitatis x reducuntur ad fluxiones, quæ nullam involvunt irrationalem quantitatem variabilis reſultantis fluxionis. 14. Traditur methodus in genere detegendi, annon fluens datæ fluxionis exprimi poſſit per finitos terminos, circulares arcus & logarithmos. 15. Datâ æquatione ſimiliter involvente X & x, unde $x = \phi : X$; & fluxione $P\dot{x}$, ubi P eſt functio quantitatis x; & in fluxione $P\dot{x}$ pro x & $\dot{x}$ ſcriptis $\phi : X$ & $\phi :\dot{} X$; reſultet $Q\dot{X} = 0$; in quâ pro X & $\dot{X}$ ſcribantur x & $\dot{x}$, exinde reſultet $Q'\dot{x}$: ſint L & l, M & m correſpondentes valores quantitatum X & x; tum erit fluens fluxionis $(P + Q)\dot{x}$ inter valores L & M quantitatis x contenta, eadem ac fluens inter valores l & m ejuſdem quantitatis. 16. Sit fluxio $(a + bx^n + \,..\, x^m)^{m+\lambda} \times x^{pn+\sigma n-1}\dot{x}$, ubi λ & σ ſunt quicunque integri affirmativi numeri; tum ex quibuſcunque $r-1$ independentibus fluentibus hujuſce formulæ detegi poſſunt omnes aliæ ejuſdem formulæ. 17. Per ſubſtitutionem erui poſſunt fluxiones, quarum fluentes per pauciores ejuſdem formulæ exprimi poſſunt. 18. Sit $x^{\vartheta+\alpha n+\beta m} \times (a + bx^n + cx^m)^{\lambda+\pi}\dot{x}$, ubi literæ α, β & π reſpective denotant affirmativos integros numeros; tum ex datis $(\alpha + \beta + \pi + 1)$ fluentibus inter ſe independentibus fluxionum hujuſce formulæ detegi poſſunt fluentes omnium fluxionum ejuſdem formulæ, &c. 19. Sit fluxio $\dot{X}\int.y\dot{x}$, ubi $\dot{X}$ eſt fluxio, cujus fluens inveniri poteſt; & ſit numerus fluentium inter ſe independentium in formulis fluxionum $Y\dot{x}$ & $Xy\dot{x}$ contentarum reſpective m & r; tum ex $(m+r)$ fluentibus fluxionum $\dot{X}\int.y\dot{x}$ inter ſe independentibus detegi poſſunt fluentes omnium fluxionum ejuſdem formulæ, &c. 20. Sit exponentialis fluxio $e^{a+bx^n\,..\,bx^{\mu n}} \times x^{r\pm\nu n}$, ubi μ & ν ſunt integri numeri, tum ex fluentibus (μ) fluxionum prædictæ formulæ independentibus acquiri poſſunt fluentes omnium fluxionum ejuſdem formulæ, &c. 21. Traditur nova generalis methodus inveniendi valorem fractionis, cum ejus numerator & denominator ſimul evaneſcant. 22. Adjiciuntur quædam de nonnullis fluxionibus in alias reducendis.

In capite tertio 1. ex datâ algebraicâ æquatione datur methodus inveniendi æquationem, cujus radix eſt data fluxionalis quantitas. 2. Si (n) fluxionales æquationes ordinum m, r, s, &c. reducantur ad unam, tum obſervatur æquationem reſultantem non majorem quam $m+r+s+$&c. ordinem habere; etiamque nonnulla de novis fluxionalibus, &c. æquationibus in æquationes reſultantes per operationem introductis. 3. Datæ fluxionales æquationes $\alpha.\beta.\gamma.\delta.$&c.$=0$ & $\pi.\rho.\sigma.\tau.$&c.$=0$ diſtingui poſſunt in ſubſequentes $\alpha=0$ & $\pi=0$; $\alpha=0$ & $\rho=0$; &c.; $\beta=0$ & $\pi=0$; $\beta=0$ & $\rho=0$; &c. 4. Ex datis duabus vel pluribus fluxionalibus æquationibus deducuntur duæ vel plures aliæ, quarum variabiles quantitates ſunt eædem. 5. Data æquatio fluentes involvens reducitur ad fluxionalem, in quâ nulla continetur fluens. 6. Datâ algebraicâ æquatione relationem inter x & y exprimente; inveniuntur quidam novi caſus, in quibus x & y facile exprimi poſſunt in terminis tertiæ z. 7. Datâ algebraicâ æquatione relationem inter x & y exprimente; invenitur, annon fluens (v) fluxionis, quæ eſt algebraica functio quantitatum x & y & earum fluxionum, in finitis terminis exprimi poteſt, viz. ex aſſumendo æquationem algebraicam, quæ neceſſario exprimit relationem inter v & x vel y; idem per infinitas ſeries perficitur. 8. Hinc datur methodus inveniendi quam plurimas æquationes ad curvas, quarum areæ, &c. in finitis terminis exprimi poſſunt. 9. Traditur nota, ex quâ ſæpe dici poteſt aream curvæ in finitis terminis non exprimi poſſe. 10. Summa e ſingulis valoribus areæ curvæ, cujus æquatio relationem inter abſciſſam x & ordinatam y deſignans ſit algebraica, in finitis terminis exprimi poteſt. 11. Sit data algebraica æquatio, cujus fluxio ducatur in quantitates æquales ex datâ æquatione deductas, tum fluens generalis æquationis reſultantis non erit data æquatio; & vice verſâ eadem æquatio præbeat plures fluxionales æquationes. 12. Nonnulla adjiciuntur de correctione fluentium fluxionalium æquationum. 13. Datur methodus detegendi, annon data æquatio ſit generalis fluens datæ fluxionalis æquationis. 14. Traduntur quædam de methodis inveniendi, annon fluxionales æquationes ſint integrabiles. 15. Datis $n . \frac{n-1}{2}$ diverſis, i. e. independentibus æquationibus hujuſce generis $a\int. y\dot{x} + b\int. y^2\dot{x} + c\int. yx\dot{x} +$ &c., exinde deduci poteſt valor cujuſcunque quantitatis hujuſce generis $p\int. v\dot{z} + q\int. v^2\dot{z} + r\int. vz\dot{z} +$ &c., ubi n eſt maxima dimenſio ad quam aſcendunt variabiles quantitates; a, b, c, &c., p, q, r, &c. ſunt invariabiles quantitates, & $a'x + b'y + c' = z$ & $p'x + q'y + r' = v$. 16. Datis generaliter contentis ſolidorum a rotatione curvæ circa tres axes generatorum, dabuntur contenta ſolidorum a rotatione ejuſdem curvæ circa quoſcunque alios axes generatorum, &c. 17. Sit data fluxio $(ay + bx + c)\dot{x} + (Ay + Bx + C)\dot{y} = 0$, ex datâ fluente fluxionis $y\dot{x}$ (quæ plerumque acquiri poteſt e datâ æquatione) erui poteſt fluens omnis fluxionis formulæ $(hy^m + (k + lx)y^{m-1} +$ &c.$)\dot{x}$; &c. 18. Datâ algebraicâ vel fluxionali æquatione $\alpha = 0$ & fluxionali quantitate π, in quibuſdam caſibus invenitur ejus fluens ope datarum. 19. Datâ fluxionali vel fluxionalibus æquationibus, inveniuntur functiones variabilium in iis contentarum & earum fluxionum, quarum fluentes innoteſcunt. 20. Si M ſit particularis valor fluxionalis æquationis, tum erit quæcunque functio quantitatis M valor fluxionalis æquationis. 21. Sint $\alpha = 0$ & $\pi = 0$ duæ fluxionales æquationes m & r ordinum, quorum m minor eſt quam r; & ſit $K\alpha + L\pi = \dot{V}$, ubi V eſt fluxionalis quantitas ordinis $(r - 1)$; duæ datæ fluxionales reducuntur ad duas fluxionales

$V =$

$V =$ conſt. & $\alpha = 0$. 22. Datâ fluente n ordinis, docetur methodus ejus fluxionalem æquationem inveniendi. 23. Sunt n diverſæ fluentes $\alpha = 0$, $\beta = 0$, $\gamma = 0$, &c. primi ordinis; $n . \frac{n - 1}{2}$ fluentes ſecundi ordinis; &c.; (n) vero diverſi multiplicatores, qui reddunt fluxionalem æquationem (n) ordinis integrabilem. 24. Generalis fluens prædictæ fluxionalis æquationis erit $\varphi : (\alpha, \beta, \gamma, \delta,$ &c.$)$ 25. Datis duabus quantitatibus p & q, quæ ſunt functiones vel algebraicæ vel fluxionales quantitatum x & y, traditur methodus detegendi, annon p ſit functio quantitatis x. 26. Cum dimenſiones fluxionis ordinis maxime ſuperioris in datâ æquatione ſint majores quam 1; tum reducendæ ſunt per extractionem ad unam dimenſionem. 27. Nonnulla adjiciuntur de methodis inveniendi fluentes fluxionalium æquationum ſuperiorum ordinum. 28. Per infinitas ſeries inveniuntur multiplicatores datarum fluxionalium æquationum. 29. Dantur caſus, in quibus fluxionales æquationes, qui habent terminos variabilium $(m$ & $m - 1)$ vel $(m, m - 1, m - 2)$, &c. dimenſionum, integrari poſſunt. 30. Fluxionalis æquatio (n) ordinis, quæ ſit homogenea, reducitur ad fluxionalem æquationem $(n - 1)$ ordinis. 31. Sint duæ vel tres vel m homogeneæ fluxionales æquationes ordinis (n), tres vel quatuor $(m + 1)$ variabiles quantitates $(x, y, z,$ &c.$)$ involventes; eæ reduci poſſunt ad fluxionalem æquationem, cujus ordo non major eſt quam $n m - 1$. 32. In ſubſtitutionibus minime pro x & y ſubſtituantur homogeneæ functiones nullarum dimenſionum quantitatum z & v, nam hæ quantitates reduci poſſunt ad functiones ejuſdem quantitatis. 33. Detur æquatio algebraica, in quâ ſimiliter involvuntur x & y; inveniatur ejus fluxio $\varphi \dot{x} + \varphi' \dot{y} = 0$; in hâc æquatione pro y in φ & pro x in φ' ſcribantur earum valores ex datâ æquatione deducti; & æquationis reſultantis generalis fluens forſan non erit data æquatio. 34. Detur fluxionalis æquatio $p \dot{x} + q \dot{y}$, & ſit $\dot{p} = \alpha \dot{x} + \beta \dot{y}$ & $\dot{q} = \pi \dot{x} + \rho \dot{y}$; tum, ſi $\frac{\beta - p}{2}$ ſit functio quantitatis x, data æquatio redditur integrabilis per multiplicatorem, qui eſt functio quantitatis x; ex conſimilibus principiis per fluxionalem æquationem inferioris ordinis invenitur, annon fluxionalis æquatio ſuperioris ordinis recipiat multiplicatorem, qui eſt functio quantitatum x & $\dot{x}$. 35. Eadem principia extenduntur ad detegendam fluentem fluxionalis æquationis $p\overset{n}{\dot{y}} + q\overset{n-1}{\dot{y}} \ldots t\overset{n-m}{\dot{y}} = o$, ubi $2m$ minor eſt quam n; & p eſt functio quantitatum $x, \dot{x}, y, \dot{y}, \ldots \overset{n-2m-1}{\dot{y}}$, &c. 36. Per methodum in cor. 4. prob. 53. traditam detegi poteſt; annon data fluxionalis æquatio, quæ eſt functio quantitatum $x, \dot{x}, y, \dot{y}, \ddot{y}$, &c. evadere poſſit integrabilis per multiplicatorem, in quâ dantur omnes functiones quantitatum $y, \dot{y}, \ddot{y}$, &c.; & in eâ continentur variabiles quantitates, quæ denotant functiones quantitatum x & $\dot{x}$ deducendas. 37. Sint m æquationes $(m+1)$ variabiles quantitates involventes, quarum formulæ ſint $a\overset{n}{\dot{x}} + b\overset{n}{\dot{y}} + \&c. + a'\overset{n-1}{\dot{x}}\,t + b'\overset{n-1}{\dot{y}}\,t + \ldots \overset{n}{a'}\,x\,t^{n} + \&c. + T\,t^{n} = o$, &c., tum earum fluentes deducuntur. 38. Traditur methodus inveniendi (l) fluxionales æquationes $(l+1)$ variabiles quantitates habentes, quarum fluentes innoteſcunt. 39. Generalis fluens fluxionalis æquationis deductæ ex transformando datam fluxionalem æquationem, in quâ x fluit uniformiter in alteram, in quâ y fluit uniformiter, eadem erit ac generalis fluens datæ fluxionalis æquationis. 40. Per infinitas ſeries datur methodus inveniendi, annon fluens fluxionalis æquationis in finitis terminis exprimi poſſit. 41. Ex aſſumptis pro x & y functionibus vel algebraicis vel fluentialibus quantitatis z deduci poſſunt fluxionales æquationes, quarum nec x per y, nec y per x, quamvis utraque per tertiam z, exprimi poteſt. 42. Inveniuntur quædam functiones duarum diverſarum fluentialium quantitatum ex datâ fluxionali æquatione deductarum, quæ erunt inter ſe æquales. 43. Sint n fluxionales æquationes $a = o$, &c., quarum ordo ſit l, $(n+1)$ variabiles quantitates $(x, y, z, \&c.)$ involventes; tum ſint m, m', &c. functiones quantitatum $(x, y, z, \&c.)$ & earum fluxionum, quarum ordines minores ſunt quam l; & ita aſſumi poſſunt multiplicatores m, m', &c., ut exoriantur n diverſæ & independentes fluxionales æquationes, quæ integrari poſſunt.

In libro ſecundo 1. traduntur nonnulla nova de inveniendis incrementis

mentis fluentialium quantitatum. 2. Sit quantitas P functio quantitatis x, cujus incrementum ſit $\mathcal{Q}$; tum erit $\dot{\mathcal{Q}}$ æqualis incremento fluxionis $\dot{P}$, ſi modo fluxio $(\dot{o})$ incrementi (o) quantitatis x æqualis ſit incremento fluxionis $\dot{x}$. 3. Incrementum $\frac{ax^n + bx^{n-1} + \&c.}{x(x+\underset{.}{x})\ldots(x+(n-1)\dot{x})}$ in primâ editione hujuſce operis reducitur ad incrementa hujuſce generis $\alpha + \frac{\beta}{x} + \frac{\gamma}{x(x+\underset{.}{x})} + \&c.$; & obſervatur, ſi $\beta = b - n \,.\, \frac{n-1}{2} a\underset{.}{x}$ non ſit $= o$, integralem in finitis terminis non exprimi poſſe. 4. Sit incrementum fractio $\frac{ax^m + bx^{m-1} +}{x(x+\underset{.}{x}).(x+(n-1)\underset{.}{x}) \times (x+p)\,..\,(x+p+}$ $\frac{\&c.}{(m-1)\underset{.}{x} \times \&c.}$; ea reducitur ad integrabiles formulas, cum formulis $\frac{\beta}{x}$, $\frac{\beta'}{x+p}$, &c. adjunctis. 5. Si numerus literarum o, p, &c. ſit λ, tum ex λ independentibus integralibus hujuſce formulæ deduci poſſunt omnes ejuſdem formulæ: & ſi dimenſiones quantitatis x in numeratore ſint minores quam ejus dimenſiones in denominatore per duas vel plures, tum $\beta + \beta' + \&c. = o$, &c. 6. Dimenſiones quantitatis x in incremento ſemper erunt minores quam dimenſiones ejuſdem quantitatis in integrali per unitatem; ni dimenſiones quantitatis in numeratore integralis æquales ſint ejus dimenſionibus in denominatore, in quo caſu minores erunt per quantitatem majorem quam unitatem: hinc, ſi dimenſiones in integrali minores ſint per unitatem quam ejus dimenſiones in incremento, tum ejus integralis in finitis terminis non exprimi poteſt. 7. Si quicunque diviſor in denominatore contineatur, qui non habeat alterum a ſe diſtantem per $r\underset{.}{x}$, ubi r eſt integer numerus; tum integralis in finitis terminis non exprimi poteſt. 8. Traditur methodus in genere inveniendi integralem dati incrementi, ſi modo finitis terminis exprimi poſſit, rejiciendo ſingulos ultimos terminos in denominatore contentos, &c. 9. Datur methodus inveniendi integrales incrementorum, quæ duas vel plures variabiles quantitates & earum incrementa involvunt. 10. Si inveniatur incrementum quantitatis V ordinis m ex hypotheſi quod x ſolummodo ſit variabilis, & reſultet quantitas W, deinde inveniatur incrementum quantitatis W ordinis n ex hypotheſi, quod y ſolummodo ſit variabilis; eadem reſultabit quantitas ac ſi modo inveniatur primo incrementum quantitatis V ordinis n ex hypotheſi quod y ſolummodo ſit variabilis, & reſultet quantitas u, deinde inveniatur incrementum m ordinis quantitatis u ex hypotheſi quod x ſolummodo ſit variabilis; &c. 11. Si A ſit integralis, quæ eſt functio quantitatis z & ejus incrementorum; & aſſumatur æquatio, in quâ x & z ſimiliter involvuntur, deinde inveniantur quantitas z & ejus incrementa in terminis quantitatis x & ejus incrementorum; ſcribantur hæ quantitates pro ſuis valoribus in A, & reſultet B; in B pro x ſcribatur z & reſultet C: ſint π & ρ, α & β correſpondentes valores quantitatum x & z, tum integralis inter valores α & β variabilis z quantitatis $A+C$ contenta eadem erit ac integralis ejuſdem quantitatis $A+C$ inter valores π & ρ ejuſdem quantitatis z. 12. Traditur methodus inveniendi, annon integralis logarith. $\frac{A}{B}$ inveniri poſſit ope logarithmorum, ubi A & B ſunt functiones variabilis x. 13. Datâ æquatione algebraicâ $y^n + (a + bx)y^{n-1} + \&c., = 0$; traditur methodus inveniendi æquationem, cujus radix eſt quæcunque algebraica functio quantitatum x & y & earum incrementorum. 14. Adjicitur nota, e quâ ſæpe dici poteſt prædictam æquationem in finitis terminis non exprimi poſſe. 15. Sit integralis æquatio $ax^r + bx^s y^t = cy^n$; & exinde conſtant quantitates x & y in terminis quantitatis z, & conſequenter quæcunque algebraica functio quantitatum x & y & earum incrementorum in algebraicis terminis quantitatis z & ejus incrementorum; & exinde annon ejus integralis inveniri poſſit. 16. Datur methodus exterminandi integrales quantitates ex datâ æquatione. 17. Nonnulla dantur de correctione integralium incrementorum. 18. Obſervatur n eſſe integrales primi ordinis incrementialis æquationis (n) ordinis; $n \cdot \frac{n-1}{2}$ diverſas integrales ſecundi ordinis; &c. 19. Prædicta æquatio n habet multiplicatores. 20. Generalis integralis incrementialis æquationis (n) ordinis erit quæcunque functio e ſingulis (n) generalibus integralibus prædictæ æquationis; quæcunque functio ex ſingulis $\left(n \cdot \frac{n-1}{2}\right)$ generalibus integralibus ſecundi ordinis prædictæ fluxionalis æquationis; & ſic deinceps. 21. Sit æquatio $y_{n|} + a y_{n|-1} + b y_{n|-2} \ldots + d y + f y = o$, & erit ejus generalis integralis $A e^{\alpha x} + B e^{\beta x} +$ &c. $= y$, ubi $e^{\alpha x} = \pi$, $e^{\beta x} = \rho$, &c., & π, ρ, σ, &c. ſunt radices æquationis $v^n + a v^{n-1} +$ &c. $= o$. Plura in hoc libro adjiciuntur de integralibus & incrementis, & incrementialibus æquationibus conſimilia iis, quæ in priori libro de fluxionibus & fluxionalibus æquationibus traduntur.

In libro tertio 1. datur ratio, quam habeant inter ſe vera & apparens convergentia. 2. Datâ lege, quam obſervant termini ſeriei in infinitum progredientis; traditur methodus dignoſcendi, annon ſeries ſit finita. 3. Dantur ſeries, quæ ſemper convergant ad omnes valores earum incognitarum quantitatum; aliæ ſeries, quæ ſemper divergant. 4. Sit infinita æquatio $A = a x^r + b x^{r+1} +$ &c.; & ſi coefficientes haud creſcant in majori quam quâcunque geometricâ ratione & ſint omnes affirmativæ; tum datur una affirmativa radix & non plures; &c. 5. Traditur æquatio, quæ habet infinitas & incognitas radices, &c. 6. Docetur methodus reducendi duas vel plures æquationes infinitas in unam, ita ut incognitæ quantitates exterminentur. 7. Traditur methodus detegendi, annon data quantitas ſit radix datæ infinitæ æquationis. 8. Æquatio $1 - x + x^2 - x^3 +$ &c. $= o$ habet radicem $x = 1$. &c. 9. Sit æquatio $o = a - bx + cx^2 - dx^3 +$ &c.; ubi $\frac{b}{a}$ multo major ſit quam $\frac{c}{b}$, & $\frac{c}{b}$ quam $\frac{d}{c}$, $\frac{d}{c}$ quam $\frac{f}{e}$, &c.; tum erunt omnes radices datæ æquationis poſſibiles: & $\frac{a}{b}$ erit approximatio ad minimam radicem; $\frac{b}{c}$ approximatio ad ſecundam; &c. 10. Sint m radices multo majores vel minores quam reliquæ; tum ex $(m+1)$

primis vel ultimis terminis deduci poſſunt approximationes ad illas radices; ſi autem una radix multo major ſit quam m radices, multo autem minor quam reliquæ, tum datur ejus approximatio; & ſic de pluribus. 11. Docetur methodus inveniendi maximam radicem datæ æquationis, ſi modo omnes ſint poſſibiles; & exinde reverſionem ſeriei $y = bx + cx^2 + \&c.$ 12. Traditur methodus inveniendi approximationes ad radicem datæ æquationis quæ multo major ſit quam m radices & multo minor quam reliquæ. 13. Si una radix multo major vel minor ſit quam quæcunque alia & vera ad r figuras; tum ex $m + 1$ primis vel ultimis terminis erui poteſt approximatio vera ad $m \times r$ figuras prope. 14. Animadvertitur in æquationibus ſuperiorum ordinum (ut maxime probabile eſt) numerum impoſſibilium radicum majorem eſſe quam numerum poſſibilium, & conſequenter maximum impoſſibilem radicem majorem eſſe quam maximum poſſibilem, & methodum inveniendi approximationes per extractionem radicum poteſtatum e ſingulis radicibus, &c. perraro in æquationibus ſuperiorum dimenſionum uſui inſervire. 15. Erit $n . \frac{n-1}{2} . \frac{n-r}{r+1} - n . n . \frac{n-1}{2} .. \frac{n-r+1}{r} + n . \frac{n-1}{2} . n . \frac{n-1}{2} .. \frac{n-r+2}{r-1} - \&c. = 0$ vel $\pm n . \frac{n-1}{2} . \frac{n-\frac{r-1}{2}}{\frac{r+1}{2}}$. 16. Sit æquatio

$$1 - (\alpha + \beta + \gamma + \&c.)\frac{1}{x} + (\alpha\beta + \alpha\gamma + \beta\gamma + \&c.)\frac{1}{x^2} - (\alpha\beta\gamma + \alpha\beta\delta + \&c.)\frac{1}{x^3} + \&c. = A = 0;$$

datur lex ſeriei, quæ ſit A^n. 17. Ex terminis ſeriei reſultantis per diviſionem numeratoris 1, &c. per datam æquationem deducuntur approximationes ad maximam radicem ſecundam, &c.; ſi modo radices ſint omnes poſſibiles. 18. Approximatio ad radicem inventa pendet ex hoc, nempe quo propius quantitas aſſumpta pro radice ſit ad unam radicem quam ad reliquas. 19.

Approximationes ſic inventæ ultimo convergent in majori quam quâvis geometricâ progreſſione. 20. Docentur caſus, in quibus approximationes inventæ convergunt. 21. Si terminorum datæ æquationis ſigna ſemel ſolummodo vel progrediantur de + in +, vel — in —; vel mutentur de + in —, vel — in +; tum aſſumatur quæcunque negativa quantitas in uno caſu, affirmativa vero in altero, pro approximatione ad radicem prædictam, & ſemper convergent approximationes per prædictam methodum inventæ. 22. Datis approximationibus ad r radices datæ æquationis, inveniuntur approximationes magis appropinquantes. 23. Detur æquatio $A = o$, ubi A ſit functio quantitatis x; fingatur x variabilis, & ſit a valor quantitatis x prope; pro x ſcribatur $a - e$ & reſultet æquatio $P - Qe + \frac{1}{2}Re^2 - \frac{1}{6}Se^3 +$ &c. $= o$; & ex terminis hujuſce æquationis deduci poſſunt continuæ approximationes ad radices datæ æquationis. 24. Ex datâ æquatione n invariabiles & incognitas quantitates involvente & datis ad eas approximationibus, per n ſubſtitutiones erui poſſunt quantitates ad earum valores magis approximantes. 25. Datis in finitis æquationibus duas vel plures incognitas quantitates habentibus, invenitur una in terminis reliquarum. 26. Sit y ſumma ſeriei ſecundum dimenſiones quantitatis x, v vero ſumma ſeriei ſecundum dimenſiones quantitatis z; & detur ſeries w ſecundum dimenſiones quantitatum z & x progrediens; traditur methodus inveniendi w in terminis quantitatum y & v. 27. Invenitur formula ſeriei $p + aqx +$ &c. $= A$, cujus poteſtates A^n vel radices $A^{\frac{n}{m}}$ ſemper eandem obſervant legem. 28. Docetur methodus inveniendi approximationes ad fluentes fluxionum reducendo datam fluxionem ad ſeriem ſecundum dimenſiones perparvarum quantitatum, vel variabilium, vel unius vel plurium progredientem. 29. Generalis fluens fluxionis $\overset{2}{\dot{y}} = y\dot{z}^2$ erit $y = Ex + Fx'$, ubi $x = z + \frac{z^3}{2 \cdot 3} + \frac{z^5}{2 \cdot 3 \cdot 4 \cdot 5} +$ &c., $x' = 1 + \frac{z^2}{1 \cdot 2} + \frac{z^4}{1 \cdot 2 \cdot 3 \cdot 4} +$ &c., & $z = \log. (x + \surd(1 + x^2)) =$

log. $(x' - \sqrt{(x'^2 - 1)})$, & E & F ſunt invariabiles quantitates ad libitum aſſumendæ. 30. Datur ſeries pro inveniendâ $\int. \frac{\sqrt{(1 - cx^2)}\dot{x}}{\sqrt{(1 - x^2)}}$, ubi c eſt perparva quantitas; i.e. petipheriâ ellipſeos, quæ haud multum differt a circulo. 31. Traduntur caſus, in quibus ſeries $a^m \pm ma^{m-1}x + \text{\&c.} = (a \pm x)^m$, &c. convergit. 32. Docetur methodus inveniendi, annon ſeries exorta ex reducendo algebraicam functionem quantitatis x ad feriem (P) ſecundum ejus dimenſiones progredientem convergat; etiamque annon ſeries, quæ exprimit $\int. P\dot{x}$, convergat. 33. In fluentibus, &c. inveſtigandis; docetur methodus interpolandi, ita ut ſeries maxime celeriter convergant. 34. Si transformetur fluxio, cujus variabilis eſt x, in alteram cujus variabilis eſt z, quæ datam habeat relationem ad x; & reducantur duæ fluxiones ad ſeries ſecundum dimenſiones quantitatum x & z reſpective progredientes; dantur convergentiæ, quas habent inter ſe duæ ſeries. 35. Transformantur datæ algebraicæ quantitates in terminos ſecundum dimenſiones quantitatis x progredientes, quarum formulæ ſunt integrabiles. 36. $\frac{a}{z - \alpha} = \frac{a}{z} + \frac{a\alpha}{z(z+1)} + \frac{a\alpha(\alpha+1)}{z.z+1.z+2} + \frac{a\alpha(\alpha+1)(\alpha+2)}{z.z+1.z+2.z+3} + \text{\&c.}$ 37. Vera & apparens convergentia geometricæ ſeriei eſt uniformis. 38. Contineantur tres incognitæ quantitates (x, y & z) in datâ æquatione, datur methodus inveniendi unam in terminis duarum reliquarum. 39. Datâ æquatione involvente duas incognitas quantitates x & y, datur ſubſequens methodus inveniendi y in terminis quantitatis x; aſſumatur $y = Ax^m + Bx^{n+m} + Cx^{2n+m} + \text{\&c.}$; deinde 1^{mo}. inveniatur approximatio ad coefficientem A; & exinde approximatio ad coefficientem B, & propior ad coefficientem A; & ſic deinceps; &c. 40. Sit fluxionalis æquatio (n) ordinis, duas variabiles quantitates x, y & earum fluxiones involvens; ex eâ deducatur ſeries, quæ exprimat y in terminis quantitatis x; & ſi modo ab initio ſeriei incipiat, ſemper occurrent in eâ (n) invariabiles quantitates ad libitum aſſumendæ. 41. In prædictâ reſolutione forſan occurrent reſolutiones

ſolutiones homogenearum fluxionalium æquationum ordinem (n) non ſuperantium. 42. Datâ fluxionali æquatione relationem inter abſciſſam, ejus correſpondentes ordinatas & earum fluxiones exprimente, invenitur fluxionalis æquatio relationem inter abſciſſam & ejus correſpondentes ordinatas ad aſymptotos deſignans. 43. Ex datis relationibus inter valores x & z incognitæ quantitatis x, & inter ſummas duarum ſerierum exinde reſultantium; traditur methodus inveniendi coefficientes ipſius ſeriei. 44. Promovetur regula vulgariter dicta falſi: viz. cum datæ approximationes ad duas vel plures radices maxime accedant, vel cum dentur plures ſubſtitutiones pro incognitâ quantitate in datâ æquatione: & eadem perficiuntur in pluribus æquationibus plures incognitas quantitates habentibus. 45. Erit

$x = \sqrt[m]{}\left(a + \frac{b}{\sqrt[m]{}(a + \&c.)}\right)$, reſolutio æquationis $x^{m+1} - ax = b$; &c.;

$x = \sqrt[n]{}(a + \sqrt[n]{}(a + \&c.))$ radix æquationis $(x^n - a)^n - x^n \&c. = a$. &c.

In libro quarto 1. ad particularem caſum, ubi $\dot{z}$ incrementum quantitatis z diſtantiæ a primo ſeriei termino eſt 1, applicantur ea, quæ prius traduntur de fluxionalibus & incrementialibus æquationibus. 2. Datur methodus transformandi æquationem inter ſummas & terminos in incrementialem æquationem. 3. Summa nullius ſeriei exprimi poteſt per algebraicam æquationem inter prædictam ſummam & quantitatem x, ſecundum cujus dimenſiones progrediatur ſeries, relationem deſignantem; ni dimenſiones quantitatis x vel earum differentiæ denotari poſſint per arithmeticas ſeries; & differentiæ inter dimenſiones quantitatis z diſtantiæ a primo ſeriei termino in numeratore & denominatore eædem ſint: & non exprimi poteſt per fluxionalem æquationem; ni prædictæ differentiæ ſint eædem, vel uniformiter creſcant vel decreſcant. 4. Sit ſeries $a + bx + cx^2 + \&c.$ ſecundum dimenſiones quantitatis x progrediens, cujus coefficientes terminorum proxime ſubſequentium ad infinitam diſtantiam $:: r : 1$;

ducatur hæc ſeries in functionem ipſius $x = o$, cum $x = \alpha$; tum ſi x major ſit quam r, ſeries diverget; ſin minor, converget. 5. Tranſformatur data ſeries in alteram, cujus termini ſint ſummæ e ſingulis (n) ſucceſſivis terminis. 6. Nova methodus adjicitur dividendi alteram quantitatem per alteram; & exinde detegendi ſeries, quarum ſummæ innoteſcant. 7. Sit ſeries $\frac{1}{\alpha . \beta . \gamma . \delta . \&c.} + \frac{1}{\alpha + 1 . \beta + 1 . \gamma + 1 . \delta + 1 . \&c.} x^b + \frac{1}{\alpha + 2 . \beta + 2 . \gamma + 2 . \delta + 2 . \&c.} x^{2b} + \&c.$, & ſi omnes reliquæ quantitates γ, δ, &c. diſtent a duabus α & β per integros numeros; tum ſummæ ſerierum inveniri poſſunt a fluentibus fluxionum $\frac{x^{b\alpha - 1}\dot{x}}{1 \pm x^b}$ & $\frac{x^{b\beta - 1}\dot{x}}{1 \pm x^b}$. 8. Inveniuntur ſeries, quæ ſunt generales vel particulares fluentes fluxionalis æquationis. 9. Ex datis convergentibus ſeriebus inveniuntur aliæ. 10. Deteguntur ſummæ ſerierum, quarum ſinguli termini dantur ex infinitis ſeriebus. 11. Conceſsâ methodo detegendi generaliter annon una data ſeries vel quantitas ſit algebraica functio aliarum, traditur methodus inveniendi, annon detur algebraica vel fluxionalis relatio inter ſummam & terminos datæ ſeriei. 12. Generalis fluens æquationis $\overset{n}{\dot{y}} = y\dot{x}^n$ erit $a(1 + \frac{x^n}{1 . 2 .. n} + \frac{x^{2n}}{1 . 2 .. 2n} + \&c.) + bx(1 + \frac{x^n}{2 . 3 .. n + 1} + \&c.) + \&c.$ 13. Invenitur quantitas, quæ in ſeriem ducta, præbeat ſeriem magis convergentem. 14. Inveniuntur caſus, in quibus differentiæ ſucceſſivæ celeriter convergant, &c. 15. In detegendâ ſummâ ſeriei, cujus generalis terminus ſit $\frac{az^m + bz^{m-1} + \&c.}{z^n + pz^{n-1} + \&c.}$ vel quæcunque alia determinata functio quantitatis x, primum invenienda eſt ſumma terminorum, uſque donec terminus z evadat major quam maxima radix æquationum $az^m + bz^{m+1} + \&c. = o$ & $z^n + pz^{n-1} + \&c. = o$, &c. 16. Transformatur æquatio relationem inter ſucceſſivas ſummas &

terminos deſignans in infinitam fluxionalem æquationem relationem inter ſummam S vel terminum t, ejus fluxiones, & z diſtantiam a primo ſeriei termino & $\dot{z}$ exprimentem; & nonnullæ ſeries traduntur. 17. Ex ſummis ſerierum primi ordinis deducuntur ſummæ omnium ſerierum ſuperiorum ordinum. 18. Inveniuntur ſummæ ſerierum, quarum termini ſunt irrationales. 19. Datur convergentia interpolationum ſerierum. 20. Traditur methodus interpolationis ſerierum, cum numerus factorum in terminis contentorum non ſit idem. 21. Dantur ſeries interpolabiles. 22. Etiamque ſummæ ſerierum $\frac{1}{1.2..m} \pm \frac{x^n}{1.2..n+m} + \frac{x^{2n}}{1.2..2n+m} \pm$ &c. 23. Ex datis fluentibus fluxionum $\int.\frac{\dot{x}}{1+ax^n}$, $\int.\frac{\dot{x}}{x}\int.\frac{\dot{x}}{1+ax^n}$, $\int.\frac{\dot{x}}{x}\int.\frac{\dot{x}}{x}\int.\frac{\dot{x}}{1+ax^n}$, &c. ad $h-1$, &c.; terminos datur ſeries pro inveniendâ ſummâ ſeriei, cujus generalis terminus eſt $\frac{\pm a^z}{nz+l.(nz+1)^{h-1}}$ &c. 24. Traduntur nonnulla problemata *Naudeanis* haud multum diſſimilia. 25. Adjicitur methodus inveniendi continuum contentum datæ quantitati æquale. 26. Dantur diverſæ ſeries pro inveniendis ſinubus & coſinubus arcuum qui inter ſe ſunt $:: n : m$; & methodus inveniendi approximationes ad radices æquationis per ſinus & coſinus. 27. Invenitur aggregatum plurium fractionum, quæ ſint infinitæ. 28. Adjungitur nova methodus inveniendi approximationem ad fluentem fluxionis $\frac{x^\alpha \dot{x}}{1 \pm x^h}$, cum $h : \alpha$ non habeat rationalem rationem; quæ ad omnes fluxiones applicari poteſt. 29. Datur facilis methodus detegendi; annon ſumma ſeriei, cujus generalis terminus eſt $\frac{a'}{\alpha+z.\beta+z.\gamma+z.\&c.}$, in finitis terminis exprimi poſſit: methodus generaliter detegendi, annon ſumma omnis ſeriei, cujus generalis terminus ſit data functio quantitatis x, in finitis terminis exprimi poſſit, in methodo incrementorum traditur. 30. Per ſimplices æquationes deducitur ſumma ſeriei, cujus generalis terminus

minus eſt $\frac{d'z^m + b'z^{m-1} + \&c.}{(\alpha + z)^b (\beta + z)^{b'} (\gamma + z)^{b''} \&c.}$, ex ſummis ſerierum quarum generales termini reſpective ſunt $\frac{1}{\alpha + z}$, $\frac{1}{(\alpha + z)^2}$, $\frac{1}{(\alpha + z)^3}$ . . $\frac{1}{(\alpha + z)^b}$; $\frac{1}{\beta + z}$, $\frac{1}{(\beta + z)^2}$ . . $\frac{1}{(\beta + z)^{b'}}$, &c. 31. Invenitur ſumma ſeriei, cujus generalis terminus eſt $\frac{d'z^m + b'z^{m-1} + \&c.}{1 . 2 . 3 . . nz \times m' + z . m'' + z . \&c.}$, ubi m' & m'',&c. ſunt inæquales quantitates, per circulares vel logarithmos,&c. 32. Ducantur ſucceſſivi termini in quaſcunque m arithmeticas independentes ſeries, & ex ſummis $(m + 1)$ ſerierum reſultantium erui poſſunt per ſimplices æquationes omnium hujuſce generis ſummæ. 33. Sit $P = A + Bx^n + Cx^{2n} + \&c.$; tum ex datis P, $\int . x^\alpha \dot{P}$, $\int . x^\beta \dot{P}$, &c. erui poteſt ſumma ſeriei, cujus generalis terminus eſt $\frac{\varphi (d'z^m + b'z^{m-1} + \&c.)}{\alpha + z . \beta + z . \gamma + z . \&c.}$; ſi modo φ ſit generalis terminus datæ ſeriei $\frac{\dot{P}}{\dot{x}}$. 34. Erit $\frac{1}{\alpha} . \frac{1}{\beta - \alpha} . \frac{1}{\gamma - \alpha} . \&c. + \frac{1}{\beta} . \frac{1}{\alpha - \beta} . \frac{1}{\gamma - \beta} . \&c. + \frac{1}{\gamma} . \frac{1}{\alpha - \gamma} . \frac{1}{\beta - \gamma} . \&c. + \&c. = \frac{1}{\alpha\beta\gamma \&c.}$ 35. Sit $(a + bx^n + cx^{2n} + . . x^{ln})^k x^\theta = x (A + Bx^n + Cx^{2n} + \&c.)$; tum ex $(l - 1)$ fluentibus independentibus fluxionum formulæ $(a + bx^n + cx^{2n} + \&c.)^{k \pm i} x^{\theta \pm hn} \dot{x}$, ubi h & i ſunt integri numeri, deduci poſſunt ſummæ omnium ſerierum, quarum generales termini ſunt $\frac{\varphi (d'z^m + b'z^{m-1}}{\theta \pm (r + zn) . \theta \pm (s + zn) .}$ $\frac{+ \&c.)}{\theta \pm (t + zn . \&c.)}$, ubi r, s, t, &c. ſunt diverſi integri numeri, & φ eſt generalis terminus ſeriei $A + Bx^n + \&c.$. &c. 36. Sit $(a + bx^n)^m = a^m + ma^{m-1}bx^n + \&c.$; tum per conicas ſectiones inveniri poteſt ſumma ſeriei, cujus generalis terminus eſt $m . \frac{m - 1}{2} . . . \frac{m - z + 1}{z} \times \frac{d'z^{m'} + b'z^{m'-1} + \&c.}{nz + rn . nz + sn . nz + tn . \&c.}$; ſi modo r, s, t, &c. ſint inæquales fractiones, quarum denominatores vel ſunt 1 vel 2: inveniri poteſt per arcus conicarum ſectionum, ſi denominatores non ſint majores quam 1, 2, 3 vel 4: & ſic e fluentibus in capite 2. libri 1. traditis, erui poſſunt plures hujuſmodi ſeries. 37. Erunt $x^{-\beta}\int . x^{\beta-\alpha-1}\dot{x} \int . x^{\alpha-1}\dot{x}P = x^{-\alpha}\int . x^{\alpha-\beta-1}\dot{x}\int . x^{\beta-1}\dot{x}P$, &c. 38. Dantur methodi inveniendi ſummabiles ſeries, quæ ſunt inter ſe æquales. 39. Traditur fluens fluxionis $\overset{(q)}{s}{}^{m} . \overset{(q)}{c}{}^{n} \times \frac{q\dot{x}}{\sqrt{(1-x^2)}}$, ubi $\overset{(q)}{s}$ & $\overset{(q)}{c}$ ſunt ſinus & coſinus arcûs qA; & A eſt arcus, cujus coſinus eſt x. 40. Datur nova methodus differentiarum, in quâ ex terminis ad diſtantias $n-1$, $n-2$, $n-3$, &c., a primo, qui ſint S^{n-1}, S^{n-2}, S^{n-3}, &c. acquiri poteſt terminus S^{n+m} ad diſtantiam $n+m$ a primo; erit enim $S^{m+n} = (m+n)S^{n-1} - (m+n) . \frac{m+n-1}{2}S^{n-2} +$ &c.; datis etiam S^{n}, S^{-n}, S^{n-1}, S^{-n+1}, S^{n-2}, S^{-n+2}, &c. traditur lex pro S^{m}. 41. Adjicitur methodus correſpondentium valorum, i. e. ſint S^{α} & α, S^{β} & β, S^{γ} & γ, &c., correſpondentes valores quantitatum x & y, invenitur $y = \frac{x-\alpha . x-\beta . \&c.}{\alpha-\beta . \alpha-\gamma} S^{\alpha} + \frac{x-\alpha . x-\gamma . \&c.}{\beta-\alpha . \beta-\gamma . \&c.} \times S^{\beta} +$ &c. in omnibus his methodis correſpondentium valorum, ſi modo inventæ quantitates fingantur nihilo æquales, reſultant æquationes, quæ præbebunt approximationes per regulas falſi datas. 42. Si modo detur formula functionis quantitatum x, y, z, &c., quarum tot correſpondentes valores dentur, quot incognitæ quantitates in datâ functione contineantur; tum ex iis deduci poteſt functio quæſita. 43. Dentur correſpondentes valores trium vel plurium quantitatum x, y, z, &c. inveniuntur quantitates, quæ habeant prædictos correſpondentes valores. 44. Datur correctio datæ quantitatis, quæ invenit veros correſpondentes valores in n caſibus, fallit vero in m. 45. Aſſeritur ſummam omnium fractionum $\frac{A}{\alpha-\beta . \alpha-\gamma . \alpha-\delta . \&c.} + \frac{B}{\beta-\alpha . \beta-\gamma . \beta-\delta} +$ &c., ubi A eſt eadem rationalis & integralis functio quantitatis α ac β eſt quantitatis β; & in A ſimiliter involvuntur quantitates β, γ, δ, &c.; & ſimiliter etiam ac quantitates α, γ, δ, &c. in B; &c. eſſe rationalem & integralem functionem quantitatum α, β, γ, &c. in quâ ſimiliter involvuntur prædictæ quantitates α, β, γ, &c., quæ habeat tot dimenſiones prædictarum quantitatum, quot ſit differentia inter dimenſiones quantitatum A & $\alpha-\beta\,.\,\alpha-\gamma\,.\,\alpha-\delta\,.$ &c.; ſi differentia ſit o, tum erit data quantitas; ſi negativa ſit, tum erit prædicta ſumma $=o$. 46. Ad finem adjungitur methodus deductionis & reductionis; cujus potius mentionem feci, quam aliquid particulariter adjecerim, ut via in hâc maxime generali ſcientiâ aliis ſternatur; ad quod maxime conferet tractatus de generalibus functionibus.

Quædam etiam alia adjiciuntur nova non alibi petenda; queri liceat opera mathematica exterarum gentium in hanc academiam perraro migraſſe; unde evenit me nunquam vidiſſe opera multorum præclarorum mathematicorum.

Pag. 147. dele Fig. 1. p. 95; pag. 467. l. 17. pro $n-1$, lege n; l. antepen. pro n, lege $n+1$.

MEDITATIONES ANALYTICÆ.

CAPUT I.

De Fluxionibus fluentium inveniendis.

PROB. I.

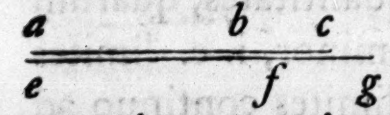
DATA methodo inveniendi ſucceſſivos terminos convergentis ſeriei $a+b+c+d+e+f+$ *&c. in infinitum progredientis, ſummam ejus invenire.*

Addatur primus terminus (a) ad ſecundum (b), & eorum ſumma $(a+b)$ ad tertium (c), trium terminorum ſumma $(a+b+c)$ ad quartum d, & ſic deinceps; & quoniam ſeries eſt convergens, hæ ſucceſſivæ ſummæ ad ſummam totius ſeriei vergunt, & ultimo propius accedunt ad eam quam pro quâvis datâ differentiâ: quæret vero aliquis, & recte quidem, quomodo cognoſci poteſt has ſummas ad ſummam totius ſeriei perpetuo vergere, & ultimo propius ad ſe invicem accedere quam pro datâ quâvis differentiâ: cui reſpondendum eſt, hanc eſſe methodum: continuo inveniantur duæ quantitates, quarum una major eſt quam quæſita ſumma, altera vero minor, i. e. limites inter quos interponitur ſumma quæſita, & ſi hi limites continuo ad ſe invicem vergant, & denique ſi hi limites ultimo probari poſſint propius ad ſe invicem accedere quam pro quâvis datâ differentiâ, tum hæ ſummæ propius ad ſe invicem convergunt, & ultimo propius accedunt quam pro datâ quâvis differentiâ.

FIG. I. Ex. Sit curva $AbcdBA$ rectâ lineâ AB & curvâ AcB comprehenſa; & ductis tangentibus ad puncta curvæ A & B, ſint interni anguli BAC & ABC ſimul aſſumpti haud majores quam duo recti; ducatur linea fci parallela lineæ AB, & tangens curvam in puncto

c, & agantur lineæ Ac, cB, erit inſcriptum triangulum vel æquale vel majus quam dimidium trapezii $ABifA$, & conſequenter majus quam dimidium areæ curvilineæ: & ſic ducantur lineæ eg & hk curvam tangentes in punctis b & d, parallelæ autem lineis Ac & Bc, & erunt triangula Abc & cdB reſpective majora quam dimidia trapeziorum $Aegc$ & $chkB$, ergo majora erunt quam dimidia curvilinearum arearum $AbcA$ & $cdBc$; & ſic continuo inſcribantur triangula, quorum baſes reſpective parallelæ ſunt lineis tangentibus curvam in eorum verticibus, & eodem modo probari poteſt hæc triangula inſcripta majora eſſe quam dimidia reliquarum curvilinearum arearum, & exinde e curvilineâ areâ continuo aufertur quantitas major quam dimidium reliquæ, &c. & conſequenter ultimo reſiduum erit minus quam quæcunque data quantitas. Sit ſeries, cujus primus terminus a ſit triangulum AcB, ſecundus vero b æqualis ſit ſummæ duorum triangulorum Abc & cdB, tertius vero terminus æqualis ſit ſummæ quatuor triangulorum ſimiliter inſcriptorum, &c.; hæc ſeries erit convergens, ſummæ enim ſucceſſivæ (a, $a+b$, $a+b+c$, &c.) perpetuo ad ſummam ſeriei, i. e. ad aream curvæ $AbcdBA$ vergunt, & ultimo propius ad eam accedunt quam pro datâ quâvis differentiâ.

Cor. 1. Omnis quantitas, quæ continuo inter limites convergentis ſeriei ponitur, æqualis erit datæ ſeriei ſummæ. Si non ſit æqualis datæ ſeriei ſummæ, ſit differentia d, & inveniantur limites ab & ac,



inter quos ponitur ſeriei ſumma & prædicta quantitas, & qui propius ad ſe invicem accedunt, quam pro differentiâ d. Sint ef & eg reſpective ſeriei ſumma & prædicta quantitas, & ef & eg majores erunt quam ab, minores vero quam ac; per hypotheſin vero differentia $fg = d$ major eſt quam bc, ergo ſumma $ef + fg = eg$ major erit quam ac, quod hypotheſi contradicit; unde ef haud major vel minor erit quam eg. Q. E. D.

Hoc modo demonſtravit Archimedes aream circuli æqualem eſſe triangulo, cujus baſis eſt ejus peripheria & perpendiculum radius.

Cor. 2. Sint duæ convergentes ſeries, & ſi ſingulus terminus unius ſeriei ſit in datâ ratione ad ſingulum alterius terminum, tum prioris ſeriei ſumma erit ad ſummam poſterioris ſeriei in eâdem ratione.

Hujus corollarii demonſtratio eadem erit ac ea in prob. 2. lib. xii. Eucl. elem. quam conſulas.

PROB. II.

Datis ſucceſſivis ſeriei ſummis, invenire terminos ſucceſſivos.

Sint S, S', S'', S''', S^4, &c. ſucceſſivæ ſummæ, & ſeries vel finita, vel in infinitum progrediens $a+b+c+d+e+$ &c. i. e. ſit

$S = a+b+c+d+e+$ &c.
$S' = \phantom{a+{}} b+c+d+e+$ &c.
$S'' = \phantom{a+b+{}} c+d+e+$ &c.
$S''' = \phantom{a+b+c+{}} d+e+$ &c.
&c. &c. &c. tum $a = S - S'$, $b = S^1 - S^2$, $c = S^2 - S^3$, $d = S^3 - S^4$, & ſic deinceps.

Ex. Sit diſtantia (x) termini quæſiti a primo, & ſint duæ ſucceſſivæ ſummæ $x \times \overline{x-e} \times \overline{x-2e} \times \overline{x-3e} \ldots \overline{x-n-2e} \times \overline{x-n-1e}$, & $\overline{x+e} \times x \times \overline{x-e} \times \overline{x-2e} \ldots \overline{x-n-2e}$. Subtrahatur prior ſumma e poſteriori, i. e. de ſummâ $\overline{x+e} \times x \times \overline{x-e} \times \overline{x-2e} \ldots\ldots \overline{x-n-2e}$ auferatur ſumma $\overline{x-n-1e} \times x \times \overline{x-e} \times \overline{x-2e} \ldots\ldots \overline{x-n-2e}$ & reſiduum erit $ne \times x \times \overline{x-e} \times \overline{x-2e} \ldots\ldots \overline{x-n-2e}$ terminus quæſitus.

Fig. 2. Cor. 1. Sit figura $abcdeIAa$ rectis Aa & AI & curvâ $abcI$ comprehenſa; inſcribantur & circumſcribantur parallelogramma ſub baſibus AB, BC, CD, &c. æqualibus, quæ dicantur e; & lateribus Aa, Bb, Cc, Dd, &c. figuræ lateri Aa parallelis, contenta; & compleantur circumſcripta & inſcripta parallelogramma; & ſit $x = AI$, &

ſumma inſcriptorum parallelogrammorum $Ab + Bc + Cd +$ &c. continuo ſit $x \times \overline{x-e} \times \overline{x-2e} \ldots \times \overline{x-\overline{n-2}e} \times \overline{x-\overline{n-1}e}$; tum ſumma circumſcriptorum parallelogrammorum erit proxima ſumma, viz. $\overline{x+e} \times x \times \overline{x-e} \times \overline{x-2e} \ldots\ldots \overline{x-\overline{n-2}e}$: circumſcriptorum & inſcriptorum parellalogrammorum differentia per exemplum erit $ne \times x \times \overline{x-e} \times \overline{x-2e} \ldots \overline{x-\overline{n-2}e}$. Sed hæc differentia æqualis eſt parallelogrammo Ab, cujus baſis eſt e; & conſequenter latus Aa, ſi parallelogrammum ſit rectangulum, erit æquale; ſin aliter proportionale contento $\frac{ne \times x \times \overline{x-e} \times \overline{x-2e} \ldots \overline{x-\overline{n-2}e}}{e} = nx \times \overline{x-e} \times \overline{x-2e} \ldots \overline{x-\overline{n-2}e}$. Area vero curvæ inter duas prædictas ſummas $x \times \overline{x-e} \ldots \overline{x-\overline{n-1}e}$ & $\overline{x+e} \times x \times \overline{x-e} \ldots \overline{x-\overline{n-2}e}$, viz. ſummas circumſcriptorum & inſcriptorum parallelogrammorum ſemper ponitur: ſit $e = o$, & hæ duæ ſummæ $x \times \overline{x-e} \times \overline{x-2e} \ldots \overline{x-\overline{n-1}e}$ & $\overline{x+e} \times x \times \overline{x-e} \times \overline{x-2e} \ldots \overline{x-\overline{n-2}e}$ fient $x \times x \times x \times$ &c. $= x^n$ inter ſe & quantitati x^n æquales: ſed curvæ area inter has duas quantitates continuo ponitur, ergo area curvæ in hoc caſu erit x^n: ordinata Aa fuit $n \times x \times \overline{x-e} \times \overline{x-2e} \ldots \overline{x-\overline{n-2}e}$; quod, cum $e = o$, fit $n \times x \times x \times x \times$ &c. $= nx^{n-1}$; ergo, ſi area curvæ continuo ſit x^n, ejus ordinata correſpondens erit nx^{n-1}.

Cor. 2. Sint ordinatæ $n \times x \times \overline{x-e} \times \overline{x-2e} \ldots \overline{x-\overline{n-2}e}$, & ſumma prædictorum inſcriptorum parallelogrammorum erit $\overline{x+e} \times x \times \overline{x-e} \times \overline{x-2e} \ldots \overline{x-\overline{n-2}e}$; & conſequenter, ſi ordinata curvæ ſit nx^{n-1}, ejus area erit x^n.

Et ſic ratiocinari liceat de pluribus hujuſcemodi quantitatibus.

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FIG. 3. Fluxio vel velocitas quantitatis ($x = Ap$) per $\dot{x}$ ſemper deſignetur, tum fluxio ejus poteſtatis ($x^n = AP$) erit $nx^{n-1}\dot{x}$; fluat enim uniformiter x, & ſint tres ſucceſſivi valores quantitatis x reſpective, $x-o$, x, $x+o$; tum tres ſucceſſivi valores quantitatis x^n erunt reſpective

$$\overline{x-o}^{n} = x^n - nx^{n-1}o + n \,.\, \frac{n-1}{2} x^{n-2}o^2 - \&c.$$

$$x^n = x^n.$$

$$\overline{x+o}^{n} = x^n + nx^{n-1}o + n \,.\, \frac{n-1}{2} x^{n-2}o^2 + \&c.$$

ſubtrahantur hi tres ſucceſſivi valores quantitatis (x^n) a ſe invicem, & reſultant duæ ſucceſſivæ differentiæ $nx^{n-1}o - n \,.\, \frac{n-1}{2} x^{n-2}o^2 + \&c.$ & $nx^{n-1}o + n \,.\, \frac{n-1}{2} x^{n-2}o^2 + \&c.$ quæ continuo creſcunt; unde, ſi x fluat uniformiter, quantitatis (x^n) motus erit acceleratus: ſi vero ſit motus acceleratus, tum ſpatium $nx^{n-1}o - n \,.\, \frac{n-1}{2} x^{n-2}o^2 + \&c.$ minus, & $nx^{n-1}o + n \,.\, \frac{n-1}{2} x^{n-2}o^2 + \&c.$ majus erit quam ſpatium, quod corpus velocitate ad punctum P eodem tempore deſcriberet; ergo velocitas quantitatis (x) erit ad velocitatem quantitatis (x^n) in minori ratione quam $o : nx^{n-1}o - n \,.\, \frac{n-1}{2} x^{n-2}o^2 + \&c$: in majori autem ratione quam $o : nx^{n-1}o + n \,.\, \frac{n-1}{2} x^{n-2}o^2 + \&c.$ i. e. in minore ratione quam $\dot{x} : nx^{n-1}\dot{x} - n \,.\, \frac{n-1}{2} x^{n-2}o\dot{x} + \&c.$ & in majori quam $\dot{x} : nx^{n-1}\dot{x} + n \,.\, \frac{n-1}{2} x^{n-2}o\dot{x} + \&c.$ ergo, ſi velocitas quantitatis (x) per $\dot{x}$ deſignetur, velocitas quantitatis x^n inter duas quantitates $n x^{n-1} \dot{x} - n \, . \, \frac{n-1}{2} x^{n-2} o \dot{x} +$ &c. & $n x^{n-1} \dot{x} + n \times \frac{n-1}{2}$ $x^{n-2} o \dot{x} +$ &c. ſemper ponetur, quicunque ſit valor quantitatis (o). Supponatur o nihilo æqualis, & duæ prædictæ quantitates fiunt $n x^{n-1} \dot{x}$ & inter ſe æquales; unde velocitas quantitatis x^n, quæ inter eas ponitur, erit etiam $n x^{n-1} \dot{x}$.

Cor. 1. Sit quantitas $x^{\frac{n}{m}}$, cujus fluxio requiritur; fingatur $x^{\frac{n}{m}} = y$, & conſequenter $x^n = y^m$, unde $n x^{n-1} \dot{x} = m y^{m-1} \dot{y}$, & per reductionem $\dot{y} = \frac{n x^{n-1} \dot{x}}{m y^{m-1}} = \frac{n}{m} x^{\frac{n}{m}-1} \dot{x}$.

Cor. 2. Sit quantitas $a x - b y + c z + d v$, &c. & ſint $\dot{x}, \dot{y}, \dot{z}, \dot{v}$, &c. reſpective fluxiones quantitatum (x, y, z, v, &c.) i. e. earum ſpatia dato tempore uniformi motu deſcripta; & erit fluxio datæ quantitatis $a \dot{x} - b \dot{y} + c \dot{z} + d \dot{v}$ &c. Velocitates vel fluxiones enim erunt in eâdem ratione ac ſpatia uniformi motu dato tempore deſcripta; hæc vero ſpatia ſunt $a \dot{x} - b \dot{y} + c \dot{z} + d \dot{v}$, &c. unde conſtat cor.

Cor. 3. Fluxio rectanguli $x y$ erit $x \dot{y} + y \dot{x}$. Fluxio enim quadrati $\overline{x+y}^2 = (x^2 + y^2 + 2 x y) = 2 \times \overline{x+y} \times \overline{\dot{x}+\dot{y}} = 2 x \dot{x} + 2 y \dot{y} + 2 y \dot{x} + 2 x \dot{y}$; ſed fluxio quantitatis $x^2 + y^2$ eſt $2 x \dot{x} + 2 y \dot{y}$, quæ ſubtrahatur e priori, reſultat fluxio quantitatis $(2 x y) = 2 x \dot{y} + 2 y \dot{x}$, unde fluxio rectanguli $(x y) = x \dot{y} + y \dot{x}$.

Cor. 4. Sit fractio $\frac{x}{y}$, cujus fluxio requiritur; fingatur $\frac{x}{y} = z$, & exinde $x = y z$, & conſequenter $\dot{x} = y \dot{z} + z \dot{y}$; unde $\dot{z} = \frac{\dot{x} - z \dot{y}}{y} =$ $\frac{\dot{x} - \frac{x}{y} \dot{y}}{y} = \frac{y \dot{x} - x \dot{y}}{y^2}$.

Cor. 5. Fluxio contenti $x y z$ erit $x y \times \dot{z} + z \times \dot{\overline{x y}}$, ſed $\dot{\overline{x y}} = x \dot{y} + y \dot{x}$; ergo fluxio contenti $x y z$ erit $x y \dot{z} + x z \dot{y} + z y \dot{x}$; & ſic fluxio contenti $x y z v w$ invenietur $x y z v \dot{w} + x y z w \dot{v} + x y v w \dot{z}$

+

$+ xvwz\dot{y} + yvwz\dot{x}$: & fluxio contenti $x^n y^m z^r v^s$ erit $nx^{n-1} y^m z^r v^s \dot{x} + my^{m-1} x^n z^r v^s \dot{y} + rz^{r-1} x^n y^m v^s \dot{z} + sv^{s-1} x^n y^m z^r \dot{v}$; & ſic deinceps.

Cor. 6. Sit quantitas data quæcunque algebraica functio fluentium quantitatum; ſcribendo y, z, v, &c. pro quantitatibus in his functionibus contentis inveniri poteſt ejus fluxio.

Ex. 1. Sit fluens $\overline{a + bx^n + cx^{2n} + dx^{3n} + \&c.}^{m}$ cujus fluxio requiritur: ſcribatur $y = a + bx^n + cx^{2n} + dx^{3n} + \&c.$ & exinde $\dot{y} = nbx^{n-1}\dot{x} + 2ncx^{2n-1}\dot{x} + 3ndx^{3n-1}\dot{x}$; & $y^m = \overline{a + bx^n + cx^{2n} + dx^{3n} + \&c.}^{m}$ ſed fluxio quantitatis y^m eſt $my^{m-1}\dot{y} = m \times \overline{a + bx^n + cx^{2n} + \&c.}^{m-1} \times \overline{nbx^{n-1}\dot{x} + 2ncx^{2n-1}\dot{x} + \&c.}$ ſi modo pro y & $\dot{y}$ ſcribantur earum valores $a + bx^n + cx^{2n} + \&c.$ & $nbx^{n-1}\dot{x} + 2ncx^{2n-1}\dot{x} + \&c.$

Ex. 2. Sit fluens $\overline{a + bx^n + cx^{2n} + \&c.} \times \overline{e + fx^n + gx^{2n} + \&c.}^{\lambda} \times \overline{k + lx^n + mx^{2n} + \&c.}^{\mu} \times \overline{p + qx^n + rx^{2n} + \&c.}^{\nu} \times \&c.$ & ejus fluxio e prædictis principiis invenietur $n\dot{x} \times \overline{e + fx^n + gx^{2n} + \&c.}^{\lambda-1}$

$$\times \overline{k + lx^n + mx^{2n} + \&c.}^{\mu-1} \times \overline{p + qx^n + rx^{2n} + \&c.}^{\nu-1} \times \overline{\overline{bekp +}}$$

$$\overline{\overline{\lambda fakp + \mu laep + \nu qaek}} \times x^{n-1} + \overline{\overline{2cekp + 2\lambda gakp + 2\mu maep}}$$

$$\overline{\overline{+ 2\nu raek + b \times \overline{ekq + epl + kpf} + \lambda f \times \overline{akq + apl + bkp} +}}$$

$$\overline{\overline{\mu l \times \overline{aeq + apf + peb} + \nu q \times \overline{ael + akf + keb}}} \times x^{2n-1} + \&c.$$

Def. Logarithmi dicuntur quantitates, quæ ſunt proportionales exponentibus poteſtatum vel radicum datæ quantitatis; e. g. ſit (A) logarithmus quantitatis x, tum $\frac{n}{m}A$ erit logarithmus quantitatis $x^{\frac{n}{m}}$.

Hæ autem exponentes ſunt menſuræ rationum quantitatum, ergo logarithmi etiam erunt menſuræ earum rationum.

Cor. 1. Hinc facile colligitur, ſi modo logarithmi quantitatum a & b dicantur reſpective A & B, logarithmum producti $a \times b$ eſſe $A + B$.

Cor. Hinc, ſi quantitas (x) creſcat proportionaliter, ejus logarithmus fluit uniformiter, i. e. ſi quantitas (x) fluat eâdem ratione ac quantitas ipſa, tum ejus logarithmus fluit uniformiter; & conſequenter quantitas x erit ad datam quantitatem $M :: \dot{x}$ ad fluxionem ejus logarithmi $\frac{M\dot{x}}{x}$; hinc fluxio logarithmi quantitatis (x) erit proportionalis fluxioni $\frac{\dot{x}}{x}$. Si vero pro x & $\dot{x}$ ſcribantur $x + a$ & $\dot{x}$, reſultat $\left(\frac{M\dot{x}}{x+a}\right)$ fluxio logarithmi quantitatis $(x + a)$.

PROB. III.

1. Sit exponentialis quantitas x^{y}, invenire ejus fluxionem: fingatur $x^{y} = X$, unde $y \times \log. x = \log. X$; hujus æquationis inveniatur fluxio, & reſultat $\log. x \times \dot{y} + y \times \frac{\dot{x}}{x} = \frac{\dot{X}}{X}$, unde $X \times \log. x, \times \dot{y} + Xy \frac{\dot{x}}{x} = \dot{X}$; pro X ſcribatur x^{y}, reſultat fluxio quæſita $\dot{X} = x^{y} \times \log. x \times \dot{y} + yx^{y-1}\dot{x}$.

2. Sit data exponentialis quantitas $x^{y} \times v$, & per præcedentem methodum inveniri poteſt ejus fluxio $x^{y}\dot{v} + vyx^{y-1}\dot{x} + v \times x^{y} \times \log. x \times \dot{y}$.

3. Sit exponentialis quantitas $x^{y^{z^{v^{w^{\&c.}}}}}$, & ejus fluxio erit $y^{z^{v^{w^{\&c.}}}} \times x^{y^{z^{v^{w^{\&c.}}}}-1}$
$\dot{x} + x^{y^{z^{v^{w^{\&c.}}}}} \times z^{v^{w^{\&c.}}} \times y^{z^{v^{w^{\&c.}}}-1} \times \log. x \times \dot{y} + x^{y^{z^{v^{w^{\&c.}}}}} \times y^{z^{v^{w^{\&c.}}}} \times v^{w^{\&c.}} \times z^{v^{w^{\&c.}}-1} \times \log. x \times$
$\log. y \times \dot{z} + x^{y^{z^{v^{w^{\&c.}}}}} \times y^{z^{v^{w^{\&c.}}}} \times z^{v^{w^{\&c.}}} \times w^{\&c.} \times v^{w^{\&c.}-1} \times \log. x \times \log. y \times \log.$
$z \times \dot{v} + x^{y^{z^{v^{w^{\&c.}}}}} \times y^{z^{v^{w^{\&c.}}}} \times z^{v^{w^{\&c.}}} \times v^{w^{\&c.}} \times \&c.\ w^{\&c.-1} \times \log. x \times \log. y \times$
$\log. z \times \log. v \times \dot{w} + \&c.$ unde facile conſtabit lex, quam obſervat hæc ſeries.

4. Fluxio exponentialis $x^{y^{z^{v^{w^{\&c.}}}}} \times V$ erit $V \times$ fluxion. exponent. $\left(x^{y^{z^{v^{w^{\&c.}}}}}\right)$ $+\, x^{y^{z^{v^{w^{\&c.}}}}} \times \dot{V}$; hæc vero fluxio ſic exprimi poteſt $x^{y^{z^{v^{w^{\&c.}}}}} \times \dot{V} +$ $\frac{V \times y^{z^{v^{w^{\&c.}}}} \times A}{x\dot{V}}\dot{x} + \frac{z^{v^{w^{\&c.}}} \times B \times x \times \text{log.}\, x}{y\dot{x}}\dot{y} + \frac{v^{w^{\&c.}} \times C \times y \times \text{log.}\, y}{z\dot{y}}\dot{z} +$ $\frac{w^{\&c.} \times D \times z \times \text{log.}\, z}{v\dot{z}}\dot{v} + \frac{\&c. \times E \times v \times \text{log.}\, v}{w\dot{v}}\dot{w} +$ &c. ubi literæ A, B, C, & præcedentes terminos reſpective denotant.

Sit V exponentialis quantitas, & facile conſtabit fluxio quæſita.

Et ſic inveniri poteſt fluxio contenti ſub pluribus exponentialibus quantitatibus.

Cor. Si vero x, y, z, &c. ſint quæcunque functiones quarumlibet incognitarum quantitatum; & pro x, y, z, &c. & earum fluxionibus in ſerie prius traditâ ſcribantur hæ functiones & earum fluxiones reſpective; invenietur fluxio quæſita.

5. Secundæ fluxiones inveniuntur e primis, tertiæ e ſecundis, &c. eodem modo, quo primæ inveniuntur e fluentibus; ergo datæ fluxiones pro fluentibus habeantur, & facile inveniri poſſunt earum fluxiones.

Ex. Sint $\dot{x}$, $\ddot{x}$, $\dddot{x}$, &c. reſpective prima, ſecunda, tertia, &c. fluxiones quantitatis (x); & ſic $\dot{y}$, $\ddot{y}$, $\dddot{y}$, &c. prima, ſecunda, tertia, &c. fluxiones quantitatis (y), &c. invenire ſecundam, tertiam, &c. fluxionem quantitatis $x^m y^n$: prima vero fluxio datæ quantitatis invenietur $m x^{m-1} y^n \dot{x} + n y^{n-1} x^m \dot{y}$; ejus vero fluxio, quæ erit ſecunda fluxio quantitatis $(x^m y^n)$, erit $m \times \overline{m - 1}\, x^{m-2} y^n \dot{x}^2 + 2mn x^{m-1} y^{n-1} \dot{y}\dot{x} +$ $m x^{m-1} y^n \ddot{x} + n \times \overline{n - 1}\, y^{n-2} x^m \dot{y}^2 + n y^{n-1} x^m \ddot{y}$; & eodem modo inveniri poteſt ejus $(x^m y^n)$ tertia fluxio, &c.

Cor. Hinc fluxio n ordinis rectanguli xy erit $\overset{n}{\dot{x}}y + n\overset{n-1}{\dot{x}}\dot{y} + n \times \frac{n-1}{2}\overset{n-2}{\dot{x}}\dot{y}^2 + n \,.\, \frac{n-1}{2} \,.\, \frac{n-2}{3}\overset{n-3}{\dot{x}}\dot{y}^3 \,.\,.\, n\dot{x}\overset{n-1}{\dot{y}} + x\overset{n}{\dot{y}}$.

SCHOLIUM.

Duæ ſunt methodi inveniendi areas curvarum, vel quod ad idem redit, inveniendi ſummas quantitatum, quarum datur infinitus numerus; altera e repetitis additionibus continuo vergit ad ſummam quæſitam, & ultimo propius ad eam accedit quam pro datâ quâvis differentiâ, quod cognoſci poteſt e limitibus ſucceſſive deductis inter quos conſiſtit quæſita ſumma: hæc fuit methodus veterum. Altera vero aſſumit ſummam tanquam quantitatem generaliter cognitam, & ejus partes exinde deducit; & quantitatibus, quarum datur infinitus numerus, fiunt prædictæ partes vel æquales vel propiores ad rationem æqualitatis quam pro datâ quâvis differentiâ; & exinde concludit ſummam quantitatum, quarum datur infinitus numerus, eſſe aſſumptam quantitatem: hæc fuit methodus recentiorum: in hâc vero methodo minime refert, quo modo generantur partes ipſæ; ſolummodo refert, utrum partes quantitatis aſſumptæ quantitatibus, quarum datur infinitus numerus, conveniant, necne: ſi vero partes conſiderentur tanquam motu generatæ, tum dicitur methodus fluxionum vel incrementorum; ſin aliter dici poteſt methodus integrationis.

C A P. II.

De inveniendis Fluxionum fluentibus.

P R O B. IV.

DATA quâcunque fluxione, quæ ſit algebraica funƈtio literæ **x** *in* $\dot{x}$ *ducta; invenire utrum ejus fluens exprimi poteſt terminis incognitæ quantitatis* **x**, *necne.*

Caſ. 1. Sit fluxio $ax^m\dot{x}$, & ejus fluens erit $\frac{ax^{m+1}}{m+1}$.

1. Caſ. 2. Sit fluxio $x^{\theta-1} \times \overline{e + fx^n + gx^{2n} + kx^{3n} + \&c.}^{\lambda-1} \times \overline{a + bx^n + cx^{2n} + dx^{3n} + \&c.} \times \dot{x}$; aſſumatur pro ejus fluente $x^\theta \times \overline{e + fx^n + gx^{2n} + kx^{3n} + \&c.}^{\lambda} \times \overline{A + Bx^n + Cx^{2n} + \&c.}$ tum per cor. 6. theor. 1. ejus fluxio invenietur $\overline{\theta eA + \overline{\theta + \lambda n} fA + \overline{\theta + n} eBx^n +}$ $\overline{\overline{\theta + 2\lambda n} gA + \overline{\theta + n + \lambda n} fB + \overline{\theta + 2n} eCx^{2n} + \&c.} \times x^{\theta-1} \times \overline{e + fx^n + gx^{2n} + \&c.}^{\lambda-1} \dot{x}$; & ex æquatis inter ſe correſpondentibus terminis reſultabunt $\theta eA = a$, $\overline{\theta + \lambda n} fA + \overline{\theta + n} eB = b$, $\overline{\theta + 2\lambda n} \times gA + \overline{\theta + n + \lambda n} \times fB + \overline{\theta + 2n} eC = c$, &c. & exinde $A = \frac{a}{\theta e}$, $B = \frac{b - \overline{\theta + \lambda n} fA}{\overline{\theta + n} e}$, $C = \frac{c - \overline{\theta + 2\lambda n} gA - \overline{\theta + n + \lambda n} fB}{(\theta + 2n) e}$, &c.

Caſ. 3. Sit fluxio $x^{\theta-1} \times \overline{e + fx^n + gx^{2n} + hx^{3n} + \&c.}^{\lambda-1} \times \overline{k + lx^n + mx^{2n} + \&c.}^{\mu-1} \times \overline{a + bx^n + cx^{2n} + dx^{3n} + \&c.} \times \dot{x}$; & ad modum præcedentem pro ejus fluente aſſumatur quantitas $x^\theta \times (e + fx^n + gx^{2n} + \&c.)^\lambda \times (k + lx^n + mx^{2n} + \&c.)^\mu \times (\alpha + \beta x^n + \gamma x^{2n} + \&c.)$; cujus inveniatur fluxio, & ex æquatis correſpondentibus datæ & reſultantis fluxionis terminis inveniri poteſt prædicta fluens $x^{\theta} \times \overline{e + fx^{n} + gx^{2n} + \&c.}^{\lambda} \times \overline{k + lx^{n} + mx^{2n} + \&c.}^{\mu} \times \frac{a}{\theta e k}(A) + \frac{\overline{b - \overline{\overline{\theta + \lambda}\, nfk + \overline{\theta + \mu}\, nelA}}}{\overline{\theta + n}\, ek} \times x^{n} + \&c.$ Et ſic deinceps.

Caſ. 4. Pro quantitatibus $a + bx^{n} + cx^{2n} + \&c.$, $d + ex^{n} + fx^{2n} + \&c.$, $g + hx^{n} + kx^{2n} + \&c.$, &c. ſcribantur reſpective R, S, T, &c. & ſit data fluxio $(A + Bx^{n} + Cx^{2n} + \&c.)\, x^{\theta-1} \times R^{\lambda-1} S^{\mu-1} T^{\nu-1} \times \&c.\, \dot{x}$, ſi vero communem habeant diviſorem nonnullæ ſubſequentes quantitates R, S, T, &c. & $A + Bx^{n} + Cx^{2n} + \&c.$ tum ita reducatur data fluxio, ut quantitates prædictæ nullum habeant communem diviſorem, i. e. ſimul colligantur ſub eodem vinculo omnes iidem diviſores, qui in datâ fluxione continentur; e. g. ſit $\alpha + \beta x^{n}$ communis diviſor quantitatum $a + bx^{n} + cx^{2n} + \&c. = R$, $d + ex^{n} + fx^{2n} + \&c. = S$, & $g + hx^{n} + kx^{2n} + \&c. = T$; i. e. $\overline{\alpha + \beta x^{n}} \times \overline{l + mx^{n} + ox^{2n} + \&c.} = a + bx^{n} + cx^{2n} + \&c. = R$, $\overline{\alpha + \beta x^{n}} \times \overline{p + qx^{n} + rx^{2n} + \&c.} = d + ex^{n} + fx^{2n} + \&c. = S$, & $\overline{\alpha + \beta x^{n}} \times \overline{s + tx^{n} + \&c.} = g + hx^{n} + kx^{2n} + \&c. = T$; & reducatur data fluxio in hanc formulam $x^{\theta-1} \times \overline{A + Bx^{n} + Cx^{2n} + \&c.} \times \overline{\alpha + \beta x^{n}}^{\lambda-1} \times \overline{l + mx^{n} + ox^{2n} + \&c.}^{\lambda-1}$ $(R^{\lambda-1})$ $\times \overline{\alpha + \beta x^{n}}^{\mu-1} \times \overline{p + qx^{n} + rx^{2n} + \&c.}^{\mu-1}$ $(S^{\mu-1})$ $\times \overline{\alpha + \beta x^{n}}^{\nu-1} \times \overline{s + tx^{n} + \&c.}^{\nu-1}$ $(T^{\nu-1})$ $= \overline{\alpha + \beta x^{n}}^{\lambda+\mu+\nu-3} \times \overline{l + mx^{n} + ox^{2n} + \&c.}^{\lambda-1} \times \overline{p + qx^{n} + rx^{2n} + \&c.}^{\mu-1} \times \overline{s + tx^{n} + \&c.}^{\nu-1} \times x^{\theta-1} \times \overline{A + Bx^{n} + \&c.}$ aſſumatur pro ejus fluente quantitas $x^{\theta} \times \overline{\alpha + \beta x^{n}}^{\lambda+\mu+\nu-2} \times \overline{l + mx^{n} + ox^{2n} + \&c.}^{\lambda} \times \overline{p + qx^{n} + rx^{2n} + \&c.}^{\mu} \times \overline{s + tx^{n} + \&c.}^{\nu} \times \overline{\pi + \varrho x^{n} + \sigma x^{2n} + \&c.}$ per caput præcedens inveniatur ejus fluxio, & ſupponantur datæ & reſultantis fluxionis termini correſpondentes inter ſe æquales; & exinde inveſtigari poſſunt coefficientes π, ϱ, σ, &c. quæſitæ.

Caſ. 5. Sit data fluxio eadem ac in præcedente caſu, & quantitatis ſub eodem vinculo contentæ ſint plures diviſores inter ſe æquales, viz. diviſores hujuſce formulæ $\overline{\alpha+\beta x^n}^{\,2}$ vel $\overline{\alpha+\beta x^n}^{\,m}$; ubi litera m vel binarium vel majorem numerum denotat; & ita reducatur data fluxio, ut quantitatum ſub vinculis nulli inveniantur communes diviſores, vel in eâdem quantitate nulli inter ſe æquales; deinde ex methodo in præcedente caſu traditâ inveſtigari poteſt fluens quæſita.

Ex. 1. Sit $\overline{\alpha+\beta x^n}^{\,\nu}\times\overline{l+mx^n+ox^{2n}+\&c.}=a+bx^n+cx^{2n}+\&c.$ aſſumatur pro fluente quantitas $x^\theta\times\overline{\alpha+\beta x^n}^{\,\nu\lambda-\nu+1}\times\overline{l+mx^n+ox^{2n}+\&c.}^{\,\lambda}$ $\times S^\mu\times T^\nu\times(\pi+\varrho x^n+\sigma x^{2n}+\&c.)$; & inveniatur ejus fluxio, quæ fiat æqualis datæ fluxioni, & exinde detegi poteſt fluens quæſita.

Ex. 2. Sint $\overline{\alpha+\beta x^n}^{\,\zeta}\times\overline{\gamma+\delta x^n}^{\,\theta}\times\overline{l+mx^n+ox^{2n}+\&c.}=a+bx^n+cx^{2n}+\&c.=R$, & $\overline{\alpha+\beta x^n}^{\,\chi}\times\overline{\varepsilon+\zeta x^n}^{\,\varphi}\times\overline{p+qx^n+rx^{2n}+\&c.}=d+ex^n+fx^{2n}+\&c.=S$, & pro fluente fluxionis prædictæ $\overline{A+Bx^n+Cx^{2n}+\&c.}\times x^{\theta-1}\times R^{\lambda-1}\times S^{\mu-1}\times T^{\nu-1}\times\&c.\times\dot{x}$ aſſumenda eſt quantitas $x^\theta\times\overline{\alpha+\beta x^n}^{\,\zeta(\lambda-1)+\chi(\mu-1)+1}\times\overline{\varepsilon+\zeta x^n}^{\,\varphi(\mu-1)+1}\times\overline{\gamma+\delta x^n}^{\,\theta(\lambda-1)+1}\times$ $\overline{l+mx^n+ox^{2n}+\&c.}^{\,\lambda}\times\overline{p+qx^n+rx^{2n}+\&c.}^{\,\mu}\times T^\nu\times\&c.$ & exinde per methodum prius traditam deduci poteſt fluens quæſita.

*Cor. 1. Si fluxio ſit fractio rationalis irreducibilis cum denominatore ex duobus vel pluribus terminis compoſito, reſolvendus eſt denominator in diviſores ſuos omnes primos: & ſi diviſor ſit aliquis, cui nullus alius eſt æqualis, curva quadrari nequit: ſin duo vel plures λ ſint diviſores æquales, viz. $\overline{\alpha+\beta x^n+\gamma x^{2n}+\&c.}^{\,\lambda}=R^\lambda$, & ſi adhuc alii duo vel plures μ ſint ſibi mutuo æquales & prioribus inæquales viz. $\overline{\pi+\varrho x^n+\sigma x^{2n}+\&c.}^{\,\mu}=T^\mu$, tum pro fluente quæſitâ fluxionis $x^{\theta-1}\times R^{-\lambda}\times T^{-\mu}\times(a+bx^n+\&c.)\times\dot{x}$ aſſumenda eſt quantitas $R^{-\lambda+1}\times T^{-\mu+1}\times x^\theta\times\&c.\times(A+Bx^n+Cx^{2n}+\&c.)$ & ex methodo prius traditâ deduci poſſunt valores incognitarum coefficientium A, B, C, &c. in hoc caſu λ & μ ſunt integri numeri.

Cor. 2. Si fluxio ſit fractio irreducibilis, & ejus denominator contentum

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* Newt. Quadr. Curv. p. 54.

tentum ſit ſub factore rationali $\mathcal{Q}$ & factore ſurdo irreducibili R^{π}; inveniendi ſunt lateris R diviſores omnes primi, & ſi duo vel plures λ diviſores ſint inter ſe æquales, viz. $\overline{\alpha + \beta x^{n} + \gamma x^{2n} + \&c.}^{\lambda}$ & ſi adhuc alii duo vel plures μ ſint ſibi mutuo æquales & prioribus inæquales, viz. $\overline{\varrho + \sigma x^{n} + \tau x^{2n} + \&c.}^{\mu}$, &c. & ſi rationalis factor $\mathcal{Q}$ ſit $= \overline{\alpha + \beta x^{n} + \gamma x^{2n} + \&c.}^{a} \times \overline{\varrho + \sigma x^{n} + \tau x^{2n} + \&c.}^{b} \times \overline{\delta + \varepsilon x^{n} + \zeta x^{2n} + \&c.}^{c} \times \&c.$ tum pro fluente quæſitâ fluxionis $x^{\theta-1} \times \mathcal{Q}^{-1} \times R^{-\pi} \times (a + bx^{n} + cx^{2n} + \&c.) \times \dot{x}$ aſſumenda eſt quantitas $\overline{\alpha + \beta x^{n} + \gamma x^{2n} + \&c.}^{1-a-\pi\lambda} \times \overline{\varrho + \sigma x^{n} + \tau x^{2n} + \&c.}^{1-b-\pi\mu} \times \overline{\delta + \varepsilon x^{n} + \zeta x^{2n} + \&c.}^{1-c} \times \&c. \times S^{1-\pi} \times (A + Bx^{n} + Cx^{2n} + \&c.)$, ubi $R^{\pi} = S^{\pi} \times \overline{\alpha + \beta x^{n} + \gamma x^{2n} + \&c.}^{\lambda\pi} \times \overline{\varrho + \sigma x^{n} + \tau x^{2n} + \&c.}^{\pi\mu} \times \&c.$ & per methodum in hoc problemate traditam deduci poſſunt valores incognitarum coefficientium A, B, C, &c.

Caſ. 6. Sit fluens quæcunque algebraica functio literæ x; tum, ſi inveniatur ejus fluxio, ea minores habebit dimenſiones quantitatis x quam fluens per unitatem; ni dimenſiones quantitatis x in prædictâ algebraicâ functione nihilo ſint æquales, i. e. maximæ dimenſiones quantitatis x in numeratore æquales ſint ejus maximis dimenſionibus in denominatore; in quo caſu maximæ dimenſiones quantitatis x in fluente majores erunt quam maximæ dimenſiones ejuſdem quantitatis in fluxione per quantitatem majorem quam unitatem. Maximæ dimenſiones quantitatis (x) in irrationalibus quantitatibus per methodum, quæ docetur in prob. 26. noſt. meditat. algebr. acquirendæ ſunt; unde e contra, datâ fluxione, quæ eſt functio algebraica quantitatis x in fluxionem $\dot{x}$ ducta; & conſequenter datâ maximâ dimenſione quantitatis x in eâ contentâ, datur etiam maxima dimenſio quantitatis x in fluente contenta; exinde facile conſtant dimenſiones quantitatis x, ad quas & haud plures aſſurgent ſeriei termini, ſi modo haud in infinitum progrediatur. e. g. Sit fluxio $(A + Bx^{n} + Cx^{2n} \ldots + Mx^{\kappa n})$

$\times x^{\theta-1}$

$x^{\theta-1} \times R^{\lambda-1} \times S^{\mu-1} \times T^{\nu-1} \times$ &c. $\times \dot{x}$, ubi $R = a + bx^n + cx^{2n} \ldots x^{\pi n}$; $S = d + ex^n + fx^{2n} \ldots x^{\varrho n}$, $T = l + hx^n + kx^{2n} \ldots x^{\sigma n}$, &c.; aſſumatur pro fluente quæſitâ quantitas $(\alpha + \beta x^n + \gamma x^{2n} + \&c.) \times x^{\theta} \times R^{\lambda} \times S^{\mu} \times T^{\nu} \times$ &c.; deinde per methodum prius traditam inveniatur quantitas $\alpha + \beta x^n + \gamma x^{2n} + \ldots x^{\kappa n - (\pi + \varrho + \sigma + \&c.)n}$; i. e. ſeries $\alpha + \beta x^n + \gamma x^{2n} +$ &c. non ultra terminum $(x^{\kappa n - (\pi + \varrho + \sigma + \&c.)n})$, ni in infinitum progrediatur; & conſequenter non habet plures quam $\kappa - \pi - \varrho - \sigma -$ &c. $+ 1$ terminos.

Caſ. 7. Si requiratur fluens prædictæ fluxionis $(A + Bx^n \ldots Mx^{\kappa n}) \times x^{\theta-1} \times R^{\lambda-1} \times S^{\mu-1} \times T^{\nu-1} \times \dot{x}$ in ſerie ſecundum dimenſiones quantitatis x^n deſcendente; quæ in pluribus caſibus converget, quam præcedens ſeries: aſſumatur $(\alpha x^{\varphi n} + \beta x^{(\varphi-1)n} + \gamma x^{(\varphi-2)n} + \&c.) \times x^{\theta} \times R^{\lambda} \times S^{\mu} \times T^{\nu} \times$ &c. pro fluente, ubi $\varphi = \kappa - \pi - \varrho - \sigma -$ &c.; & deinde inveniatur fluxio fluentis aſſumptæ, & æquentur correſpondentes datæ & reſultantis fluxionis termini, & exinde deduci poſſunt coefficientes α, β, γ, &c.

Hæc ſeries ſemper terminat, cum ſeries ex priori methodo deducta terminat.

Hìc excipienda ſunt exempla nonnullarum fluxionum, quarum fluentium termini progrediuntur, uſque donec dimenſiones quantitatis (x) evadunt nihilo æquales.

Caſ. 8. Sit fluxio rationalis functio literæ x in fluxionem $\dot{x}$ ducta, & maximæ dimenſiones quantitatis x in denominatore ſuperent maximas dimenſiones quantitatis x in numeratore per unitatem, tum ejus fluens haud inveniri poteſt in finitis algebraicis terminis literæ x.

e. g. Sit fluxio $\dfrac{a + bx + \overline{c + dx^3}^{\frac{1}{2}}}{e + fx + \overline{g + hx^5}^{\frac{1}{2}}} \times \dot{x}$; maximæ dimenſiones quantitatis x in numeratore datæ fluxionis ſunt $3 \times \frac{1}{2} = 1\frac{1}{2}$; maximæ autem dimenſiones ejuſdem quantitatis (x) in denominatore ſunt $5 \times \frac{1}{2} = 2\frac{1}{2}$; & exinde dimenſiones quantitatis (x) in denominatore majores ſunt quam dimenſiones ejuſdem quantitatis in numeratore per unitatem;

tatem; & conſequenter fluens fluxionis in finitis algebraicis terminis non exprimi poteſt.

Caſ. 9. Omnis fluxio duobus modis in ſeriem reſolvi poteſt; nam index n quantitatis x vel affirmativus eſſe poteſt vel negativus. Proponatur fluxio $\frac{\overline{3k-lxx}\dot{x}}{x^2\sqrt{(kx-lx^3+mx^4)}}$, in quâ $n=1$: hæc vel ſic ſcribi poteſt $x^{-\frac{5}{2}}\times\overline{3k-lx^2}\times\overline{k-lx^2+mx^3}^{-\frac{1}{2}}\dot{x}$; vel ſic $x^{-2}\times\overline{-l+3kx^{-2}}\times\overline{m-lx^{-1}+kx^{-3}}^{-\frac{1}{2}}\dot{x}$. Tentandus eſt caſus uterque per præcedentes methodos, & ſi ſerierum alterutra ob terminos tandem deficientes abrumpitur ac terminatur, habebitur area curvæ in finitis terminis.

Si vero una ſeries terminet; tum neceſſario, rebus recte diſpoſitis, terminat altera. e. g. Sit fluxio $\frac{ax^{(\lambda-1)n-1}\dot{x}}{(\alpha+\beta x^n)^\lambda}$, cujus fluens eſt $\frac{a}{n(\lambda-1)\alpha}\times\frac{x^{(\lambda-1)n}}{(\alpha+\beta x^n)^{\lambda-1}}$; transformetur hæc fluxio in formulam $\frac{ax^{-n-1}\dot{x}}{(\alpha x^{-n}+\beta)^\lambda}$, cujus fluens eſt $\frac{a}{n\times(\lambda-1)\alpha}\times\frac{1}{(\alpha x^{-n}+\beta)^{\lambda-1}}$: hæ autem duæ fluentes ſunt eædem & eâdem facilitate fere per methodum hic traditam deducuntur.

Omnia hæc etiam applicari poſſunt ad ſubſequentem caſum.

Caſ. 10. Datâ fluxione $\overline{A+Bx^n+Cx^{2n}+\&c.+\overline{D+Ex^n+Fx^{2n}+\&c.}}$ $\times\overline{p+qx^n+rx^{2n}+\&c.}^{\lambda-1}$ $(\alpha)+\overline{G+Hx^n+Ix^{2n}+\&c.}\times\overline{s+tx^n+\&c.}^{\mu-1}$ $(\beta)+\overline{K+Lx^n+\&c.}\times\alpha\beta\times\overline{a+bx^n+cx^{2n}+\&c.+\overline{d+ex^n+fx^{2n}+\&c.}}\times$ $\overline{\overline{p+qx^n+rx^{2n}+\&c.}^{\nu}}^{\lambda}\times\overline{g+hx^n+\&c.+\overline{k+lx^n+mx^{2n}+\&c.}}\times$ $\overline{\overline{s+tx^n+\&c.}^{\pi}}^{\mu}$, invenire, utrum ejus fluens exprimi poteſt in finitis algebraicis terminis quantitatis x^n, necne.

Aſſumatur pro ejus fluente quantitas $\overline{a+bx^n+cx^{2n}+\&c.+}$ $d+$

$$\overline{\overline{d+ex^n+fx^{2n}+\&c.}\times\overline{p+qx^n+rx^{2n}+\&c.}^{\lambda}}^{\pi+1}\times\overline{g+hx^n+\&c.}$$

$$+\overline{\overline{k+lx^n+mx^{2n}+\&c.}\times\overline{s+tx^n+\&c.}^{\mu}}^{\pi+1}\times\overline{\alpha'+\beta'x^n+\gamma'x^{2n}+\&c.}$$

hujus quantitatis inveniatur fluxio, quæ fiat æqualis datæ fluxioni, & exinde deduci poſſunt coefficientes α', β', γ', &c. quæſitæ.

Sit fluens $(a+bx^n+cx^{2n}+\&c.)x^{\theta}\times R^{\lambda}S^{\mu}T^{\nu}\times$ &c. ubi in quantitatibus R, S, T, &c. contineantur irrationales functiones quantitatis x^n, viz. α, β, γ, &c. & fluentis fluxio ſit $R^{\lambda-1}S^{\mu-1}T^{\nu-1}\times$ &c. $\times x^{\theta-1}\times Q$; tum in quantitate Q involvuntur irrationales quantitates α, β, γ, &c.; & rectangula ſub quibuſque duabus $\alpha\beta$, $\alpha\gamma$, $\beta\gamma$, &c.; contenta ſub quibuſque tribus, quatuor, &c.; $\alpha\beta\gamma$, &c.; &c.

Ea, quæ dicta fuerunt in præcedentibus caſibus de fluxionibus prius datis ad omnes algebraicas fluxiones applicari poſſunt. Quibus etiam adjici poteſt ſubſequens.

Caſ. 11. Sit data fluxio $\overline{A+Bx^n+Cx^{2n}+\&c.+\overline{d+ex^n+fx^{2n}+\&c.}\times\overline{p+qx^n+rx^{2n}+\&c.}^{\lambda}}^{\pi}\times\overline{a+bx^n+cx^{2n}+\&c.+\overline{h+kx^n+\&c.}\times\overline{p+qx^n+rx^{2n}+\&c.}^{\lambda-1}}\times\dot{x}$; & ſi fluxio quantitatis $A+Bx^n+Cx^{2n}+\&c.$ (P) diviſa per quantitatem $\overline{d+ex^n+fx^{2n}+\&c.}\times\overline{p+qx^n+rx^{2n}+\&c.}^{\lambda}$ (Q) æqualis ſit fluxioni Q per quantitatem (P) $A+Bx^n+Cx^{2n}+\&c.$ diviſæ, i. e. ſi fluxio prioris partis per poſteriorem diviſa æqualis ſit fluxioni poſterioris partis per priorem diviſæ; tum haud aſſumi debet quantitas $\overline{A+Bx^n+Cx^{2n}+\&c.+\overline{d+ex^n+fx^{2n}+\&c.}\times\overline{p+qx^n+rx^{2n}+\&c.}^{\lambda}}^{\pi+1}\times\overline{\alpha'+\beta'x^n+\&c.}$ ſed quantitas $\overline{A+Bx^n+Cx^{2n}+\&c.+\overline{d+ex^n+fx^{2n}+\&c.}\times\overline{p+qx^n+rx^{2n}+\&c.}^{\lambda}}^{\pi}$

$\times\, \alpha' + \beta' x^n + \&c.\, \gamma' \times \overline{d + e x^n + \&c.} \times \overline{p + q x^n + \&c.}^{\lambda}$

Ex. Sit data fluxio $\overline{x + \surd(x^2 + a^2)}^{n}\, \dot{x}$, cujus fluens requiritur: quantitatis $(x + \surd(x^2 + a^2))$ fluxio eſt $\dot{x} + \frac{x\dot{x}}{\surd(x^2 + a^2)}$: fluxio $(\dot{x})$ vero quantitatis x per quantitatem $(\surd(x^2 + a^2))$ diviſa æqualis eſt fluxioni $\left(\frac{x\dot{x}}{\surd(x^2 + a^2)}\right)$ quantitatis $\surd(x^2 + a^2)$ per x diviſæ, viz. $\frac{\dot{x}}{\surd(x^2 + a^2)} = \frac{x\dot{x}}{\surd(x^2 + a^2) \times x}$; unde aſſumatur pro fluente quæſitâ quantitas $\overline{x + \surd(x^2 + a^2)}^{n} \times \overline{\alpha' \surd(x^2 + a^2) + \beta' x}$: & ejus fluxio invenietur $\overline{x + \surd(x^2 + a^2)}^{n} \times \left(n\alpha' + \frac{n\beta' x}{\surd(x^2 + a^2)} + \frac{\alpha' x}{\surd(x^2 + a^2)} + \beta'\right) \times \dot{x}$.

Fiat igitur $n\alpha' + \frac{n\beta' x}{\surd(x^2 + a^2)} + \frac{\alpha' x}{\surd(x^2 + a^2)} + \beta' = 1$; reſultant $n\beta' + \alpha' = 0$, & $n\alpha' + \beta' = 1$; unde $\beta' - n^2\beta' = 1$; & conſequenter $\beta = \frac{1}{1 - n^2}$ & $\alpha = \frac{n}{n^2 - 1}$; & fluens quæſita $= (x + \surd(x^2 + a^2))^n \times \left(\frac{-n}{n^2 - 1} \surd(x^2 + a^2) + \frac{1}{n^2 - 1}\right)$.

Conſimilia etiam affirmari poſſunt de trinomialibus $P + Q + R$, &c. quantitatibus.

Hinc e præcedentibus principiis inveniri poteſt fluens cujuſcunque datæ algebraicæ fluxionis, ſi modo exprimi poſſit in finitis algebraicis terminis literæ x.

Sæpe vero facilius inveſtigari poteſt fluens prædicta, ſi abjiciantur irrationales quantitates ſub vinculis contentæ; & fluxionis reſultantis inveniatur fluens; e quâ etiam ſæpe erui poteſt fluens quæſita.

Et ſic de pluribus hujuſcemodi quantitatibus.

Caſ. 12. In reſolutione caſuum prius traditorum nonnunquam denominator termini deducti nihilo evadat æqualis, in quo caſu fluens fluxionis quæſitæ in finitis terminis non exprimi poteſt.

Caſ. 13. Fluentes quarumcunque algebraicarum fluxionum etiam per infinitas ſeries terminorum ſecundum dimenſiones quantitatis (x) progredientium deduci poſſunt.

1. Ex irrationalibus quantitatibus facile conſtat numerus valorum, quos habet quæſita fluens; per infinitas ſeries inveniantur ſinguli valores (P, Q, R, S, T, &c.) quæſitæ fluentis, & exinde ſumma ſingulorum valorum quæſitæ fluentis; & ſic ſumma rectangulorum ſub ſingulis duobus; contentorum ſub ſingulis tribus; &c.: per medit. algebr. inveniri poteſt, utrum hæ ſummæ per rationalem functionem incognitæ quantitatis x, & exinde utrum fluens quæſita per prædictam functionem exprimi poteſt.

Hæc methodus invenit fluentes, ſi modo nulli valores fluentium correctionem exigant.

2. Si vero correctionem exigant, tum corrigendi ſunt ſinguli (n) valores (P, Q, R, S, &c.) addendo ad ſingulos P, Q, R, S, &c. invariabiles quantitates generaliter aſſumptas A, B, C, D, &c.; & deinde inveniendo ſummam (α) e ſingulis valoribus reſultantibus $P+A$, $Q+B$, $R+C$, &c.; & (β) ſummam eorum rectangulorum ſub quibuſque duobus, (γ) contentorum ſub quibuſque tribus, (δ) ſub quibuſque quatuor; &c. deinde per meditat. algebr. i. e. per methodum communes diviſores detegendi inveniatur, annon ita aſſumi poſſunt quantitates A, B, C, &c. ut prædictæ ſummæ evadant finitæ rationales functiones incognitæ quantitatis x; ſi hoc fieri poſſit, tum invenietur æquatio $v^n - \alpha v^{n-1} + \beta v^{n-2} - \gamma v^{n-3} + \&c. = 0$, cujus ($n$) radices erunt ($n$) diverſi valores fluentis quæſitæ.

Ex. Sit fluxio $\surd(a^2 - x^2)x\dot{x}$; conſtat duos eſſe valores irrationalis quantitatis $\surd(a^2 - x^2)$, & conſequenter duos eſſe valores fluentis (y); reducatur hæc fluxio in infinitas ſeries, & reſultant duo valores $ax\dot{x} - \frac{1}{2}\frac{x^3\dot{x}}{a} - \frac{1}{8}\frac{x^5\dot{x}}{a^3} + \&c.$ $-ax\dot{x} + \frac{x^3\dot{x}}{2a} + \frac{x^5\dot{x}}{8a^3} - \&c.$ quorum fluentes erunt reſpective $\frac{ax^2}{2} - \frac{x^4}{8a} - \frac{x^6}{48a^3} +$ &c. & $-\frac{ax^2}{2} + \frac{x^4}{8a} + \frac{x^6}{48a^3} -$ &c. corrigantur hæ fluentes addendo quamcunque quantitatem $A - \frac{a^3}{3}$ ad unum valorem, & $A + \frac{a^3}{3}$ ad alterum; & reſultant duo valores $A - \frac{a^3}{3} + \frac{ax^2}{2} - \frac{x^4}{8a} - \frac{x^6}{48a^3} +$ &c., & $A + \frac{a^3}{3} - \frac{ax^2}{2} + \frac{x^4}{8a} + \frac{x^6}{48a^3} -$ &c., quorum ſumma eſt $2A$, & rectangulum ſub duobus valoribus erit $A^2 - \frac{1}{9}a^6 + \frac{a^4x^2}{3} - \frac{a^2x^4}{3} + \frac{x^6}{9}$; &c.

Cor. 1. Si modo ſint (n) valores prædicti, & corrigatur unus valor addendo quantitatem A; tum ex quantitate A & datâ fluxione deduci poſſunt correctiones ſingulorum ($n-1$) reliquorum.

Cor. 2. Si ſit fluxio ſecundi vel m ordinis, cujus fluens requiritur; tum in correctione prædictorum (n) valorum aſſumi poſſunt (m) quantitates ad libitum.

PROB. V.

Datâ fluxione, quæ in ſe continet fluentem (v), *quæ haud exprimi poteſt in finitis algebraicis terminis variabilis quantitatis* (x); *invenire utrum fluens datæ fluxionis ſit finita algebraica functio quantitatis* (x) *& prædictæ fluentis* (v).

1. Collocentur termini ſecundum dimenſiones fluentis v, ita ut illi primum locum occupent, in quibus maxima invenitur dimenſio fluentialis quantitatis v; & ſic deinceps.

Terminus, in quo maximæ inveniuntur dimenſiones fluentialis quantitatis (v), ſit rectangulum quantitatis *(W)*, quæ ſit functio literæ v, in quantitatem $B\dot{x}$ ductæ, in quâ haud continetur litera v:

inveniatur fluens fluxionis $B\dot{x}$, quæ dicatur X; deinde inveniatur fluxio rectanguli $X \times W$, & erit $W\dot{X}$ ($WB\dot{x}$ prædictus terminus) $+ X\dot{W}$: in hâc vero poſteriori parte $X\dot{W}$ ſæpe quantitas v haud tam multas habet dimenſiones quam prædictus terminus W; & ſic his operationibus repetitis ſæpe continuo deprimi poſſunt dimenſiones quantitatis v, ita ut tandem evaneſcet, & exinde inveniri poteſt fluens fluxionis quæſita.

Ex. 1. Sit $v =$ fluen. flux. $\frac{\dot{x}}{1 - x}$, invenire fluentem fluxionis $v \times x^n \dot{x}$; ubi n eſt integer numerus. Hic v eſt unica dimenſio fluentialis quantitatis v, quæ dicatur W; & erit $x^n \dot{x} = B\dot{x}$, cujus fluens eſt $\frac{x^{n+1}}{n+1} = X$; unde fluctio rectanguli $X \times W = \frac{v \times x^{n+1}}{n+1}$ erit $v\, x^n \dot{x}$ (data fluxio) $+ \frac{x^{n+1}}{n+1} \times \frac{\dot{x}}{1-x}$ $(\dot{v})$; ſed fluens fluxionis $- \frac{x^{n+1}}{(n+1)} \times \frac{\dot{x}}{(1-x)} = \frac{x^{n+1}}{(n+1)^2} + \frac{x^n}{(n+1) \times n} + \frac{x^{n-1}}{(n+1) \times (n-1)} \ldots \frac{x}{n+1} - \frac{v}{n+1}$, & conſequenter fluens quæſita erit $\frac{v\, x^{n+1}}{n+1} + \frac{x^{n+1}}{(n+1)^2} + \frac{x^n}{(n+1) . n} + \frac{x^{n-1}}{(n+1)(n-1)} \ldots \frac{x}{n+1} - \frac{v}{(n+1)}$: ſit n negativus numerus; & erit fluens quæſita $\frac{v\, x^{n+1}}{n+1} - \frac{1}{n+1}\left(\frac{x^{n+2}}{n+2} + \frac{x^{n+3}}{n+3} \ldots \frac{x^{-1}}{-1} + \int . \frac{\dot{x}}{x} + v\right)$.

Ex. 2. Sit $\dot{z} = \frac{\dot{x}}{1+x^2}$, & erit fluens fluxionis $z\, x^n \dot{x}$ per præcedentem methodum inventa $\frac{1}{n+1} \times \overline{x^{n+1} z - \frac{x^n}{n} + \frac{x^{n-2}}{n-2} - \frac{x^{n-4}}{n-4} + \frac{x^{n-6}}{n-6} \,\&c. \mp z}$; ſi fuerit $\frac{n+1}{2}$ par numerus, erit $-z$; ſin impar, $+z$: etiamque 2^{do}. ſi $\frac{n}{2}$ fuerit par numerus, pro $\mp z$ ſcribatur $-\frac{1}{2}$ log. $(1+x^2)$; ſin impar, ſcribatur $+\frac{1}{2}$ log. $(1+x^2)$. Si n ſit negativus numerus, tum fluens fluxionis $z x^n \dot{x} = \frac{1}{n+1} (x^{n+1} z - \frac{x^{n+2}}{n+2} + \frac{x^{n+4}}{n+4} - \&c. \ldots \frac{x^{-1}}{-1}$ vel $\left(\frac{x^{-2}}{-2}\right) \pm z)$; 1^{mo}. erit $+z$, ſi $\frac{n+1}{2}$ ſit impar negativus numerus; ſin par, $-z$: 2^{do}. ſi autem $\frac{n}{2}$ ſit par negativus numerus, tum pro $\pm z$ ſcribatur $\int. \frac{\dot{x}}{x} - \frac{1}{2} \log. (1 + x^2)$; ſin impar, $+ \frac{1}{2} \log. (1 + x^2) - \int. \frac{\dot{x}}{x} (\log. x)$.

Et ſic ratiocinari liceat de fluentibus fluxionis $z x^n \dot{x}$, ubi $\dot{z} = \frac{\dot{x}}{1 - x^2}$.

Ex. 3. Sit $\dot{v} = \frac{\dot{x}}{x}$, & data fluxio $v^m x^n \dot{x}$, cujus fluens requiritur; tum $v^m = W$ & $x^n \dot{x} = B\dot{x}$, unde $X = \frac{x^{n+1}}{n+1}$, & $X \times W = v^m \times \frac{x^{n+1}}{n+1}$, cujus fluxio erit $v^m x^n \dot{x}$ (data fluxio) $+ \frac{m}{n+1} v^{m-1} \times x^n \dot{x}$; aſſumatur $\frac{m}{n+1} v^{m-1} x^n \dot{x}$ tanquam data fluxio, & repetatur operatio, & erit $\frac{m}{n+1} v^{m-1} = W'$ & $x^n \dot{x} = B'\dot{x}$, cujus fluens eſt $\frac{x^{n+1}}{n+1} = X'$, unde $X' \times W' = \frac{m v^{m-1} \times x^{n+1}}{(n+1) \times (n+1)}$, cujus fluxio erit $m v^{m-1} \times \frac{x^n \dot{x}}{n+1}$ (data fluxio) $+ \frac{m}{(n+1)^2} \times (m-1) v^{m-2} \dot{x} \times x^n$; & ſic deinceps e repetitis operationibus invenietur fluens quæſita $v^m \times \frac{x^{n+1}}{n+1} - m \frac{v^{m-1} \times x^{n+1}}{(n+1)^2} + m . (m-1) \times v^{m-2} \times \frac{x^{n+1}}{(n+1)^3} - m . (m-1) . (m-2) \times v^{m-3} \times \frac{x^{n+1}}{(n+1)^4} + \&c.$ quæ terminat, cum m ſit integer poſitivus numerus.

Ex. 4. Sit $\dot{z} = \frac{a \dot{y}}{\surd(1 \mp y^2)}$; ſcribatur $u = \surd(1 \mp y^2)$; & erit fluens fluxionis $z^n\dot{y} = yz^n \pm nauz^{n-1} \mp n.(n-1)a^2yz^{n-2} \mp n.(n-1) \times (n-2)a^3uz^{n-3}$. Per problema enim aſſumatur pro fluente yz^n, cujus fluxio eſt $z^n\dot{y} + nz^{n-1} \times \frac{a\dot{y}}{\surd(1 \mp y^2)} \times y$; deinde pro fluente quantitatis $+ nz^{n-1} \times \frac{ay\dot{y}}{\surd(1 \mp y^2)}$ aſſumenda eſt quantitas $\mp naz^{n-1}\surd(1 \mp y^2)$, cujus fluxio erit $naz^{n-1} \times \frac{y\dot{y}}{\surd(1 \mp y^2)} \mp n \times (n-1)az^{n-2}a\dot{y}$, & ſic deinceps; unde erit fluens fluxionis $(z^n\dot{y}) = yz^n \pm nauz^{n-1} \mp n.(n-1)a^2z^{n-2}y \mp$ &c.

Ex. 5. Sint $\dot{z} = \frac{a\dot{y}}{\surd(1-y^2)}$ & $u = \surd(1-y^2)$, invenire fluentem fluxionis $z^nu^ry^m\dot{y}$: per problema $W = z^n$ & $B\dot{x} = u^ry^m\dot{y}$, cujus fluens dicatur V; & erit fluxio rectanguli $(VW) = W\dot{V}$ ($z^nu^ry^m\dot{y}$ data fluxio) $+ nz^{n-1} \times V \times \frac{a\dot{y}}{u}$, &c. Scribantur pro fluentibus fluxionum $u^ry^m\dot{y}$, $\frac{A\dot{y}}{u}$, $\frac{B'\dot{y}}{u}$, $\frac{C\dot{y}}{u}$, &c. reſpective A, B', C, D, &c. Et fluens quæſita per problema invenietur $Az^n - naB'z^{n-1} + n.(n-1)a^2Cz^{n-2} - n.(n-1).(n-2)a^3Dz^{n-3} +$ &c. Aliter, ſi fluens fluxionis $B\dot{x}$ non detegi poſſit, aſſumatur pro fluente quæſitâ $\frac{1}{(n+1)a} \times z^{n+1}u^{r+1}y^m - x$, cujus fluxio eſt $z^nu^ry^m\dot{y}$ (data fluxio) $- \frac{1}{(n+1)a}z^{n+1}u^{r-1}y^{m-1}(\overline{r+1}y^2 - mu^2)\dot{y} - \dot{x}$, unde $\dot{x} = -\frac{1}{(n+1)a}z^{n+1}u^{r-1}y^{m-1}(\overline{r+1}y^2 - mu^2)\dot{y} = -\frac{1}{(n+1)a}z^{n+1}u^{r-1}y^{m-1}(\overline{r+1+m}y^2 - m)\dot{y}$; & ſic deinceps, &c. hic autem animadvertendum eſt in omnibus fluentibus inveſtigandis, ſi $\int.B\dot{y}$, $\int.B'\dot{y}$, &c. inveniri poſſit, tum prior; ſin non, tum poſterior methodus adhibenda eſt.

Ex. 6. Sit $\dot{z} = \overline{b + cy^n}^{\,m}\,\dot{y}$; tum erit fluens fluxionis $(az^2y^{n-1}\dot{y})$ $= \frac{a}{n-1}z^2y^{n-1} - \frac{2a}{n-1} \times \frac{1}{(m+1).nc}\overline{b+cy^n}^{\,m+1}\,z + \frac{2a}{n-1} \times \frac{1}{(m+1).nc}$ $\times \int .\, \overline{b + cy^n}^{\,2m+1}\,.\,\dot{y}$.

Et ſic de plurimis hujuſcemodi exemplis.

Ex. 7. Sint $x\dot{A} = P$, $P\dot{A} = \mathcal{Q}$, $\mathcal{Q}\dot{A} = \dot{R}$, $R\dot{A} = \dot{S}$, &c. & fluens fluxionis $A^n\dot{x}$ invenietur $xA^n - nA^{n-1}P + n \times (n-1)A^{n-2}\mathcal{Q} - n \times (n-1) \times (n-2)A^{n-3}R +$ &c.

Ex. 8. Datis fluentibus fluxionum $y\dot{x}$, $yx\dot{x}$, $yx^2\dot{x}$, $yx^3\dot{x}$, . . $yx^{n-1}\dot{x}$: ſint $y\dot{x} = \dot{A}$, $A\dot{x} = \dot{B}$, $B\dot{x} = \dot{C}$, $C\dot{x} = \dot{D}$, &c. . . . $L\dot{x} = \dot{M}$, $M\dot{x} = \dot{N}$, $N\dot{x} = \dot{O}$, $O\dot{x} = \dot{P}$, $P\dot{x} = \dot{\mathcal{Q}}$, $\mathcal{Q}\dot{x} = \dot{R}$, $R\dot{x} = \dot{S}$, $S\dot{x} = \dot{\mathcal{T}}$, quarum æquationum numerus ſit n; invenire fluentem fluxionis $S\dot{x} = \dot{\mathcal{T}}$; datâ fluxionali æquatione $S\dot{x} = \dot{\mathcal{T}}$, aſſumenda eſt $Sx - \alpha = \mathcal{T}$ pro fluente, cujus fluxio eſt $S\dot{x} + x\dot{S} - \dot{\alpha} = \dot{\mathcal{T}}$, unde $x\dot{S} = Rx\dot{x} = \dot{\alpha}$: in fluxionali æquatione $Rx\dot{x} = \dot{\alpha}$ aſſumenda eſt æquatio $\frac{Rx^2}{2} - \beta = \alpha$ pro fluente, cujus fluxio $Rx\dot{x} + \frac{x^2}{2}\dot{R} - \dot{\beta} = \dot{\alpha}$, unde $\dot{\beta} = \frac{x^2}{2}\dot{R} = \frac{x^2}{2}\mathcal{Q}\dot{x}$; & ſic continuo repetitis operationibus invenietur $\mathcal{T} = Sx - R \times \frac{x^2}{2} + \mathcal{Q} \times \frac{x^3}{2.3} - P\frac{x^4}{2.3.4} +$ &c. . . . $\pm$ $\frac{1}{2.3.4\ldots(n-1)} \times (x^{n-1}A - \int .\, x^{n-1}y\dot{x})$.

Ex. 9. Datis fluentibus fluxionum $y\dot{x}$, $y^2\dot{x}$, $y^3\dot{x}$, $y^4\dot{x}$, &c. quæ dicantur reſpective A, B, C, D, &c. ſint $A\dot{y} = \dot{K}$, $K\dot{y} = \dot{L}$, $L\dot{y} = \dot{M}$, &c. & ſic $B\dot{y} = \dot{k}$, $k\dot{y} = \dot{l}$, $l\dot{y} = \dot{m}$, &c. etiamque $C\dot{y} = \dot{p}$, $p\dot{y} = \dot{q}$, $q\dot{y} = \dot{r}$, &c. & $D\dot{y} = \dot{P}$, $P\dot{y} = \dot{\mathcal{Q}}$, $\mathcal{Q}\dot{y} = \dot{R}$, &c. &c. tum erit $K = Ay - \int .\, y\dot{A}\,(y^2\dot{x} = \dot{B})$: $L = Ky - \int .\, y\dot{K}\,(yA\dot{y})$; ſed fluens fluxionis $(Ay\dot{y}) = \frac{Ay^2}{2} - \int .\, \frac{y^2}{2}\dot{A}\left(\frac{y^3\dot{x}}{2} = \frac{\dot{C}}{2}\right)$; unde $L = Ky -$

Ay^2

$\frac{Ay^2}{2} + \frac{\dot{C}}{2} = \frac{Ay^2 - 2By + C}{2}$: & $M = Ly - \text{ſ}.\, y\dot{L}\,(yK\dot{y})$; ſed fluens $\text{ſ}.\, yK\dot{y} = \frac{y^2}{2} \times K - \text{ſ}.\, \frac{y^2}{2} \times \dot{K}\left(\frac{y^2}{2} \times A\dot{y}\right)$; & fluens $\text{ſ}.\, \frac{y^2}{2}A\dot{y} = \frac{y^3A - D}{2 \cdot 3}$; fluens igitur fluxionis $yK\dot{y} = \frac{y^2}{2}K - \frac{y^3A - D}{2 \cdot 3}$, & fluens quæſita $M = Ly - \frac{y^2}{2}K + \frac{y^3A - D}{2 \cdot 3}$; ſcribantur pro K & L earum reſpectivi valores, & reſultat $M = \frac{Ay^3 - 2By^2 + Cy}{2}$ $(Ly) - \frac{Ay^3 - By^2}{2}\left(\frac{y^2K}{2}\right) + \frac{y^3A - D}{2 \cdot 3} = \frac{y^3A - 3y^2B + 3Cy - D}{2 \cdot 3}$: &c. & in genere terminus H, cujus diſtantia a primo ſit r, erit

$$\frac{y^rA - ry^{r-1}B + r \cdot \frac{r-1}{2}y^{r-2}C - \&c.}{1 \cdot 2 \cdot 3 \ldots r} = H.$$

Eodem modo aſſumenda eſt pro fluente fluxionis $(\dot{k} = B\dot{y})$ quantitas $By - \lambda$, & exinde deduci poteſt $\dot{\lambda} = y\dot{B} = \dot{C}$, unde $k = By - C$; & ſic $l = \frac{y^2B - 2yC + D}{2}$, $m = \frac{y^3B - 3y^2C + 3yD - E}{2 \cdot 3}$, &c.: per eandem methodum detegi poſſunt $p = Cy - D$, $q = \frac{y^2C - 2yD + E}{2}$, $r = \frac{y^3C - 3y^2D + 3yE - F}{2 \cdot 3}$, &c.; etiamque $P = Dy - E$, $Q = \frac{y^2D - 2yE + F}{2}$, &c.; & ſic deinceps.

2. Iiſdem literis eaſdem quantitates denotantibus; ſit $\dot{z} = (a-y)^r \times \dot{A} = a^r\dot{A} - ra^{r-1}y\dot{A} + r \cdot \frac{r-1}{2}a^{r-2}y^2\dot{A} - r \cdot \frac{r-1}{2} \cdot \frac{r-2}{3}a^{r-3}y^3\dot{A} + \&c.$ unde $z = a^rA - ra^{r-1}B + r \cdot \frac{r-1}{2}a^{r-2}C - \&c.$

Cor. Ex præcedentibus duobus caſibus conſtat, quodſi $y = a$, $H = \frac{z}{1 . 2 . 3 \ldots r}$.

Ex. 10. Sint $\dot{T} = S\dot{\alpha}$. $\dot{S} = R\dot{\beta}$, $\dot{R} = Q\dot{\gamma}$, $\dot{Q} = P\dot{\delta}$, $\dot{P} = O\dot{\varepsilon}$, &c. Sint etiam $\dot{\alpha} = \dot{p}$, $p\dot{\beta} = \dot{q}$, $q\dot{\gamma} = \dot{r}$, $r\dot{\delta} = \dot{s}$, $s\dot{\varepsilon} = \dot{t}$, &c. & erit per prob. $T = Sp - Rq + Qr - Ps + Ot -$ &c. ad finem terminorum.

Et ſic ſint $\dot{\beta} = \dot{p}_{\prime}$, $p_{\prime}\dot{\gamma} = \dot{q}_{\prime}$, $q_{\prime}\dot{\delta} = \dot{r}_{\prime}$, $r_{\prime}\dot{\varepsilon} = \dot{s}_{\prime}$, &c. & erit $S = Rp_{\prime} - Qq_{\prime} + Pr_{\prime} - Os_{\prime} +$ &c.

Etiamque ſint $\dot{\gamma} = \dot{p}_{\prime\prime}$, $p_{\prime\prime}\dot{\delta} = \dot{q}_{\prime\prime}$, $q_{\prime\prime}\dot{\varepsilon} = \dot{r}_{\prime\prime}$, &c. & erit $R = Qp_{\prime\prime} - Pq_{\prime\prime} + Or_{\prime\prime} -$ &c. unde e datis quantitatibus p, q, r, s, t, &c. $p_{\prime}, q_{\prime}, r_{\prime}, s_{\prime}$, &c. $p_{\prime\prime}, q_{\prime\prime}, r_{\prime\prime}, s_{\prime\prime}$, &c. facile deduci poſſunt fluentes T, S, R, Q, P, &c.

2. Sint $\dot{A} = y\dot{\alpha}$, $\dot{B} = A\dot{\beta}$, $\dot{C} = B\dot{\gamma}$, $\dot{D} = C\dot{\delta}$, $\dot{E} = D\dot{\varepsilon}$, &c. etiamque $y\beta\dot{\alpha} = \dot{P}$, erit $B = A\beta - P$; ſint $\gamma\dot{\beta} = \dot{p}$, $py\dot{\alpha} = \dot{Q}$, & erit $C = B\gamma - pA + Q = A\beta\gamma - P\gamma - pA + Q$; ſint etiam $\delta\dot{\gamma} = \dot{p}_{\prime}$, $p_{\prime}\dot{\beta} = \dot{q}_{\prime}$, $yq_{\prime}\dot{\alpha} = \dot{R}$, & erit $D = C\delta - p_{\prime}B + q_{\prime}A - R$; ſint etiam $\varepsilon\dot{\delta} = \dot{p}_{\prime\prime}$, $p_{\prime\prime}\dot{\gamma} = \dot{q}_{\prime\prime}$, $q_{\prime\prime}\dot{\beta} = \dot{r}_{\prime\prime}$, $yr_{\prime\prime}\dot{\alpha} = \dot{S}$, & erit $E = D\varepsilon - Cp_{\prime\prime} + Bq_{\prime\prime} - Ar_{\prime\prime} + S$; & ſic deinceps.

Cor. Sint $\alpha = \beta = \gamma = \delta = \varepsilon =$ &c. & erunt $\dot{p} = \dot{p}_{\prime} = \dot{p}_{\prime\prime} =$ &c. $= \alpha\dot{\alpha}$, unde $p = p_{\prime} = p_{\prime\prime} =$ &c. $= \frac{\alpha^2}{2}$, & ſic inveniri poteſt $q = q_{\prime} = q_{\prime\prime} =$ &c. $= \frac{\alpha^3}{2 . 3}$, $r = r_{\prime} = r_{\prime\prime} =$ &c. $= \frac{\alpha^4}{2 . 3 . 4}$, & ſic deinceps; & erunt $\dot{A} = y\dot{\alpha}$, $P = y\alpha\dot{\alpha}$, $\dot{Q} = y\frac{\alpha^2}{2}\dot{\alpha}$, $\dot{R} = \frac{y\alpha^3}{6}\dot{\alpha}$, $\dot{S} = \frac{y\alpha^4}{2 . 3 . 4}\dot{\alpha}$, $\dot{T} = \frac{y\alpha^5}{2 . 3 . 4 . 5}\dot{\alpha}$, &c. & $\dot{B} = A\dot{\alpha}$, $\dot{C} = B\dot{\alpha}$, $\dot{D} = C\dot{\alpha}$, &c. & per exemplum erunt $B = A\alpha - P$, $C = B\alpha - \frac{\alpha^2}{2}A + Q = \frac{\alpha^2}{2}A - P\alpha + Q$,

$D = C\alpha$

$D = C\alpha - \frac{\alpha^2}{2}B + \frac{\alpha^3}{1.2.3}A - R = \frac{\alpha^3}{1.2.3}A - \frac{\alpha^2}{1.2}P + \alpha Q - R$, $E = D\alpha - \frac{\alpha^2}{2}C + \frac{\alpha^3}{1.2.3}B - \frac{\alpha^4}{1.2.3.4}A + S = \frac{\alpha^4 A}{1.2.3.4} - \frac{\alpha^3}{1.2.3}P + \frac{\alpha^2}{1.2}Q - \alpha R + S$, & ſic invenitur $F = \frac{\alpha^5}{1.2.3.4.5}A - \frac{\alpha^4}{1.2.3.4}P + \frac{\alpha^3}{1.2.3}Q - \frac{\alpha^2}{1.2}R + \alpha S - T$; & ſic deinceps.

Ex. 11. Fluens fluxionis $\left(\frac{x^{b\beta-1}}{x^{b\alpha}}\dot{x}\int. \frac{x^{b\alpha-1}}{1 \pm x^b}\dot{x}\right) = -\frac{1}{b}\left(\frac{1}{\alpha-\beta} x^{b(\beta-\alpha)}\int. \frac{x^{b\alpha-1}}{1 \pm x^b}\dot{x} + \left(\frac{1}{\beta-\alpha}\int. \frac{x^{b\beta-1}}{1 \pm x^b}\dot{x}\right)\right) = A$: fluens fluxionis $\left(\frac{x^{b\gamma-1}}{x^{b\beta}} \times A\dot{x}\right) = \frac{1}{b^2}\left(\frac{1}{(\alpha-\beta)\times(\alpha-\gamma)} x^{b(\gamma-\alpha)}\int. \frac{x^{b\alpha-1}}{1 \pm x^b}\dot{x} + \frac{1}{(\beta-\alpha)\times(\beta-\gamma)} x^{b(\gamma-\beta)}\int. \frac{x^{b\beta-1}}{1 \pm x^b}\dot{x} + \left(\frac{1}{(\gamma-\alpha)\times(\gamma-\beta)}\int. \frac{x^{b\gamma-1}}{1 \pm x^b}\dot{x}\right)\right) = B$; fluens fluxionis $\left(\frac{x^{b\delta-1}}{x^{b\gamma}}\dot{x} \times B\right) = -\frac{1}{b^3}\left(\frac{1}{(\alpha-\beta)(\alpha-\gamma)(\alpha-\delta)} x^{b(\delta-\alpha)}\int. \frac{x^{b\alpha-1}}{1 \mp x^b}\dot{x} + \frac{1}{(\beta-\alpha)(\beta-\gamma)(\beta-\delta)} x^{b(\delta-\beta)}\int. \frac{x^{b\beta-1}}{1 \pm x^b}\dot{x} + \frac{1}{(\gamma-\alpha)(\gamma-\beta)(\gamma-\delta)} x^{b(\delta-\gamma)}\int. \frac{x^{b\gamma-1}}{1 \pm x^b} + \left(\frac{1}{(\delta-\alpha)\times(\delta-\beta)\times(\delta-\gamma)}\int. \frac{x^{b\delta-1}}{1 \pm x^b}\dot{x}\right)\right) = C$ & in genere ſint $\int. \frac{x^{b\beta-1}}{x^{b\alpha}}\dot{x}\int. \frac{x^{b\alpha-1}\dot{x}}{1 \pm x^b} = A$, $\int. \frac{x^{b\gamma-1}}{x^{b\beta}}\dot{x} \times A = B$, $\int. \frac{x^{b\delta-1}\dot{x}}{x^{b\gamma}} B = C$, $\int. \frac{x^{b\varepsilon-1}\dot{x}}{x^{b\delta}} \times C = D$, $\int. \frac{x^{b\zeta-1}\dot{x}}{x^{b\varepsilon}} \times D = E$, & ſic deinceps, uſque ad terminos $\int. \frac{x^{b\lambda-1}\dot{x}}{x^{b\kappa}} \times P = Q$, $\int. \frac{x^{b\mu-1}\dot{x}}{x^{b\lambda}} \times Q = R$; etiamque $\int. \frac{x^{b\alpha-1}\dot{x}}{1 \pm x^b} = A'$, $\int. \frac{x^{b\beta-1}\dot{x}}{1 \pm x^b} = B'$, $\int. \frac{x^{b\gamma-1}\dot{x}}{1 \pm x^b} = C'$, $\int. \frac{x^{b\delta-1}\dot{x}}{1 \pm x^b} = D'$, &c. uſquæ ad $\int.\frac{x^{b\lambda-1}\dot{x}}{1\pm x^b}=L'$, $\int.\frac{x^{b\mu-1}\dot{x}}{1\pm x^b}=M'$; & ſit (n) numerus literarum $(\alpha, \beta, \gamma, \delta, \varepsilon, \&c.\ \varkappa, \lambda\ \&\ \mu)$; tum erit

$$R=\pm\frac{1}{b^{n-1}}\left(\frac{x^{b(\mu-\alpha)}A'}{(\alpha-\beta)(\alpha-\gamma)(\alpha-\delta)\,..\,(\alpha-\lambda)(\alpha-\mu)}+\frac{x^{b(\mu-\beta)}B'}{(\beta-\alpha)(\beta-\gamma)(\beta-\delta)\ldots(\beta-\lambda)(\beta-\mu)}+\frac{x^{b(\mu-\gamma)}C'}{(\gamma-\alpha).(\gamma-\beta).(\gamma-\delta)\ldots(\gamma-\mu)}+\frac{x^{b(\mu-\delta)}D'}{(\delta-\alpha)\times(\delta-\beta)\times(\delta-\gamma)\,..\,(\delta-\mu)}\ldots+\frac{x^{b(\mu-\lambda)}L'}{(\lambda-\alpha)(\lambda-\beta)(\lambda-\gamma)\,..\,(\lambda-\mu)}+\frac{M'}{(\mu-\alpha)(\mu-\beta)(\mu-\gamma)\,..\,(\mu-\chi)}\right).$$

Signum erit + vel —, prout n ſit impar vel par numerus.

Hoc facile conſtat ex eo quod aggregatum ſingulorum contentorum, viz. $(\beta-\gamma)\times(\beta-\delta)\times(\gamma-\delta)\times(\beta-\varepsilon)\times(\gamma-\varepsilon)\times(\delta-\varepsilon)\times(\beta-\zeta)\times(\gamma-\zeta)\times(\delta-\zeta)\times\&c. - (\alpha-\gamma)\times(\alpha-\delta)\times(\gamma-\delta)\times(\alpha-\varepsilon)\times(\gamma-\varepsilon)\times(\delta-\varepsilon)\times(\alpha-\zeta)\times(\gamma-\zeta)\times(\delta-\zeta)\times(\varepsilon-\zeta)\times\&c. + (\alpha-\beta)\times(\alpha-\delta)\times(\beta-\delta)\times(\alpha-\varepsilon)\times(\beta-\varepsilon)\times(\delta-\varepsilon)\times(\alpha-\zeta)\times(\beta-\zeta)\times(\delta-\zeta)\times\&c. - (\alpha-\beta)\times(\alpha-\gamma)\times(\beta-\gamma)\times(\alpha-\varepsilon)\times(\beta-\varepsilon)\times(\gamma-\varepsilon)\times(\alpha-\zeta)\times(\beta-\zeta)\times(\gamma-\zeta)\ \&c. + \&c.$ nihilo erit æquale. In primo contento non continetur litera α, in ſecundo non continetur litera β, in tertio non continetur litera γ; & ſic deinceps.

1. 2. Eadem methodus etiam deteget fluentes fluxionum, quæ duas vel plures diverſas fluentes involvunt. e. g. Sint A, B, C, &c. fluentes datarum fluxionum; & data fluxio, cujus fluens requiritur, ſit $ABC\ \&c.\times\dot{v}$; tum per prob. pro fluente quæſitâ aſſumenda eſt quantitas $ABC\ \&c.\times v-\alpha$, cujus fluxio eſt $ABC\ \&c.\times\dot{v}$ (data fluxio) $+vBC\ \&c.\ \dot{A}+vAC\ \&c.\ \dot{B}+vAB\ \&c.\ \dot{C}+\&c.-\dot{\alpha}$: deinde per eandem methodum inveniantur fluentes fluxionum $vBC\ \&c.\ \dot{A}$, $vAC\ \&c.\ \dot{B}$, $vAB\ \&c.\ \dot{C}$, &c. & tandem invenietur fluens datæ fluxionis quæſita.

2. Sit fluxio $z^m\dot{v}$, ubi z denotat fluentem datæ fluxionis; ejus fluens nonnunquam etiam deduci poteſt e ſubſequentibus principiis.

Aſſumatur pro fluente quantitas $z^m A + z^{m-1} B + z^{m-2} C + z^{m-3} D + \&c.$ Hujus quantitatis inveniatur fluxio, & datæ fluxioni fiat æqualis, & exinde inveniri poſſunt quantitates aſſumptæ A, B, C, &c..

3. Sit data fluxio $WB\dot{x}$, ubi B ſit data functio quantitatis x; & W functio quantitatis v, cujus $\dot{v}$ fluxio ſit $C\dot{x}$, ubi C denotat functionem quantitatis x; & vel per præcedentem methodum aſſumatur quantitas $W\int. B\dot{x} - \alpha$ pro fluente, & ſic per prædictam methodum progredi liceat: vel data fluxio $WB\dot{x}$ ita ſcribi poteſt $W \times \frac{B}{C} \times C\dot{x} = W \times \frac{B}{C}\dot{v}$; & exinde ſi modo pro fluente fluxionis $(W\dot{v})$ ſcribatur Y, per methodum in problemate contentam aſſumatur quantitas $Y \times \frac{B}{C} - \alpha$ pro fluente quæſitâ, cujus fluxio eſt $W \times \frac{B}{C}\dot{v}$ (data fluxio) $+ Y \times \dot{\overline{\frac{B}{C}\Big|}} - \dot{\alpha}$, unde $\dot{\alpha} = Y \times \dot{\overline{\frac{B}{C}\Big|}} = Y \times \frac{\dot{B}C - \dot{C}B}{C^2} = Y\dot{Z} = Y \times \frac{\dot{Z}}{C\dot{x}} \times C\dot{x} = Y \times \frac{\dot{Z}}{C\dot{x}}\dot{v}$; & eodem modo pro fluente fluxionis $(Y \times \frac{\dot{Z}}{C\dot{x}} \times \dot{v})$ aſſumatur $\frac{\dot{Z}}{C\dot{x}}\int.(Y\dot{v}) - \beta$, cujus fluxio erit $Y \times \frac{\dot{Z}}{C\dot{x}}\dot{v}$ (data fluxio) $+ \dot{\overline{\frac{\dot{Z}}{C\dot{x}}\Big|}} \times \int.(Y\dot{v}) - \dot{\beta}$, unde pro fluente β aſſumatur quantitas $\dot{\overline{\frac{\dot{Z}}{C\dot{x}}\Big|}} \times \frac{1}{C\dot{x}} \times \int.\dot{v}\,(\int.(Y\dot{v}) - \gamma$; & ſic deinceps.

Ex. 1. Sit fluxio $\frac{X\dot{x}}{(lx)^n}$, ubi X ſit functio quantitatis x, & $lx = v$ hyper. log. quantitatis x; hæc vero quantitas hoc modo ſcribi poteſt $Xx \frac{\dot{x}}{x(lx)^n} = Xx\frac{\dot{v}}{v^n}$; aſſumatur quantitas $-Xx \times \frac{1}{(n-1)v^{n-1}} - \alpha$ pro ejus fluente, cujus fluxio erit $X x \frac{\dot{v}}{v^{n}}$ (data fluxio) $- (\dot{X} x) \times \frac{1}{(n-1)v^{n-1}} - \dot{\alpha}$, unde $\dot{\alpha} = -(\dot{X} x) \times \frac{1}{(n-1)v^{n-1}} = -(\dot{X} x) \times \frac{x}{\dot{x}} \times \frac{\dot{x}}{(n-1)\, x\, v^{n-1}} = -(\dot{X} x) \frac{x}{\dot{x}} \times \frac{\dot{v}}{(n-1)\, v^{n-1}}$; deinde aſſumatur pro fluente fluxionis $\dot{\alpha}$ quantitas $(\dot{X} x) \frac{x}{\dot{x}} \times \frac{1}{(n-1)(n-2)\, v^{n-2}} - \gamma$, & ſic deinceps: unde ſi modo ſcribantur $(\dot{X} x) = P \dot{x}$, $(\dot{P} x) = Q \dot{x}$, $(\dot{Q} x) = R \dot{x}$, &c. $V \dot{x}$, erit

$$f.\left(\frac{X\dot{x}}{(lx)^{n}}\right) = \frac{-Xx}{(n-1)(lx)^{n-1}} - \frac{Px}{(n-1)(n-2)(lx)^{n-2}} - \frac{Qx}{(n-1)(n-2)(n-3)(lx)^{n-3}} - \ldots \frac{1}{(n-1)(n-2)\ldots 1} f.\left(\frac{V\dot{x}}{lx}\right).$$

E. g. Sit $X = x^{m}$, & erit

$$f.\left(\frac{x^{m}\dot{x}}{(lx)^{n}}\right) = \frac{-x^{m+1}}{(n-1)(lx)^{n-1}} - \frac{\overline{m+1}\, x^{m+1}}{(n-1)(n-2)(lx)^{n-2}} - \frac{\overline{m+1}^{2}\, x^{m+1}}{(n-1)(n-2)(n-3)(lx)^{n-3}} \ldots \frac{\overline{m+1}^{n-1}}{(n-1).(n-2)..1} \times f.\frac{x^{m}\dot{x}}{lx}, \text{ \&c.}$$

Ex. 2. Sit data fluxio $a^{x}X\dot{x}$, cujus fluens $f.\, a^{x}X\dot{x} = \frac{1}{la} \times a^{x}X - \frac{1}{la} f.\, a^{x}\dot{X}$: ſcribatur $\dot{X} = P\dot{x}$, $\dot{P} = Q\dot{x}$, $\dot{Q} = R\dot{x}$, &c.; tum e præcedente methodo deduci poteſt $f.\, a^{x}X\dot{x} = m a^{x}X - m^{2}a^{x}P + m^{3}a^{x}Q -$ &c. ubi $m = \frac{1}{la}$ & $la = \log.\, a$: aliter hoc problema reſolvi poteſt e principiis prius traditis, ſint enim $X\dot{x} = \dot{P}$, $P\dot{x} = \dot{Q}$, $Q\dot{x} = \dot{R}$, &c. & erit $f.\, a^{x}X\dot{x} = a^{x}P - (la)\, a^{x}Q + (la)^{2}\, a^{x}R -$ &c.

4. Sit $\beta\dot{x} \times \phi f.\, W\dot{x} \times \pi f.\, V\dot{x} \times F f.\, y\dot{x}$, vel $\beta\dot{x}\, \phi f.\, W\dot{x}\, \pi f.\, V\dot{x}\, F f.\, y\dot{x}$ &c. &c., ubi in priore fluxione per $\phi f.\, W\dot{x}$, $\pi f.\, V\dot{x}$, $F f.\, y\dot{x}$, &

&c. deſigno datas functiones fluentium $\int.W\dot{x}$, $\int.V\dot{x}$, $\int.y\dot{x}$, &c. in poſteriori caſu per $F\int.y\dot{x}$ intelligo datam functionem fluentis $\int.y\dot{x}$; per $\pi\int.V\dot{x}F\int.y\dot{x}$ intelligo datam functionem fluentis $\int.V\dot{x}F\int.y\dot{x}$; & ſic deinceps. Si fluentes $\int.W\dot{x}$ & $\int.V\dot{x}$ & $\int.y\dot{x}$, &c., & $\int.V\dot{x}F\int.y\dot{x}$, &c. inveniri poſſint vel in finitis terminis quantitatis (x), vel in terminis reliquarum fluentium, &c. cum finitis terminis; tum primo inveniantur prædictæ fluentes in terminis reliquarum fluentium, &c.; & in datâ fluxione ex iis exprimantur: deinde fluentes prædictarum fluxionum per methodos in hoc & præced. problem. traditas deduci poſſunt; e. g. fluentiales plerumque per eundum modum ac irrationales quantitates tractandæ ſunt, i. e. ſit $Q\times R^{\lambda-1}\times S^{\mu-1}\times$ &c. $\times x^{\theta-1}\dot{x}$ data fluxio, ubi literæ Q, R, S, &c. denotant fluentes fluxionum, quæ ſunt functiones quantitatis x in $\dot{x}$ ductæ; tum aſſumenda eſt pro fluente datæ fluxionis quantitas $P\times R^{\lambda}\times S^{\mu}\times$ &c.; deinde inveniatur ejus fluxio, & fiant correſpondentes termini datæ & reſultantis æquationis inter ſe æquales, & ex æquationibus reſultantibus deduci poteſt fluens quæſita, ſi modo ea in prædictis terminis exprimi poſſit; &c. Horum caſuum facile infinita dari poſſunt exempla.

Ex iiſdem principiis deduci poſſunt fluentes omnium hujuſce generis fluxionum.

PROB. VI.

Datâ exponentiali fluxione, invenire utrum ejus fluens exprimi poteſt in finitis algebraicis & exponentialibus terminis, necne.

Obſervatâ lege fluxionis exponentialis in prob. 3. traditâ, facile reſolvi poteſt hoc problema. Nullæ aliæ enim in fluente continentur exponentiales quantitates præter eas, quæ in datâ fluxione dantur. Exponentialibus vero quantitatibus datis & earum fluxionibus inventis, facile per ſubſtitutiones in præcedentibus problematibus traditas erui poſſunt algebraicæ quantitates: ſi fluxio contineat exponentiales ſuperiorum ordinum quantitates, tum ejus fluens, terminis ipſis inter ſe comparatis, facile deduci poteſt: ſi vero ſit exponentialis primi ordinis, tum vel e terminis ipſis inter ſe comparatis; vel ſi hoc non valeat in quibuſdam caſibus, ex continuis approximationibus deduci poteſt fluens quæſita.

E principiis pro detegendis irrationalium & fluentialium fluxionum fluentibus, datis; ſemper inveniri poteſt fluens cujuſcunque fluxionis, quæ continet irrationales, fluentiales & exponentiales quantitates.

Ex. 1. Sit data fluxio $(y+n-1)\times\overline{y-1}^{\,n-1}\,y^y\dot{y}+\log. y\times y^y\times(y^n-ny^{n-1}+n.\frac{n-1}{2}y^{n-2}-n.\frac{n-1}{2}.\frac{n-2}{3}y^{n-3}+\&c.)\times\dot{y}$.

In hâc fluxione unica exponentialis invenitur, viz. y^y; ergo nulla alia continetur in quæſitâ fluente: ſed $\log. y\times y^y\dot{y}$ ducitur in $y^n-ny^{n-1}+n.\frac{n-1}{2}y^{n-2}-\&c.=\overline{y-1}^{\,n}$; ergo per problema fluens, ſi modo exprimi poſſit finitis terminis, erit $y^y\times\overline{y-1}^{\,n}$.

Ex. 2. Sit data fluxio $\mathcal{Q}^x x^n\dot{x}$; fluxio vero quantitatis $\mathcal{Q}^x$ eſt $\log.\mathcal{Q}\,(m)\,\mathcal{Q}^x\dot{x}$; ergo pro fluente quæſitâ aſſumatur $N\mathcal{Q}^x\times x^n-\alpha$, cujus fluxio erit $Nm\mathcal{Q}^x x^n\dot{x}$ (data fluxio, ſi modo $N=\frac{1}{m}$) $+nN\mathcal{Q}^x x^{n-1}\dot{x}-\dot{\alpha}$, unde $\dot{\alpha}=nN\mathcal{Q}^x x^{n-1}\dot{x}$; & ſic iteratâ operatione aſſumatur pro α quantitas $\frac{n}{m^2}\mathcal{Q}^x x^{n-1}-\beta$, & ſic deinceps; & tandem reſultabit fluens quæſita $\frac{\mathcal{Q}^x}{m}\times(x^n-\frac{nx^{n-1}}{m}+\frac{n\times(n-1)}{m^2}x^{n-2}-\frac{n.(n-1).(n-2)}{m^3}x^{n-3}+\&c.)$ quæ ſeries terminat, ſi n ſit integer poſitivus numerus: aliter pro fluente fluxionis $\mathcal{Q}^x x^n\dot{x}$ aſſumatur ſumma $\mathcal{Q}^x\frac{x^{n+1}}{n+1}-\alpha$, cujus ſummæ inveniatur fluxio, & reſultat $\mathcal{Q}^x x^n\dot{x}$ (data fluxio) $+m\mathcal{Q}^x\frac{x^{n+1}}{n+1}\dot{x}-\dot{\alpha}$, unde $\dot{\alpha}=\frac{m}{n+1}\mathcal{Q}^x x^{n+1}\dot{x}$; & ſic iteratis operationibus invenietur ſeries

$\mathcal{Q}^x\times$

$$\mathcal{Q}^x \times \left(\frac{x^{n+1}}{n+1} - \frac{m\,x^{n+2}}{(n+1).(n+2)} + \frac{m^2\,x^{n+3}}{(n+1).(n+2).(n+3)} - \&c.\right)$$
quæ ſemper in infinitum progreditur.

Subſequenti modo hoc exemplum aliter reſolvi poteſt: aſſumatur pro quæſitâ fluente quantitas $\mathcal{Q}^x \times (Ax^n + Bx^{n-1} + Cx^{n-2} + \&c.)$ & ſi æquetur ejus fluxio datæ, exinde reſultabunt coefficientes quæſitæ A, B, C, &c.

Ex. 3. Sit fluxio $\overline{a+bx}^{\,\overline{f+gx}^{m}} \times \overline{c+dx+ex^2}^{\,l+mx+nx^2} \times \overline{h+kx}^{\,s} \times$
$(m\overline{f+gx}^{\,m-1} \times g \times \log.\ \overline{a+bx} + \overline{f+gx}^{\,m} \times \frac{b}{a+bx} + \overline{m+2nx}$
$\times \log.\ \overline{c+dx+ex^2} + \overline{l+mx+nx^2} \times \frac{d+2ex}{c+dx+ex^2} + \frac{sk}{h+kx})\,\dot{x}$
$+ \&c.$: pro ejus fluente aſſumenda eſt quantitas $A\,\overline{a+bx}^{\,\overline{f+gx}^{m}} \times$
$\overline{c+dx+ex^2}^{\,l+mx+nx^2} \times \overline{h+kx}^{\,s} + \&c.$; & ex æquatâ ejus fluxione datæ, erui poteſt fluens fluxionis datæ.

Cor. Sit data fluxio $X\dot{x}$, ubi X eſt functio quantitatis x; in eâ pro x ſcribatur quæcunque fluentialis & exponentialis quantitas, & ejus fluxio pro $\dot{x}$; & reſultat fluxio, quæ reduci poteſt ad priorem $X\dot{x}$.

THEOR. II.

1. Datâ quantitate A, in quâ continentur duæ variabiles quantitates x & y, ſit ejus fluxio $\dot{A} = a\dot{x} + b\dot{y}$; inveniantur fluxiones quantitatum a & b, quæ ſint reſpective $\dot{a} = \alpha\dot{x} + \beta\dot{y}$, $\dot{b} = \pi\dot{x} + \varrho\dot{y}$, ubi $a, b, \alpha, \beta, \pi, \varrho$ ſunt functiones literarum x & y; tum erit $\pi = \beta$, i. e. π erit eadem quantitas ac β.

Cor. Hinc, datâ fluxione ($\dot{A} = a\dot{x} + b\dot{y}$) duas variabiles quantitates x & y involvente, inveniri poteſt; utrum ejus fluens exprimi poteſt, necne.

Inveniantur enim fluxiones quantitatum a & b, & ſi $\dot{a} = \alpha\dot{x} + \beta\dot{y}$,

& $\dot{b} = \pi\dot{x} + \varrho\dot{y}$, & $\pi = \beta$, tum exprimi poteſt fluens; ſin aliter vero non.

2. Sint tres vel plures variabiles quantitates (x, y, z, &c.) in datâ quantitate (A) contentæ, & ſit $\dot{A} = a\dot{x} + b\dot{y} + c\dot{z} +$ &c. & $\dot{a} = \alpha\dot{x} + \beta\dot{y} + \gamma\dot{z} +$ &c. $\dot{b} = \lambda\dot{x} + \mu\dot{y} + \nu\dot{z} +$ &c. $\dot{c} = \pi\dot{x} + \varrho\dot{y} + \sigma\dot{z} +$ &c. tum erit per præcedentem caſum $\beta = \lambda$, $\gamma = \pi$, $\nu = \varrho$, &c.

Cor. Datâ fluxione $\dot{A} = a\dot{x} + b\dot{y} + c\dot{z} +$ &c. tres, &c. variabiles quantitates involvente, inveniantur fluxiones quantitatum a, b, c, &c. quæ ſint reſpective $\dot{a} = \alpha\dot{x} + \beta\dot{y} + \gamma\dot{z} +$ &c. $\dot{b} = \lambda\dot{x} + \mu\dot{y} + \nu\dot{z} +$ &c. $\dot{c} = \pi\dot{x} + \varrho\dot{y} + \sigma\dot{z} +$ &c. & ſi $\beta = \lambda$, $\gamma = \pi$, $\nu = \varrho$, &c. etiamque ſi fluxiones quantitatis A inveniantur per hanc methodum, viz. 1$^{\text{mo}}$. una quantitas vel x vel y vel z &c. in quantitate A contenta ſolummodo ſupponatur variabilis; & $\dot{A}$ evadat B; deinde in quantitate B tantummodo ſupponatur quæcunque reliqua vel y vel z vel x &c. variabilis, & $\dot{B}$ evadat quantitas C; tertio quæcunque adhuc reliqua quantitas, i. e. quæ non prius ſuppoſita fuit variabilis, & evadat $\dot{C}$ quantitas D; & ſic deinceps: & ſi ultima quantitas ſemper evadat eadem; annon hæc vel illa vel quæcunque alia 1$^{\text{mo}}$. ſupponitur eſſe invariabilis, & ſic deinceps; i. e. ſi non refert, quo ordine quantitates ſupponuntur variabiles; ultima enim quantitas ſemper eadem reſultat; tum ejus fluens exprimi poteſt: ſin aliter vero non.

3. 1. Sit fluxio $\overset{n}{\dot{A}}$ ordinis (n) duas vel tres vel plures variabiles quantitates ($x, y, z,$ &c.) & earum fluxiones n ordinis continens, i. e. ſit

$$\begin{array}{l} \overset{n}{\dot{A}} = a\overset{n}{\dot{x}} + b\dot{x}\overset{n-1}{\dot{x}} + c\dot{x}^2\overset{n-2}{\dot{x}} + d\dot{x}^3\overset{n-3}{\dot{x}} + \text{\&c.} \\ \qquad\qquad\qquad + B\ddot{x}\overset{n-2}{\dot{x}} + C\ddot{x}\dot{x}\overset{n-3}{\dot{x}} + \text{\&c.} \\ \qquad\qquad\qquad\qquad + D\dddot{x}\overset{n-3}{\dot{x}} + \text{\&c.} \\ \qquad\qquad\qquad\qquad\qquad \text{\&c.} \end{array} \left(\begin{array}{c} n \\ \dot{P} \end{array}\right) + p\overset{n-1}{\dot{x}}\dot{y} + q\overset{n-2}{\dot{x}}\dot{y}^2 + \text{\&c.}$$

$$\begin{array}{l} + \alpha\overset{n}{\dot{y}} + \beta\dot{y}\overset{n-1}{\dot{y}} + \gamma\dot{y}^2\overset{n-2}{\dot{y}} + \text{\&c.} \\ \qquad\qquad + \Gamma\ddot{y}\overset{n-2}{\dot{y}} + \text{\&c.} \\ \qquad \text{\&c.} \qquad\qquad \text{\&c.} \end{array} \left(\begin{array}{c} n \\ \dot{\mathcal{Q}} \end{array}\right) + \pi\overset{n-1}{\dot{y}}\dot{x} + \varrho\overset{n-2}{\dot{y}}\dot{x}^2 + \text{\&c.}$$

$$\text{\&c.}$$

Supponantur omnes quantitates præter x eſſe invariabiles & erit $\overset{n}{\dot{P}} =$ fluxi. $n - 1$ ordinis quantitatis $a\dot{x}$; deinde ſupponantur omnes quantitates præter y eſſe invariabiles, & erit $\overset{n}{\dot{Q}} =$ flux. $n - 1$ ordinis quantitatis $\alpha\ddot{y}$; & ſic de reliquis variabilibus quantitatibus.

3. 2. Sit $\dot{a} = \delta\dot{x} + \varepsilon\dot{y} +$ &c. & $\dot{\alpha} = \lambda\dot{x} + \mu\dot{y} +$ &c. & erit coefficiens p termini $(\overset{n-1}{\dot{x}}\,\dot{y}) = \overline{n - 1}\,\varepsilon + \lambda$, & ſic invenitur π coefficiens termini $(\overset{n-1}{\dot{y}}\,\dot{x}) = \overline{n - 1}\,\lambda + \varepsilon$; ſed e theor. conſtat $\lambda = \varepsilon$, ergo $p = \pi = n\lambda$; & ſic e methodo inveniendi fluxiones datarum fluentium facile conſtant reliqui termini; & vice verſâ e datis terminis inveniri poteſt, utrum fluens datæ fluxionis ſit integrabilis, necne.

3. 3. Inveniatur fluxio $(n - 1)$ ordinis fluxionis $a\dot{x} + \alpha\dot{y} +$ &c.; & ſi haud eadem evadat ac data fluxio (n) ordinis; tum fluens datæ fluxionis non integrari poteſt: ſi vero eadem ſit, tum per præcedentem caſum inveniatur, annon fluens fluxionis $a\dot{x} + \alpha\dot{y} +$ &c. integrari poteſt; ſi integrari poſſit, tum fluens datæ fluxionis etiam integrari poteſt; ſin aliter non.

4. Datâ quantitate (A) continente duas variabiles quantitates x & y: 1mo. aſſumatur x tanquam invariabilis quantitas, & inveniatur (α) fluxio ordinis m quantitatis (A); deinde in fluxione (α) ordinis m reſultante aſſumantur $y, \dot{y}, \ddot{y}, \ldots$ & $\overset{m}{\dot{y}}$ tanquam invariabiles quantitates, & inveniatur fluxio ordinis r fluxionis (α): fluxio reſultans eadem erit ac fluxio inventa e ſubſequenti methodo, viz. aſſumatur y tanquam invariabilis quantitas, & inveniatur fluxio (β) ordinis r quantitatis A; deinde in fluxione (β) ordinis (r) reſultante aſſumantur $x, \dot{x}, \ddot{x}$, &c. tanquam invariabiles quantitates, & inveniatur fluxio ordinis m fluxionis (β).

Idem etiam verum eſt, i. e. fluxiones reſultantes erunt inter ſe æquales, ſi modo fluxiones ordinis $(m + r)$ quantitatis (A) inveniantur, i. e. inveniantur reſpective $(m + r)$ fluxiones $\dot{A}, \ddot{A}, \dddot{A}, \ldots \overset{m+r}{\dot{A}}$, quarum quæcunque r fluxiones inveniantur ex hypotheſi quod $y, \dot{y}, \ddot{y}$, &c.

ſint

ſint invariabiles quantitates; (m) vero reliquæ, quod x, $\dot{x}$, $\ddot{x}$, &c. ſint invariabiles quantitates. Minime refert, quo ordine inveniuntur fluxiones prædictæ.

5. Eadem etiam affirmari poſſunt de fluxionibus quantitatis A tres vel plures variabiles quantitates habentis.

5. 2. Conſimilia etiam prædicari poſſunt de fluxionibus quantitatis A, in quâ continentur fluxiones ſuperiorum (m) ordinum quantitatum variabilium $(x, y, z, \&c.)$. Minime enim refert ordo, in quo deteguntur fluxiones; ex hypotheſi quod x vel y vel z &c. ſit invariabilis.

6. Invenire generaliter, annon data fluxio cujuſcunque ordinis m ſit integrabilis; 1$^{\text{mo}}$. aſſumatur quantitas (A) tanquam generalis functio quantitatum $(x, y, z, v, \&c.)$; deinde inveniatur fluxio ordinis (m) quantitatis A; & æquentur correſpondentes datæ & inventæ fluxionis termini: ſi hoc non fieri poſſit, tum fluxio data non erit integrabilis.

Hic animadvertendum eſt, quodſi $M \times \overset{\pi}{\dot{x}}{}^{b} \times \overset{\pi'}{\dot{x}}{}^{b'} \times \&c. \times \overset{\rho}{\dot{y}}{}^{k} \times \overset{\rho'}{\dot{y}}{}^{k'} \times \&c. \times \overset{\sigma}{\dot{z}}{}^{l} \times \overset{\sigma'}{\dot{z}}{}^{l'} \times \&c. = \Gamma$ & $N \times \overset{e}{\dot{x}}{}^{\epsilon} \times \overset{e'}{\dot{x}}{}^{\epsilon'} \times \&c. \times \overset{f}{\dot{y}}{}^{\zeta} \times \overset{f'}{\dot{y}}{}^{\zeta'} \times \&c. \times \overset{g}{\dot{z}}{}^{\theta} \times \overset{g'}{\dot{z}}{}^{\theta'} \times \&c. = \Delta$ ſint duo termini reſultantis fluxionis ordinis (m); & utriuſque $(\Gamma$ & $\Delta)$ inveniantur fluxiones ex hypotheſi aliquando ut x ſolummodo ſit variabilis, & aliquando ut $\dot{x}$ vel $\ddot{x}$, vel $\dddot{x}$, vel &c., vel y vel $\dot{y}$ vel $\ddot{y}$ vel &c., vel z vel $\dot{z}$ vel $\ddot{z}$ vel &c. ſolummodo ſit variabilis, ita ut duæ reſultantes fluxiones contineant eundem fluxionalem terminum $\overset{\lambda}{\dot{x}}{}^{\vartheta} \times \overset{\lambda'}{\dot{x}}{}^{\vartheta'} \times \&c. \times \overset{\mu}{\dot{y}}{}^{\chi} \times \overset{\mu'}{\dot{y}}{}^{\chi'} \times \&c. \times \overset{\nu}{\dot{z}}{}^{\iota} \times \overset{\nu'}{\dot{z}}{}^{\iota'} \times \&c. = H$, & ſint fluxiones reſultantes $R \times H$ & $S \times H$, ubi R & S ſunt algebraicæ functiones quantitatum $(x, y, z, \&c.)$; tum erit $R : S$ in datâ ratione, quæ ratio facile deduci poteſt: ſi hanc ſemper obſervent rationem conſimiles quantitates ex iis in datâ fluxione contentis deductæ; tum fluxio integrabilis erit; ſin aliter vero non.

Si $\overset{\pi}{\dot{x}}{}^{b}$, $\overset{\rho}{\dot{x}}{}^{\epsilon}$, &c.; $\overset{\rho}{\dot{y}}{}^{k}$, &c. habeant maximas dimenſiones prædictarum fluxionum $\overset{\pi}{\dot{x}}$, $\overset{\rho}{\dot{x}}$, &c.; $\overset{\rho}{\dot{y}}$, &c. in duobus terminis Γ & Δ; tum in fluxionali termino *H* haud neceſſe eſt, ut fluxiones $\overset{\scriptscriptstyle\top}{\dot{x}}$, $\overset{\scriptscriptstyle\prime}{\dot{x}}$, &c.; $\overset{\scriptscriptstyle\prime}{\dot{y}}$, &c. ad majores dimenſiones aſcendant.

PROB. VII.

1. *Datâ fluxione duas variabiles* (x & y) *quantitates involvente, invenire ejus fluentem.*

Aſſumatur x tanquam invariabilis quantitas, & inveniatur fluens quantitatis reſultantis; deinde aſſumatur y tanquam invariabilis quantitas, & inveniatur fluens quantitatis exinde reſultantis; & ſi hæ duæ fluentes omnes functiones, in quibus continentur duæ incognitæ quantitates (x & y), eaſdem habent; tum fluens ejus invenitur; aliter vero haud inveniri poteſt.

Ex. 1. Sit fluxio $\overline{(6y^5 + 6y\sqrt{(a^2 + x^2)}) \times \dot{y} + 3x^2 + \frac{3y^2}{\sqrt{(a^2 + x^2)}}} \times \dot{x}$, invenire ejus fluentem.

1^mo^. Supponatur x eſſe invariabilis; tum transformatur data fluxio in fluxionem $\overline{6y^5 + 6y\sqrt{(a^2 + x^2)}}\,\dot{y}$, cujus fluens erit $y^6 + 3y^2\sqrt{(a^2 + x^2)}$: 2^do^. vero ſupponatur y eſſe invariabilis, & fluxio reſultans erit $\overline{3x^2 + \frac{3y^2}{\sqrt{(a^2 + x^2)}}}\,\dot{x}$, cujus fluens invenitur $x^3 + 3y^2\sqrt{(a^2 + x^2)}$; ſed functio $3y^2\sqrt{(a^2 + x^2)}$, in quâ continentur duæ incognitæ quantitates x & y in utriſque fluentibus eadem eſt; ergo fluens inveniri poteſt, & erit $3y^2\sqrt{(a^2 + x^2)} + y^6 + x^3$.

Ex. 2. Sit data fluxio $9x^2y^3\dot{x} + 16x^3y^3\dot{y}$; ſupponatur y eſſe invariabilis, & reſultat fluxio $9x^2y^3\dot{x}$, cujus fluens eſt $3x^3y^3$; deinde ſupponatur x eſſe invariabilis, & reſultat fluxio $16y^3x^3\dot{y}$, cujus fluens eſt $4x^3y^4$; ſed $3x^3y^3$ eſt functio in quâ continentur literæ x & y in unâ fluxione, & $4x^3y^4$ in alterâ: hæ autem duæ functiones haud ſunt eædem, ergo fluens datæ quantitatis haud inveniri poteſt.

2. Datâ fluxione tres vel plures variabiles quantitates (x, y, z, v, &c.) habente, cognoſci poteſt e præcedente methodo, utrum ejus fluens

inveniri poteſt, necne: aſſumantur omnes variabiles quantitates præter unam tanquam invariabiles, & quantitatis reſultantis inveniatur fluens, & ſic de reliquis; & ſi hæ fluentes reſultantes eaſdem habeant functiones duarum vel plurium variabilium quantitatum (x, y, z, &c.) tum ejus fluens inveniri poteſt, ſin aliter vero non.

Ex. 1. Sit fluxio $2x^4y^3z\dot{z} + \overline{3x^4z^2y^2 + 2ax^2y + \frac{2y}{\sqrt{(x^2+y^2)}} + 2by}\,\dot{y} + \overline{4x^3y^3z^2 + 2ay^2x + \frac{2x}{\sqrt{(x^2+y^2)}} + 3ex^2 + 2fx} \times \dot{x}$.

Primo ſupponantur omnes quantitates (x, y, &c.) præter z eſſe invariabiles, & reſultat fluxio $2x^4y^3z\dot{z}$, cujus fluens eſt $x^4y^3z^2$; deinde ſupponantur omnes præter y invariabiles eſſe, & reſultat fluxio $\overline{3x^4z^2y^2 + 2ax^2y + \frac{2y}{\sqrt{(x^2+y^2)}} + 2by}\,\dot{y}$, cujus fluens eſt $x^4z^2y^3 + ax^2y^2 + 2\sqrt{(x^2+y^2)} + by^2$; tertio ſupponatur x ſolummodo eſſe variabilis, & reſultat fluxio $\overline{4x^3y^3z^2 + 2ay^2x + \frac{2x}{\sqrt{(x^2+y^2)}} + 3ex^2 + 2fx}\,\dot{x}$, cujus fluens eſt $x^4y^3z^2 + ay^2x^2 + 2\sqrt{(x^2+y^2)} + ex^3 + fx^2$: in his tribus fluentibus terminus (in quo continentur omnes literæ z, y, x) idem eſt, viz. $x^4y^3z^2$: in duabus poſterioribus fluentibus termini, in quibus continentur duæ literæ x & y, iidem ſunt; viz. ay^2x^2 & $2\sqrt{(x^2+y^2)}$; unde fluens datæ fluxionis invenitur $x^4y^3z^2 + ax^2y^2 + 2\sqrt{(x^2+y^2)} + by^2 + ex^3 + fx^2 + A$, ubi litera A denotat quantitatem ad libitum aſſumendam.

Cor. Hæc methodus haud ſolummodo deteget, annon fluentes exprimi poſſunt; ſed etiam fluentes ipſas; exhinc etiam facilius inveniri poſſunt fluentes fluxionum ſuperiorum ordinum, maxime vero ex iis obſervatis, quæ in theor. 2. tradita fuere.

3. Si vero, cum aſſumantur omnes quantitates præter unam tanquam invariabiles, fluentes reſultantium fluxionum haud per vulgares methodos cognoſci poſſint; ſingula reſultans fluxio in infinitas ſeries

reducatur, quarum termini progrediuntur ſecundum dimenſiones ejuſdem literæ, & ex hoc problemate inveniantur fluentes terminorum ſerierum reſultantium; & id, quod requiritur, peractum erit.

Ex. Sit $\frac{\dot{x}+\dot{y}}{x+y}$; aſſumatur x invariabilis, & reſultat $\frac{\dot{y}}{x+y} = \frac{\dot{y}}{x} - \frac{y\dot{y}}{x^2} + \frac{y^2\dot{y}}{x^3}$ — &c. cujus fluens erit $\frac{y}{x} - \frac{y^2}{2x^2} + \frac{y^3}{3x^3}$ — &c. deinde aſſumatur y invariabilis, & reſultat fluxio $\frac{\dot{x}}{x+y} = \frac{\dot{x}}{x} - \frac{y\dot{x}}{x^2} + \frac{y^2\dot{x}}{x^3} - \frac{y^3\dot{x}}{x^4}$ + &c. cujus fluens eſt log. $ax + \frac{y}{x} - \frac{y^2}{2x^2} + \frac{y^3}{2x^3}$ — &c. termini vero utriuſque ſeriei, in quibus continentur literæ (x & y), iidem ſunt; ergo fluens datæ fluxionis inveniri poteſt.

Et ſic de fluxionibus tres vel plures variabiles quantitates involventibus. Hæc principia etiam ad detegendum, annon fluentes fluxionum ſuperiorum ordinum exprimi poſſunt, facile applicari poſſunt.

PROB. VIII.

1. *Datis correſpondentibus valoribus datæ fluentis & ſingularum variabilium quantitatum in eâ contentarum, eam corrigere.*

Scribantur dati valores ſingularum variabilium quantitatum pro ſuis valoribus in datâ fluente; & quantitas reſultans ſit B; ſit vero correſpondens valor fluentis A; & addatur differentia $A - B$ ad datam fluentem, & ſumma erit correcta fluens.

Ex. 1. Sit fluxio $ax^n\dot{x}$, cujus fluens eſt $\frac{ax^{n+1}}{n+1}$; cum vero x fiat nihilo æqualis, fiat prædicta fluens A: ſcribatur pro x ejus valor o in datâ fluente $\frac{ax^{n+1}}{n+1}$, & reſultat $B = o$; & exinde fluens correcta erit $\frac{ax^{n+1}}{n+1} + A$.

Ex. 2. Sit fluxio $3\surd(x^2+y^2)\,\overline{x\dot{x}+y\dot{y}}$, cujus fluens eſt $\overline{x^2+y^2}^{\,1\frac{1}{2}}$. Sint correſpondentes valores quantitatum x & y, & datæ fluentis reſpective α, β & A; ſcribantur in datâ fluente pro incognitis quantitatibus x & y reſpective α & β, & reſultat $\overline{\alpha^2+\beta^2}^{\,1\frac{1}{2}}=B$; & conſequenter fluens erit $\overline{x^2+y^2}^{\,1\frac{1}{2}}+A-B$.

Ex. 3. Sit fluxio $sby^{s-1}\ddot{y}+s.\overline{s-1}\,by^{s-2}\dot{y}^2+r.\overline{r-1}\,ax^{r-2}\dot{x}^2$, ejus fluens erit $sby^{s-1}\dot{y}+rax^{r-1}\dot{x}$, ſi modo fluat uniformiter x; prædictâ methodo ſcribantur pro $x, y, \dot{x}, \dot{y}$ earum correſpondentes valores, & quantitas reſultans ſit B; ſit A correſpondens valor datæ fluentis, & erit correcta fluens $sby^{s-1}\dot{y}+rax^{r-1}\dot{x}+\overline{A-B}\,(C\dot{x})$ quoniam $\dot{x}$ eſt data quantitas. Fluxionis vero $sby^{s-1}\dot{y}+rax^{r-1}\dot{x}+C\dot{x}$ fluens erit by^s+ax^r+Cx, corrigatur hæc fluens; & erit correcta fluens by^s+ax^r+Cx+D, ubi D invariabilem quantitatem prædictâ methodo acquiſitam denotat. Et ſic in genere de correctionibus fluentium hujuſce generis.

Cor. 1. Sit m minor quam n; & data fluxio n ordinis, cujus fluens (π) ordinis (m) erit fluxio ordinis $(n-m)$: ſit v quantitas, quæ fluit uniformiter; & literæ $A, B, C, D, \ldots P, Q$ invariabiles reſpective denotent quantitates; & erit correcta fluens (m) ordinis $=\pi+(Av^{m-1}+Bv^{m-2}+Cv^{m-3}\ldots Pv+Q)\times\dot{v}^{n-m}$.

Ex. Sit fluxio $n\times(n-1)\times(n-2)\ldots(n-m+1)\,Lx^{n-m}\dot{x}^m$, ubi x fluit uniformiter, cujus fluens erit x^n; fluens vero correcta erit $x^n+Ax^{m-1}+Bx^{m-2}+Cx^{m-3}\ldots Px+Q$.

2. Erit generalis fluens fluxionis $(y\dot{z})=\int.y\dot{z}+A$; & generalis fluens fluxionis $(\dot{x}\int.y\dot{z})=\int.\dot{x}\int.y\dot{z}+Ax+B$; erit generalis fluens fluxionis $(\dot{w}\int.\dot{x}\int.y\dot{z})=\int.\dot{w}\int.\dot{x}\int.y\dot{z}+A\int.x\dot{w}+Bw+C$, & fluens fluxionis $(\dot{v}\int.\dot{w}\int.\dot{x}\int.y\dot{z})=\int.\dot{v}\int.\dot{w}\int.\dot{x}\int.y\dot{z}+A\int.\dot{v}\int.x\dot{w}+B\int.w\dot{v}+Cv+D$, & fluens fluxionis $(\dot{u}\int.\dot{v}\int.\dot{w}\int.\dot{x}\int.y\dot{z})=\int.\dot{u}\int.\dot{v}\int.\dot{w}\int.\dot{x}\int.y\dot{z}+A\int.\dot{u}\int.\dot{v}\int.x\dot{w}+B\int.\dot{u}\int.w\dot{v}+C\int.v\dot{u}+Du+E$,

& ſic deinceps: hìc literæ A, B, C, D, E, &c. reſpective denotant invariabiles coefficientes ad libitum aſſumendas.

PROB. IX.

Datis m *valoribus datæ fluentis* (π) & *correſpondentibus valoribus ſingularum variabilium* (y, z, &c.) *quantitatum in eâ contentarum, invenire ejus fluentis* m *ordinis correctionem.*

Scribantur dati m correſpondentes valores ſingularum variabilium quantitatum in datâ fluente pro quantitatibus ipſis, & ſint quantitates reſultantes reſpective $\varrho, \sigma, \tau, \upsilon$, &c. Sint vero m dati valores datæ fluentis reſpective a, b, c, d, &c. & aſſumatur pro correctâ fluente (ſi modo x fluat uniformiter) quantitas $\pi + Ax^{m-1} + Bx^{m-2} + Cx^{m-3}$ .. $Px + \mathfrak{Q}$; ubi literæ A, B, C, &c. incognitas & invariabiles denotant quantitates.

Sint m valores quantitatis x, qui correſpondent (m) datis valoribus (a, b, c, &c.) datæ fluentis (π) reſpective $\alpha, \beta, \gamma, \delta$, &c. & reſultant ($m$) ſimplices æquationes $A\alpha^{m-1} + B\alpha^{m-2} + C\alpha^{m-3} +$ &c. $= a - \varrho$; $A\beta^{m-1}$ $B\beta^{m-2} + C\beta^{m-3} +$ &c. $= b - \sigma$; $A\gamma^{m-1} + B\gamma^{m-2} + C\gamma^{m-3} +$ &c. $=$ $c - \tau$, &c. totidem (m) incognitas quantitates (A, B, C, &c.) habentes; quæ ita reduci poſſunt, ut inveniantur valores incognitarum quantitatum (A, B, C, &c.); & conficitur problema.

THEOR. III.

In corrigendis fluentibus fluxionum $X\dot{x}$ eædem radices ſemper adhibendæ ſunt; e. g. ſi $+\sqrt{(A)}$ (ubi A eſt functio variabilis quantitatis x) in fluxione occurrat; tum in fluentibus correctis etiam adhibenda eſt, ſed in ejus locum minime ſubſtituenda eſt radix $-\sqrt{(A)}$; in calculo enim fluentium non datur ſaltus, nempe nulla datur mutatio in radicibus.

In corrigendis fluentibus, i. e. cum ſubſtituantur a & b, &c. pro variabili x in generali fluente, in quantitatibus reſultantibus eædem radices ſemper correſpondenter uſurpandæ ſunt.

Ex. Requiratur fluens datæ fluxionis $(3+3x\sqrt{(1-x^2)})\,(e+3x-(1-x^2)^{1\frac{1}{2}})^{\frac{1}{m}}\times\dot{x}$ inter duos valores (a & b) quantitatis x contenta: ejus fluens generalis erit $\frac{m}{m+1}\times(e+3x-(1-x^2)^{1\frac{1}{2}})^{\frac{m+1}{m}}\,(P)+E$, ubi E ſit invariabilis ad libitum aſſumenda; fluens correcta quæſita erit $\frac{m}{m+1}\,((e+3a-(1-a^2)^{1\frac{1}{2}})^{\frac{m+1}{m}}-(e+3b-(1-b^2)^{1\frac{1}{2}})^{\frac{m+1}{m}})$; vel $\frac{m}{m+1}\,((e+3a+(1-a^2)^{1\frac{1}{2}})^{\frac{m+1}{m}}-(e+3b+(1-b^2)^{1\frac{1}{2}})^{\frac{m+1}{m}})$; vel $\frac{m}{m+1}\,(\alpha(e+3a+(1-a^2)^{1\frac{1}{2}})^{\frac{m+1}{m}}-\alpha(e+3b+(1-b^2)^{1\frac{1}{2}})^{\frac{m+1}{m}})$; &c.; minime autem $\frac{m}{m+1}\,(e+3a+(1-a^2)^{1\frac{1}{2}})^{\frac{m+1}{m}}-(e+3b-(1-b^2)^{1\frac{1}{2}})^{\frac{m+1}{m}})$; vel $\frac{m}{m+1}\,(\alpha(e+3a+(1-a^2)^{1\frac{1}{2}})^{\frac{m+1}{m}}-\beta(e+3b+(1-b^2)^{1\frac{1}{2}})^{\frac{m+1}{m}})$; vel &c.; ubi 1, α, β, &c. ſunt diverſæ radices æquationis $y^m-1=0$; & $+1$ & -1 duæ radices æquationis $z^2-1=0$; & in priori caſu iidem valores ſemper earundem radicum (y & z), in poſteriori haud ſemper iidem valores prædicti in correſpondentibus terminis ejuſdem correctæ fluentis continentur.

Et ſic de fluxionibus, in quibus plures continentur variabiles quantitates & earum fluxiones.

THEOR. IV.

Datâ fluxione ($A\dot{x}$), quæ eſt functio quantitatis x in $\dot{x}$; ſcribatur data functio quantitatis z pro x, & ejus fluxio pro $\dot{x}$; & reſultet fluxio ($B\dot{z}$); inveniantur valores quantitatis z, cum x evadat vel $=a$, vel $=b$, qui ſint reſpective α, β, γ, δ, &c.; & π, ϱ, σ, τ, &c.; inveniantur etiam valores fluentis datæ fluxionis inter duos valores a & b quantitatis (x) poſiti; iiſdem radicibus, ut in præcedente regulâ docetur, in utroque caſu ſemper adhibitis: ſint α & π duo valores prædicti quantitatis z, in quibus eædem ſemper adhibentur radices, eædemque etiam ac eæ in prædictis duobus caſibus uſurpatæ, cum x evadat vel $= a$, vel $= b$; tum erit fluens fluxionis $(A\dot{x})$ inter valores a & b quantitatis x contenta, eadem ac fluens fluxionis $(B\dot{z})$ inter valores α & π quantitatis $\dot{z}$ contenta.

Ex. Sit fluxio $(2x + 3x\surd(20^2 - x^2))\dot{x} = A\dot{x}$, cujus fluens eſt $x^2 - (20^2 - x^2)^{1\frac{1}{2}} + E$ quantitas invariabilis; ſit $z^2 - 5z = x$, unde $(2z^2 - 10z + 3 \times (z^2 - 5z)(20^2 - \overline{z^2 - 5z}^{\,2})^{\frac{1}{2}})(2z - 5) \times \dot{z} = B\dot{z}$: ſint 6 & 14 valores quantitatis x, inter quos fluens datæ fluxionis ponitur; tum erit fluens datæ fluxionis inter valores 14 & 6 quantitatis x reſpective $14^2 + (20^2 - 14^2)^{1\frac{1}{2}} - (6^2 + (20^2 - 6^2)^{1\frac{1}{2}}) = P$, & $14^2 - (20^2 - 14^2)^{1\frac{1}{2}} - (6^2 - (20^2 - 6^2)^{1\frac{1}{2}}) = \mathfrak{Q}$: cum ſubſtituantur 14 & 6 pro x in quantitate P adhibentur $+ (20^2 - 14^2)^{1\frac{1}{2}}$ & $+ (20^2 - 6^2)^{1\frac{1}{2}}$ pro $- (20^2 - x^2)^{1\frac{1}{2}}$, & in poſteriori quantitate $\mathfrak{Q}$ uſurpantur $- (20^2 - 14^2)^{1\frac{1}{2}}$ & $- (20^2 - 6^2)^{1\frac{1}{2}}$ pro eâdem quantitate $- (20^2 - x^2)^{1\frac{1}{2}}$; i. e. in priori caſu ſemper applicatur radix $\surd(1) = -1$; in poſteriori vero caſu $\surd(1) = +1$: deinde inveniantur valores quantitatis z, cum evadat $x = 14$ vel $= 6$; i. e. radices æquationum $z^2 - 5z = 14$ & $z^2 - 5z = 6$; radices prioris æquationis erunt $\pm\surd\left(\frac{25}{4} + 14\right) + \frac{5}{2} = 7$, cum adhibeatur ſignum $+$; & $= -2$, ſi modo adhibeatur ſignum $-$: & ſimiliter radices æquationis $z^2 - 5z = 6$ erunt $\pm\surd\left(\frac{25}{4} + 6\right) + \frac{5}{2} = 6$ cum adhibeatur ſignum $+$; & $= -1$, ſi modo uſurpetur ſignum $-$: unde inter valores 7 & 6 quantitatis z duo valores fluentis fluxionis $(B\dot{z})$ viz. P & $\mathfrak{Q}$; etiamque inter valores -2 & -1 quantitatis z continentur duo valores prædictis (P & $\mathfrak{Q}$) æquales fluentis fluxionis $(B\dot{z})$.

Cor. Hinc, ſi modo $A\dot{x}$ ſit fluxio, cujus fluens vel in finitis terminis exprimi poteſt, vel non; vel etiam ſit fluentialis vel exponentialis, &c. fluxio; & deducatur fluxio $B\dot{z}$, cujus variabilis quantitas eſt z, ubi z eſt functio quantitatis x; tum ex hoc theoremate deduci poſſunt valores quamplurimi fluentis fluxionis $(B\dot{z})$ inter α & π, β & ϱ, γ & σ, &c: valores quantitatis z contenti, qui erunt inter ſe æquales, ſi modo correſpondentes radices in α & π, β & ϱ, γ & σ, &c. irrationalium quantitatum in fluente $\int. B\dot{z}$ contentæ, reſpective, ut prius docetur, ſint eædem.

Cor. 2. Conſimilia etiam applicari poſſunt ad caſus, in quibus x eſt functio quantitatis z e datâ æquatione relationem inter x & z exprimente deducenda; etiamque ad caſus in quibus plures variabiles quantitates & earum fluxiones primi, ſecundi, &c. ordinum in datâ fluxione continentur.

PROB. X.

Invenire fluentem (W) *datæ fluxionis* $(X\dot{x} = y\dot{x})$, *quæ ſit functio quantitatis* x *in* $\dot{x}$ *inter valores* a & b *quantitatis* x *contentam; cum inter valores* a & b *contineatur valor vel valores* $(\alpha, \beta, \gamma, \delta$, &c.) *quantitatis* x, *in quibus evadat* $X = 0$; *vel valores* $(\pi, \varrho, \sigma, \tau$, &c.), *in quibus* X *evadat infinita quantitas, & conſequenter* $\frac{1}{X} = 0$: *ubi in* $X = 0$ & $\frac{1}{X} = 0$ *ſemper eædem adhibentur radices* (*e. g. quadraticæ* $+\sqrt{\ }$ *vel* $-\sqrt{\ }$, *&c.*), *eædemque etiam ac eæ in quantitatibus ex ſubſtitutione quantitatum* a & b *pro* x *in prædictâ quantitate* X *exortis; ut in theor.* 3, & 4. *docetur.*

Primo inveniantur radices æquationum $X = 0$ & $\frac{1}{X} = 0$, quæ ſint reſpective $\alpha, \beta, \gamma, \delta$, &c.; & $\pi, \varrho, \sigma, \tau$, &c.; in quibus eædem prædictæ radices adhibentur; & ſint $\gamma, \varrho, \sigma, \delta$, &c. ſucceſſivæ radices inter a & b inventæ, & conſequenter $a, \gamma, \varrho, \sigma, \delta$, &c. b ſucceſſivæ quantitates.

1^mo^. 1. Si fluens vel quantitas W continua, i. e. quæ in eâ eaſdem radices involvit, ex valore (a) quantitatis x ad valorem b, ſemper ſit finita & poſſibilis; ſcribantur $a, \gamma, \varrho, \sigma$, &c., b ſucceſſive pro x in fluente W, & reſultent quantitates A, P, R, S, T, &c., B; inveniantur differentiæ $A - P = \lambda$, $P - R = \mu$, $R - S = \nu$, $S - T = \pi$, &c.: ſumma omnium quantitatum λ, μ, ν, &c. affirmative ſumptarum ſit Σ; tum erit Σ fluens quæſita.

2^do^. Scribantur $v + \gamma$, $v + \delta$, &c.; $v + \varrho$, $v + \sigma$, &c.; pro x in datâ quantitate W, & reſultent quantitates Γ, Δ, &c.; P, Σ, &c.; reducantur hæ quantitates in convergentes ſeries ſecundum dimenſiones quantitatis v progredientes, viz. $Cv^{\frac{m}{n}}$ + &c., $Dv^{\frac{m'}{n'}}$ + &c., + &c.; $Rv^{\frac{r}{s}}$ + &c., $Sv^{\frac{r'}{s'}}$ + &c. + &c.; ubi $\frac{m}{n}, \frac{m'}{n'}$, &c.; $\frac{r}{s}, \frac{r'}{s'}$, &c. ſunt fractiones ad minimos terminos reductæ: in his ſeriebus & quantitatibus ut in theor. 3, & 4. docetur, ſemper adhibendæ ſunt eædem radices: 1. ſi omnes numeratores m, m', m'', m''', &c., r, r', &c. ſint pares, & conſequenter denominatores impares; tum in fluente W pro x ſcribantur a & b; & differentia inter quantitates reſultantes A & B erit fluens quæſita.

2. Si omnes numeratores m'^{b} vel r'^{l}, $m'^{b+1}\ r'^{l+1}$, m'^{b+2}, … m'^{b+s} vel r'^{l+t}, prædictarum fractionum inter m'^{b} vel r'^{l}, & m'^{b+s} vel r'^{l+t} poſitarum, ſint pares; quorum correſpondentes valores quantitatis x ſint reſpective $e, f, g, \ldots k$: tum in fluente W pro x ſcribantur e & k; & differentia $E - K$ inter quantitates reſultantes E & K erit fluens quæſita. Si denominator ſit par, tum evadit fluens impoſſibilis.

Si numerator & denominator ſit impar; tum inveniendæ ſunt fluentes utrinque, quæ incipiunt ab eodem puncto.

FIG. α. 3. Sit fractio $1 + \frac{r}{s}$ vel $1 + \frac{r'}{s'}$, &c. vel $= 0$, vel affirmativa quantitas; tum erit fluens inter a & b contenta, infinita: duæ autem hujuſmodi fluentes ſemel, vel bis vel ter, &c. occurrent, quarum differentia non erit infinita. e. g. Sit curva $bafgbl$, cujus crura in infinitum pergentia haf & gbl, ſint continua; & conſequenter in functione abſciſſæ, quæ deſignat ordinatas am & bn ad prædicta crura, eædem radices involvuntur; tum erit differentia inter duas areas $famok$ & $gbnok$ ſemper finita quantitas.

4. Si vero continuus valor quantitatis X; i. e. quæ eaſdem radices ſemper involvit, inter valores a & b variabilis quantitatis vel abſciſſæ x evadat impoſſibilis; tum inter valores a & b non continetur poſſibilis fluens vel area.

FIG. β. 5. Quamvis curva $a\pi\varrho b$ continue progreditur a puncto a ad punctum b; tamen nec ordinata am; nec area $am\alpha\mu$, $am\beta\nu$, &c. proprie dici poteſt continuari a linea am ad lineam bn.

6. In inveſtigandis diverſis ordinatis $\alpha\mu$, $\alpha\mu'$ & $\alpha\mu''$; vel $\beta\nu$, $\beta\nu'$, $\beta\nu''$, &c. ad eandem abſciſſam ſemper uſurpantur diverſæ radices in functione abſciſſæ, quæ exprimit ordinatam: ab ordinata am ad ordinatam $\pi h'$ eædem radices ſemper uſurpandæ ſunt in functione abſciſſæ, quæ exprimit ordinatas; & in functione abſciſſæ, quæ exprimit areas curvæ: in functionibus abſciſſæ, quæ exprimunt ordinatas & areas curvæ ab ordinatâ ϱh ad ordinatam $\pi h'$ eædem ſemper continentur radices, at non omnes eædem cum iis, quæ uſurpantur in iiſdem functionibus ab ordinatâ am ad ordinatam $\pi h'$: ordinatâ $\pi h'$ evadet eadem, utrum hæ vel illæ radices uſurpantur: ab ordinatâ $\varrho' h$ ad ordinatam bn eædem radices, ſed non omnes eædem cum iis, quæ in hoc vel illo præcedente caſu, i. e. in prædictis functionibus uſurpantur.

Eadem, quæ aſſerimus de areis & ordinatis, æque ad fluxiones & fluentes applicari poſſunt.

FIG. γ. Ad has res magis illuſtrandas: ſit curva continua, quæ tria habet continua crura; nempe $mbcdefghklpqrn$ & $\alpha\beta\gamma$ & $\pi\varrho\sigma$; requiratur area inter ordinatam am & ordinatam bn contenta. 1$^{\text{mo}}$. Inveniatur area $amba$, deinde $bcdb$, tum area $defd$ & $fghf$; deinde area inter aſymptoton oi crus hk & abſciſſam ho contenta; etiamque area inter aſymptoton eandem oi, crus lp & abſciſſam op; ſi hæ duæ areæ ſint infinitæ, tum ſumma areæ inter am & bn erit infinita: differentia autem inter has duas areas $iohk$ & $lpoi$ ſemper erit finita;

ultimo) inveniantur areæ pqr & rnb; ſumma harum arearum erit ſumma quæſita.

7. Si fluens inveniri poſſit aliarum fluentium & finitorum terminorum ope; & dentur caſus, in quibus finiti termini & quædam prædictæ fluentes evaneſcunt; tum in iis caſibus detegi poteſt fluens ope reliquarum fluentium.

8. Si diverſæ radices ejuſdem irrationalis quantitatis ſimiliter involvantur in fluente, tum ſimiliter etiam involventur in fluxione; & vice verſâ ſi ſimiliter involvantur in fluxione, tum ſimiliter etiam involventur in generali fluente.

9. Si unum crus vel valor fluxionis, quæ eſt functio quantitatis x in $\dot{x}$, habeat dimenſiones quantitatis (x) in denominatore majores per unitatem quam dimenſiones ejuſdem quantitatis (x) in numeratore, tum area prædicti cruris vel fluens prædictæ fluxionis non exprimi poteſt in finitis algebraicis terminis quantitatis (x).

PROB. XI.

Invenire, quando fluens datæ fluxionis $X\dot{x}$; *vel area curvæ, cujus ordinata eſt* X, *evadat impoſſibilis.*

Inveniatur, quando ordinata X evadat impoſſibilis; in quibus caſibus fluxio evadit impoſſibilis, in iiſdem caſibus fluens etiam evadit impoſſibilis; ſi quantitas X (iiſdem radicibus ſemper adhibitis) perpetuo maneat poſſibilis inter duos valores a & b quantitatis x; tum fluens correſpondens inter eoſdem valores a & b quantitatis x, etiam ſemper poſſibilis evadet.

Eædem radices irrationalium quantitatum in ordinata X ſemper uſurpandæ ſunt.

2. Nonnullæ exponentiales, &c. quantitates (X), etiamque fluentes earundem quantitatum X in $\dot{x}$ ductarum, &c. continuo evadunt poſſibiles & impoſſibiles. e.g. ſit $e^x = \overline{-a}^x$, ubi e eſt negativa quantitas: hæc quantitas continuo mutatur de poſſibili in impoſſibilem, & vice

verſâ de impoſſibili in poſſibilem quantitatem: fluxionis log. $(-a) \times \overline{-a}^{\,x}\,\dot{x}$ fluens erit $\overline{-a}^{\,x}$, quæ etiam continuo mutatur de poſſibili in impoſſibilem, & de impoſſibili in poſſibilem quantitatem.

PROB. XII.

Invenire, utrum fluens fluxionis ($X\dot{x}$), *ubi* X *eſt functio quantitatis* x *inter datam & infinitam diſtantiam contenta*; *i. e. inter valores finitos & infinite magnos quantitatis variabilis* (x); *ſit finita, necne.*

Si maximæ dimenſiones variabilis (x) in numeratore quantitatis X ſint minores (per quantitatem majorem quam unitatem) quam dimenſiones ejuſdem quantitatis (x) in denominatore; tum fluens prædicta erit finita, ſin aliter non.

2. Si correſpondentes exponentiales vel quantitates cujuſcunque generis ad infinitam diſtantiam habeant rationem, i. e. ſingula præcedens ad ejus ſubſequentem, quæ major eſt quam ratio ſucceſſivarum quantitatum ad eaſdem diſtantias, cum dimenſiones variabilis quantitatis x in numeratore ſint minores per unitatem quam dimenſiones ejuſdem quantitatis in denominatore; tum area erit finita; ſin aliter vero non. e. g. Sit data fluxio $e^{-x}\dot{x} = X\dot{x}$, etiamque duo ſucceſſivi valores quantitatis X reſpective e^{-x} & e^{-x-1}: correſpondentes autem valores fractionum ad eaſdem diſtantias x & $x+1$, cum dimenſiones quantitatis x in numeratore ſint minores per unitatem quam dimenſiones ejuſdem quantitatis in denominatore erunt ad infinitam diſtantiam $\frac{1}{x}$ & $\frac{1}{x+1}$; ſed $e^{-x} : e^{-x-1} :: e : 1$: ſi e major ſit quam 1; tum e^{-x} habet majorem rationem ad e^{-x-1}, cum x ſit infinita quantitas, quam $:: \frac{1}{x} : \frac{1}{x+1}$; ergo fluens contenta inter finitos & infinite magnos valores quantitatis x in fluxione $e^{-x}\dot{x}$ erit finita; cum e ſit major quam 1, & x ſit affirmativus numerus: etiamque ex iiſdem principiis

conſtat prædictam fluentem eſſe finitam, cum e minor ſit quam 1, & x ſit negativa quantitas, i. e. X ſit e^x: ſin aliter non.

3. Invenire, utrum area vel fluens datæ fluxionis $X\dot{x}$ inter duas finitas diſtantias vel valores (a & b) abſciſſæ (x) ſit finita, necne.

Primo inveniatur, annon inter duos prædictos valores a & b infinita evadat ordinata X; ſi non evadat infinita inter prædictos valores, tum area non erit infinita: ſi vero evadat infinita, cum $x = \alpha$, tum $x - \alpha$ erit radix denominatoris fractionis X, quæ exprimit ordinatam; pro x in datâ fractione X ſcribatur $z + \alpha$, & ſi minimæ dimenſiones quantitatis (z) in numeratore contentæ ſint minores per quantitatem minorem quam unitatem quam minimæ dimenſiones ejuſdem quantitatis (z) in denominatore; tum area quoad hoc crus in infinitum pergens erit finita; ſin aliter vero non.

Et ſic de pluribus cruribus in infinitum pergentibus.

4. Si exponentiales, &c. quantitates exprimentes ordinatas X, cum evadant infinitæ, ad correſpondentes diſtantias abſciſſæ (z) evadant minores quam omnes quantitates $\frac{d}{z}$, ubi d eſt data quantitas & z fere $= 0$; tum area vel fluens $f.\ X\dot{x}$ eſt finita quoad hoc crus, ſin aliter vero non.

5. In omnibus his caſibus a valore a ad valorem b ſemper uſurpandæ ſunt eædem radices irrationalium quantitatum in ordinatâ X contentarum.

Ex. Sit fluxio $\frac{x^n\dot{x}}{(ax^m + bx^r + (a^2x^{2m} + dx^s)^{\frac{1}{2}})^{\frac{1}{2}}}$; hæc fluxio diſtingui poteſt in quatuor fluxiones, quarum prima $\frac{x^n\dot{x}}{(ax^m + bx^r + (a^2x^{2m} + dx^s)^{\frac{1}{2}})^{\frac{1}{2}}}$

$$= \frac{x^n\dot{x}}{(ax^m + bx^r + (ax^m + \frac{d}{2a}x^{s-m} + \&c.))^{\frac{1}{2}}} = \frac{x^n\dot{x}}{(2ax^m + bx^r + \frac{1}{2}\frac{d}{a}x^{s-m} + \&c.)^{\frac{1}{2}}}$$

$$= \frac{x^n \dot{x}}{\surd(2a)\, x^{\frac{m}{2}} + \dfrac{b x^r + \frac{1}{2}\frac{d}{a} x^{s-m}}{2\surd(2a)\, x^{\frac{m}{2}}} + \&c.}$$; in his fluxionibus (m) ſupponitur major quam r vel s; tum, ſi n minor ſit quam $\frac{m}{2}$ per quantitatem majorem quam unitatem, fluens inter finitum & infinitum valorem quantitatis x contenta, erit finita; ſin aliter non: 2^da^. fluxio

$$\frac{x^n \dot{x}}{(a x^m + b x^r - (a^2 x^{2m} + d x^s)^{\frac{1}{2}})^{\frac{1}{2}}} = \frac{x^n \dot{x}}{(a x^m + b x^r - a x^m - \frac{d}{2a} x^{s-m} + \&c.)^{\frac{1}{2}}}$$

$$= \frac{x^n \dot{x}}{b^{\frac{1}{2}} x^{\frac{r}{2}} \pm \&c.,\ \text{vel}\ \surd\left(-\frac{d}{2a}\right) x^{\frac{s-m}{2}} \pm \&c.,\ \text{vel}\ \surd\left(b - \frac{d}{2a}\right) x^{\frac{s-m \pm r}{2}} \pm \&c.}:$$

ſi n minor ſit per quantitatem majorem quam unitatem quam major quantitas vel $\frac{r}{2}$ vel $\frac{s-m}{2}$; tum fluens inter prædictos valores contenta erit finita, ſin aliter non. Et ſimiliter ratiocinari liceat de duobus reliquis fluxionibus $\frac{x^n \dot{x}}{-(a x^m + b x^r + (a^2 x^{2m} + d x^s)^{\frac{1}{2}})^{\frac{1}{2}}}$ & $\frac{x^n \dot{x}}{-(a x^m + b x^r - (a^2 x^{2m} + d x^s)^{\frac{1}{2}})^{\frac{1}{2}}}$.

THEOR. V.

Datâ fluxione, quæ eſt algebraica functio literæ x in $\dot{x}$ ducta, & quæ nullos in ſe habet tranſcendentales terminos; ſumma ejus fluentis valorum ſemper exprimi poteſt per finitos terminos, circulares arcus & logarithmos.

Summa enim e ſingulis valoribus cujuſcunque irrationalis quantitatis (P) nihilo ſemper erit æqualis, & exinde ſumma e ſingulis valoribus fluentis $(P\dot{x})$: ſummam vero e ſingulis valoribus fluentis $(Q\dot{x})$,

$(\mathcal{Q}\dot{x})$, quæ eſt rationalis functio quantitatis x in $\dot{x}$ ducta, ſemper inveniri poſſe finitorum terminorum, circularium arcuum & logarithmorum ope, e ſubſequentibus conſtabit; ergo conſtat theorema.

E capite primo medit. algebr. conſtat ſummam ſingulorum valorum cujuſcunque algebraicæ quantitatis haud tranſcendentalis eſſe rationalem quantitatem.

PROB. XIII.

Datâ quantitate vel algebraicâ vel fluentiali, quæ exprimit ſummam ſeriei ſecundum dimenſiones literæ x *progredientis; invenire quantitatem alternis ſeriei terminis æqualem; vel denique ſeriei terminis, quorum diſtantia a ſemetipſis ſit* n.

1. Si modo ſeries requiratur alternis ſeriei terminis æqualis; pro x, $\dot{x}$, $\ddot{x}$, &c. in datâ quantitate ſcribantur reſpective x, $\dot{x}$, $\ddot{x}$, &c. & $-x$, $-\dot{x}$, $-\ddot{x}$, &c. emergent duæ quantitates, quarum ſemiſumma & ſemidifferentia erunt quantitates deſideratæ.

2. Si requiratur ſumma terminorum, quorum diſtantia a ſe invicem ſit n. Sint α, β, γ, δ, ε, &c. reſpectivæ radices æquationis $\pi^n - 1 = 0$; in datâ quantitate pro x, $\dot{x}$, $\ddot{x}$, &c. ſcribantur reſpective αx, $\alpha\dot{x}$, $\alpha\ddot{x}$, &c. βx, $\beta\dot{x}$, $\beta\ddot{x}$, &c. γx, $\gamma\dot{x}$, $\gamma\ddot{x}$, &c. &c. & reſultant n quantitates: ſumma harum n quantitatum reſultantium per n diviſa erit ſumma quæſita, i. e. erit ſumma primi, $n+1$, $2n+1$, $3n+1$, &c. terminorum. Et ſic e principiis in prob. 27. tertiæ edit. & in præfatione noſt. medit. algebr. traditis inveniri poſſunt ſumma ſecundi, $n+2$, $2n+2$, $3n+2$, &c. terminorum; ſumma tertii, $n+3$, $2n+3$, $3n+3$, &c. terminorum; & ſic deinceps: methodus enim, quæ invenit ſummam prædictorum terminorum ſeriei ab expanſione irrationalium vel fractionalium quantitatum in terminos ſecundum dimenſiones quantitatis x progredientes, etiam deteget ſummam conſimilium terminorum ſeriei progredientis ſecundum dimenſiones quantitatis x, quæ eſt fluens ſeriei ex expanſione irrationalium vel fractionalium quantitatum in terminos ſecundum dimenſiones quantitatis x progredientes in $\dot{x}$ ductæ, &c.

Conſimilia etiam applicari poſſunt ad ſeries, quæ ex expanſione irrationalium, &c. quantitatum in terminos ſecundum dimenſiones quantitatum x, y, z, &c. progredientes, exoriuntur; etiamque ad ſeries, quæ ſunt fluentes præcedentium ſerierum in $\dot{x}$, $\dot{y}$, &c. ductarum.

PROB. XIV.

Datam fluxionem, quæ ſit functio variabilium quantitatum (x, y, z, &c.) *& earum fluxionum, in alteram transformare; quæ ſit functio variabilium quantitatum* (v, w, u, &c.) *& earum fluxionum, & cujus variabiles quantitates datam habeant relationem ad variabiles datæ fluxionis quantitates.*

Supponatur data fluxio $= \dot{X}$, & reſultans æquatio & datæ æquationes inter variabiles quantitates datæ (x, y, &c.) & quæſitæ (v, w, &c.) æquationis relationem exprimentes ita transformentur ut reſultet æquatio relationem inter fluxionem $\dot{X}$ & variabiles quantitates v, w, &c. & earum fluxiones exprimens; ex hâc æquatione, ſi modo fieri poſſit, inveniatur quantitas $\dot{X}$ terminis vero variabilium quantitatum v, w, &c. & earum fluxionum, & conſit problema.

Ex hâc methodo ſæpe reduci poteſt data fluxio in magis ſimplicem.

Ex. 1. Sit data fluxio $\frac{-x^{-n}\dot{x}}{a+2bx+cx^2} = \dot{X}$; & $x = \frac{1}{v}$; transformabitur data fluxio in fluxionem $\frac{v^n\dot{v}}{av^2+2bv+c} = \dot{X}$.

Ex. 2. Sit data fluxio $\frac{x^{\frac{m}{r}}\dot{x}}{e+fx^n} = \dot{X}$, & $v^r = x$, & transformari poteſt data fluxio in fluxionem $\frac{rv^{m+r-1}\dot{v}}{e+fv^{rn}} = \dot{X}$.

Hinc conſtat, ſi modo numerus dimenſionum variabilis quantitatis in datâ fluxione (e. g. ſit data fluxio $\frac{Ax^{\overline{r+\frac{l}{s}n-1}}\dot{x}}{\overline{e+fx^n+gx^{2n}+\&c.}^{\lambda}\times P}$) ſine vinculo ſit aliquota pars vel partes $\left(\frac{l}{s}\right)$ dimenſionum (n) ejuſdem quantitatis ſub vinculo, transformari poſſe fluxionem in meliorem formulam, ſi modo pro $x^{\frac{n}{s}}$ ſcribatur v, &c. Si hâc methodo haud bene proceſſit transformatio, nonnunquam transformari poteſt data fluxio in præſtantiorem formulam, ſcribendo $v = x^{\frac{1}{sn}}$, & reducendo datam fluxionem in alteram, cujus variabilis quantitas eſt v.

Ex. 3. Sit data fluxio $\overline{a+bx^n}^m x^{rn-1}\dot{x}$, ubi litera r integrum denotat numerum; ſcribatur $a+bx^n = v$, & erit $x^n = \frac{v-a}{b}$; unde fluxio data $\overline{a+bx^n}^m x^{rn-1}\dot{x} = \frac{1}{n}v^m \times \left(\frac{v-a}{b}\right)^{r-1} \times \frac{\dot{v}}{b}$, & quoniam $r-1$ eſt integer numerus, facile reduci poteſt $(v-a)^{r-1}$ per binomiale theorema in ſimplices terminos, & exinde fluens datæ fluxionis inveniri poteſt.

Si r haud ſit integer numerus, ſed $-r-m-1$ ſit integer numerus; & data fluxio $\overline{a+bx^n}^m x^{rn-1}\dot{x} = \overline{\frac{a}{x^n}+b}^m \times x^{rn+nm-1}\dot{x}$; ſcribatur $\frac{a}{x^n}+b = v$, & reſultat $\frac{a}{v-b} = x^n$; unde $\overline{\frac{a}{x^n}+b}^m \times x^{nr+nm-1}\dot{x} = -v^m \times \frac{1}{n}(v-b)^{-m-r-1} \times a^{r+m}\dot{v}$.

Cor. Facile exhinc fingi poſſunt fluxiones, quæ reduci poſſunt in magis ſimplices: aſſumatur enim ſimplex fluxio, & pro variabilibus quantitatibus & earum fluxionibus in eâ contentis ſcribantur quantitates magis compoſitæ & earum fluxiones; reſultat fluxio, quæ facile reduci poteſt in prædictam ſimplicem.

Ex. 1. Sit data fluxio $x^\theta \times \overline{a+bx^s}^m \dot{x}$; aſſumatur $x = \overline{ev^r + fv^{r+n} + gv^{r+2n} + \&c.}^\pi$; ſubſtituatur hic valor & ejus fluxio pro quantitate x & ejus fluxione in datâ fluxione, & reſultat fluxio magis compoſita

$a+b$

$\overline{a + b\,\overline{e v^{\nu} + f v^{\nu+n} + g v^{\nu+2n} + \&c.}^{\,\pi s}}^{\,m} \times \overline{e + f v^{n} + g v^{2n} + \&c.}^{\,\theta\pi+\pi-1} \times \pi v^{(\theta+1)\pi\nu-1} \times (\nu e + (n+\nu) f v^{n} + \&c.) \times \dot{v}$, quæ facile reduci poteſt ad prædictam ſimplicem fluxionem.

Ex. 2. Sit data eadem fluxio $x^{\theta} \times \overline{a + b x^{s}}^{\,m} \times \dot{x}$; aſſumatur $\overline{e v^{\nu} + f v^{\nu+n} + g v^{\nu+2n} + \&c.}^{\,\pi} \times \overline{h + k v^{n} + l v^{2n} + \&c.}^{\,\lambda} = x$, ſcribatur hic valor & ejus fluxio pro x & $\dot{x}$ in datâ fluxione, & reſultat fluxio magis compoſita.

Sit fluxio $X\dot{x}$, & ſint dimenſiones quantitatis x in numeratore quantitatis X minores per unitatem quam ejus dimenſiones in denominatore; transformetur hæc fluxio in alteram $V\dot{v}$, ubi v eſt quæcunque algebraica functio quantitatis (x), & V conſimilis functio quantitatis v; tum erunt dimenſiones quantitatis v in numeratore fractionis V minores per unitatem quam ejus dimenſiones in denominatore prædictæ fractionis.

PROB. XV.

Datam fluxionem irrationales quantitates habentem in alteram ſæpe transformare; ſi modo fieri poſſit, quæ nullas habet irrationales quantitates.

Per caput 5. medit. algebr. inveniatur, annon ſingula irrationalis, &c. quantitas in datâ fluxione contenta exprimi poteſt in rationalibus functionibus ejuſdem literæ v; ſi vero hujuſce generis inveniantur rationales functiones, ſubſtituantur hæ rationales functiones, &c. pro ſuis valoribus in datâ fluxione, & reſultat fluxio quæſita.

Ex. 1. Sit prædicta fluxio rationalis functio quantitatum x^{n} & $(a+bx^{n})^{\frac{1}{s}}$ (ubi litera s integrum denotat numerum) in fluxionem $x^{n-1}\dot{x}$ ducta; eam transformare in alteram, quæ nullas habet irrationales quantitates. Scribatur $(a + b x^{n})^{\frac{1}{s}} = v$, unde $\frac{v^{s} - a}{b} = x^{n}$, quibus quantitatibus pro ſuis valoribus in datâ fluxione reſpective ſubſtitutis, & $\frac{s v^{s-1} \dot{v}}{n b}$ pro $x^{n-1} \dot{x}$; reſultat fluxio quæſita.

Ex. 2. Sit prædicta fluxio rationalis functio quantitatum x^n & $\left(\frac{a + b x^n}{c + d x^n}\right)^{\frac{1}{s}}$, in fluxionem $x^{n-1} \dot{x}$ ducta; ſcribatur $\left(\frac{a + b x^n}{c + d x^n}\right)^{\frac{1}{s}} = v$, & exinde $v^s = \frac{a + b x^n}{c + d x^n}$, etiamque $\frac{c v^s - a}{b - d v^s} = x^n$, & $x^{n-1} \dot{x} = \frac{s c v^{s-1} \times (b - d v^s) + s d v^{s-1} \times (c v^s - a)}{n (b - d v^s)^2} \times \dot{v}$; quibus quantitatibus pro ſuis valoribus in datâ fluxione ſubſtitutis, reſultat fluxio quæſita.

Cor. Sit fluxio rationalis functio quantitatum x^n & $(a + b x^n)^{\frac{1}{s}} \times (c + d x^n)^{\frac{s-1}{s}}$; pro $(a + b x^n)^{\frac{1}{s}} \times (c + d x^n)^{\frac{s-1}{s}}$ ſcribatur ejus valor $(c + d x^n) \times \left(\frac{a + b x}{c + d x^n}\right)^{\frac{1}{s}}$, & reducitur fluxio in fluxionem præcedentis formulæ.

Ex. 3. Sit data fluxio rationalis functio quantitatum x^n & $\sqrt[2]{(a^2 + b x^n + c x^{2n})}$ in $x^{n-1} \dot{x}$ ducta: ſcribatur $(a^2 + b x^n + c x^{2n})^{\frac{1}{2}} = a + p x^n v$, & exinde $b x^n + c x^{2n} = 2 a p x^n v + p^2 x^{2n} v^2$, unde $x^n = \frac{b - 2 a p v}{p^2 v^2 - c}$, & $(a^2 + b x^n + c x^{2n})^{\frac{1}{2}} = a + p x^n v = \frac{p b v - p^2 a v^2 - c a}{p^2 v^2 - c}$; quibus quantitatibus pro ſuis valoribus in datâ fluxione ſubſtitutis, & fluxione $\frac{2 p^3 a v^2 - 2 p^2 b v + 2 a p c}{(p^2 v^2 - c)^2} \dot{v}$ pro $n x^{n-1} \dot{x}$; reſultat fluxio quæſita.

Sit data fluxio functio rationalis quantitatum x^n & $(a + b x^n + c x^{2n})^{\frac{1}{2}}$ in $x^{n-1} \dot{x}$; ſcribatur $(a + b x^n + c x^{2n})^{\frac{1}{2}} = \surd(c) x^n + v$, & reſultat $c x^{2n} + 2 \surd(c) v x^n + v^2 = a + b x^n + c x^{2n}$; unde $x^n = \frac{v^2 - a}{b - 2 \surd(c) v}$ &

(a +

$(a+bx^n+cx^{2n})^{\frac{1}{2}}=\surd(c)x^n+v=\frac{\surd(c)bv-cv^2-ca}{b\surd(c)-2cv}$; & fluxio $x^{n-1}\dot{x}$ $=\frac{1}{n}\times\frac{2bv-2a\surd(c)-2\surd(c)v^2}{(b-2\surd(c)\times v)^2}\times\dot{v}$; quibus quantitatibus pro ſuis valoribus in datâ fluxione ſubſtitutis, reſultat fluxio quæſita.

Et ſimiliter ſcribatur $(a+bx^n+cx^{2n})^{\frac{1}{2}}=\surd(a)+\surd(c)x^n+v$, vel $(a+bx^n+cx^{2n})^{\frac{1}{2}}=c^{\frac{1}{2}}(x^n+\alpha)^{\frac{1}{2}}(x^n+\beta)^{\frac{1}{2}}=v(x^n+\alpha)=(b^2x^{2n}+l(x^n+\gamma)(x^n+\delta))^{\frac{1}{2}}=bx^n+v(x^n+\gamma)=(k^2+t(x^n+\varepsilon)(x^n+\zeta))^{\frac{1}{2}}=k+v(x^n+\varepsilon)$, &c. & facile transformari poteſt data in fluxionem, cujus variabilis quantitas eſt v.

Eædem ſubſtitutiones etiam transformant multas alias formulas in magis ſimplices.

Ex. 3. Sit data fluxio $x^{rn-1}\dot{x}\left(a+(e+fx^n)^{\frac{1}{m}}\right)^{\frac{1}{s}}$ in rationalem functionem quantitatum x^n & $(e+fx^n)^{\frac{1}{m}}$; ubi r & m ſunt integri numeri; ſcribatur $(e+fx^n)^{\frac{1}{m}}=v$, & erit $x^n=\frac{v^m-e}{f}$, & data fluxio transformatur in alteram $\left(\frac{v^m-e}{f}\right)^{r-1}\times\frac{m}{nf}v^{m-1}\dot{v}\times\sqrt[s]{(a+v)}\times$ rationalem functionem quantitatum (v) & v^m-e; cujus fluens facile inveniri poteſt.

Ex. 4. Sit fluxio $x^{rn-1}\dot{x}\times P$, ubi P ſit rationalis functio quantitatum $\left(a+(e+fx^n)^{\frac{1}{m}}\right)^{\frac{1}{s}}$, $(e+fx^n)^{\frac{1}{m}}$ & x^n; & r, m & s integri ſint numeri; ſcribantur v^s, v^s-a, & $\frac{(v^s-a)^m-e}{f}$: & fluxione $\frac{msv^{s-1}\times(v^s-a)^{m-1}}{f}\dot{v}$ pro $a+(e+fx^n)^{\frac{1}{m}}$, $(e+fx^n)^{\frac{1}{m}}$, x^n, & $nx^{n-1}\dot{x}$ in datâ fluxione; & reſultat fluxio quæſita.

Ex. 5. Sit data fluxio $x^{rn-1}\dot{x} \times \left(a + bx^{n} + (e + fx^{n})^{\frac{1}{m}}\right)^{\frac{1}{s}}$, ubi literæ m & r integros denotant numeros; ſcribatur $e + fx^{n} = v^{m}$, unde $x^{n} = \frac{v^{m} - e}{f}$; & transformari poteſt data fluxio in fluxionem $\left(\frac{v^{m} - e}{f}\right)^{r-1}$ $\times \frac{mv^{m-1}\dot{v}}{nf} \times \left(a + b \times \frac{v^{m} - e}{f} + v\right)^{\frac{1}{s}}$.

Ex. 6. Sint literæ m & r integri numeri, & fluxio data $nx^{rn-1}\dot{x}$ $\left(a + \left(\frac{e + fx^{n}}{g + hx^{n}}\right)^{\frac{1}{m}}\right)^{\frac{1}{s}}$ × in rationalem functionem quantitatum x^{n} & $\left(\frac{e + fx^{n}}{g + hx^{n}}\right)^{\frac{1}{m}}$; ſcribatur $\frac{e + fx^{n}}{g + hx^{n}} = v^{m}$, unde $\frac{gv^{m} - e}{f - hv^{m}} = x^{n}$, & transformari poteſt data fluxio in fluxionem $\left(\frac{gv^{m} - e}{f - hv^{m}}\right)^{r-1} \times (a + v)^{\frac{1}{s}} \times$ in flux. quantitatis $\left(\frac{gv^{m} - e}{f - hv^{m}}\right)$ × functionem quantitatum v & $\frac{gv^{m} - e}{f - hv^{m}}$.

Ex. 7. Sit data fluxio $\left(a + (e^{2} + fx^{n} + gx^{2n})^{\frac{1}{2}}\right)^{\frac{1}{s}} x^{rn-1}\dot{x}$ × quamcunque rationalem functionem quantitatum x^{n} & $(e^{2} + fx^{n} + gx^{2n})^{\frac{1}{2}}$, ubi r ſit integer numerus; ſcribatur $e^{2} + 2evx^{n} + v^{2}x^{2n} = e^{2} + fx^{n} + gx^{2n}$, unde $x^{n} = \frac{2ev - f}{g - v^{2}}$; & data fluxio facile transformatur in alteram $\left(a + e + \frac{2ev^{2} - fv}{g - v^{2}}\right)^{\frac{1}{s}} \times \frac{1}{n} \times \left(\frac{2ev - f}{g - v^{2}}\right)^{r-1}$ × in flux. quantitatis $\frac{2ev-f}{g-v^2}$ × rationalem functionem quantitatum v & $e+\frac{2ev^2-fv}{g-v^2}$.

Ex. 8. Si vero ſit data fluxio $\dot{x}x^{rn-1}\times(e+fz+gz^2+\&c.)^{\frac{1}{s}}\times P$, ubi z vel $=(e+fx^n)^{\frac{1}{s}}$ vel $\left(\frac{e+fx^n}{h+kx^n}\right)^{\frac{1}{s}}$ &c. & P ſit rationalis functio quantitatis x^n; tum facile transformari poteſt data fluxio in alteram magis ſimplicem ſcribendo pro $(e+fx^n)^{\frac{1}{s}}$ vel $\left(\frac{e+fx^n}{h+kx^n}\right)^{\frac{1}{s}}$ ejus valorem (v); & pro x^n in priori caſu $\frac{v^s-e}{f}$, in poſteriori vero $\frac{hv^s-e}{f-kv^s}$; & earum fluxiones reſpective pro $nx^{n-1}\dot{x}$, in datâ fluxione; &c.

Ex. 9. Sit data fluxio $x^{n-1}\dot{x}$ × in rationalem functionem quantitatum $\sqrt[m]{\left(a+b\sqrt[r]{\left(c+d\sqrt[s]{\left(e+\&c.+\left(\frac{g+fx^n}{h+kx^n}\right)^{\frac{1}{t}}\right)}\right)}\right)}=P$, $\sqrt[r]{\left(c+d\sqrt[s]{\left(e+\&c.+\left(\frac{g+fx^n}{h+kx^n}\right)^{\frac{1}{t}}\right)}\right)}=\mathcal{Q}$, $\sqrt[s]{\left(e+\&c.+\left(\frac{g+fx^n}{h+kx^n}\right)^{\frac{1}{t}}\right)}=R$, &c., $\left(\frac{g+fx^n}{h+kx^n}\right)^{\frac{1}{t}}=U$, & x^n; aſſumantur $v=P$, $\frac{v^m-a}{b}=\mathcal{Q}$, $\frac{\left(\frac{v^m-a}{b}\right)^r-c}{d}=R$, &c., $\left(\frac{\left(\left(\frac{v^m-a}{b}\right)^r-c\right)^s}{d}-e,\ \&c.\right)^t=U$, $\frac{hU^t-g}{f-kU^t}=x^n$, & ejus fluxio pro $nx^{n-1}\dot{x}$; ſcribantur hæ quantitates pro ſuis valoribus; & transformatur data fluxio in alteram, quæ erit rationalis functio quantitatis v in $\dot{v}$.

Ex. 10. Sit data fluxio rationalis functio quantitatum $(x^n+$

$\sqrt{}(1+x^{2n}))^{\frac{1}{m}}$ & $\sqrt{}(1+x^{2n})$ & x^n in $x^{n-1}\dot{x}$, ubi m ſit integer numerus; ſcribatur $x^n+\sqrt{}(1+x^{2n})=y^m$, & reſultat $x^n=\frac{y^{2m}-1}{2y^m}$ & $\sqrt{}(1+x^{2n})=y^m-x^n$; unde facile reduci poteſt data fluxio in rationalem functionem quantitatis y in $\dot{y}$.

Et ſic de quamplurimis hujuſcemodi exemplis; facile enim deduci poſſunt infinitæ conſimiles ſubſtitutiones, quæ transformant fluxionem irrationales quantitates involventem in fluxionem rationales vel minus irrationales quantitates ſolummodo habentem; e contra inveniri poſſunt infinitæ hujuſmodi irrationales fluxiones, quæ in rationales transformari poſſunt; aſſumatur enim quæcunque rationalis fluxio, & pro x^n vel quâcunque aliâ quantitate ſcribatur irrationalis quantitas, & pro ejus fluxione fluxio prædictæ quantitatis, &c. & exterminetur quantitas x & ejus fluxio; frequenter reſultabit irrationalis fluxio, quæ facile reduci poteſt in aſſumptam rationalem.

Conſtat nullam aſſignari poſſe fluxionem, cujus fluens finitis terminis algebraicis inveniri poteſt, vel finitis ſolummodo terminis logarithmorum exprimi poteſt, &c. quæ haud transformari poteſt per algebraicarum quantitatum ſubſtitutiones in rationalem fluxionem; ſed dantur fluxiones, quarum fluentes exprimuntur algebraicis & logarithmicis terminis finitis conjunctim, quæ haud reduci poſſunt per prædictas ſubſtitutiones in rationales fluxiones: ſed de his ſatis.

LEMMA.

1. Sit n integer numerus, & æquatio $x^n-px^{n-1}+qx^{n-2}-rx^{n-3}+$ &c. $=o$, &c. ſint etiam a, b, c, d, &c. producta differentiarum unicuique radicum & radicibus reliquis interjacentium, hoc eſt, ſit $a=(\alpha-\beta)\times(\alpha-\gamma)\times(\alpha-\delta)\times$ &c. $=n\alpha^{n-1}-(n-1)p\alpha^{n-2}+(n-2)q\alpha^{n-3}-$ &c. & $b=(\beta-\alpha)\times(\beta-\gamma)\times(\beta-\delta)\times$ &c. $=n\beta^{n-1}-(n-1)p\beta^{n-2}+(n-2)q\beta^{n-3}-$ &c. & $c=(\gamma-\alpha)\times(\gamma-\beta)\times(\gamma-\delta)\times$ &c.

$= n\gamma^{n-1} - (n-1)p\gamma^{n-2} + (n-2)q\gamma^{n-3} -$ &c. & $d = (\delta-\alpha) \times (\delta-\beta) \times (\delta-\gamma) \times$ &c. $= n\delta^{n-1} - (n-1)p\delta^{n-2} + (n-2)q\delta^{n-3} -$ &c. &c. & erit fractio $\frac{1}{x^n - px^{n-1} + qx^{n-2} - \&c.} = \frac{\frac{1}{a}}{x-\alpha} + \frac{\frac{1}{b}}{x-\beta} + \frac{\frac{1}{c}}{x-\gamma} + \frac{\frac{1}{d}}{x-\delta} +$ &c.

2. Sit $\frac{x^s}{x^n - px^{n-1} + qx^{n-2} - \&c.} = \frac{a}{x-\alpha} + \frac{b}{x-\beta} + \frac{c}{x-\gamma} + \frac{d}{x-\delta}$ + &c. ubi exponens s denotat integrum poſitivum numerum, minorem vero quam n; tum erit $a = \frac{\alpha^s}{(\alpha-\beta) \times (\alpha-\gamma) \times (\alpha-\delta) \times \&c.} = \frac{\alpha^s}{n\alpha^{n-1} - (n-1)p\alpha^{n-2} + (n-2)q\alpha^{n-3} - \&c.}$, $b = \frac{\beta^s}{(\beta-\alpha) \times (\beta-\gamma) \times (\beta-\delta) \times \&c.}$, $c = \frac{\gamma^s}{(\gamma-\alpha) \times (\gamma-\beta) \times (\gamma-\delta) \times \&c.}$, &c.

Cor. Sint α & β radices duæ quadraticæ ex eâdem quadraticâ ortæ, quas ideo radices cognatas appellare licet, addanturque in unam fractiones $\frac{a}{x-\alpha}$ & $\frac{b}{x-\beta}$; nunc quicquid imaginarii eſt in utrâque fractione ſeorſim ſumptâ ſemper ex earum ſummâ $\frac{(a+b)x - b\alpha - a\beta}{(x-\alpha) \times (x-\beta)}$ evaneſcet.

3. Sit $\frac{1}{(x+\alpha)^m \times (x+\beta)^n} = \frac{a}{(x+\alpha)^m} + \frac{b}{(x+\alpha)^{m-1}} + \frac{c}{(x+\alpha)^{m-2}} \ldots\ldots \frac{A}{(x+\beta)^n} + \frac{B}{(x+\beta)^{n-1}} + \frac{C}{(x+\beta)^{n-2}} +$ &c. & erit $a = \frac{1}{(\beta-\alpha)^n}$, $b = \frac{-n}{(\beta-\alpha)^{n+1}}$, $c = \frac{n \times \frac{n+1}{2}}{(\beta-\alpha)^{n+2}}$, &c. $A = \frac{1}{(\alpha-\beta)^m}$, $B = \frac{-m}{(\alpha-\beta)^{m-1}}$, $C =$

$m \times$

$\frac{m\times\frac{m+1}{2}}{(\alpha-\beta)^{m-2}}$, &c. eodem modo ſupponatur $\frac{1}{(x+\alpha)^n\times(x+\beta)^m\times(x+\gamma)^r}$ $=\frac{a}{(x+\alpha)^n}+\frac{b}{(x+\alpha)^{n-1}}+\frac{c}{(x+\alpha)^{n-2}}\ldots\ldots\frac{A}{(x+\beta)^m}+\frac{B}{(x+\beta)^{m-1}}+\frac{C}{(x+\beta)^{m-2}}\ldots\ldots\frac{P}{(x+\gamma)^r}+\frac{Q}{(x+\gamma)^{r-1}}+\frac{R}{(x+\gamma)^{r-2}}+$ &c. & exinde deduci poſſunt coefficientes $a, b, c,$ &c. $A, B, C,$ &c. $P, Q, R,$ &c. ex reductione harum fractionum in communem denominatorem.

4. Literæ $\alpha, \beta, \gamma,$ &c. $a, b, c,$ &c. eaſdem quantitates ac in caſu 1. denotent: & erit quantitas $\frac{Q}{(x^n-px^{n-1}+\&c.)\times R}=\frac{Q}{a\times(x-\alpha)\times R}+\frac{Q}{b\times(x-\beta)\times R}+\frac{Q}{c\times(x-\gamma)\times R}+$ &c. ubi literæ Q & R quaſcunque quantitates variabiles vel invariabiles deſignare poſſunt.

5. Datâ fluxione, cujus denominator continetur ſub diviſoribus $a, b, c, d,$ &c. i. e. ſit data fluxio $\frac{A\dot{x}}{P}$, ubi $P=a\times b\times c\times d\times$ &c. ea in alias transformari poteſt hujuſce formulæ $\frac{l\dot{x}}{a}+\frac{m\dot{x}}{b}+\frac{n\dot{x}}{c}+\&c.=\frac{A\dot{x}}{P}$; reducantur hæ fractiones in communem denominatorem, & reſultat $\frac{lbc\,\&c.}{abc\,\&c.}+\frac{mac\,\&c.}{abc\,\&c.}+\frac{nab\,\&c.}{abc\,\&c.}+\&c.=\frac{A}{P}$: ita vero ſæpe aſſumi poſſunt coefficientes $l, m, n,$ &c. ut evaneſcant quidam irrationales termini.

6. Sit data fluxio $\frac{ax^m+bx^{m\pm1}+cx^{m\pm2}+\&c.}{px^n+qx^{n\pm1}+rx^{n\pm2}+\&c.}\times\dot{x}$, ubi m major eſt quam n; tum reduci poteſt in alteram, ubi maximus index quantitatis x in numeratore minor eſt quam ejus maximus index (n) in

denominatore. Dividatur numerator per denominatorem, viz. $p x^n + q x^{n \pm 1} + r x^{n \pm 2} + \&c.$ $(a x^m + b x^{m \pm 1} + c x^{m \pm 2} + \&c.)$ $\frac{a}{p} x^{m-n} + \frac{bp - aq}{p^2} x^{m-n \pm 1} + \&c. = Q$; uſque donec maximus index reſidui (R) minor ſit quam index n, & id, quod requiritur, perficietur; erit enim data fluxio $= Q\dot{x} + \frac{R\dot{x}}{p x^n + q x^{n \pm 1} + \&c.}$.

7. Erit fluxio $\frac{a x^{n-1} + b x^{n-2} + c x^{n-3} + \&c.}{p x^n + q x^{n-1} + r x^{n-2} + \&c.} \times \dot{x} = \frac{a}{np} \times \frac{np x^{n-1} + (n-1) q x^{n-2} + (n-2) r x^{n-3} + \&c.}{p x^n + q x^{n-1} + r x^{n-2} + \&c.} \dot{x}$ (fluxio, cujus fluens eſt $\frac{a}{np} \times \log. (p x^n + q x^{n-1} + r x^{n-2} + \&c.)$ $+ \frac{\left(b - (n-1)\frac{aq}{np}\right) x^{n-2} + \left(c - (n-2)\frac{ar}{np}\right) x^{n-3} + \&c.}{p x^n + q x^{n-1} + r x^{n-2} + \&c.} \dot{x}$.

LEMMA.

1. Sit æquatio $x^{2n} - 2p x^n + 1 = 0$, & p minor quam 1.

FIG. Sit circulus $ABEF$ &c.P, cujus centrum ſit C, & radius unitas. E tabulis ſinuum inveniatur arcus AG, cujus ſinus ſit p. E prædictis tabulis inveniantur ſinus arcuum $\frac{(n-1) \times 90° + AG}{n}$, $\frac{(n-5) \times 90° + AG}{n}$, $\frac{(n-9) \times 90° + AG}{n}$, $\frac{(n-13) \times 90° + AG}{n}$ .... $\frac{-3 \times (n-1) 90° + AG}{n}$; qui ſint reſpective $\alpha, \beta, \gamma, \delta, \varepsilon$, &c. tum $x^2 - 2\alpha x + 1$, $x^2 - 2\beta x + 1$, $x^2 - 2\gamma x + 1$, $x^2 - 2\delta x + 1$, &c. erunt quadratici diviſores quantitatis $x^{2n} - 2p x^n + 1$.

2. Sit data quantitas $x^n - 1$, cujus quadratum $x^{2n} - 2x^n + 1$ erit quantitas præcedentis formulæ. In hoc caſu $p = 1$ ſinus arcûs ($AG = 90^\circ$): inveniantur igitur ſinus α, β, γ, &c. arcuum 90°, $\frac{(n-4)\times 90^\circ}{n}$, $\frac{(n-8)\times 90^\circ}{n}$, $\frac{(n-12)\times 90^\circ}{n} \ldots\ldots - \frac{(3n-4)90^\circ}{n}$; tum $x^2 - 2\alpha x + 1$, $x^2 - 2\beta x + 1$, $x^2 - 2\gamma x + 1$, &c. erunt quadratici diviſores quadrati $x^{2n} - 2x^n + 1$; & $x - 1$ ſimplex diviſor datæ quantitatis $x^n - 1$, cum n ſit impar numerus; etiamque $x - 1$ & $x + 1$ erunt ſimplices diviſores datæ quantitatis $x^n - 1$, cum n ſit par numerus: ſi vero $n = 4m$ ſit numerus pariter par, tum data quantitas recipiet ſimplices diviſores $x + 1$ & $x - 1$ & quadraticum $x^2 + 1$.

3. Sit data quantitas $x^n + 1 = 0$, cujus quadratum $x^{2n} + 2x^n + 1$ erit quantitas præcedentis formulæ; in hoc caſu $p = -1$ ſinus arcûs ($AG = -90^\circ$); inveniantur igitur ſinus arcuum $\frac{(n-2)\times 90^\circ}{n}$, $\frac{(n-6)\times 90^\circ}{n}$, $\frac{(n-10)\times 90^\circ}{n} \ldots\ldots \frac{-(3n-2)\times 90^\circ}{n}$: ſi vero $n = 4m + 2$ ſit impariter par, tum unus ſinus arcûs erit o: ſint ſinus inventi, cum n vel $= 2m + 1$, vel $= 4m$, vel $4m + 2$ reſpective, in primo caſu $-1, \alpha, \beta, \gamma, \delta$, &c. in ſecundo $\alpha, \beta, \gamma, \delta$, &c. in tertio $o, \alpha, \beta, \gamma, \delta$, &c. tum data quantitas conſtat in primo caſu e quadraticis diviſoribus $x^2 - 2\alpha x + 1$, $x^2 - 2\beta x + 1$, $x^2 - 2\gamma x + 1$, &c. & e ſimplici diviſore $x + 1$; etiamque e prædictis & quadratico diviſore $x^2 + 1$ in poſtremo caſu; &c.

4. Sit n impar numerus, & $x^n - 1 = (x - 1) \times (x^2 - 2\alpha x + 1) \times (x^2 - 2\beta x + 1) \times$ &c. & erit $\frac{1}{x^n - 1} = \frac{\frac{1}{n}}{x - 1} + \frac{\frac{2}{n}\alpha x - \frac{2}{n}}{x^2 - 2\alpha x + 1} + \frac{\frac{2}{n}\beta x - \frac{2}{n}}{x^2 - 2\beta x + 1} +$ &c.

Sit n vero par numerus, & $x^n - 1 = (x - 1) \times (x + 1) \times (x^2 - 2\alpha x + 1)$

$\times (x^2 - 2\beta x + 1) \times$ &c. & erit $\frac{\frac{1}{n}}{x-1} - \frac{\frac{1}{n}}{x+1} + \frac{\frac{2}{n}\alpha x - \frac{2}{n}}{x^2 - 2\alpha x + 1} +$ &c. $= \frac{1}{x^n - 1}$.

Sit n pariter par numerus, & erit $x^n - 1 = (x-1) \times (x+1) \times (x^2+1) \times (x^2 - 2\alpha x + 1) \times$ &c. & $\frac{1}{x^n - 1} = \frac{\frac{1}{n}}{x-1} - \frac{\frac{1}{n}}{x+1} - \frac{\frac{2}{n}}{x^2+1} + \frac{\frac{2}{n}\alpha x - \frac{2}{n}}{x^2 - 2\alpha x + 1} +$ &c.

Sit n impar numerus & $x^n + 1 = (x+1) \times (x^2 - 2\alpha x + 1) \times (x^2 - 2\beta x + 1) \times$ &c. & erit $\frac{-1}{x^n + 1} = \frac{-\frac{1}{n}}{x+1} + \frac{\frac{2}{n}\alpha x - \frac{2}{n}}{x^2 - 2\alpha x + 1} \times$ &c.

Sit n par numerus, & $x^n + 1 = (x^2+1) \times (x^2 - 2\alpha x + 1) \times (x^2 - 2\beta x + 1) \times$ &c. & erit $\frac{-1}{x^n + 1} = \frac{-\frac{2}{n}}{x^2+1} + \frac{\frac{2}{n}\alpha x - \frac{2}{n}}{x^2 - 2\alpha x + 1} + \frac{\frac{2}{n}\beta x - \frac{2}{n}}{x^n - 2\beta x + 1} +$ &c. Hæc facile deduci poſſunt e lemmate præced.

5. Sit $x^{2n} \pm 2px^n + 1$, ubi p minor ſit quam 1; & $x^{2n} \pm 2px^n + 1 = (x^2 - 2\alpha x + 1) \times (x^2 - 2\beta x + 1) \times (x^2 - 2\gamma x + 1) \times$ &c. i. e. ſint α, β, γ, &c. coſinus arcuum $\frac{A}{n}$, $\frac{360^\circ - A}{n}$, $\frac{360^\circ + A}{n}$, $\frac{2 \times 360^\circ - A}{n}$, $\frac{2 \times 360^\circ + A}{n}$, $\frac{3 \times 360^\circ - A}{n}$, &c. ubi A ſit arcus, cujus coſinus ſit $\pm p$, itemque ſint e, f, g, h, &c. coſinus arcuum, quorum unuſquiſque ad unumquemque arcum antecedentem eodem ordine ſumptum eam rationem habeat, quæ eſt numeri $n - 1$ ad 1. Et erit $\frac{1}{x^{2n} \pm 2px^n + 1} = - \frac{\frac{\alpha \pm pe}{n - np^2} x - \frac{1}{n}}{x^2 - \alpha x + 1} - \frac{\frac{\beta \pm pf}{n - p^2 n} x - \frac{1}{n}}{x^2 - \beta x + 1} - \frac{\frac{\gamma \pm pg}{n - np^2} x - \frac{1}{n}}{x^2 - \gamma x + 1} -$ &c. ubi affirmativum ſignum affixum eſt quantitati pe, &c.; ſi modo ſit $+p$, ſin aliter negativum.

6. Sit $(x^n \mp \alpha) \times (x^n \mp \beta) = x^{2n} \mp 2px^n + 1$; ubi p major eſt quam 1; ſcribatur $z = x\sqrt[n]{(\alpha)}$ & $y = x\sqrt[n]{(\beta)}$, & erit $\frac{1}{x^{2n} - 2px^n + 1}$ $= \frac{\alpha}{\beta - \alpha} \times \frac{1}{z^n - 1} + \frac{\beta}{\alpha - \beta} \times \frac{1}{y^n - 1}$; & $\frac{1}{x^{2n} + 2px^n + 1} = \frac{\alpha}{\alpha - \beta} \times \frac{1}{1 + z^n}$ $+ \frac{\beta}{\beta - \alpha} \times \frac{1}{1 + y^n}$.

7. Sit $\frac{e\dot{x}}{e + fx^n + gx^{2n}}$; ſcribatur $x^n = v^n \sqrt{\left(\frac{e}{g}\right)}$, & $f = 2p\sqrt{(ge)}$; & erit $\frac{e\dot{x}}{e + fx^n + gx^{2n}} = \frac{\sqrt[2n]{\left(\frac{e}{g}\right)}\dot{v}}{v^{2n} + 2pv^n + 1}$.

8. Sit $\frac{z^{m-1}\dot{z}}{(1 + z)^n \pm (1 - z)^n}$; pro z ſcribatur $\frac{x - a}{x + a}$, & reſultat

$$\frac{z^{m-1}\dot{z}}{(1 + z)^n \pm (1 - z)^n} = \frac{2a\dot{x} \times (x - a)^{m-1}}{(x + a)^{m+1}} \times \frac{1}{\frac{(2x)^n \pm (2a)^n}{(x + a)^n}} = \frac{2a\dot{x} \times (x - a)^{m-1}}{(x + a)^{m-n+1} \times ((2x)^n \pm (2a)^n)}.$$

Plura autem de diviſoribus inveniendis adjiciuntur, quam ratio noſtri inſtituti exigit.

PROB. XVI.

Invenire quantitatem, quæ in datam irrationalem quantitatem ducta, rationale productum facit.

Datæ irrationalis quantitatis inveniantur ſinguli diverſi valores, quibus in ſeſe continuo ductis, contentum erit rationalis quantitas; dividatur hoc contentum per datam quantitatem, & quotiens erit quantitas quæſita. Vid. pag. 152. Medit. Algebr.

Cor. Hinc transferri poſſunt e numeratore in denominatorem, & vice verſâ e denominatore in numeratorem, quæcunque irrationales quantitates (A, &c.); inveniatur rationalis quantitas B, cujus data irrationalis A eſt diviſor; ſit vero $C = \frac{B}{A}$, pro A ſubſtituatur $\frac{B}{C}$ & conſit corollarium.

Ex. 1. Sit fluxio $\frac{Q\dot{x}}{+R\sqrt[2]{(a)}+S\sqrt[2]{(b)}}$; ubi literæ Q, R, S, a, b reſpective denotent quaſcunque rationales functiones literæ x; ducantur & numerator & denominator in quantitatem $+R\sqrt{(a)}-S\sqrt{(b)}$; & reſultat fluxio $\frac{+RQ\sqrt{(a)}-SQ\sqrt{(b)}}{R^2a-S^2b}\dot{x}$.

Ex. 2. Sit fluxio $\frac{Q\dot{v}}{P+R\sqrt{(a)}+S\sqrt{(b)}}$; ducantur & numerator & denominator in quantitatem $(P+R\sqrt{(a)}-S\sqrt{(b)})\times(P-R\sqrt{(a)}+S\sqrt{(b)})\times(P-R\sqrt{(a)}-S\sqrt{(b)})$; & reſultat quantitas quæſita.

Fluens fluxionis $\frac{a^2\dot{x}}{a^2+x^2}=\dot{z}$ erit circularis arcus, cujus radius eſt a & tangens x; hæc vero fluens erit rectangulum impoſſibilis quantitatis $\frac{\sqrt{(-a^2)}}{2}\times$ in hyperb. logarith. quantitatis impoſſibilis $\frac{x+\sqrt{(-a^2)}}{x-\sqrt{(-a^2)}}$, i. e. erit logarith. impoſſibilis quantitatis $\left(\frac{x+\sqrt{(-a^2)}}{x-\sqrt{(-a^2)}}\right)^{\frac{\sqrt{(-a^2)}}{2}} = \left(\frac{y+\sqrt{(y^2-a^2)}}{y-\sqrt{(y^2-a^2)}}\right)^{\frac{\sqrt{(-a^2)}}{2}}$, ubi y eſt ſinus ejuſdem arcûs.

Ex hac æquatione deduci poteſt $y = a\times\frac{e^{\frac{z\sqrt{(-1)}}{a}}+e^{\frac{-z\sqrt{(-1)}}{a}}}{2}$ & $\sqrt{(a^2-y^2)} = a\times\frac{e^{\frac{z\sqrt{(-1)}}{a}}-e^{\frac{-z\sqrt{(-1)}}{a}}}{2\sqrt{(-1)}}$.

Logarithmus autem quantitatis $\left(\frac{x+\surd(-a^2)}{x-\surd(-a^2)} = \frac{(x+\surd(-a^2))^2}{x^2+a^2}\right)$ $= 2$ logar. $(x+\surd(-a^2)) -$ log. $(x^2+a^2) = \frac{2}{\surd(-a^2)} \times$ in circularem arcum prædictum; unde log. quantitatis impoſſibilis $x + \surd(-a^2) = \frac{1}{\surd(-a^2)} \times$ circularem arcum prædictum $+ \frac{1}{2}$ log. quantitatis (x^2+a^2). Logarithmus autem quantitatis $x - \surd(-a^2) = \frac{1}{2}$ log. quantitatis $(x^2+a^2) - \frac{1}{\surd(-a^2)} \times$ circularem arcum prædictum.

P R O B. XVII.

Invenire fluentem cujuſcunque fluxionis, quæ eſt algebraica functio literæ x *in fluxionem* $\dot{x}$ *ducta; ſi modo ea fluens per finitos terminos literæ* x, *ejus circulares arcus & logarithmos exprimi poſſit.*

1. Sit fluxio $\frac{x^s\dot{x}}{x^n - px^{n-1} + \&c.}$; ubi literæ p, q, r, &c. cognitas & invariabiles coefficientes reſpective denotant; per lemma conſtat $\frac{x^s\dot{x}}{x^n - px^{n-1} + \&c.} = \frac{a\dot{x}x^s}{x-\alpha} + \frac{b\dot{x}x^s}{x-\beta} + \frac{c\dot{x}x^s}{x-\gamma} +$ &c. ubi literæ α, β, γ, &c. ſunt radices datæ æquationis $x^n - px^{n-1} + qx^{n-2} -$ &c. $= 0$; quantitas vero $\frac{ax^s\dot{x}}{x-\alpha}$ reduci poteſt in ſubſequentem $ax^{s-1}\dot{x} + a\alpha x^{s-2}\dot{x} + a\alpha^2x^{s-3}\dot{x} \ldots + a\alpha^{s-1}\dot{x} + \frac{a\alpha^s\dot{x}}{x-\alpha}$; fluentes autem harum fluxionum erunt $\frac{ax^s}{s} + \frac{a\alpha x^{s-1}}{s-1} + \frac{a\alpha^2x^{s-2}}{s-2} \ldots + \alpha a^{s-1}x + a\alpha^s \times$ log. $(x-\alpha)$. Et ſic de reliquis terminis $\frac{bx^s\dot{x}}{x-\beta}$, $\frac{cx^s\dot{x}}{x-\gamma}$, &c. Si vero aliquid imaginarii

narii in duabus correſpondentibus fractionibus $\frac{bx^s\dot{x}}{x-\alpha}$ & $\frac{cx^s\dot{x}}{x-\beta}$ ſeorſim ſumptis contineatur, addantur in unam hæ duæ fractiones, & reſultat fractio $\frac{(b+c)x^{s+1}-(b\beta+c\alpha)x^s}{(x-\alpha)\times(x-\beta)}\dot{x}$ ex impoſſibilitate libera: dividatur numerator per denominatorem uſque donec ſolummodo reſtat $(Ax+B)\dot{x}$; quotientis terminorum inveniantur fluentes; fluens vero reſidui $\frac{(Ax+B)'\times\dot{x}}{(x-\alpha)\times(x-\beta)}$ erit $\frac{1}{2}A\times\log.(x-\alpha)\times(x-\beta)-\frac{B+(\alpha+\beta)\times\frac{1}{2}A}{\frac{1}{4}(\alpha-\beta)^2}\times$ arc. circul. cujus radius ſit $\frac{1}{2}\times(\alpha-\beta)\times\sqrt{(-1)}$, & cujus tangens ſit $x-\frac{1}{2}\alpha-\frac{1}{2}\beta$; ubi $\frac{1}{2}(\alpha-\beta)\sqrt{(-1)}$ eſt affirmativa poſſibilis quantitas.

Cor. 1. Sit data fluxio $\frac{ax^s+bx^{s-1}+\&c.}{x^n-px^{n-1}+qx^{n-2}-\&c.}\dot{x}$, e prædictis conſtat fluentem ſingularium fluxionum $\frac{ax^s\dot{x}}{x^n-px^{n-1}+\&c.}$ inveniri poſſe circularium arcuum, logarithmorum & finitorum terminorum ope; ergo fluens ex omnibus conjunctim; acquiſitis prædictis radicibus α, β, γ, &c. facile acquiri poteſt fluens quæſita.

Hinc fluens omnis fluxionis, quæ eſt rationalis algebraica functio literæ x in fluxionem $\dot{x}$ ducta, inveniri poteſt circularium arcuum, logarithmorum & finitorum terminorum ope.

2. Sit data fluxio quæcunque rationalis functio quantitatum x^n & $(a+bx^n)^{\frac{1}{m}}$, ducta in $x^{n-1}\dot{x}$; tum ejus fluens ſemper inveniri poteſt ope finitorum terminorum, circularium arcuum & logarithmorum.

Data fluxio enim transformari poteſt in rationalem fluxionem per methodum in ex. 1. prob. 15. traditam; unde per prob. ejus fluens prædictâ methodo inveniri poteſt.

Ex. Sit data fluxio $\frac{ax^n+bx^{2n}+\&c.+A(\alpha+\beta x^n)^{\frac{1}{m}}+B\times(\alpha+\beta x^n)^{\frac{2}{m}}+\&c.}{p+qx^n+rx^{2n}+\&c+P\times(\alpha+\beta x^n)^{\frac{1}{m}}+\mathcal{Q}(\alpha+\beta x^n)^{\frac{2}{m}}+\&c}$ $x^{n-1}\dot{x}$: per prædictum exemplum ſcribatur $(\alpha+\beta x^n)^{\frac{1}{m}}=v$, & erit $\frac{v^m-\alpha}{\beta}=x^n$, & exinde $x^{n-1}\dot{x}=\frac{mv^{m-1}\dot{v}}{n\beta}$; unde data fluxio erit rationalis, viz. $\frac{a\times\frac{v^m-\alpha}{\beta}+b\times\frac{(v^m-\alpha)^2}{\beta^2}+A\times v+Bv^2+\&c.}{p+q\times\frac{v^m-\alpha}{\beta}+r\times\frac{(v^m-\alpha)^2}{\beta^2}+Pv+\mathcal{Q}v^2+\&c.}\times$ $\frac{mv^{m-1}\dot{v}}{n\beta}$; & exinde per prob. ejus fluens inveniri poteſt ope finitorum terminorum, circularium arcuum & logarithmorum.

3. Sit data fluxio quæcunque rationalis functio quantitatis x^n & irrationalis quantitatis $\left(\frac{a+bx^n}{c+dx^n}\right)^{\frac{1}{m}}=v$ in $x^{n-1}\dot{x}$ ducta; & per exemp. 2. prob. 15. transformari poteſt in rationalem functionem literæ v in fluxionem $\dot{v}$ ductam; unde conſtat ejus fluentem inveniri poſſe ope finitorum terminorum, circularium arcuum & logarithmorum.

4. Sit data fluxio rationalis functio quantitatum x^n & $(a+bx^n)^{\frac{1}{s}}\times(c+dx^n)^{\frac{s-1}{s}}$ in $x^{n-1}\dot{x}$ ducta. Pro $(a+bx^n)^{\frac{1}{s}}\times(c+dx^n)^{\frac{s-1}{s}}$ ſcribatur ejus valor $(c+dx^n)\times\left(\frac{a+bx^n}{c+dx^n}\right)^{\frac{1}{s}}$, & reducitur data fluxio in fluxionem præcedentis formulæ; unde ejus fluens ope prædictarum quantitatum inveniri poteſt.

5. 1. Sit data fluxio rationalis functio quantitatum x^n & $(e+fx^n+gx^{2n})^{\frac{1}{2}}$ in $x^{n-1}\dot{x}$ ducta; & $gx^{2n}+fx^n+e=g\times(x^n-\alpha)\times(x^n-\beta)$; unde $gx^{2n}+fx^n+e=g\times\frac{(x^n-\alpha)\times(x^n-\beta)^2}{x^n-\beta}$, & exinde $(gx^{2n}+$

$fx^n+e)^{\frac{1}{2}} = g^{\frac{1}{2}} \times \left(\frac{x^n-\alpha}{x^n-\beta}\right)^{\frac{1}{2}} \times (x^n-\beta)$. Supponatur $\left(\frac{x^n-\alpha}{x^n-\beta}\right)^{\frac{1}{2}} = v$, & exinde $\frac{\beta v^2-\alpha}{v^2-1} = x^n$; quo valore pro x^n & ejus fluxione pro $nx^{n-1}\dot{x}$ in datâ fluxione ſubſtituto; reſultabit fluxio, quæ erit rationalis functio literæ v in fluxionem $\dot{v}$ ducta.

2. Si vero radices æquationis $x^{2n} + \frac{f}{g}x^n + \frac{e}{g} = 0$ ſint impoſſibiles, & conſequenter prædictæ quantitates ſubſtitutæ impoſſibiles; tum impoſſibiles quantitates evitandi gratiâ ad ſubſtitutiones ſubſequentes recurrere præſtat. Supponatur $(gx^{2n}+fx^n+e(b^2))^{\frac{1}{2}} = b+px^nv$, & exinde $gx^{2n}+fx^n+b^2 = b^2+2pbx^nv+p^2v^2x^{2n}$; unde $x^n = \frac{f-2bpv}{p^2v^2-g}$, & $(b^2+fx^n+gx^{2n})^{\frac{1}{2}} = b+px^nv = \frac{pfv-bp^2v^2-gb}{p^2v^2-g}$; quibus quantitatibus pro ſuis valoribus in datâ fluxione ſubſtitutis, & fluxione $\frac{2p^3bv^2-2p^2fv+2bpg}{n(p^2v^2-g)^2}\dot{v}$ pro $x^{n-1}\dot{x}$; transformatur data fluxio in rationalem fluxionem.

6. Sit data fluxio rationalis functio quantitatum x^n, & $\sqrt[m]{(a+b\sqrt[r]{(c+d\sqrt[s]{(e+\&c.+(f+gx^n)^{\frac{1}{t}})}})} = P$, $\sqrt[r]{(c+d\sqrt[s]{(e+\&c.+(f+gx^n)^{\frac{1}{t}})})} = Q$, $\sqrt[s]{(e+\&c.+(f+gx^n)^{\frac{1}{t}})} = R$, $(f+gx^n)^{\frac{1}{t}} = V$; ubi literæ r, s, &c.; t ſunt integri numeri; pro P, Q, R, &c. &c. & V & gx^n ſcribantur reſpective $v=P$, $\frac{v^m-a}{b} = Q$, $\frac{\left(\frac{v^m-a}{b}\right)^r-c}{d} = R$, &c.; $\left(\frac{\left(\frac{v^m-a}{b}\right)^r-c}{d}\right)^s - e$, &c. $= (f+gx^n)^{\frac{1}{t}}$, & $\left(\left(\frac{\left(\frac{v^m-a}{b}\right)^r-c}{d}\right)^s \&c.\right)^t - f = gx^n$; & pro $x^{n-1}\dot{x}$ ejus valor exinde deductus; unde facile

quæcunque rationalis functio quantitatum P, Q, R, &c., V & x^n in $x^{n-1}\dot{x}$ ducta transformari poteſt in rationalem functionem quantitatum v^m & v in $\dot{v}$.

7. Sit data fluxio rationalis functio quantitatum x^n, $\sqrt[m]{(a+b\sqrt[r]{(c+d\sqrt[s]{(e+\&c.+(\frac{f+gx^n}{h+kx^n})^{\frac{1}{t}})})})}=P$, $\sqrt[r]{(c+d\sqrt[s]{(e+\&c.+(\frac{f+gx^n}{h+kx^n})^{\frac{1}{t}})})}=Q$, $\sqrt[s]{(e+\&c.+(\frac{f+gx^n}{h+kx^n})^{\frac{1}{t}})}=R$, &c., $(\frac{f+gx^n}{h+kx^n})^{\frac{1}{t}}=V$ in $x^{n-1}\dot{x}$ multiplicata; ubi r, s, &c., t ſunt integri numeri; pro P, Q, R, &c., V, & x^n, & $x^{n-1}\dot{x}$ ſcribantur reſpective $v=P$, $\frac{v^m-a}{b}=Q$, $\frac{(\frac{v^m-a}{b})^r-c}{d}=R$, &c., $\left(\frac{(\frac{v^m-a}{b})^r\,\&c.}{d\,\&c.}\,\&c.\right)=V$, & $\frac{f-hV^t}{kV^t-g}=x^n$, & $\frac{thg-tkf}{n(kV^t-g)^2}V^{t-1}\dot{V}$; & exinde facile transformari poteſt rationalis functio quantitatum P, Q, R, &c., V & x^n, in $x^{n-1}\dot{x}$ ducta, in rationalem functionem quantitatum v^m & v in $\dot{v}$; & conſequenter e methodis prius traditis conſtat ejus fluentem inveniri poſſe ope finitorum terminorum, circularium arcuum & logarithmorum.

8. Sit data fluxio rationalis functio quantitatum x^n & $(a+bx^n+(e+fx^n)^{\frac{1}{2}})^{\frac{1}{2}}$, & $(e+fx^n)^{\frac{1}{2}}$ in $x^{n-1}\dot{x}$ ducta: ſcribatur $e+fx^n=v^2$ & exinde $(a+bx^n+(e+fx^n)^{\frac{1}{2}})^{\frac{1}{2}}=(a+b\times\frac{v^2-e}{f}+v)^{\frac{1}{2}}$, & $\frac{v^2-e}{f}=x^n$; quibus quantitatibus pro ſuis valoribus ſubſtitutis, & $\frac{2v\dot{v}}{nf}$ pro $x^{n-1}\dot{x}$ in datâ fluxione; reſultat fluxio, quæ erit rationalis functio

quantitatum v & $\left(a + b \times \frac{v^2 - e}{f} + v\right)^{\frac{1}{2}}$, cujus fluens etiam inveniri poteſt ope finitorum terminorum, circularium arcuum & logarithmorum.

9. Sit data fluxio rationalis functio quantitatum x^n & $\left(\frac{e + fx^n}{h + kx^n}\right)^{\frac{1}{2}} = v$, & $(p + qv + rv^2)^{\frac{1}{2}}$ in $x^{n-1}\dot{x}$ ducta; ſcribantur in datâ fluxione pro $\left(\frac{e + fx^n}{h + kx^n}\right)^{\frac{1}{2}}$ & $(p + qv + rv^2)^{\frac{1}{2}}$, & pro x^n & $x^{n-1}\dot{x}$; reſpective v, $(p + qv + rv^2)^{\frac{1}{2}}$, $\frac{hv^2 - e}{f - kv^2}$ & $\frac{2hv\dot{v}}{n(f - kv^2)} + \frac{2kv\dot{v} \times (hv^2 - e)}{n(f - kv^2)^2}$; & reſultat fluxio, quæ eſt rationalis functio quantitatum v & $(p + qv + rv^2)^{\frac{1}{2}}$ in $\dot{v}$, cujus fluens e methodis prius traditis innoteſcit.

10. Sit data fluxio rationalis functio quantitatum x^n & $\frac{\left(a + l\left(\frac{e + fx^n}{h + kx^n}\right)^{\frac{1}{s}}\right)^{\frac{1}{m}}}{\left(b + c\left(\frac{e + fx^n}{h + kx^n}\right)^{\frac{1}{s}}\right)^{\frac{1}{m}}}(P)$ & $\left(\frac{e + fx^n}{h + kx^n}\right)^{\frac{1}{s}}$ in $x^{n-1}\dot{x}$, ubi s eſt integer numerus: ſcribatur $P = v$, & erit $z = \left(\frac{bv^m - a}{l - cv^m}\right)^s = \frac{e + fx^n}{h + kx^n}$, unde $\frac{hz - e}{f - kz} = x^n$; quibus quantitatibus pro P, $\left(\frac{e + fx^n}{h + kx^n}\right)$, x^n, & ejus $\left(\frac{hz - e}{f - kz}\right)$ fluxione pro $nx^{n-1}\dot{x}$, in datâ fluxione ſubſtitutis; reſultat fluxio, quæ erit rationalis functio quantitatis v^m & v in $\dot{v}$, cujus fluens e prædictis methodis innoteſcet.

Et ſic de fluentibus plurium hujuſce generis fluxionum inveniendis:

dis: e. g. cum data fluxio ſit rationalis functio quantitatum x^n & $x^n + \surd(1+x^{2n}) = v$ & $\surd(1+x^{2n})$ in $x^{n-1}\dot{x}$; &c.

10. Reducere datas fluxiones in alias, quarum fluentes innoteſcunt e multiplicatione earum numeratorum & denominatorum in aſſumptas quantitates.

Caſ. 1. Datâ fluxione $\dfrac{Px^{n-1}\dot{x}}{Q\overset{2}{\surd}(e+fx^n+gx^{2n})+R\overset{2}{\surd}(b+kx^n+lx^{2n})}$; ubi literæ P, Q, & R reſpective denotant rationales functiones quantitatis x^n; ducantur & numerator & denominator hujuſce fluxionis in quantitatem $Q\surd(e+fx^n+gx^{2n})-R\surd(b+kx^n+lx^{2n})$, & reſultat fluxio $\dfrac{PQ\surd(e+fx^n+gx^{2n})-PR\surd(b+kx^n+lx^{2n})}{Q^2\times(e+fx^n+gx^{2n})-R^2\times(b+kx^n+lx^{2n})}\times x^{n-1}\dot{x}$, cujus fluens e regulis prius traditis innoteſcit.

2. Sit data fluxio

$$\frac{Px^{n-1}\dot{x}}{Q\times\left(R^{m-1}(a+bx^n)^{\frac{m-1}{m}}+R^{m-2}S(a+bx^n)^{\frac{m-2}{m}}\times(d+ex^n)^{\frac{1}{m}}+R^{m-3}S^2(a+bx^n)^{\frac{m-3}{m}}\times(d+ex^n)^{\frac{2}{m}}\ldots S^{m-1}\times(d+ex^n)^{\frac{m-1}{m}}\right)};$$

ubi P, Q, R, & S denotant rationales functiones quantitatis x^n, & m eſt integer numerus; ducatur hæc fluxio in $R\times(a+bx^n)^{\frac{1}{m}}-S(d+ex^n)^{\frac{1}{m}}$; & reſultat fluxio $\dfrac{Px^{n-1}\dot{x}}{Q\times R^m(a+bx^n)\pm S^m(d+ex^n)}\times\left(R(a+bx^n)^{\frac{1}{m}}-S\times(d+ex^n)^{\frac{1}{m}}\right)$; cujus fluens in formulis prius traditis continetur.

3. Datâ fluxione $\dfrac{Px^{n-1}\dot{x}}{Q\surd(e+fx^n)+R\surd(g+bx^n)+S\surd(l+mx^n)}$ ubi literæ P, Q, R & S quaſcunque rationales functiones quantitatis x^n reſpective denotant: ducantur & numerator & denominator datæ fluxionis in quantitatem $(Q\surd(e+fx^n)-R\surd(g+bx^n)+S\surd(l+mx^n))\times(Q\surd(e+fx^n)-R\surd(g+bx^n)-S\surd(l+mx^n))\times(Q\surd(e+fx^n)+R\surd(g+bx^n)-S\surd(l+mx^n))$; & reſultat fluxio, cujus fluens in-

veniri poteſt, ſi modo fluens fluxionis $\frac{Ax^{n-1}\dot{x}}{B} \times (e+fx^n)^{\frac{1}{2}} \times (g+hx^n)^{\frac{1}{2}} \times (l+mx^n)^{\frac{1}{2}}$ detegi poſſit, ubi literæ A & B denotant rationales functiones quantitatis x^n.

4. Sit data fluxio $\dot{A}$, cujus numerator ſit $\dot{x}$ in quamcunque rationalem functionem irrationalium algebraicarum functionum α, β, γ, δ, &c. quantitatis x ducta, quæ functiones α, β, γ, δ, &c. haud in ſeſe multiplicantur; & cujus denominator ſit quæcunque rationalis functio irrationalium functionum algebraicarum π, ϱ, σ, τ, &c. quantitatis (x): tum e datis fluentibus fluxionum, quæ exoriuntur ex quantitatibus $\pi\alpha$, $\pi\beta$, $\pi\gamma$, &c.; $\varrho\alpha$, $\varrho\beta$, $\varrho\gamma$, &c.; $\sigma\alpha$, $\sigma\beta$, $\sigma\gamma$, &c.; &c.; $\pi\varrho\alpha$, $\pi\varrho\beta$, $\pi\varrho\gamma$, &c.; $\pi\sigma\alpha$, $\pi\sigma\beta$, &c.; $\varrho\sigma\alpha$, &c.; &c.; $\pi\varrho\sigma\alpha$, &c.; &c.; (quæ ſunt quantitates π, ϱ, σ, &c. in quantitates α, β, γ, δ, &c. reſpective ductæ; rectangula ſub quibuſque duabus prædictis ($\pi\varrho$, $\pi\sigma$, $\varrho\sigma$, &c.) in quantitates α, β, γ, δ, &c. reſpective ductis; etiamque contenta ſub quibuſque tribus, quatuor, &c. quantitatibus (π, ϱ, σ, τ, &c.) reſpective ductis in prædictas quantitates (α, β, γ, δ, &c.)) ductis in quaſcunque vel eaſdem vel diverſas rationales functiones quantitatis x in $\dot{x}$; ſemper detegi poteſt fluens fluxionis $(\dot{A})$.

Conſtat ex ducendo numeratorem & denominatorem datæ fluxionis in quantitatem acquiſitam per methodum in prob. 26. medit. algebr. traditam, ita ut denominator evadat rationalis quantitas.

4. Datâ fluxione $\frac{Px^{n-1}\dot{x}}{\mathfrak{Q} \times \alpha}$, ubi literæ P, $\mathfrak{Q}$, & A, B, C, D, &c. rationales functiones quantitatis x^n reſpective denotant, & litera α denotat quantitatem, quæ in $A\sqrt{(e+fx^n+gx^{2n})}+B\sqrt{(h+kx^n+lx^{2n})}+C\sqrt{(m+n'x^n+ox^{2n})}+D\sqrt{(p+qx^n+rx^{2n})}+\&c.$ (T) ducta, creat rationalem functionem quantitatis x^n; ducantur numerator & denominator datæ fluxionis in quantitatem T; & reſultat fluxio, cujus fluens e prædictis methodis inveniri poteſt; &c.

THEOR.

THEOR. VI.

Rationalis functio cujuſcunque literæ $\dot{x}$ ſemper dividi poteſt in ſimplices diviſores $x+a$, &c.: hinc etiam multæ fluxiones, quæ apparent tanquam irrationales, transformari poſſunt in rationales fluxiones.

Ex. 1. Sit data fluxio quæcunque algebraica & rationalis functio quantitatum $\sqrt[m]{(a+bx^n)}$ & x^n in $x^{n-1}\dot{x}$; & facile conſtat methodus transformandi datam fluxionem in rationalem functionem quantitatis $\sqrt[m]{(a+bx^n)}$, &c.; & exinde conſtat denominatorem dividi poſſe in ſuos ſimplices diviſores hujuſce generis $\sqrt[m]{(a+bx^n)}+p$, $\sqrt[m]{(a+bx^n)}+q$, &c. ubi literæ p & q invariabiles quantitates reſpective denotant.

Ex. 2. Sit data fluxio quæcunque algebraica functio & rationalis quantitatum $\dfrac{\sqrt[m]{(a+bx^n)}}{\sqrt[m]{(d+ex^n)}}$ & x^n in $x^{n-1}\dot{x}$ ducta; & erit data fluxio rationalis functio quantitatis $\dfrac{\sqrt[m]{(a+bx^n)}}{\sqrt[m]{(c+dx^n)}}$; & conſequenter ejus denominator dividi poteſt in ſimplices diviſores hujuſce formulæ $\sqrt[m]{\left(\dfrac{a+bx^n}{c+dx^n}\right)}+p$, &c. & ſic de reliquis exemplis prius traditis; &c.

PROB. XVIII.

Datâ fluxione, quæ ſit algebraica functio quantitatis x^n; *invenire, annon ejus fluens inveſtigari poteſt logarithmorum poſſibilium vel impoſſibilium quantitatum, i. e. circularium arcuum & finitorum terminorum ope.*

1. Inveniantur omnes diviſores denominatoris datæ fluxionis maxime ſimplices, qui haud minores recipiunt dimenſiones incognitæ quantitatis x; quam eas, quas habet data fluxio; & ſint hi diviſores

reſpective $A, B, C, D, E,$ &c. i. e. ſit $A \times B \times C \times D \times E \times$ &c. $=$ datæ fluxionis denominatori, ubi nullus datur diviſor hujuſce formulæ $(a + bx^n + cx^{2n} + \text{\&c.})^{\frac{r}{s}}$, & litera s haud diviſor eſt literæ r; tum fluens datæ fluxionis forſan erit aggregatum $\alpha \times \log. A + \beta \times \log. B + \gamma \times \log. C + \delta \times \log. D +$ &c. ubi $\alpha, \beta, \gamma, \delta,$ &c. invariabiles quantitates reſpective denotant.

2. Sint vero duo diviſores A & B impoſſibiles & correſpondentes, i. e. ſint $p + \surd(-q^2)$ & $p - \surd(-q^2)$; tum $\alpha \log. A + \beta \log. \beta$ denotat aream circuli, ut prius docetur.

3. Denominator, &c. finitorum terminorum deducendus eſt e denominatore dato, &c. per methodum in prob. 4. traditam; & conſequenter ſummæ fluxionum finitorum terminorum per prædictam methodum deductorum, & fluxionum logarithmorum & circularium arcuum prius inventorum æquentur datæ fluxioni, ſi modo poſſibile ſit; & exinde deduci poteſt fluens quæſita.

Ex. Sit data fluxio

$$\frac{bx^{n-1} + \frac{2}{5} \times (e + fx^n)^{-\frac{3}{5}} \times fx^{n-1} + (a + bx^n + (e + fx^n)^{\frac{2}{5}} + 1)(\frac{4}{3} dx^{n-1} \times (g + bx^n) + bx^{n-1} \times (c + dx^n)) \times (c + dx^n)^{\frac{1}{3}}}{(a + bx^n + (e + fx^n)^{\frac{2}{5}} + 1)} \times \dot{x}.$$

Primo animadvertendum eſt denominatorem habere ſimplicem diviſorem $(a + bx^n + (e + fx^n)^{\frac{2}{5}} + 1) = H$, qui in ſimpliciores diviſores haud reſolvi poteſt; & conſequenter ſi fluens datæ fluxionis inveniri poſſit finitorum terminorum, logarithmorum, & circularium arcuum ope; tum quædam pars fluentis erit $\alpha \times \log. H + \beta \times \log. (c + dx^n) + \gamma \log. (e + fx^n)$; ubi literæ $\alpha, \beta,$ & γ conſtantes quantitates inveniendas denotant; & $(c + dx^n)^{\frac{1}{3}}$ & $(e + fx^n)^{\frac{1}{5}}$, &c. ſunt irrationales quantitates in prædicto ſimplici diviſore (H) contentæ: & ſic de pluribus ſimplicibus diviſoribus in denominatore datæ fluxionis contentis argumentari liceat: 2^{do}. pro ſingulis irrationalibus quantitatibus $(k+lx^n)^{\frac{1}{s}}$ in datâ fluxione ſubſtituere liceat $a \times$ log. $(A+B(k+lx^n)^{\frac{1}{s}}+C(k+lx^n)^{\frac{2}{s}}+D(k+lx^n)^{\frac{3}{s}}\ldots+(k+lx^n)^{\frac{s}{s-1}})$, ubi litera s integrum denotat numerum, & A, B, C, D; &c. ſunt rationales functiones quantitatis x^n; pro parte fluentis quæſitâ: 3^{tio}. ſi vero ſit H^t, ubi t ſit integer numerus, eædem quantitates pro logarithmicis partibus fluentis aſſumendæ ſunt.

Hactenus de logarithmicis partibus fluentis; quarum, ſi aliquis evadat impoſſibilis, ea ad circulares arcus reducenda eſt.

Nunc quoad partes fluentis, quæ finitos terminos recipiunt; 1^{mo}. obſervandum eſt in iis haud contineri ſimplicem diviſorem datæ fluxionis denominatoris, qui non habet alterum ſibi ipſi æqualem; ergo, ſi talis diviſor in denominatore contineatur, numerator partis fluxionis, quæ per finitos terminos detegi poteſt, per eum diviſorem dividi queat: quoad reliquos denominatoris terminos, & irrationales datæ fluxionis numeratoris terminos, fluentes aſſumendæ ſunt per methodos in prob. 4. traditas; e. g. ſint quantitates in denominatore $(a'+b'x^n+\&c.)^{\pi}$, &c. & in numeratore $(c'+d'x^n+\&c.)^{\rho}$, &c. tum pro parte fluentis quæſitâ ad eas refertâ, & per finitos terminos inveſtigandâ, aſſumatur $(a'+b'x^n+\&c.)^{1-\pi}\times(c'+d'x^n+\&c.)^{1+\rho}\times(\lambda+\mu x^n+\nu x^{2n}+\&c.)$; ubi λ, μ, ν, &c. ſunt conſtantes quantitates inveſtigandæ: & ſic de pluribus quantitatibus hujuſce generis.

Inveniantur fluxiones ſingularum prædictarum fluentium, deinde fiat earum ſumma datæ fluxioni æqualis; & ex æquatis correſpondentibus datæ fluxionis & prædictæ ſummæ terminis deduci poteſt ejus fluens; (quæ in hoc ex. erit $\frac{1}{n}$ log. $(a+bx^n+(e+fx^n)^{\frac{2}{5}}+1)+\frac{1}{n}(c+dx^n)^{\frac{4}{3}}\times(g+bx^n))$; ſi modo per finitos terminos quantitatis x^n, logarithmos & circulares arcus algebraicarum functionum prædictæ quantitatis x^n exprimi poſſit.

In pleriſque caſibus præſtat, ut negativi indices ex denominatore & numeratore exterminentur.

Sit $\frac{P\dot{x}}{\mathcal{Q}}$ fluxio logarithmi fractionis $\frac{\alpha}{\beta}$; ubi P & $\mathcal{Q}$, α & β ſunt algebraicæ functiones quantitatis x; tum erunt dimenſiones quantitatis x in denominatore $\mathcal{Q}$ majores quam ejus dimenſiones in numeratore P per unitatem; ni dimenſiones quantitatis x in numeratore α æquales ſint ejus dimenſionibus in β, in quo caſu dimenſiones denominatoris $\mathcal{Q}$ ſuperant dimenſiones numeratoris per quantitatem majorem quam unitatem.

THEOR. VII.

1. Sit $\frac{\dot{x}}{x} = \frac{\dot{y}}{y}$; erit $x = ay$, ubi a ſit invariabilis coefficiens ad libitum aſſumenda.

Cor. Hinc fluxio $(\dot{x})$ nullius quantitatis per ſuam quantitatem (x) diviſa eandem habet quotientem, ac fluxio $(\dot{y})$ alterius quantitatis per ſuam quantitatem y diviſa; ni $x = ay$.

2. Sint duæ prædictæ variabiles quantitates x & y, & ſit $\frac{\dot{x}}{y} = \frac{\dot{y}}{x}$; & erit $x\dot{x} = y\dot{y}$, & exinde $x^2 = y^2 + a$.

Ex. Sit $\frac{\dot{y}}{\sqrt{(y^2+a)}}$, ejus fluens inveniri poteſt ope logarithmorum: aſſumatur enim $x^2 = y^2 + a$, & erit $\frac{\dot{x}}{y} = \frac{\dot{y}}{x}$ i. e. $\frac{\dot{x}}{y} = \frac{y\dot{y}}{y\sqrt{(y^2+a)}} = \frac{\dot{y}}{\sqrt{(y^2+a)}}$; ergo, ſi aſſumatur pro fluente $y + x = y + \sqrt{(y^2+a)}$, erit

$$\frac{\dot{y}+\dot{x}}{x+y} = \frac{\dot{y}}{x} = \frac{\dot{x}}{y} = \frac{\dot{y} + \frac{y\dot{y}}{\sqrt{(y^2+a)}}}{\sqrt{(y^2+a)}+y} = \frac{\dot{y}}{\sqrt{(y^2+a)}}.$$

Cor. Hinc fluxio $(\dot{x})$ nullius quantitatis (x) per alteram quantitatem (y) diviſa eandem dabit quotientem, ac fluxio $(\dot{y})$ alterius quantitatis per priorem quantitatem (x) diviſa; ni $x^2 = y^2 + a$.

3. Sint tres variabiles quantitates x, y & z; & ſit $\frac{\dot{x}}{z} = \frac{\dot{y}}{x} = \frac{\dot{z}}{y}$; & per reductionem, viz. e primâ æquatione $\frac{\dot{x}}{z} = \frac{\dot{y}}{x}$, reſultat $x\dot{x} = z\dot{y}$: inveniatur ejus fluxio $x\ddot{x} + \dot{x}^2 = \dot{z}\dot{y}$, & conſequenter $\dot{z} = \frac{x\ddot{x} + \dot{x}^2}{\dot{y}}$, & exinde ob $\left(\frac{\dot{y}}{x} = \frac{\dot{z}}{y}\right)$ invenitur fluxionalis æquatio $y\dot{y}^2 = x^2\ddot{x} + x\dot{x}^2$, ſi modo y fluat uniformiter.

4. Sit $\frac{\dot{x}}{v} = \frac{\dot{y}}{x} = \frac{\dot{z}}{y} = \frac{\dot{v}}{z}$; & per reductionem invenitur fluxionalis æquatio $x^3\dddot{x} + 4x^2\dot{x}\ddot{x} + x\dot{x}^3 = y\dot{y}^3$; & in genere ſint n variabiles quantitates, & $n - 1$ æquationes prædicti generis, & ſit fluxionalis æquatio relationem inter x & y exprimens prædicti generis $A = y\dot{y}^{n-1}$; tum ſint $n + 1$ variabiles quantitates & n æquationes prædicti generis, & erit æquatio inter x & y & earum fluxiones reſultans $x\dot{A} = y\dot{y}^n$, ſi modo y fluat uniformiter.

Ex reſolutione fluxionalium æquationum quærendæ ſunt fluentes prædictarum æquationum.

5. E diviſoribus denominatoris ſemper detegi poteſt, annon fluens datæ fluxionis inveniri poteſt ope finitorum terminorum, circularium arcuum, & logarithmorum; conſtat enim e prob. 4. ſubſtitutio, quæ neceſſario continet finitos terminos; & e diviſoribus denominatoris, &c. ſemper conſtabunt termini, qui involvunt logarithmos & circulares arcus, ni in caſibus hoc theorem. datis, i. e. ubi $\alpha + \beta$ ſit diviſor quæſitus & $\frac{\dot{\alpha}}{\beta} = \frac{\dot{\beta}}{\alpha}$, & conſequenter $\beta = \surd(\alpha^2 + a)$, ubi a denotat invariabilem quantitatem; & ſic ubi $\alpha + \beta + \gamma$ ſit diviſor quæſitus & $\frac{\dot{\alpha}}{\beta} = \frac{\dot{\beta}}{\gamma} = \frac{\dot{\gamma}}{\alpha}$; & ſic deinceps.

In hoc caſu facile conſtat (ſi modo α ſit rationalis algebraica functio, & β & γ finitæ algebraicæ functiones literæ x) β & γ eſſe reſpective $pP^{\frac{1}{3}}$ & $qP^{\frac{2}{3}}$; ubi literæ p, q & P denotant reſpective rationales functiones literæ x: &c.

In detegendis prædictis diviſoribus denominatoris: ſæpe e denominatore in numeratorem; & vice versâ e numeratore in denominatorem, transformari debent irrationales quantitates.

Ni igitur reſolutioni obſtet vulgaris algebra; ſemper inveniri poteſt, utrum fluens cujuſcunque datæ fluxionis inveſtigari poteſt ope finitorum terminorum, circularium arcuum & logarithmorum; necne.

PROB. XIX.

1. *Invenire, annon data fluxio, quæ eſt functio quantitatis* x *in* $\dot{x}$; *ſit rationalis functio quantitatis* (z), *quæ eſt data functio quantitatis* x, *in* $\dot{z}$.

Aſſumatur rationalis functio $\frac{a + bz + cz^2 + dz^3 + \&c.}{A + Bz + Cz^2 + Dz^3 + \&c.} \times \dot{z}$: in hâc functione pro z & $\dot{z}$ ſcribatur data functio quantitatis x & ejus fluxio; & ex æquatis correſpondentibus terminis datæ & reſultantis fluxionis detegi poteſt, annon data fluxio ſit rationalis functio quantitatis z in $\dot{z}$.

2. *Invenire, utrum prædicta fluxio, quæ eſt functio quantitatis* x *in* $\dot{x}$; *ſit rationalis functio quantitatum* z, v, w, *&c.*; *in fluxione rationalis functionis quantitatum* x, v, w, *&c.*; *ubi literæ* z, v, w, *&c. reſpective denotant datas functiones quantitatis* x; *necne.*

Aſſumantur duæ rationales functiones

$$\frac{a + bz + cv + dw + \&c. + ez^2 + fv^2 + \&c. + gzv + hzw + \&c. + kz^3 + \&c.}{A + Bz + Cv + Dw + \&c. + Ez^2 + Fv^2 + Gzv + \&c.} = P, \text{ \& }$$

$$\frac{a' + b'z + c'v + d'w + \&c. + e'z^2 + \&c.}{A' + B'z + C'v + D'w + \&c. + E'z^2 + \&c.} = Q;$$

in fluxione $P \times Q$ pro z, v, w, &c.; & earum fluxionibus ſcribantur reſpective earum valores in datis functionibus quantitatis x & ejus fluxionis; & ex æquatis correſpondentibus terminis datæ & reſultantis fluxionis detegi poteſt, annon data fluxio ſit rationalis functio prædicta.

PROB. XX.

Datis fluentibus fluxionum datæ formulæ; invenire alias fluxiones, quarum fluentes e datis deduci poſſunt.

In fluxionibus, quarum fluentes dantur, pro variabili & ejus fluxione ſcribantur quæcunque functio novæ variabilis & ejus fluxio; tum ſequitur fluxio, cujus fluens e datis innoteſcit.

Ex. 1. Sit data fluxio $(A) y^r \dot{y} \times y^{-\frac{1}{2}} \times (cy^2 + by + a)^{-\frac{1}{2}}$, ubi r eſt integer numerus poſitivus vel negativus; tum ex duabus independentibus fluentibus hujuſce formulæ deduci poſſunt fluentes omnium fluxionum ejuſdem formulæ, i. e. utcunque variatur litera r, ſi modo invariabiles maneant coefficientes a, b, & c; pro y & $\dot{y}$ ſcribantur in datâ fluxione $x + p$ & $\dot{x}$; & ita aſſumantur coefficientes a, b, c & p ut evadat fluxio $(B) \frac{b'(x^b + \&c.)\dot{x}}{\surd(x^3 + a')}$, ubi litera b denotat integrum numerum: unde, ſi fluentes omnium fluxionum formulæ $y^r \dot{y} \times y^{-\frac{1}{2}} \times (cy^2 + by + a)^{-\frac{1}{2}}$ detegi poſſint, fluentes omnium fluxionum formulæ $\frac{x^b \dot{x}}{\surd(x^3 + a')}$ detegi poſſunt: 2. in fluxione $\frac{x^b \dot{x}}{\surd(x^3 + a')}$ pro $\surd(x^3 + a')$ ſcribatur z, &c. & reſultat fluxio formulæ $(z^2 - a')^{\frac{b+1}{3}} \times \dot{z}$; unde $(C)\, z^l \times (z^2 - a)^{\pm\frac{1}{3} \text{ vel } \pm\frac{2}{3}} \dot{z}$, ubi l eſt integer numerus: deinde pro y, x & z in prædictis fluxionibus A, B, & C ſcribatur $\frac{e + fv}{l' + mv}$; & reſultant fluxiones formulæ $v^b \times (l + mv)^{\pm\frac{1}{2}} \times (p + qv + r'v^2 + sv^3)^{\pm\frac{1}{2}} \dot{v}$, &c.: & ſimiliter ex ſcribendo in his fluxionibus pro v quantitatem $\frac{e' + f'w}{l'' + m'w}$, vel pro quâcunque irrationali quantitate in prædictis

dictis fluxionibus contentâ ſcribendo conſimilem quantitatem $\frac{e'+f'w}{l'+m'w}$; reſultabunt formulæ fluxionum, quarum fluentes e prædictis deduci poſſunt; & ſic deinceps.

Ex. 2. Sit fluxio $\frac{\dot{y}\times A}{\surd(f\pm\surd(g+y^2))}$, ubi A eſt rationalis functio quantitatis $\surd(g+y^2)$; pro $\surd(g+y^2)$ ſcribatur x, & reſultat æquatio formulæ $\frac{x\dot{x}\times A'}{\surd(x^2-g)\times\surd(f\pm x)}$, ubi A' eſt functio quantitatis x.

P R O B. XXI.

Invenire, annon fluens datæ fluxionis ex fluentibus quarundam datarum deduci poteſt.

1. Ex formulâ irrationalitatis datæ fluxionis ſæpe deduci poſſunt formulæ quantitatum, quæ transformabunt prædictas fluxiones in alias ejuſdem irrationalitatis, quam habet data fluxio; deinde tranſformentur generaliter hæ fluxiones in alias, quæ eandem habent irrationalitatem ac data fluxio, tum ducantur fluxiones reſultantes in invariabiles coefficientes, & ſimul addantur; deinde fiat aggregatum reſultans datæ fluxioni æquale, ſi modo poſſibile ſit; & perficitur problema.

Ex. 1. Transformare quantitatem irrationalitatem $\sqrt[3]{}(a+z)$ habentem, ita ut eandem irrationalitatem habeat ac quantitas $\sqrt[2]{}(c+dx^2)$. Supponatur $\sqrt[3]{}(a+z)=\sqrt[2]{}(c+dx^2)$, & exinde $(a+z)^2=(c+dx^2)^3$, unde $z=(c+dx^2)^{\frac{3}{2}}-a$; ſcribatur hæc quantitas pro z in quantitate $\sqrt[3]{}(a+z)$, & ea transformabitur in quantitatem $\sqrt[2]{}(c+dx^2)$. Aliter magis generaliter ſupponatur

$e+$

$\frac{e + fz + gz^2 + \&c.}{e' + f'z + g'z^2 + \&c.} + \frac{b + kz + lz^2 + \&c.}{b' + k'z + l'z^2 + \&c.} \times \sqrt[3]{(a+z)} + \&c. =$ $\frac{\alpha + \beta x + \gamma x^2 + \&c.}{\alpha' + \beta' x + \gamma' x^2 + \&c.} + \frac{\lambda + \mu x + \nu x^2 + \&c.}{\lambda' + \mu' x + \nu' x^2 + \&c.} \times \sqrt{(c + dx^2)}$; deinde inveniatur z in terminis quantitatis x, & ſubſtituatur hæc quantitas pro z in datâ fluxione; & perficietur exemplum.

Ex. 2. Transformare quantitates x & $\sqrt{(a + bx + cx^2)}$ in rationales functiones quantitatis z: aſſumatur rationalis functio $\frac{d + ez}{D + Ez}$ quantitatis z pro x; ſcribatur hæc functio pro x in quantitate $\sqrt{(a + bx + cx^2)}$; & reſultat $a + b \times \frac{d + ez}{D + Ez} + c\left(\frac{d + ez}{D + Ez}\right)^2 = \frac{Hz^2 + Iz + K}{(D + Ez)^2}$; fiat $4HK = I^2$, & exinde deduci poſſunt diverſi valores quantitatum D, E, d, &c. & conſequenter functiones quæſitæ.

Ex. 3. Invenire; annon fluens datæ fluxionis, quæ eſt functio quantitatis V in $\dot{V}$ algebraice inveniri poteſt ope fluxionum $\dot{x}$, $\frac{\dot{y}}{y}$, $\frac{\dot{z}}{a^2 + z^2}$, $\left(\frac{a^2 - b^2 v^2}{t^2 - c^2 v^2}\right)^{\frac{1}{2}} \dot{v}$, $\left(\frac{a^2 + b^2 w^2}{t^2 + c^2 w^2}\right)^{\frac{1}{2}} \dot{w}$, &c. i. e. finitorum terminorum, logarithmorum, circularium, ellipticorum & hyperbolicorum arcuum, &c. pro x, y, z, v, w, &c. in prædictis fluxionibus $\dot{x}$, $\frac{\dot{y}}{y}$, &c. ſubſtituantur tales functiones quantitatis V; & pro $\dot{x}$, $\dot{y}$, $\dot{z}$, $\dot{v}$, $\dot{w}$, &c. harum functionum fluxiones; quales eandem præbebunt irrationalitatem in reſultantibus ac in datâ fluxione: tum ex his fluxionibus generaliter aſſumptis & ſimul adjunctis detegi poteſt; annon ita aſſumi poſſunt coefficientes fluxionum aſſumptarum, ut ſumma evadat data fluxio.

Facile conſtat arcum hyperbolæ deſignari poſſe per arcum impoſſibilem ellipſeos; ſcribatur enim $\sqrt{(-1)}\, w$ pro v in fluxione

$\left(\frac{a^2 - b^2 v^2}{t^2 - c^2 v^2}\right)^{\frac{1}{2}} \dot{v}$, & reſultat $\left(\frac{a^2 + b^2 w^2}{t^2 + c^2 w^2}\right)^{\frac{1}{2}} \times \surd(-1)\, \dot{w} = \surd(-1)$ $\times$ fluxionem arcus prædicti hyperbolici.

Ex. 4. Sint tres fluxiones $\left(\frac{(a-b)^2 - x^2}{(a+b)^2 - x^2}\right)^{\frac{1}{2}} \dot{x} = \dot{P}$, $\left(\frac{(d'+b')^2 - z^2}{(d'-b')^2 - z^2}\right)^{\frac{1}{2}} \dot{z}$ $= \dot{Q}$, & $\left(\frac{e^2 - cy^2}{e^2 - y^2}\right)^{\frac{1}{2}} \dot{y} = \dot{R}$; invenire, annon ita aſſumi poſſunt coefficientes, &c., ut fluens fluxionis $m\dot{P} + n\dot{Q} + r\dot{R}$; ubi m, n & r ſunt invariabiles quantitates, in finitis terminis exprimi poſſit; ſi modo aſſumatur $a = d'$, $b = b'$, & $z = x$; & $\dot{R}' = 2\dot{x} + \left(\frac{(a-b)^2 - x^2}{(a+b)^2 - x^2}\right)^{\frac{1}{2}} \dot{x}$ $+ \left(\frac{(a+b)^2 - x^2}{(a-b)^2 - x^2}\right)^{\frac{1}{2}} \dot{x} = 2\dot{x} + \frac{2(a^2 + b^2)\dot{x} - 2x^2\dot{x}}{\surd((a-b)^2 - x^2) \times \surd((a+b)^2 - x^2)}$ $= \dot{T}$: in hâc fluxione $(\dot{T})$ pro x ſcribatur $\frac{a^2 - b^2}{a^2} \times y \times$ $\frac{(a^2 - y^2)^{\frac{1}{2}}}{\left(a^2 - \frac{(a^2 - b^2)}{a^2} y^2\right)^{\frac{1}{2}}}$ & reſultat fluxio $\dot{T} = 4 \times \left(\frac{a^2 - \frac{a^2 - b^2}{a^2} y^2}{a^2 - y^2}\right)^{\frac{1}{2}} \dot{y}$ formulam datam habens.

THEOR. VIII.

Sit data fluxio $X\dot{x}$; ita transformetur data fluxio, ut evadet $Z\dot{z}$, ubi X & Z ſunt functiones quantitatum x & z; cum vero x evadat α & β, & z reſpective π & ρ; tum fluens $\int. X\dot{x}$ inter valores α & β quantitatis x contenta æqualis erit fluenti $\int. Z\dot{z}$ inter valores π & ρ quantitatis z.

Ex. 1. Sit $\frac{x^{m-1}\dot{x}}{(1-x^n)^{\frac{n-k}{n}}}$; & ſtatuatur $1 - x^n = z^n$, ut prodeat $-\int.$ $\frac{z^{k-1}\dot{z}}{(1-z^n)^{\frac{n-m}{n}}}$, & erit $\int. \frac{x^{m-1}\dot{x}}{(1-x^n)^{\frac{n-k}{n}}} = \int. \frac{-z^{k-1}\dot{z}}{(1-z^n)^{\frac{n-m}{n}}}$, quarum utræque inter valores *o* & 1 quantitatum x & z continentur; cum enim x fiat *o*, z evadet 1; & vice verſâ cum x fiat 1, z evadet *o*.

Ex. 2. Sit $\frac{x^{k-1}\dot{x}}{1+x^{n}}$; ſtatuatur $z=\frac{1}{x}$, & evadet $\frac{-z^{n-k-1}\dot{z}}{1+z^{n}}$; & erit $\int.\frac{x^{k-1}\dot{x}}{1+x^{n}}=-\int.\frac{z^{n-k-1}\dot{z}}{1+z^{n}}$, quarum utræque fluentes inter valores *o* & infinitum quantitatum x & z continentur: cum enim x fiat infinita, z evadet *o*; &c.

Ex. 3. 1. Sit $x^{m-1}\dot{x}(1-x^{n})^{k}$; ſcribatur $x=\frac{y}{(1+y^{n})^{\frac{1}{n}}}$, & erit $1-x^{n}=\frac{1}{1+y^{n}}$, & $x^{m}=\frac{y^{m}}{(1+y^{n})^{\frac{m}{n}}}$; & exinde $x^{m-1}\dot{x}(1-x^{n})^{k}=\frac{y^{m-1}\dot{y}}{(1+y^{n})^{k+1+\frac{m}{n}}}$; cujus fluens $\int.x^{m-1}\dot{x}(1-x^{n})^{k}$ inter valores *o* & 1 quantitatis x contenta æqualis erit fluenti $\int.\frac{y^{m-1}\dot{y}}{(1+y^{m})^{k+1+\frac{m}{n}}}$ inter valores *o* & infinitum quantitatis y poſitæ.

2. Ponatur $x=(1-z^{n})^{\frac{1}{n}}$, & exinde reſultat $\int.x^{m-1}\dot{x}(1-x^{n})^{k}=-\int.(1-z^{n})^{\frac{m-n}{n}}\times z^{kn+n-1}\dot{z}$; unde hæ fluentes inter valores *o* & 1 quantitatum x & z poſitæ, evadunt æquales.

Pro x vel y, &c. in his & ſubſequentibus fluxionibus ſcribatur v; & erui poſſunt fluxiones, quarum fluentes inter prædictos valores innoteſcunt.

THEOR. IX.

1. Sint fluxiones $\dot{A}$, $\dot{B}$, $\dot{C}$, &c.; quarum variabilis quantitas ſit x; in iis pro x ſubſtituantur quæcunque functiones literæ (x), & reſultent fluxiones $\dot{A}'$, $\dot{A}''$, &c.; $\dot{B}'$, $\dot{B}''$, &c.; $\dot{C}'$, &c.; &c.: ducantur hæ fluxiones in invariabiles quantitates m, m', &c.; n, n', &c.; r, r', &c.; &c.;

& ſi earum ſummæ $m\dot{A}' + m'\dot{A}'' + \&c. + n\dot{B}' + n'\dot{B}'' + \&c. + r\dot{C}' +$ &c. fluens detegi poſſit per quaſcunque datas fluentes; tum ex iis fluentibus detegi poteſt fluens functionis fluxionum $\dot{A}$, $\dot{B}$, $\dot{C}$, &c.

In his functionibus acquirendis ſæpe præſtat; ut tales functiones literæ x pro x in diverſis fluxionibus aſſumantur, quales prebent fluxiones reſultantes conſimiles irrationales quantitates involventes.

2. Sit X talis functio quantitatis x; qualis, ſi modo in eâ (X) pro x ſcribatur etiam prædicta functio X, præbet quantitatem (x): tum in data fluxione $P\dot{x}$, ubi P eſt functio quantitatis x, pro x ſubſtituatur X, & pro $\dot{x}$ ſcribatur $\dot{X}$; & reſultet fluxio $Q\dot{x}$; tum, ſi in fluxione $P\dot{x} + Q\dot{x}$ pro x & ejus fluxione vel ſcribatur x & ejus fluxio, vel X & ejus fluxio; in utroque caſu eadem reſultabit fluxio: & conſequenter, ſi duo valores quantitatis x ſint l & m, & iis correſpondentes valores quantitatis X ſint reſpective L & M; tum (correſpondentibus radicibus adhibitis) fluens fluxionis $P\dot{x} + Q\dot{x}$ inter valores l & m quantitatis x, eadem erit ac fluens ejuſdem fluxionis $P\dot{x} + Q\dot{x}$ inter valores L & M ejuſdem quantitatis (x) contenta.

Hæ quantitates x & X hoc modo ſemper reciprocant; ſi modo in æquatione relationem inter quantitates X & x deſignante, ſimiliter involvantur prædictæ quantitates X & x.

Ex. 1. Sit $a(x + X) + bXx - ab = 0$ æquatio relationem inter X & x exprimens, unde $x = \frac{(b - X)a}{a + bX}$; & fluxio $P\dot{x} = \frac{b^{\frac{1}{2}}\dot{x}x^{\frac{1}{2}}}{(a + \frac{b^2 - a}{b}x - x^2)^{\frac{1}{2}}}$; in eâ pro x ſcribatur $a \times \frac{b - x}{a + bx}$ (ob $(b - x) \times \left(\frac{a}{b} + x\right) = a + \frac{b^2 - a}{b}x - x^2$), & reſultat fluxio $Q\dot{x} = -\frac{ba \times (b - x)^{\frac{1}{2}}}{x^{\frac{1}{2}}(a + bx)^{1\frac{1}{2}}}\dot{x}$ eaſdem irrationales quantitates $(b - x)^1$ & $\left(x + \frac{a}{b}\right)^{\frac{1}{2}}$

tantummodo continens: fluens ſummæ fluxionum $\frac{1}{2}P\dot{x}+\frac{1}{2}\mathcal{Q}\dot{x}=\frac{\frac{1}{2}b^{\frac{1}{2}}x^{\frac{1}{2}}\dot{x}}{(a+\frac{b^2-a}{b}x-x^2)^{\frac{1}{2}}}-\frac{\frac{1}{2}ba(b-x)^{\frac{1}{2}}\dot{x}}{x^{\frac{1}{2}}(a+bx)^{1\frac{1}{2}}}$ erit $-b\times\frac{(b-x)^{\frac{1}{2}}\times x^{\frac{1}{2}}}{(a+bx)^{\frac{1}{2}}}=L$.

Fluens fluxionis $(P+\mathcal{Q})\dot{x}$ inter valores α & β quantitatis x eadem erit ac fluens ejuſdem fluxionis inter valores $a\times\frac{b-\alpha}{a+b\alpha}$ & $a\times\frac{b-\beta}{a+b\beta}$ quantitatis x contenta.

Cor. Si in fluxione $\frac{x^{-\frac{1}{2}}\dot{x}}{(a+\frac{b^2-a}{b}x-x^2)^{\frac{1}{2}}}=\dot{R}$ pro x ſcribatur $a\times\frac{b-x}{a+bx}=z$, reſultabit $-\dot{R}$; deinde ſumma $P\dot{x}+\frac{a}{b^{\frac{1}{2}}}\dot{R}=+\frac{b^{\frac{1}{2}}x^{\frac{1}{2}}\dot{x}}{(a+\frac{b^2-a}{b}x-x^2)^{\frac{1}{2}}}+\frac{ab^{-\frac{1}{2}}x^{-\frac{1}{2}}\dot{x}}{(a+\left(\frac{b^2-a}{b}\right)x-x^2)^{\frac{1}{2}}}=\frac{(bx+a)\times x^{-\frac{1}{2}}\dot{x}}{b^{\frac{1}{2}}\times(a+\frac{b^2-a}{b}x-x^2)^{\frac{1}{2}}}$ $=b^{\frac{1}{2}}x^{-\frac{1}{2}}\dot{x}\times\left(\frac{x+\frac{a}{b}}{b-x}\right)^{\frac{1}{2}}$; tum in ſummâ $P\dot{x}+\frac{a}{b^{\frac{1}{2}}}\dot{R}=b^{\frac{1}{2}}x^{-\frac{1}{2}}\dot{x}\times\left(\frac{x+\frac{a}{b}}{b-x}\right)^{\frac{1}{2}}$ pro x ſcribatur $a\times\frac{b-x}{a+bx}=z$, & reſultabit quantitas $\mathcal{Q}\dot{x}-\frac{a}{b^{\frac{1}{2}}}\dot{R}=b^{\frac{1}{2}}z^{-\frac{1}{2}}\dot{z}\times\left(\frac{z+\frac{a}{b}}{b-z}\right)^{\frac{1}{2}}$, unde $b^{\frac{1}{2}}x^{-\frac{1}{2}}\dot{x}\times\left(\frac{x+\frac{a}{b}}{b-x}\right)^{\frac{1}{2}}+b^{\frac{1}{2}}z^{-\frac{1}{2}}\dot{z}\times\left(\frac{z+\frac{a}{b}}{b-z}\right)^{\frac{1}{2}}=P\dot{x}+\frac{a}{b^{\frac{1}{2}}}\dot{R}+\mathcal{Q}\dot{x}-\frac{a}{b^{\frac{1}{2}}}\dot{R}=P\dot{x}+\mathcal{Q}\dot{x}=$ fluxioni, cujus fluens eſt $2L$.

Ex. 2. Sit fluxio $\frac{x^{m-1}\dot{x}}{(a-x^m)^{\frac{1}{m}}}$; aſſumatur æquatio $x^m + z^m = a$, in quâ literæ x & z ſimiliter involvuntur; unde reſultabit conſimilis fluxio $\frac{-z^{m-1}\dot{z}}{(a-z^m)^{\frac{1}{m}}}$; & conſequenter fluens prædictæ fluxionis inter duos valores k & l quantitatis x, æqualis erit fluenti ejuſdem fluxionis inter valores $(a-k^m)^{\frac{1}{m}}$ & $(a-l^m)^{\frac{1}{m}}$ quantitatis (x) contenta.

PROB. XXII.

Datis fluxionibus, quarum fluentes cognoſcuntur; invenire utrum datæ fluxionis fluens earum ope inveniri poteſt, necne.

Ducantur ſingulæ datæ fluxiones in incognitas & invariabiles coefficientes, & fluxiones reſultantes addantur vel detrahantur de datâ fluxione; reſultantis fluxionis inveniatur per prob. 4. fluens, i. e. per prob. 4. conſtant dimenſiones ultimi quæſitæ fluentis termini; aſſumatur igitur pro fluente quæſitâ quantitas, quæ neceſſario continet fluentem reſultantis fluxionis, ſi modo ea in finitis algebraicis terminis exprimi poſſit; aſſumptæ quantitatis inveniatur fluxio, quæ fiat æqualis prædictæ reſultanti fluxioni, ſi modo fieri poſſit; & conſit problema.

Ex. 1. Datâ fluente fluxionis $(a+bx^n)^m\dot{x}$; invenire fluentem fluxionis $(a+bx^n)^m x^n\dot{x}$: ducatur data fluxio $(a+bx^n)^m\dot{x}$ in coefficientem incognitam & invariabilem A, & reſultat $A \times (a+bx^n)^m\dot{x}$; addatur hæc reſultans fluxio ad fluxionem, cujus fluens requiritur, & fit $(A+x^n) \times (a+bx^n)^m\dot{x}$; ſed per prob. 4. fluens hujuſce fluxionis hanc habet formulam $B \times x \times (a+bx^n)^{m+1}$; aſſumatur igitur hæc quantitas pro quæſitâ fluente, & ejus fluxio eſt $(Ba + (bB + (m+1)nBb)x^n) \times (a+bx^n)^m\dot{x}$; fiat hæc fluxio eadem ac prædicta fluxio, i. e. $Ba = A$, & $(bB + (m+1)nBb)x^n = x^n$; unde $B =$

$(b+$

$\frac{1}{b+(m+1)nb}$, & $A=\frac{a}{b+(m+1)nb}$; & fluens quæſita erit $Bx \times (a+bx^n)^{m+1} - A \times$ fluen. flux. $(a+bx^n)^m \dot{x}$. Cum $1+(m+1)\times n = 0$; tum ex datâ fluente haud deduci poteſt fluens quæſita.

Ex. 2. Sit fluxio $(a+bx^n)^m x^{pn-1}\dot{x}$, cujus fluens V datur; & fluens fluxionis $(a+bx^n)^m \times x^{pn+n-1}\dot{x}$ invenietur $\frac{(a+bx^n)^{m+1}\times x^{pn}}{(m+p+1)\times nb} - \frac{paV}{(m+p+1)\times b}$. Cum $m+p+1=0$, tum etiam ex datâ fluente haud acquiri poteſt quæſita.

In hoc & ſubſequentibus exemplis litera *(p)* denotat vel integrum numerum vel fractionem vel denique quamcunque invariabilem quantitatem.

Ex. 3. Datâ fluente fluxionis $(a+bx^n)^m \times x^{pn-1}\dot{x}$; invenire fluentem fluxionis $(a+bx^n)^m \times x^{pn+vn-1}\dot{x}$; per prob. ducatur fluxio $(a+bx^n)^m x^{pn-1}\dot{x}$ in incognitam coefficientem b, & ſubducatur quantitas reſultans de datâ fluxione $(a+bx^n)^m x^{pn+vn-1}\dot{x}$, cujus fluens requiritur; & exinde inveniatur fluens differentiæ $(a+bx^n)^m \times (x^{(p+v)n-1} - bx^{pn-1}) \times \dot{x}$, ubi b eſt quantitas aſſumenda, ita ut ſeries terminet: pro fluente quæſitâ aſſumatur quantitas $(a+bx^n)^{m+1} \times (A \times x^{(p+v-1)n} + Bx^{(p+v-2)n} + Cx^{(p+v-3)n} + \&c.\ Lx^{pn})$, cujus fluxio invenietur $(a+bx^n)^m \times ((p+v+m)\times nbAx^{(p+v)n-1} + ((p+v+m-1)\times nBb + (p+v-1)nAa)\,x^{(p+v-1)n-1} + ((p+v+m-2)\,nCb + (p+v-2)\,nBa)\,x^{(p+v-2)n-1} + \&c.) \times \dot{x}$; fiat hæc fluxio æqualis fluxioni $(a+bx^n)^m \times (x^{(p+v)n-1} - bx^{pn-1})\dot{x}$: i. e. $(p+v+m)\times nbA = 1$, & $(p+v+m-1)\times nBb + (p+v-1)\,nAa = 0$, &c. unde $A=\frac{1}{(p+v+m)\times nb}$,

$$B = -\frac{(p+v-1)\times naA}{(p+v+m-1)\times nb} = \frac{-(p+v-1)a}{(p+v+m)\times(p+v+m-1)\,nb^2},$$

& ſic deinceps: lex progreſſionis hujus ſeriei, ſi modo ſcribantur $p+v=q$ & $q+m=r$, erit $(a+bx^n)^{m+1} \times \left(\frac{1}{r\times nb}x^{(q-1)n} - \frac{(q-1)aA}{(r-1)b}\right.$

$x^{(q-2)n} + \frac{(q-2)aB}{(r-2)b} x^{(q-3)n} - \frac{(q-3)aC}{(r-3)b} x^{(q-4)n} +$ &c. ad v terminos) $= \pm \frac{p}{p+m+1} \times \frac{p+1}{p+m+2} \times \frac{p+2}{p+m+3} \ldots \frac{p+v-1}{p+m+v} \times \frac{a^v}{b^v} \times$ fluen. flux. $(a+bx^n)^m x^{pn-1}\dot{x}$: ſignum affixum erit $-$, ſi v ſit impar; ſin aliter $+$. Cum $p+m+l=0$, ubi l eſt quicunque integer numerus haud major quam v; tum ex datâ fluente non acquiri poteſt quæſita.

Ex. 4. Datâ prædictâ fluxione $(a+bx^n)^m x^{pn-1}\dot{x}$, invenire fluentem fluxionis $(a+bx^n)^m \times x^{pn-vn-1}\dot{x}$: aſſumatur $(a+bx^n)^{m+1} \times (Ax^{(p-v)n} + Bx^{(p-v+1)n} + Cx^{(p-v+2)n} +$ &c.) pro fluente quæſitâ; ſcribantur $q=p-v-1$, $s=m+q$, $t=p+m+1$; fluens quæſita per methodum prius traditam invenietur $(a+bx^n)^{m+1} \times \Big(\frac{1}{(q+1)\times na} x^{(p-v)n} - \frac{(s+2)bA}{(q+2)a} \times x^{(p-v+1)n} + \frac{(s+3)bB}{(q+3)a} x^{(p-v+2)n} - \frac{(s+4)bC}{(q+4)a} x^{(p-v+3)n}$ $+$ &c. ad v terminos) $\pm \frac{t-1}{p-1} \times \frac{t-2}{p-2} \times \frac{t-3}{p-3} \ldots \frac{t-v}{p-v} \times \frac{b^v}{a^v} \times$ fluen. flux. $(a+bx^n)^m x^{pn-1}\dot{x}$; ſignum affixum erit $+$, ſi v ſit par, ſin aliter $-$. Cum $p-v+l=0$, ubi l eſt quicunque integer numerus haud major quam $v-1$; tum ex datâ fluente haud detegi poteſt quæſita.

Ex. 5. Datâ fluente fluxionis $(a+bx^n)^m \times x^{pn-1}\dot{x}$; invenire fluentem fluxionis $(a+bx^n)^{m+r} \times x^{pn-1}\dot{x}$, ubi r eſt integer poſitivus numerus: ſcribatur $a+bx^n=Q$; fluens quæſita erit $Q^{m+1} \times x^{pn} \times \Big(\frac{Q^{r-1}}{(p+m+r)\times n} + \frac{(m+r)a \quad Q^{r-2}}{(p+m+r)\times(p+m+r-1)n} + \frac{(m+r)\times(m+r-1) \quad a^2 \quad Q^{r-3}}{(p+m+r)\times(p+m+r-1)\times(p+m+r-2)n} +$ &c. ad r terminos) $+ \frac{m+1}{p+m+1} \times \frac{m+2}{p+m+2} \times \frac{m+3}{p+m+3} \ldots \frac{m+n}{p+m+r} a^r \times$ fluent. fluxionis $(a+bx^n)^m x^{pn-1}\dot{x}$.

Ex. 6. Ex iiſdem principiis e prædictâ fluente datâ inveniri poteſt fluens fluxionis $(a+bx^n)^{m-r}\times x^{pn-1}\dot{x}$ eſſe $Q^{m+1}\times x^{pn}\times\Big(\frac{-Q^{-r}}{(m+r+1)\times na}$ $-\frac{(p+m-r+1)\ Q^{1-r}}{(m-r+1)\times(m-r+2)na^2}-\frac{(p+m-r+1)\times(p+m-r+2)\,Q^{2-r}}{(m-r+1)\times(m-r+2)\times(m-r+3)na^3}$ $-$ &c. ad r terminos$\Big)+\frac{p+m}{m}\times\frac{p+m-1}{m-1}\times\frac{p+m-2}{m-3}\ldots\frac{p+m-r+1}{m-r+1}$ $\times\frac{1}{a^r}\times$ fluent. flux. $(a+bx^n)^m\times x^{pn-1}\dot{x}$. Cum m ſit integer numerus, & minor quam r; tum ex datâ fluente non detegi poteſt quæſita.

Et in genere in omnibus ſubſequentibus caſibus ex datâ fluente non detegi poteſt quæſita, cum ullus factor in denominatore nihilo evadat æqualis.

Ex hiſce ſeriebus conjunctim deducendis conſequitur fluentem fluxionis $(a+bx^n)^{m+r}\times x^{pn+vn-1}\dot{x}$ eſſe $\frac{Q^{m+r}\times x^{(p+v)n}}{(p+m+v+r)\times n}+$ $\frac{(m+r)\times aA}{(p+m+v+r-1)\times Q}+\frac{(m+r-1)\times aB}{(p+m+v+r-2)\times Q}+\frac{(m+r-2)aC}{(p+m+v+r-3)Q}$ ad r terminos $+\frac{m+1}{m+p+v}\times\frac{aH}{bx^n}-\frac{p+v-1}{m+p+v-1}\times\frac{aI}{bx^n}-\frac{p+v-2}{m+p+v-2}$ $\times\frac{aK}{bx^n}$ ad v terminos $\pm\frac{(m+1)\times(m+2)\times(m+3)\,..\,(m+r)}{(p+v+m+1).(p+v+m+2)\ldots(p+v+m+r)}$ $\frac{\times p\times(p+1)\times(p+2)\,..\,(p+v-1)}{\times(p+m+1).(p+m+2)\,..\,(p+m+v)}\times\frac{a^{v+r}}{b^v}\times$ fluen. flux. $(a+bx^n)^m$ $x^{pn-1}\dot{x}$: ubi literæ A, B, C, &c. H, I, K, &c. reſpective denotant præcedentes terminos.

2. Eodem modo deduci poteſt fluens fluxionis $(a+bx^n)^{m-r}\times x^{pn+vn-1}\dot{x}$ $=-\frac{Q^{m-r+1}x^{(p+v)n}}{(m-r+1)na}+\frac{(p+m+v-r+1)\times QA}{(m-r+2)\times a}+\frac{(p+m+v-r+2)\times QB}{(m-r+3)\times a}$ ad r terminos $-\frac{QH}{bx^n}-\frac{p+v-1}{m+p+v-1}\times\frac{aI}{bx^n}-\frac{p+v-2}{m+p+v-2}\times\frac{aK}{bx^n}$ ad v ter-

v terminos $+\frac{m+p+v}{m}\times\frac{m+p+v-1}{m-1}\ldots\frac{m+p+v-r+1}{m-r+1}\times\frac{p}{p+m+1}\times\frac{p+1}{p+m+2}\times\frac{p+2}{p+m+3}\ldots\frac{p+v-1}{p+m+v}\times\frac{a^{v-r}}{(-b)^{v}}\times$ fluen. flux. $(a+bx^{n})^{m}x^{pn-1}\dot{x}$, ubi literæ A, B, C, D, &c. H, I, K, &c. etiam præcedentes terminos reſpective denotant.

3. Fluens fluxionis $(a+bx^{n})^{m+r}\times x^{(p-v)\times n-1}\dot{x}$ invenietur $\frac{Q^{m+r+1}\times x^{(p-v)n}}{(p-v)na}-\frac{m+p-v+r+1}{(p-v+1)}\times\frac{Abx^{n}}{a}-\frac{m+r+p-v+2}{p-v+2}\times\frac{Bbx^{n}}{a}$ ad v terminos $-\frac{Hbx^{n}}{Q}+\frac{m+r}{p+m+r-1}\times\frac{aI}{Q}+\frac{m+r-1}{p+m+r-2}\times\frac{aK}{Q}+$ &c. ad r terminos $+\frac{m+r+p-v+1}{p-v}\times\frac{m+r+p-v+2}{p-v+1}\ldots\frac{m+r+p}{p-1}\times\frac{m+1}{m+p+1}\times\frac{m+2}{m+p+2}\ldots\frac{m+r}{m+p+r}\times\frac{(-b)^{v}}{a^{v-r}}\times$ fluent. fluxionis $(a+bx^{n})^{m}x^{pn-1}\dot{x}$: literis A, B, C, &c. H, I, K, &c. præcedentes terminos reſpective denotantibus.

4. Fluens fluxionis $(a+bx^{n})^{m-r}\times x^{(p-v)n-1}\dot{x}$ invenietur $=-\frac{Q^{m-r+1}x^{(p-v)n}}{(m-r+1)na}+\frac{m+p-r-v+1}{m-r+2}\times\frac{QA}{a}+\frac{m+p-r-v+2}{m-r+3}\times\frac{QB}{a}$ ad r terminos $-\frac{m+p-v}{p-v}\times\frac{QH}{a}-\frac{m+p-v+1}{p-v+1}\times\frac{Ibx^{n}}{a}-\frac{m+p-v+2}{p-v+2}\times\frac{Kbx^{n}}{a}$ ad v terminos $+\frac{(p+m).(p+m-1).(p+m-2)..(p+m-r-v+1)}{m\times(m-1)\times(m-2)..(m-r+1)\times(p-1)\times(p-2)..(p-v)}\times\frac{(-b)^{v}}{a^{r+v}}\times$ fluent. flux. $(a+bx^{n})^{m}x^{pn-1}\dot{x}$, literis A, B, C, &c. H, I, K, &c. præcedentes terminos reſpective denotantibus.

Et ſic ad trinomiales, &c. terminos progredi liceat.

Caſ. 2. Datis fluentibus fluxionum $(a+bx^{n}+cx^{2n}+dx^{3n}+\text{\&c.})^{m}\times x^{pn-1}\dot{x}$

$\times x^{pn-1}\dot{x}$, $(a+bx^n+cx^{2n}+dx^{3n}+$ &c.$)^m \times x^{pn+n-1}\dot{x}$, $(a+bx^n+cx^{2n}+dx^{3n}+$ &c.$)^m \times x^{pn+2n-1}\dot{x}$, quarum numerus eſt r, ſi modo r ſit numerus terminorum in quantitate $a+bx^n+cx^{2n}+$ &c. contentorum: ſcribantur literæ A, B, C, D, &c. pro prædictis fluentibus, & erit $(a+bx^n+cx^{2n}+dx^{3n}+$ &c.$)^{m+1} \times x^{pn} = pnaA + (p+m+1)nbB + (p+2m+2)ncC + (p+3m+3)ndD +$ &c. Hoc facile demonſtrari poteſt, ex inveniendo utriuſque æquationis partis fluxiones; hæ fluxiones enim evadunt eædem.

Si igitur dentur quæcunque $r-1$ prædictæ fluentes, ex hâc æquatione ſequitur reliqua.

Sit terminus in prædictâ ſerie $(p+lm+l)\times F$, ubi l eſt integer numerus, & $p+lm+l=0$; cujus fluens F eſt reliqua; tum ex prædictis $(r-1)$ fluentibus non deduci poteſt reliqua (F).

Caſ. 2. Iiſdem datis; ſcribatur $\mathcal{Q}$ pro fluente fluxionis $(a+bx^n+cx^{2n}+dx^{3n}+$ &c.$)^{m+1} x^{pn-1}\dot{x}$, & erit $(a+bx^n+cx^{2n}+dx^{3n}+$ &c.$)^{m+1} x^{pn} = (m+1)\times(nbB+2ncC+3ndD+$ &c.$)+pn\mathcal{Q}$; unde ſi modo dentur $r-1$ fluentes B, C, D, &c. dabitur etiam fluens $\mathcal{Q}$.

Cor. Si fluxio conſtet ex r nominibus vel terminis in vinculo radicali, & dentur $r-1$ fluentes prædicti generis; dabuntur etiam omnes fluentes fluxionum, quæ addendo vel ſubducendo quantitates n, $2n$, $3n$, $4n$, &c. de indice $pn-1$; vel unitatem, duo, tres, quatuor, &c.; de indice m; generari poſſunt.

Excipiendi ſunt nonnulli caſus, ut e ſubſequentibus conſtabit.

Caſ. 3. Si pro $e+fx^n+gx^{2n}+$ &c. . $x^{\alpha n}$, & $h+kx^n+lx^{2n}+$ &c. . $x^{\beta n}$, $q+rx^n+sx^{2n}+$ &c. . $x^{\gamma n}$, &c. ſcribantur R, S, T, &c. & in fluxione $x^{pn\pm\sigma n-1}\dot{x}R^{m\pm\lambda}S^{o\pm\mu}T^{t\pm\nu}\times$ &c. maneant quantitates datæ $pn-1$, n, m, o, t, e, f, g, h, k, l, q, r, s, &c. & pro σ, λ, μ, ν, &c. ſcribantur in datâ fluxione ſucceſſive quicunque integri numeri; & ſi dentur $\alpha+\beta+\gamma+$ &c. fluentes independentes reſultantium fluxionum, ubi literæ $\alpha+1$, $\beta+1$, $\gamma+1$, &c. reſpective denotant numerum terminorum in quantitatibus R, S, T, &c. contentorum; tum dabuntur fluentes omnium prædictarum fluxionum reſultantium.

Conſtat ex principiis in priori exemplo traditis.

Caſ. 4. Datis ſingulis fluentibus fluxionum $\overline{a+bx^n+cx^{2n}+\&c.+\overline{e+fx^n+gx^{2n}+\&c.}^{\,m}}^{\,l}\times x^{\theta\pm rn-1}\dot{x}$, ubi r integrum denotat numerum; datis etiam fluentibus $(s-1)$ fluxionum hujuſce formulæ $\overline{a+bx^n+cx^{2n}+\&c.+\overline{e+fx^n+gx^{2n}+\&c.}^{\,m}}^{\,l}\times\overline{e+fx^n+gx^{2n}+\&c.}^{\,m\pm t}\times x^{\theta\pm vn-1}\dot{x}$, ubi s ſit numerus terminorum in quantitate $e+fx^n+gx^{2n}+\&c.$ contentorum, & t & v integros denotent numeros; exinde inveniri poſſunt fluentes ſingularum fluxionum hujuſce formulæ.

Eadem etiam principia facile applicari poſſunt ad omnes algebraicas fluxiones utcunque compoſitas.

5. 1. Sint $a+bx^n+cx^{2n}+\&c.=R$, $d+ex^n+fx^{2n}+\&c.=S$, $i+hx^n+kx^{2n}+\&c.=T$, &c. tum per methodos in prob. 4. traditas ita reducatur data fluxio $(A+Bx^n+Cx^{2n}+\&c.)\times R^{\lambda-1}\times S^{\mu-1}\times T^{\nu-1}\times\&c.$, ut in eâ quantitates prædictæ (R, S, T, &c.) nullum habeant communem diviſorem, & in nullâ quantitate contineantur plures diviſores inter ſe æquales; aliter generalis fluens prædictæ fluxionis ex paucioribus fluentibus independentium fluxionum, quam ex numero hâc regulâ dato, detegi poteſt. e. g. Sit $R=a+bx^n+cx^{2n}+\ldots x^{pn}=(\alpha+\beta x^n)^l\times(a'+b'x^n+\&c.)$; $S=d+ex^n+fx^{2n}+\&c.\ldots x^{qn}=(\alpha+\beta x^n)^h\times(\gamma+\delta x^n)^k\times(d'+e'x^n+\&c.)$; &c.: & data fluxio $R^\lambda\times S^\mu\times x^\theta\dot{x}=(\alpha+\beta x^n)^{\lambda l+\mu h}\times(\gamma+\delta x^n)^{\mu k}\times(a'+b'x^n\ldots x^{(p-l)n})^\lambda\times(d'+e'x^n\ldots x^{(q-h-k)n})^\mu\times x^\theta\dot{x}$; per regulam datam ex $(p+q)$ fluentibus formulæ $R^{\lambda\pm\pi}\times S^{\mu\pm\varrho}\times x^{\theta\pm\sigma}\times\dot{x}$ inter ſe independentibus ubi literæ π, ϱ & σ quoſcunque integros denotant numeros, deduci poſſunt omnes aliæ ejuſdem formulæ; at quoniam $R^{\lambda\pm\pi}\times S^{\mu\pm\varrho}\times x^{\theta\pm\sigma}\times\dot{x}=(\alpha+\beta x^n)^{(\lambda\pm\pi)l+(\mu\pm\varrho)h}\times(\gamma+\delta x^n)^{(\mu\pm\varrho)k}\times(a'+b'x^n\ldots x^{(p-l)n})^{\lambda\pm\pi}\times(d'+e'x^n\ldots x^{(q-h-k)n})^{\mu\pm\varrho}\times x^{\theta\pm\sigma}\dot{x}$, exinde conſtat omnes fluentes prædictæ formulæ deduci poſſe ex $(p+q-l-h-k+2)$ fluentibus ejuſdem formulæ inter ſe independentibus.

2. Sit $R^m\times S^o\times T^t\times x^{pn-1}$ quantitas ita reducta, ut quantitates $R=e+fx^n\ldots+x^{\alpha n}$, $S=h+kx^n\ldots+x^{\beta n}$, $T=q+rx^n+sx^{2n}\ldots+x^{\gamma n}$, &c.,

&c., & x^{pn-1} nullum habeant inter ſe communem diviſorem; etiamque ſingulæ nullum habeant diviſorem, qui eſt quadratus $(x+a)^2$, vel cubus $(x+a)^3$, &c.: tum raro ex paucioribus quam $(\alpha+\beta+\gamma+$ &c.$)$ fluentibus fluxionum inter ſe independentibus, formulæ $R^{m\pm\lambda} \times S^{o\pm\mu} \times T^{t\pm\nu} \times x^{pn\pm\sigma n-1} \times \dot{x} \times (a+bx^n+cx^{2n}+$ &c.$)$, ubi literæ λ, μ, ν, &c., & σ integros denotant numeros, ſemper deduci poſſunt fluentes omnium fluxionum ejuſdem formulæ.

Huc etiam applicari poſſunt conſimilia iis, quæ in theor. præcedentibus traduntur.

6. Sint $(a+bx^n+cx^{2n}+$ &c. . $x^{rn})^{m+\lambda} \times x^{pn\pm\sigma n-1}\dot{x}$, ubi σ eſt quicunque integer, & m & λ quicunque integri affirmativi numeri; tum ejus fluens ſemper vel inveniri poteſt, vel ſolummodo exigit fluentem fluxionis $\frac{\dot{x}}{x}$.

Conſtat ex reducendo fluxionem datam in ſeriem ſecundum dimenſiones quantitatis x progredientem.

7. Si vero m affirmativa quantitas haud ſit integer numerus; at p, λ & σ ſint integri affirmativi numeri; & tum ex $(r-1)$ fluentibus fluxionum formulæ $(a+bx^n+cx^{2n}+\ldots+x^{rn})^{m+\lambda} \times x^{pn+\sigma n-1}\dot{x}$ datis, deduci poſſunt omnes reliquæ fluentes ejuſdem formulæ: ſed ex prædictis $(r-1)$ fluentibus non deduci poſſunt omnes fluentes formulæ $(a+bx^n+cx^{2n}+\ldots+x^{rn})^{m\pm\lambda} \times x^{pn\pm\sigma n-1}\dot{x}$; ſed ex (r) fluentibus inter ſe independentibus, i. e. quæ nec in finitis terminis, nec a ſe invicem exprimi poſſunt, formulæ $(a+bx^n+cx^{2n}+\ldots+x^{rn})^{m\pm\lambda} \times x^{pn\pm\sigma n-1}\dot{x}$ deduci poſſunt omnes fluentes ejuſdem formulæ: etiamque ex omnibus, i. e. infinitis fluentibus fluxionum formulæ $(a+bx^n+cx^{2n}\ldots+x^{rn})^{m+\lambda} \times x^{pn+\sigma n-1}\dot{x}$ non deduci poſſunt fluentes omnium fluxionum formulæ $(a+bx^n+cx^{2n}\ldots+x^{rn})^{m\pm r} \times x^{pn\pm\sigma n-1}\dot{x}$.

Literis R, S, T, &c. eaſdem quantitates ac in caſ. 5. 2. denotantibus; ſint m, &c. integri affirmativi numeri; & o, t, &c., non integri numeri; tum fluens omnis fluxionis formulæ $x^{pn\pm\sigma n-1}\dot{x} \times R^{m+\lambda} \times S^{o\pm\mu} \times T^{t\pm\nu} \times$ &c., ubi λ eſt affirmativus numerus, colligi poteſt ex

$(\beta + \gamma + \&c.)$ diverſis fluentibus prædictæ formulæ inter ſe independentibus: 2. etiamque ex ſingulis, viz. infinitis fluentibus formulæ $R^{m+\lambda} \times S^{o\pm\mu} \times T^{t\pm\nu} \times \&c. \times x^{pn\pm\sigma n-1}\dot{x}$ non colligi poteſt fluens omnis fluxionis formulæ $R^{m-\lambda} \times S^{o\pm\mu} \times T^{t\pm\nu} \times \&c. \times x^{pn\pm\sigma n-1}\dot{x}$.

8. Sit p integer affirmativus numerus, tum ex $(\alpha + \beta + \gamma + \&c. - 1)$ fluentibus inter ſe independentibus formulæ $x^{pn+\sigma n-1} \times R^{m+\lambda} \times S^{o+\mu} \times T^{t+\nu} \times \&c. \times \dot{x}$; ubi literæ $\sigma, \lambda, \mu, \nu$, &c. ſunt affirmativi numeri, & m, o, t, &c. affirmativæ quantitates, deduci poſſunt fluentes omnium fluxionum ejuſdem formulæ; at ex fluentibus omnium fluxionum hujuſce formulæ non deduci poſſunt fluentes omnium fluxionum formularum, in quibus prædicti numeri $\sigma, \lambda, \mu, \nu$, &c. ſunt negativi.

THEOR. X.

Sæpe per ſubſtitutionem acquiri poſſunt particulares formulæ generalis fluxionis, quarum fluentes e paucioribus inter ſe independentibus quam per regulam aſſignatis, deduci poſſunt; e. g. ſit fluxio $(a + bx^n)^{\lambda\pm\alpha} x^{(\theta\pm\beta)n} x^{n-1}\dot{x}$, quæ ex unâ datâ fluente hujuſce formulæ acquiri poteſt, ubi literæ α & β integros denotant numeros: in eâ pro x^n ſcribatur $c + dz^n + ez^{2n}$, & reſultat fluxio $(a + bc + bdz^n + bez^{2n})^{\lambda\pm\alpha} \times (c + dz^n + ez^{2n})^{\theta\pm\beta}\dot{x}\,(dz^{n-1} + 2ez^{2n-1})\dot{z}$: unde fluentes omnium fluxionum hujuſce poſterioris formulæ erui poſſunt ex fluente unius datæ fluxionis ejuſdem formulæ.

THEOR. XI.

1. Sit $x^{\theta+\alpha n+\beta m} \times (a + bx^n + cx^m)^{\lambda+\pi}\dot{x}$; ubi literæ α, β & π reſpective denotant affirmativos integros numeros: tum ex datis fluentibus $(N + 1)$ inter ſe independentibus fluxionum hujuſce formulæ, ubi ſumma $\alpha + \beta + \pi$ nunquam major eſt quam N; detegi poſſunt fluentes omnium fluxionum prædictæ formulæ.

Hoc theorema ex iiſdem principiis; ac ea, quæ in prob. 22. traduntur; deduci poteſt: e. g. ſint $x^{\theta}(a + bx^n + cx^m)^{\lambda}\dot{x} = x^{\theta}A^{\lambda}\dot{x} = \dot{P}$,

$= \dot{P}$, $x^{\theta+n} \times A^{\lambda}\dot{x} = \dot{\mathcal{Q}}$; tum erit $\int . \, x^{\theta+m} A^{\lambda} \dot{x} = \frac{1}{(\theta+1)c + (\lambda+1)mc}$ $(x^{\theta+1} \times A^{\lambda+1} - (\theta+1)aP - ((\lambda+1)nb + (\theta+1)b)\mathcal{Q})$; & ſic deinceps.

2. Sint n & m integri numeri, & N eorum maximus communis diviſor; etiamque ſit ν major numerus $\frac{n}{N}$ vel $\frac{m}{N}$; tum ex ν independentibus fluentibus fluxionum formulæ $x^{\theta \pm \alpha n \pm \beta m}(a + bx^{n} + cx^{m})^{\lambda \pm \pi}\dot{x}$ datis ſemper acquiri poſſunt fluentes omnium fluxionum ejuſdem formulæ, viz. $x^{\theta \pm \alpha n \pm \beta m}(a + bx^{n} + cx^{m})^{\lambda \pm \pi}\dot{x}$.

2. 2. Si vero n & m ſint fractiones, &c.; reducendæ ſunt ad minimum communem denominatorem (σ), & evadant $\frac{\mu}{\sigma}$ & $\frac{\mu'}{\sigma}$; inveniatur (N) maximus communis diviſor numeratorum μ & μ', & ſit ν major numerus $\frac{\mu}{N}$ vel $\frac{\mu'}{N}$; tum ex ν independentibus fluentibus fluxionum prædictæ formulæ ſemper deduci poſſunt fluentes omnium fluxionum ejuſdem formulæ.

3. Sit fluxio $x^{\theta + \alpha m + \beta n + \gamma r + \delta s + \&c.} \times (a + bx^{m} + cx^{n} + dx^{r} + ex^{s} + \&c.)^{\lambda + \pi}\dot{x}$, in quâ α, β, γ, δ, &c. & π reperiuntur integri affirmativi numeri: ſit $\alpha + \beta + \gamma + \&c. + \pi$ numerus non major quam b; tum fluentes omnium fluxionum hujuſce formulæ inveniri poſſunt ex $s \times \frac{s+1}{2} \times \frac{s+2}{3} \times \frac{s+3}{4} \ldots \frac{s+b-1}{b}$ inter ſe independentibus fluentibus fluxionum ejuſdem formulæ: ubi s eſt numerus indicum m, n, r, s, &c.

Conſimiles propoſitiones de fluentibus fluxionum formulæ $x^{\theta + \alpha n + \beta m + \&c.}(a + bx^{n} + cx^{2n} + \ldots + ex^{\delta n} + kx^{m} + lx^{2m} + \ldots + x^{\nu m} + px^{n+m} + qx^{n+2m} + rx^{m+2n} + \&c.)^{\lambda+\pi}\dot{x}$, ubi literæ α, β, &c., δ, ν, &c.; & π reſpective integros denotant affirmativos numeros; facile deduci poſſunt.

THEOR. XII.

1. Sit fluxio $\dot{X}\int . Y\dot{x}$, ubi $\dot{X}$ eſt fluxio, cujus fluens ſemper inveniri poteſt; & ſit numerus fluentium inter ſe independentium in formulis fluxionum $Y\dot{x}$ & $XY\dot{x}$ contentarum, reſpective m & r; e quibus detegi poſſunt omnes reliquæ earundem formularum; tum ex $(m+r)$ fluentibus fluxionum $\dot{X}\int . Y\dot{x}$ inter ſe independentibus detegi poſſunt omnes fluentes ejuſdem formulæ.

Conſtat ex eo, quod fluens datæ fluxionis $= X\int . Y\dot{x} - \int . Xy\dot{x}$; & ſi modo dentur m independentes fluentes fluxionis $Y\dot{x}$, per hypotheſin dabuntur omnes; & ſimiliter ſi modo dentur (r) fluentes independentes fluxionis $Xy\dot{x}$, dabuntur omnes ejus formulæ.

Ex. Sit fluxio $(A + Bx^{n} + Cx^{2n} + \&c.) \times x^{\theta \pm \sigma n - 1}\dot{x} \times \int . R^{m \pm \lambda} \times S^{o \pm \mu} \times T^{\iota \pm \nu} \times \&c. \times x^{\varkappa \pm \tau n - 1}\dot{x}$, ubi literæ $\sigma, \lambda, \mu, \nu$, &c. & τ quoſcunque integros denotant numeros; & ſumma $\theta \pm \sigma n$, vel $\theta \pm \sigma n + n$, vel $\theta \pm \sigma n + 2n$, vel $\theta \pm \sigma n + 3n$, &c., non evadat nihilo æqualis; etiamque $R = a + bx^{n} + cx^{2n} + \,.\,.\, x^{\alpha n}$, $S = h + kx^{n} + lx^{2n} + \,.\,.\, x^{\beta n}$, $T = q + rx^{n} + sx^{2n} + \,.\,. + x^{\gamma n}$, &c.; tum erit $X = \frac{1}{\theta \pm \sigma n} A x^{\theta \pm \sigma n} + \frac{1}{\theta \pm \sigma n + n} B x^{\theta \pm \sigma n + n} + \frac{1}{\theta \pm \sigma n + 2n} C x^{\theta \pm \sigma n + 2n} + \&c.$; & $Y = R^{m \pm \lambda} \times S^{o \pm \mu} \times T^{\iota \pm \nu} \times \&c. \times x^{\varkappa \pm \tau n - 1}$: numeri independentium fluentium in formulis fluxionum $Y\dot{x}$ & $XY\dot{x}$ contentarum erunt reſpective $\alpha + \beta + \gamma + \&c.$, & $\alpha + \beta + \gamma + \&c.$; ergo numerus fluentium inter ſe independentium, quarum fluxiones datam habent formulam, non poteſt eſſe major quam $2(\alpha + \beta + \gamma + \&c.)$.

Si $\varkappa = \theta \pm ln$, ubi l eſt integer numerus; tum prædictus numerus non poteſt eſſe major quam $\alpha + \beta + \gamma + \&c.$

2. Sit fluxio $r\dot{x} = \dot{s}$, ubi $\dot{s} = (a + bx^{n} + cx^{2n} \ldots hx^{\alpha n})^{l \pm \pi} \times (p + qx^{n} + rx^{2n} \ldots tx^{\beta n})^{m \pm \rho} \times x^{k \pm \sigma}\dot{x}$; aſſumatur $y = (A + Bx^{n} + Cx^{2n} \ldots Hx^{\delta n})^{L' \pm \tau} \times (P + Qx^{n} + Rx^{2n} \ldots Tx^{\epsilon n})^{m' \pm \upsilon} \times x^{k' \pm \xi}$; ubi $\pi, \rho, \sigma, \tau, \upsilon, \xi$ ſunt quicunque integri numeri; tum ex datis fluentibus $\delta + \epsilon + \&c.$ fluxionum formulæ $y\dot{x}$ haud a ſe invicem pendentibus, in quibus

τ, υ, &c. & ξ haud eoſdem habent valores, erui poſſunt fluentes ſingularum fluxionum formulæ prædictæ $y\dot{x}$; deinde ex ſingulis fluentibus fluxionis $y\dot{x}$ datis & ex fluentibus $\alpha + \beta +$ &c. $+ \delta + \varepsilon +$ &c. fluxionum formulæ $r\dot{x}\int . y\dot{x}$ haud a ſe invicem pendentibus, in quibus π, ϱ, &c. & σ; τ, υ, &c. haud eoſdem habent valores, erui poſſunt fluentes ſingularum fluxionum formulæ $r\dot{x}\int . y\dot{x}$.

3. Conſimiles propoſitiones etiam deduci poſſunt de fluentibus ſuperiorum ordinum; ſæpe enim reduci poſſunt in fluentes primi ordinis. e. g. Sit fluxio $\dot{X} \times \int . Y\dot{x} \times \int . Z\dot{x}$; & $\dot{X}$, $XY\dot{x}$ & $XZ\dot{x}$ fluxiones, quarum fluentes detegi poſſunt, i. e. ſint $\int . \dot{X} = X$, $\int . XY\dot{x} = \alpha$, $\int . XZ\dot{x} = \beta$; tum erit $\int . \dot{X} \times \int . Y\dot{x} \times \int . Z\dot{x} = X \times \int . Y\dot{x} \times \int . Z\dot{x} - (\alpha \times \int . Z\dot{x} + \beta \times \int . Y\dot{x}) + (\int . \alpha Z\dot{x} + \int . \beta Y\dot{x})$. Inveniantur numeri fluentium inter ſe independentium, ex quibus detegi poſſunt omnes valores fluentium $\int . Y\dot{x}$, $\int . Z\dot{x}$, $\int . \alpha Z\dot{x}$ & $\int . \beta Y\dot{x}$; qui ſint reſpective π, ϱ, σ & τ; tum ad plurimum ex $(\pi + \varrho + \sigma + \tau)$ fluentibus inter ſe independentibus datæ fluxionis detegi poſſunt omnes fluentes ejuſdem formulæ.

4. Sit data fluxio $\dot{X}\int . Y\dot{x}\int . Z\dot{x}$; cujus fluens $X\int . Y\dot{x}\int . Z\dot{x} - \int . XY\dot{x}\int . Z\dot{x}$: ſint X, $Y\dot{x}$ & $XY\dot{x}$ fluxiones, quarum fluentes detegi poſſunt, i. e. ſint reſpective X, α & β. Inveniatur numerus fluentium inter ſe independentium, ex quibus detegi poſſunt omnes valores fluentium $Z\dot{x}$, $\alpha Z\dot{x}$ & $\beta Z\dot{x}$; qui ſint reſpective π, ϱ & σ; tum ex $(\pi + \varrho + \sigma)$ ad plurimum fluentibus datæ fluxionis inter ſe independentibus ſemper detegi poſſunt fluentes omnium fluxionum ejuſdem formulæ.

Cor. 1. Sit fluxio $A\dot{x}\int . B\dot{x}\int . C\dot{x}\int . D\dot{x} \ldots . \int . P\dot{x}\int . Q\dot{x}\int . R\dot{x}\int . Y\dot{x} = p\dot{x}$ in quâ ſit $A\dot{x}\int . B\dot{x}\int . C\dot{x}\int . D\dot{x} \ldots . \int . P\dot{x}\int . Q\dot{x}\int . R\dot{x} = \dot{X}$; ubi per X & Y denotentur eædem ac prius quantitates; tum quod prius traditum fuit de fluentibus fluxionum $\dot{X}\int . Y\dot{x}$, etiam ad fluentes fluxionis $p\dot{x}$ applicari poteſt.

Facile conſtant caſus horum theorematum, in quibus e minore numero independentium fluentium quam per has regulas aſſignato deduci poſſunt omnes fluentes datam formulam habentes.

THEOR. XIII.

1. Pro $a + bx^{n} + cx^{2n} + dx^{3n} \ldots hx^{(\mu-1)n} + kx^{\mu n}$ ſcribatur p, & pro fluentibus fluxionum $e^{p}x^{\alpha-1}\dot{x}$, $e^{p}x^{\alpha+n-1}\dot{x}$, $e^{p}x^{\alpha+2n-1}\dot{x}$, $e^{p}x^{\alpha+3n-1}\dot{x}$, ... $e^{p}x^{\alpha+(\mu-1)n-1}\dot{x}$, $e^{p}x^{\alpha+\mu n-1}\dot{x}$ ſubſtituantur reſpective $A, B, C, D, E, \ldots H, K$; tum, quoniam fluxio quantitatis $e^{p}x^{\alpha}$ erit $(\alpha x^{\alpha-1} + nbx^{\alpha+n-1}l + 2ncx^{\alpha+2n-1}l + 3ndx^{\alpha+3n-1}l \ldots (\mu-1)nhx^{\alpha+(\mu-1)n-1}l + \mu nkx^{\alpha+\mu n-1}l)\dot{x} \times e^{p}$, conſequitur fluens (K) fluxionis $e^{p}x^{\alpha+\mu n-1}\dot{x} = \frac{1}{\mu nkl}(x^{\alpha} - \alpha A - nbBl - 2ncCl - 3ndDl \ldots - (\mu-1)nhHl)e^{p}$.

Hìc logarithmus hyper. quantitatis e ſupponitur l.

2. Sit exponentialis fluxio $e^{a+bx^{n}+cx^{2n}\ldots hx^{\mu n}} \times x^{r\pm\nu n}\dot{x}$, ubi μ & ν ſunt integri numeri; tum ex fluentibus (μ) fluxionum prædictæ formulæ independentibus, i. e. quæ habent μ diverſos valores quantitatis ν, datis; acquiri poſſunt fluentes omnium fluxionum ejuſdem formulæ.

3. Sit exponentialis fluxio $\dot{x}e^{a+bx^{n}+cx^{2n}\ldots hx^{\mu n}}(\alpha + \beta x^{n} + \gamma x^{2n} \ldots \varepsilon x^{\rho n})^{k\pm\sigma} \times (A + Bx^{n} + Cx^{2n} \ldots Ex^{\xi n})^{l\pm\tau} \times x^{r\pm\upsilon n}$; ubi σ & υ ſunt quicunque integri numeri; tum e fluentibus $\mu + \varrho + \xi$ fluxionum ſupradictæ formulæ independentibus erui poſſunt fluentes omnium fluxionum ejuſdem formulæ.

4. Sit exponentialis fluxio $e^{(a+bx^{n}+cx^{2n}\ldots hx^{\pi n})^{m}} \times (\alpha + \beta x^{n} + \gamma x^{2n} \ldots \varepsilon x^{\rho n})^{k\pm\sigma} \times (A + Bx^{n} + Cx^{2n} \ldots Ex^{\xi n})^{l\pm\tau} \times x^{r\mp\upsilon n}\dot{x} = P\dot{x}$; ubi σ, τ, υ, &c. ſunt quicunque integri numeri; tum ex omnibus fluentibus formulæ $P\dot{x}$ datis; & ex $(\pi + \varrho + \xi)$ independentibus fluentibus fluxionum formulæ $(a + bx^{n} + cx^{2n} \ldots + hx^{\pi n})^{m\pm r} \times P\dot{x}$, ubi r eſt etiam quicunque integer numerus; acquiri poſſunt fluentes omnium fluxionum ejuſdem formulæ.

Facile conſtant hæ propoſitiones.

Cor. 1. Sit exponentialis fluxio $e^{x}x^{a}\dot{x}$, cujus fluens (P) detur; tum fluens fluxionis $e^{x}x^{a+\pi}\dot{x}$ erit $\frac{1}{l}e^{x}(x^{a+\pi} - \frac{1}{l}(a+\pi)x^{a+\pi-1} + \frac{1}{l^{2}}(a+\pi)$.

.(a +

$\cdot (a+\pi-1)\, x^{a+\pi-2} - \frac{1}{l^3}(a+\pi)\cdot(a+\pi-1)\cdot(a+\pi-2)\, x^{a+\pi-3} \ldots\ldots \frac{1}{l^{\pi-1}}(a+\pi)\cdot(a+\pi-1)\cdot(a+\pi-2)\ldots(a+2)\cdot x^{a+1}) \pm \frac{1}{l^\pi}(a+\pi)\cdot(a+\pi-1)\cdot(a+\pi-2)\ldots(a+1)\times P$, ſi π ſit integer affirmativus numerus: ſignum $\pm$ erit $+$, ſi π ſit par; ſin aliter $-$.

2. Sit π negativus numerus, & erit $\int.\, x^{a-\pi} e^x \dot{x} = \Big(\frac{1}{a-\pi+1} \times x^{a-\pi+1} - \frac{l}{a-\pi+1} \times \frac{1}{a-\pi+2} x^{a-\pi+2} + \frac{l}{a-\pi+1} \cdot \frac{l}{a-\pi+2} \cdot \frac{1}{a-\pi+3} x^{a-\pi+3} \ldots\ldots \frac{l}{a-\pi+1} \cdot \frac{l}{a-\pi+2} \cdot \frac{l}{a-\pi+3} \ldots \frac{1}{a} x^a\Big) e^x \pm \frac{l}{a-\pi+1} \cdot \frac{l}{a-\pi+2} \cdot \frac{l}{a-\pi+3} \ldots \frac{l}{a} \times P$: ſignum $\pm$ in hoc caſu etiam erit $+$, ſi π ſit par; ſin aliter $-$.

Ea, quæ prius tradita fuere de algebraicis fluxionibus, æque ad fluentiales applicari poſſunt.

P R O B. XXIII.

1. Sit fluxio $(a+bx^n)^m \times x^{pn+vn-1}\dot{x}$, invenire fluentem inter duos valores $\left(o \;\&\; -\frac{a}{b}\right)$ quantitatis x^n contentam.

In fluente ex. 3. prob. 22. inventâ pro x^n ſcribantur reſpective o & $-\frac{a}{b}$; & erit reſultantium differentia $\frac{p}{p+m+1} \cdot \frac{p+1}{p+m+2} \cdot \frac{p+2}{p+m+3} \ldots \frac{p+v-1}{p+m+v} \times \frac{a^v}{b^v} \times$ fluen. flux. $(a+bx^n)^m \times x^{pn-1}\dot{x}$ inter duos valores o & $-\frac{a}{b}$ quantitatis x^n contentam, quæ erit fluens quæſita.

2. Sit data fluxio $(a+bx^n)^m \times x^{pn-vn-1}\dot{x}$; & erit ejus fluens inter duos valores o & $-\frac{a}{b}$ quantitatis x^n contenta $= \frac{p+m}{p-1} \times \frac{p+m-1}{p-2}$

$\times \ldots \frac{p+m-v+1}{p-v} \times \frac{b^v}{a^v} \times$ fluen. flux. $(a+bx^n)^m \times x^{pn-1}\dot{x}$ (A) inter duos prædictos valores quantitatis x^n contentam. Hoc conſtat ab ex. 4. prob. 22.

3. Sit data fluxio $(a+bx^n)^{m+r} x^{pn-1}\dot{x}$; & ejus fluens inter duos valores o & $-\frac{a}{b}$ quantitatis x^n contenta, erit $= \frac{m+1}{p+m+1} \times \frac{m+2}{p+m+2} \times \frac{m+3}{p+m+3} \ldots \frac{m+r}{p+m+r} \times a^r \times A$ fluent. prædictam.

4. Sit data fluxio $(a+bx^n)^{m-r} \times x^{pn-1}\dot{x}$; & ejus fluens inter duos valores o & $-\frac{a}{b}$ quantitatis x contenta, erit $= \frac{p+m}{m} \times \frac{p+m-1}{m-1} \times \frac{p+m-2}{m-2} \ldots \frac{p+m-r+1}{m-r+1} \times \frac{A}{a^r}$.

5. Sit data fluxio $(a+bx^n)^{m+r} \times x^{pn+vn-1}\dot{x}$; & ejus fluens inter duos valores o & $-\frac{a}{b}$ quantitatis x^n contenta, erit $= \frac{(m+1)\times(m+2)}{(p+m+1).(p+m+2)}$

$$\frac{\times(m+3)\ldots(m+r)\times p\times(p+1)\times(p+2)\ldots(p+v-1)}{.(p+m+3)\ldots(p+m+v).(p+m+v+1)\ldots(p+v+m+r)} \times \frac{a^{v+r}}{b^v}$$

$\times$ fluent. flux. $(a+bx^n)^m \times x^{pn-1}\dot{x}$ (A).

6. Fluens fluxionis $(a+bx^n)^{m-r} x^{pn+vn-1}\dot{x}$, inter duos prædictos valores o & $-\frac{a}{b}$ quantitatis x^n contenta, erit $= \frac{m+p+v}{m} \times \frac{m+p+v-1}{m-1} \times \ldots \frac{m+p+v-r+1}{m-r+1} \times \frac{p}{p+m+1} \times \frac{p+1}{p+m+2} \times \frac{p+2}{p+m+3} \ldots \frac{p+v-1}{p+m+v} \times \frac{a^{v-r}}{(-b)^v} \times$ fluent. flux. $(a+bx^n)^m x^{pn-1}\dot{x}$ (A).

7. Fluens fluxionis $(a+bx^n)^{m+r} x^{pn-vn-1}\dot{x}$ inter valores o & $-\frac{a}{b}$ quantitatis x^n contenta, erit $= \frac{m+r+p-v+1}{p-v} \times \frac{m+r+p-v+2}{p-v+1}$

$\ldots \frac{m+r+p}{p-1} \times \frac{m+1}{m+p+1} \times \frac{m+2}{m+p+2} \ldots \frac{m+r}{m+p+r} \times \frac{(-b)^v}{a^{v-r}} \times$ fluentem prædictam A.

8. Fluens fluxionis $(a+bx^n)^{m-r} x^{pn-vn-1} \dot{x}$, inter valores o & $-\frac{b}{a}$ quantitatis x^n contenta, erit $= \frac{(p+m) \times (p+m-1) \times (p+m-2)}{m \times (m-1) \times (m-2) \ldots (m-r+1) \times} \frac{\ldots (p+m-r-v+1)}{(p-1) \times (p-2) \ldots (p-v)} \times \frac{-b^v}{a^{r+v}} \times$ fluentem A.

Hæc facile conſtant e prob. 22. & conſimilia de trinomialibus $(a+bx^n+cx^{2n})^m \times x^{pn-1}\dot{x}$, &c. fluxionibus facile detegi poſſunt. e. g. fluens fluxionis $(a+bx^n+cx^{2n})^{m \pm u} \times x^{pn \pm vn-1} \dot{x}$ erit $P \times (a+bx^n+cx^{2n})^{m \pm u'} \times x^{pn \pm v'n-1} - \mathcal{Q} \times \int . (a+bx^n+cx^{2n})^{m \pm \alpha} \times x^{pn \pm \beta n-1} \dot{x} - R \times \int . (a+bx^n+cx^{2n})^{m \pm \alpha'} \times x^{pn \pm \beta' n-1} \dot{x}$; ubi α, β; α' & β', u & v ſunt dati integri numeri; & fluentes duarum fluxionum $(a+bx^n+cx^{2n})^{m \pm \alpha} \times x^{pn \pm \beta n-1} \dot{x}$ & $(a+bx^n+cx^{2n})^{m \pm \alpha'} \times x^{pn \mp \beta' n-1} \dot{x}$ dantur, attamen nec exprimi in finitis terminis, nec terminis finitis a ſe invicem deduci poſſunt; & P, $\mathcal{Q}$ & R ſunt rationales functiones quantitatis x^n.

9. 1. Si plures ſint hujuſce generis $(a+bx^n)^{m \pm r} x^{(p \pm v)n-1}$ fluxiones, quarum aggregatum fluentium inter valores o & $-\frac{a}{b}$ quantitatis (x) contentarum requiritur: inveniatur per problema ſingula fluens, & exinde deduci poteſt earum ſumma.

2. Si vero plures prædictæ quantitates per quantitates irrationales denotentur; transformandæ ſunt irrationales quantitates in terminos ſecundum dimenſiones quantitatis x progredientes; vel in tales terminos, quorum fluentes per methodos prius traditas deduci poſſunt; deinde inveniantur fluentes ſingularum fluxionum reſultantium, & exinde aggregatum quæſitum.

Ex. 1. Invenire fluentem fluxionis $\frac{(a+bx^n)^m}{(h+kx^n)^\beta} x^{pn-1} \dot{x}$ inter duos valores o & $-\frac{a}{b}$ quantitatis x^n contentam, ubi A ſit fluens fluxionis $(a+bx^n)^m \times x^{pn-1}\dot{x}$ inter duos prædictos valores quantitatis x^n contenta.

Per binomiale theorema $(b+kx^n)^{-\beta} = b^{-\beta} \times (1 - \frac{\beta k}{b} x^n + \frac{\beta \times (\beta+1) \times k^2}{1.2b^2} x^{2n} - \frac{\beta \times (\beta+1) \times (\beta+2) \times k^3}{1.2.3b^3} x^{3n} +$ &c.); ducatur hæc quantitas in $(a+bx^n)^m x^{pn-1}\dot{x}$, & fluxionum reſultantium inter valores o & $-\frac{a}{b}$ quantitatis x inveniantur fluentes; & erunt reſpective $\frac{A}{b^\beta} \times (1 + \frac{p}{p+m+1} \times \frac{a}{b} \times \frac{\beta k}{b} + \frac{p \times (p+1)}{(p+m+1) \times (p+m+2)} \times \frac{a^2}{b^2} \times \frac{\beta \times (\beta+1)k^2}{2b^2} + \frac{p \times (p+1) \times (p+2)}{(p+m+1) \times (p+m+2) \times (p+m+3)} \times \frac{a^3}{b^3} \times \frac{\beta \times (\beta+1) \times (\beta+2) \times k^3}{1.2.3b^3} +$ &c.)

Ex. 2. Erit fluens fluxionis $\dot{x}x^{pn-1} \times (a+bx^n)^{m-1} \int. (b+kx^n)^\beta \times x^{qn-1}\dot{x} = \int. \frac{1}{b^m d^{qn}} \times \dot{x} \times x^{(p+q)n-1} \times (a+bx^n)^{m-1} \times \int. x^{qn-1} \times (b+kx^n)^{m+\beta}\dot{x}$ inter duos valores (o & $-\frac{a}{b} = d$) quantitatis x^n contenta, cum $p+q+m+\beta=o$.

Per binomiale theorema erit $\dot{y} = (b+kx^n)^\beta \times x^{qn-1}\dot{x} = \dot{x}x^{qn-1}(b^\beta + \beta b^{\beta-1}kx^n + \beta \,.\, \frac{\beta-1}{2} b^{\beta-2}k^2x^{2n} +$ &c.), cujus fluens erit $x^{qn}(\frac{b^\beta}{qn} + \frac{\beta b^{\beta-1}}{(q+1) \times n} kx^n + \beta \times \frac{(\beta-1)}{2} \times \frac{b^{\beta-2}}{(q+2)n} k^2x^{2n} +$ &c.); ducatur hæc fluens in $\dot{z} = \dot{x}x^{pn-1} \times (a+bx^n)^{m-1}$, reſultat fluxio $\dot{x} \times (a+bx^n)^{m-1} \times x^{(p+q)n-1}(\frac{b^\beta}{qn} + \frac{\beta b^{\beta-1}}{(q+1)n} kx^n + \beta \times \frac{\beta-1}{2} \times \frac{b^{\beta-2}}{(q+2)n} k^2x^{2n} +$

$k^2 x^{2n} + \&c.$); per methodos prius traditas fluens hujuſce fluxionis inter valores o & $-\frac{a}{b}$ quantitatis x^n contenta, invenitur $= \frac{h^\beta}{qn} \times G \times (1 + \frac{p+q}{p+q+m} \times \frac{q}{q+1} \times \frac{\beta k}{h} \times d^n + \frac{p+q}{p+q+m} \times \frac{p+q+1}{p+q+m+1} \times \frac{q}{q+2} \times \beta \times \frac{\beta-1}{2} \times \frac{k^2}{h^2} \times d^{2n} + \&c.)$, ubi $G =$ fluen. flux. $x^{(p+q)n-1}\dot{x} \times (a + bx^n)^{m-1}$ inter valores o & $-\frac{a}{b}$ quantitatis x contentæ. In hâc ſerie pro $p+q+m$ ſcribatur ejus valor $-\beta$, & pro $p+q$ ejus valor $-\beta-m$; & evadet $= \frac{h^\beta}{qn} \times G \times (1 + \frac{\beta+m}{1} \times \frac{q}{q+1} \times \frac{k}{h} d^n + (\beta+m) \times \frac{\beta+m-1}{2} \times \frac{q}{q+2} \times \frac{k^2}{h^2} d^{2n} + \&c.)$: per prædictum theorema erit $x^{qn-1} \times (h + kx^n)^{m+\beta} \times \dot{x} = h^{m+\beta} \times x^{qn-1}\dot{x} + (m+\beta)\, h^{m+\beta-1} x^{(q+1)n-1} k\dot{x} + (m+\beta) \times \frac{m+\beta-1}{2} \times h^{m+\beta-2} x^{(q+2)n-2} k^2 \dot{x} + \&c.$, cujus fluens erit (P) $\frac{h^{m+\beta}}{qn} x^{qn} + \frac{(\beta+m)}{q+1} \times \frac{h^{m+\beta-1}}{n} k x^{(q+1)n} + (\beta+m) \times \frac{\beta+m-1}{2n \times (q+2)} h^{\beta+m-2} \times k^2 \times x^{q+2n} + \&c.$; ducatur hæc quantitas in $\frac{qn}{h^{\beta+m} x^{qn}}$, & pro x^n ſcribatur d^n, & reſultabit ſeries prius inventa $\frac{qn}{h^{\beta+m} d^{qn}} \times P = 1 + \frac{\beta+m}{1} \times \frac{q}{q+1} \times \frac{k}{h} d^n + (\beta+m) \times \frac{\beta+m-1}{2} \times \frac{q}{q+2} \times \frac{k^2}{h^2} d^{2n} + \&c.$; ducatur hæc quantitas $\frac{qn}{h^{\beta+m} d^{qn}} \times P$ in $\frac{h^\beta}{qn} \times G$; & reſultat quantitas prius tradita $\frac{1}{h^m d^{qn}} \times P \times G = \frac{h^\beta}{qn} \times G \times (1 + \frac{\beta+m}{1} \times \frac{q}{q+1} \times \frac{k}{h} d^n + \&c.) =$ fluen. fluxionis $x^{pn-1}\dot{x} \times (a + bx^n)^{m-1} \times \int. (h + kx^n)^\beta x^{qn-1}\dot{x} = \int. y\dot{z}$. Q. E. D.

Cor. 1. Si vero $q+p+m+\beta$ haud nihilo ſit æqualis, ſed p augeatur vel diminuatur per quemcunque integrum numerum, ex hoc exemplo detegi poteſt fluens fluxionis $y\dot{z}$, inter duos valores o & $-\frac{a}{b}$ quantitatis x^n contenta; e. g. ſit $\dot{Q} = \dot{z} \times$ fluen. flux. $\dot{y}x^n$, ſed $\dot{y} = (b+kx^n)^\beta \times x^{qn-1}\dot{x}$, & exinde $x^n\dot{y}$ erit $= (b+kx^n)^\beta x^{(q+1)n-1}\dot{x}$; ergo fluens flux. $x^n\dot{y}$ erit $= \frac{(b+kx^n)^{\beta+1} \times x^{qn}}{nk \times (\beta+1+q)} -$ fluent. flux. $\frac{(b+kx^n)^\beta \times x^{qn-1}\dot{x} \times qb}{k \times (\beta+1+q)}$ $\left(\frac{qb\dot{y}}{k \times (\beta+1+q)}\right)$, unde $\dot{Q} = \frac{(b+kx^n)^{\beta+1} x^{qn}}{nk \times (\beta+1+q)} \dot{z} - \frac{qby\dot{z}}{k \times (\beta+1+q)}$.

Cor. 2. Iiſdem literis eaſdem quantitates denotantibus, ſint Y, Z & V reſpective fluentes fluxionum $\dot{y}$, $\dot{z}$ & $z\dot{y}$ cum $b+kx^n = o$, i. e. inter valores o & $-\frac{b}{k}$ (D) quantitatis x^n contentæ; ſint etiam γ & π ſimultanea incrementa fluentium fluxionum $x^{(p+q)n-1}\dot{x} \times (b+kx^n)^\beta \dot{x}$, & $\dot{x}x^{pn-1} \times (a+bx^n)^{m+\beta}$, & $p+q+m+\beta = o$; tum ex hoc exemplo conſtat $V = \frac{\gamma\pi}{a^{\beta+1}D^{pn}}$, & conſequenter fluens fluxionis $y\dot{z} = YZ - V$ $\left(\frac{\gamma\pi}{a^{\beta+1}D^{pn}}\right)$.

Cor. 3. Sit $\dot{z} = -\dot{x}x^{n-r-2} \times (a+bx^n)^{m-1}$, & $\dot{y} = -\dot{x}x^r \times (b+kx^n)^\beta$; pro x in hiſce duabus fluxionibus ſcribatur $\frac{1}{v}$, & reſultant fluxiones $\dot{z} = \frac{\dot{v}}{v^2} \times \frac{1}{v^{mn-r-2}} \times (av^n+b)^{m-1} = (av^n+b)^{m-1} \times v^{r-mn}\dot{v}$, & $\dot{y} = \frac{\dot{v}}{v^2} \times \frac{1}{v^{r+\beta n}} \times (bv^n+k)^\beta = (bv^n+k)^\beta \times v^{-r-n\beta-2}\dot{v}$; unde, ſi $\frac{r-mn+1}{n} - \frac{r+\beta n+1}{n} + \beta + m = o$, fluens fluxionis $y\dot{z}$ inter duos valores o & $-\frac{b}{a}$ (d) quantitatis v^n contenta erit per exemplum $\frac{GP}{k^m d^{-(r+\beta n+1)}}$, ubi literæ P & G reſpective denotant fluentes (inter duos prædictos valores

lores a & $-\frac{b}{a}$ quantitatis v^n contentas) fluxionum $\dot{v}v^{-r-\beta n-2} \times (bv^n + k)^{\beta+m}$ & $\dot{v} \times v^{-(\beta+m)n-1} \times (av^n + b)^{m-1}$; cum autem fluens fluxionis $v\dot{z}$ inter duos valores $\left(o \;\&\; -\frac{b}{a}\right)$ quantitatis v^n contineatur, fluens fluxionis $y\dot{z}$ inter duos valores $\left(\frac{1}{o} \;\&\; -\frac{a}{b}\right)$ quantitatis x^n continebitur, quoniam $x = \frac{1}{v}$; ergo fluens fluxionis $y\dot{z}$ inter $-\frac{a}{b}$ & infinitum erit $\frac{GP}{k^m d^{-(r+\beta n+1)}}$.

Et ſic e præcedente coroll. invenietur fluens fluxionis $y\dot{z}$ inter duos valores $\left(-\frac{b}{k} (D) \;\&\; \text{infinitum}\right)$ quantitatis x^n contenta $= YZ - \gamma\pi \times \frac{D^{r+1-nm}}{b^{\beta+1}}$, ubi literæ Y, Z, γ, π reſpective denotant ſimultaneas fluentes fluxionum $-\dot{x}x^r(kx^n+b)^\beta$, $-\dot{x}x^{n-r-2} \times (bx^n+a)^{m-1}$, $\dot{v}v^{-(m+\beta)n-1} \times (bv^n + k)^\beta$ & $\dot{v}v^{r-mn} \times (b + av^n)^{m+\beta}$.

Ex iiſdem principiis conſimilia proferre liceat de trinomialibus, &c. quantitatibus; etiamque ex aliis transformationibus; ſed de his ſatis.

P R O B. XXIV.

In inveſtigandis fluxionum fluentibus nonnunquam occurrunt caſus, in quibus numerator & denominator datæ fractionis $\left(\frac{P}{Q}\right)$ *nihilo evadunt reſpective æquales; invenire ejus valorem.*

1. Reducatur data fractio ad minimos terminos; i. e. dividatur & numerator & denominator per maximum eorum communem diviſorem, & ſit quotiens reſultans $\frac{p}{q}$; tum erit $\frac{p}{q}$ valor fractionis quæſitæ: ſi numerator & denominator p & q ſimul nihilo evadant æquales,

tum

tum $\frac{\dot{p}}{\dot{q}}$ erit valor fractionis quæſitæ: ſi $\dot{p}$ & $\dot{q}$ etiam nihilo evadant æquales, tum erit prædictus valor $= \frac{\ddot{p}}{\ddot{q}}$: ſi vero $\ddot{p}$ & $\ddot{q}$ nihilo fiant æquales, tum prædictus valor $= \frac{\dddot{p}}{\dddot{q}}$; & ſic deinceps: quodſi nihilo ſemper evadant æquales & numeratores & denominatores ex hâc methodo inventi; tum reducenda eſt data fractio in alteram, ita ut omnes æquales diviſores in unam ſummam colligantur, eodem modo, qui prius docetur in prob. 4. & exinde per hanc methodum inveniri poteſt valor fractionis $\frac{P}{Q}$ quæſitus.

Subſequentia in nonnullis caſibus uſui inſervire poterunt.

2. Sint P & Q algebraicæ quantitates; & nihilo evadant æquales, cum $x = a$; etiamque prædictæ quantitates P & Q habeant diviſores $(x - a)^m$ & $(x - a)^n$, ubi literæ m & n denotant maximas poteſtates quantitatis $x - a$, per quas dividi poſſunt quantitates P & Q reſpective; tum, ſi m major ſit quam n, erit fractio $\frac{P}{Q} = 0$; ſi m minor ſit quam n, erit $\frac{P}{Q}$ infinita quantitas: ſi $m = n$, erit $\frac{P}{Q}$ finita quantitas.

3. Sit fractio $\frac{Ax^b + Bx^{b'} + Cx^{b''} + \&c.}{A'x^k + B'x^{k'} + C'x^{k''} + \&c.} = \frac{P}{Q}$ quæcunque rationalis functio quantitatis x, quæ nihilo evadat æqualis, cum $x = a$; ſit $(x - a)^m$ communis diviſor quantitatum P & Q, etiamque maxima poteſtas quantitatis $x - a$, per quam dividi poteſt vel P vel Q: ſit $\frac{P}{Q} = \frac{(x - a)^m}{(x - a)^m} \times \frac{\alpha}{\beta}$; tum erit $\frac{P}{Q} = \frac{\alpha}{\beta}$.

Cor. 1. Per hanc methodum ſemper detegi poteſt valor fractionis $\frac{P}{Q}$;

cum P & Q ſint rationales functiones quantitatis x; idem etiam perfici poteſt per methodum prius traditam inveniendi fluxionem (m ordinis) quantitatum P & Q; i. e. $\frac{\overset{m}{\dot{P}}}{\overset{m}{\dot{Q}}}$; evadet enim $\frac{\overset{m}{\dot{P}}}{\overset{m}{\dot{Q}}} = \frac{(x-a)^{m-m}\times\alpha}{(x-a)^{m-m}\times\beta}$ $=\frac{\alpha}{\beta}$; & ſimiliter erit $\frac{\overset{r}{\dot{P}}}{\overset{r}{\dot{Q}}} = \frac{(x-a)^{m-r}}{(x-a)^{m-r}}\times\frac{\alpha}{\beta} = \frac{\alpha}{\beta}$, ſi modo r minor ſit quam m; & $x = a$.

Cor. 2. Conſtat methodum prius traditam inveniendi $\frac{\overset{m}{\dot{P}}}{\overset{m}{\dot{Q}}}$ detegere valorem fractionis $\frac{P}{Q}$, cum m ſit integer numerus; ſin aliter non.

4. Sit $\frac{P}{Q} = \frac{p}{q}\times\frac{\alpha}{\beta}$; ubi $P = 0$ & $Q = 0$, & α & β ſint finitæ quantitates; tum inveniatur valor fractionis $\frac{p}{q} = \gamma$, & erit $\frac{P}{Q} = \frac{p}{q}\times\frac{\alpha}{\beta}$ $= \gamma\times\frac{\alpha}{\beta}$.

5. Si $(x-a)^m$ ſit diviſor quantitatis A; tum $(x-a)^{m-1}$ erit diviſor quantitatis $\frac{\dot{A}}{\dot{x}}$; & ſic deinceps.

6. Hoc problema generaliter reſolvi poteſt algebraice ex ſubſequente propoſitione.

Pro x in datâ quantitate (P) ſcribatur $v + a$, & reducatur quantitas (P) in alteram (B), quæ eſt functio quantitatis (v); deinde reducatur quantitas (B) in ſeriem $Av^l + Bv^{l'} +$ &c. ſecundum dimenſiones quantitatis v aſcendentem; tum erunt (l), viz. dimenſiones quantitatis (v) in primo termino, numerus diviſorum, qui ſunt $(x-a)$, in quantitate P contentorum.

Et ſimiliter pro x ſcribatur $v + a$ in quantitate Q, & reducatur ea in ſeriem ſecundum dimenſiones quantitatis v aſcendentem $A'v^{\lambda}+B'v^{\lambda'}+\&c.$: tum primo, ſi l major ſit quam λ, erit $\frac{P}{\mathcal{Q}}=0$; 2$^{\text{do}}$. ſi $l=\lambda$, tum erit $\frac{P}{\mathcal{Q}}=\frac{A}{A'}$; & denique, ſi l minor ſit quam λ, erit $\frac{P}{\mathcal{Q}}$ infinita quantitas.

Ex. Sit fractio $\frac{x-1-\sqrt[3]{(3x^2-3x-9)}}{5x-\sqrt[2]{(25x^2+7x-28)}}$, cujus numerator & denominator nihilo evadunt reſpective æquales, cum $x=4$; invenire valorem fractionis, cum $x=4$.

Pro x in numeratore & denominatore datæ fractionis ſcribatur reſpective $v+4$; & reſultant $v+3-\sqrt[3]{(3v^2+21v+27)}=P$, & $5v+20-\sqrt{(25v^2+207v+400)}=\mathcal{Q}$; reducantur irrationales quantitates in ſeriem ſecundum dimenſiones quantitatis x aſcendentem, i.e. erunt $\alpha=(27+21v+3v^2)^{\frac{1}{3}}=3+\frac{1}{3}\times\frac{21v}{9}\left(\frac{7}{9}v\right)+\&c.$, & $\beta=(400+207v+25v^2)^{\frac{1}{2}}=20+\frac{207}{40}v+\&c.$; & conſequenter $P=v+3-\alpha=v+3-3-\frac{7}{9}v-\&c.=\frac{2}{9}v-\&c.$, & $\mathcal{Q}=5v+20-20-\frac{207}{40}v-\&c.=\frac{-7}{40}v-\&c.$; unde $\frac{\frac{2}{9}v}{-\frac{7}{40}v}=\frac{-80}{63}$.

7. Sint A & B quæcunque functiones quantitatis x, tum $\frac{A-A}{B-B}$ non cuicunque quantitati dici poteſt æqualis.

Eadem principia etiam ad exponentiales, integrales, &c. quantitates applicari poſſunt; nam reduci poſſunt omnes hæ quantitates in ſeries ſecundum dimenſiones quantitatis v progredientes.

SCHO-

SCHOLIUM.

Datis fluentibus n datarum fluxionum; invenire, utrum fluens cujuſcunque datæ fluxionis, earum & finitorum terminorum ope inveſtigari poteſt, necne.

1^mo^. Quærendæ ſunt ſubſtitutiones, quæ neceſſario continent ſingulas fluxiones, quarum fluentes dantur; quod plerumque e doctrinâ irrationalium quantitatum prius datâ, facile innoteſcit; etiamque innoteſcit ſubſtitutio, quæ neceſſario continet finitos terminos; quibus quantitatibus pro ſuis valoribus ſubſtitutis, & quantitatum reſultantium aggregato datæ fluxioni æquale eſſe ſuppoſito, ſi modo poſſibile ſit; exinde deduci poteſt problematis reſolutio.

2^do^. Sæpe vero in detegendâ datæ fluxionis fluente, ſi transformetur data fluxio in alteram, cujus variabilis quantitas quandam habet relationem ad variabilem datæ fluxionis quantitatem; reſultabit fluxio, cujus fluens per notas regulas inveſtigari poteſt.

3^tio^. Si transformentur etiam irrationales quantitates e denominatore fluxionis in numeratorem, & vice verſâ e numeratore in denominatorem; ſæpe reſultabunt fluxiones, quarum fluentes innoteſcunt.

E.g. Literæ $\pi, \varrho, \sigma, \tau,$ &c. integros denotent numeros vel affirmativos vel negativos, & $a, b, c, d, e,$ &c. h; $p, q, r, s, t,$ &c. $A, B, C, D, E,$ &c. $P, Q, R, S, T,$ &c. $\alpha, \beta, \gamma, \delta, \eta,$ &c. $\varkappa, \lambda, \mu, \nu,$ &c. quaſcunque invariabiles quantitates; & $x, y, z, v,$ &c. variabiles quantitates; & dentur fluentes omnium fluxionum hujuſce formulæ $\frac{x^{\pi}\dot{x}}{\sqrt{(a+bx+cx^2+dx^3+ex^4)}}$, ubi π vel $= o$ vel ſit affirmativus vel negativus integer numerus; pro x ſcribatur $z-\alpha$, & conſtat inveniri poſſe fluentes omnium fluxionum formulæ $\frac{(z-\alpha)^{\pi}\dot{z}}{\sqrt{(p+qz+rz^2+sz^3+tz^4)}}$; ſit $\pi = -1$ & fit fluxio formulæ $\frac{\dot{z}}{(z-\alpha)\times\sqrt{(p+qz+rz^2+sz^3+tz^4)}}$; etiamque e pluribus fluxionibus prædictæ formulæ in unam ſummam collectis conſtabunt fluentes omnium fluxionum formulæ (*A*) $\frac{\dot{z}}{(bz-\alpha)\times\sqrt{}(p+qz+rz^2+sz^3+tz^4)}$; & exinde deduci poſſunt fluentes omnium fluxionum formulæ $\frac{x^{\pi}}{(x^n+px^{n-1}+qx^{n-2}+\&c.)\times}$ $\frac{\dot{x}}{\sqrt{}(a+bx+cx^2+dx^3+ex^4)}$ (*B*): ſit enim $x^n+px^{n-1}+\&c.=(x-\alpha)\times(x-\beta)\times(x-\gamma)\times\&c.$ per lem. 1. dividi poteſt hæc fluxio in plures fluxiones formulæ *A*; ſit $x^n+px^{n-1}+qx^{n-2}+\&c.=(x-\alpha)^{\pi}\times(x-\beta)^{\rho}\times(x-\gamma)^{\sigma}\times\&c.$ & per idem lem. reduci poteſt hæc fluxio in plures prædictæ formulæ; pro x in fluxione (*B*) ſcribatur z^m, & pro $\dot{x}$, $mz^{m-1}\dot{z}$; & reſultat fluxio formulæ $\frac{z^{(\pi+1)m-1}}{(z^{nm}+pz^{(n-1)m}+\&c.)\sqrt{}(a+}$ $\frac{\dot{z}}{bz^m+cz^{2m}+dz^{3m}+ez^{4m})}$, cujus fluens per datas formulas innoteſcet.

2. In fluxione $\frac{(Px^{\pi m}+Qx^{(\pi-1)m}+Rx^{(\pi-2)m}+\&c.)}{(x^{nm}+px^{nm-m}+qx^{nm-2m}+\&c.)\times\sqrt{}(a+bx^m+cx^{2m}+}$ $\frac{x^{m-1}\dot{x}}{dx^{3m}+ex^{4m})}$ (*C*), cujus fluens per prædictas fluentes acquiri poteſt, ſit $a+bx^m+cx^{2m}+dx^{3m}+ex^{4m}=(ex^{2m}-\alpha x^m+\beta)\times(x^{2m}-\gamma x^m+\delta)=(ex^m-\varepsilon)\times(x^{3m}-\zeta x^{2m}+\eta x^m-\theta)$; & in fluxione *C* pro $\sqrt{}(a+bx^m+cx^{2m}+dx^{3m}+ex^{4m})$ ſcribatur ejus valor, vel $\frac{a+bx^m+cx^{2m}+dx^{3m}+ex^{4m}}{\sqrt{}(a+bx^m+cx^{2m}+dx^{3m}+ex^{4m})}$, vel $\frac{(ex^{2m}-\alpha x^m+\beta)\times\sqrt{}(x^{2m}-\gamma x^m+\delta)}{\sqrt{}(ex^{2m}-\alpha x^m+\beta)}$, vel $\frac{(ex^m-\varepsilon)\times\sqrt{}(x^{3m}-\zeta x^{2m}+\eta x^m-\theta)}{\sqrt{}(ex^m-\varepsilon)}$, vel $\frac{(x^{3m}-\zeta x^{2m}+\eta x^m-\theta)\times}{\sqrt{}(x^{3m}-\zeta x^{2m}+}$ $\frac{\sqrt{}(ex^m-\varepsilon)}{\eta x^m-\theta)}$ & reſultant fluxiones ſubſequentium formularum $\frac{(Px^{\pi m}+Qx^{(\pi-1)m}+Rx^{(\pi-2)m}+\&c.)}{(x^{nm}+px^{nm-m}+qx^{nm-2m}+\&c.)}\times x^{m-1}\dot{x}\times\sqrt{}(a+bx^m+cx^{2m}+$ $dx^{3m}+$

$dx^{3m}+ex^{4m})$; vel $\dfrac{(Px^{\pi m}+\mathcal{Q}x^{(\pi-1)m}+Rx^{(\pi-2)m}+\&c.)}{x^{nm}+px^{(n-1)m}+qx^{(n-2)m}+\&c.}\times(x^{2m}-\gamma x^{m}+\delta)\times\surd\left(\dfrac{ex^{2m}-\alpha x^{m}+\beta}{x^{2m}-\gamma x^{m}+\delta}\right)x^{m-1}\dot{x}$; vel $\dfrac{(Px^{\pi m}+\mathcal{Q}x^{(\pi-1)m}+Rx^{(\pi-2)m}+\&c.)}{(x^{nm}+px^{(n-1)m}+qx^{(n-2)m}+\&c.)}\times(x^{3m}-\daleth x^{2m}+nx^{m}-\theta)\times\surd\dfrac{(ex^{m}-\varepsilon)\times\dot{x}x^{m-1}}{x^{3m}-\daleth x^{2m}+nx^{m}-\theta}$; vel $\dfrac{(Px^{\pi m}+\mathcal{Q}x^{(\pi-1)m}+Rx^{(\pi-2)m}+\&c.)}{x^{nm}+px^{(n-1)m}+qx^{(n-2)m}+\&c.}\times(\varepsilon x^{m}-\varepsilon)\times\surd\left(\dfrac{x^{3m}-\daleth x^{2m}+nx^{m}-\theta}{ex^{m}-\varepsilon}\right)\times x^{m-1}\dot{x}$; unde omnes fluentes harum formularum per prædictas fluentes inveniri poſſunt.

3. Sit data fluxio hujus formulæ $\dfrac{(Px^{\pi m}+\mathcal{Q}x^{(\pi-1)m}+\&c.)\times x^{m-1}\dot{x}}{x^{nm}+px^{(n-1)m}+qx^{(n-2)m}+\&c.}\times\sqrt[4]{(ax^{2m}+bx^{m}+c)}$; ſcribatur $\surd(ax^{2m}+bx^{m}+c)=vx^{m}+\surd(c)$ (d); & exinde $ax^{2m}+bx^{m}+c=v^{2}x^{2m}+2dvx^{m}+c$, & conſequenter $x^{m}=\dfrac{2dv-b}{a-v^{2}}$, & $\sqrt[4]{(ax^{2m}+bx^{m}+c)}=\surd(vx^{m}+\surd(c))=\surd\left(\dfrac{2dv^{2}-bv}{a-v^{2}}+d\right)$; quibus valoribus pro x^{m} & $\sqrt[4]{(ax^{2m}+bx^{m}+c)}$ in datâ fluxione ſubſtitutis, & fluxione quantitatis $\dfrac{2dv-b}{a-v^{2}}$ pro $mx^{m-1}\dot{x}$; reſultat fluxio formulæ prius traditæ: ſi vero a & c ſint negativæ quantitates, & ſint duæ radices æquationis $aw^{2}+bw+c=0$ poſſibiles; viz. $\surd\left(\dfrac{b^{2}-4ac}{4a^{2}}\right)$ (λ) ſit poſſibilis quantitas, & μ quæcunque invariabilis quantitas inter duas radices prædictas $\dfrac{-b}{2a}+\lambda$ & $\dfrac{-b}{2a}-\lambda$ poſita; ſcribatur $z+\mu$ pro x in datâ fluxione, & $\dot{z}$ pro $\dot{x}$; & reſultat fluxio hujuſce formulæ $\Gamma\sqrt[4]{(az^{2m}+kz^{m}+l^{2})}\,z^{m-1}\dot{z}$, ubi Γ ſit rationalis functio quantitatis z^{m}, quæ fluxio reduci poteſt per præcedentem ſubſtitutionem: ſi vero c ſit negativa quantitas

& radices æquationis $aw^2+bw+c=0$ impoſſibiles; tum fluxio ſemper erit impoſſibilis quantitas, & conſequenter ejus fluens impoſſibilis quantitas. Sit data fluxio $\Gamma\times\frac{x^{m-1}\dot{x}}{\sqrt[4]{(ax^{2m}+bx^m+c)}}$, ubi Γ in hâc fluxione & ſubſequentibus eſt rationalis functio quantitatis x^m; eadem ſubſtitutio hanc fluxionem in fluxionem formularum prius traditarum ſemper transformabit: ſit fluxio $\Gamma\times\sqrt[4]{\left(\frac{a+bx^m}{c+dx^m}\right)}\times x^{m-1}\dot{x}$, ſcribatur $\sqrt[4]{\left(\frac{a+bx^m}{c+dx^m}\right)}=v$; & transformari poteſt hæc fluxio in fluxionem, cujus variabilis quantitas ſit v, præcedentis formulæ. Sint formulæ $\Gamma\times(a+bx^m)^{\frac{1}{4}}\times(c+dx^m)^{\frac{3}{4}}x^{m-1}\dot{x}$, $\Gamma\times(a+bx^m)^{\frac{3}{4}}\times(c+dx^m)^{\frac{3}{4}}x^{m-1}\dot{x}$, $\Gamma\times\frac{x^{m-1}\dot{x}}{(a+bx^m)^{\frac{3}{4}}\times(c+dx^m)^{\frac{1}{4}}}$, $\Gamma\times\frac{x^{m-1}\dot{x}}{(a+bx^m)^{\frac{3}{4}}\times(c+dx^m)^{\frac{3}{4}}}$, $\Gamma\times\frac{x^{m-1}\dot{x}\times(a+bx^m)^{\frac{3}{4}}}{(c+dx^m)^{\frac{3}{4}}}$, $\Gamma\times\frac{x^{m-1}\times\dot{x}\times(a+bx^m)^{\frac{1}{4}}}{(c+dx^m)^{\frac{3}{4}}}$, $\Gamma\times\frac{x^{m-1}\dot{x}\times(a+bx^m)^{\frac{3}{4}}}{(c+dx^m)^{\frac{1}{4}}}$. In his formulis pro $(a+bx^m)^{\frac{3}{4}}$ & $(c+dx^m)^{\frac{3}{4}}$ ſcribantur reſpective $\frac{a+bx^m}{(a+bx^m)^{\frac{1}{4}}}$ & $\frac{c+dx^m}{(c+dx^m)^{\frac{1}{4}}}$, & evadunt fluxiones formularum prius deductarum.

4. Sit data fluxio $\Gamma\times\sqrt[4]{(a+bx^m)}\times\sqrt[2]{(c+dx^m)}\times x^{m-1}\dot{x}$, ſcribatur $(a+bx^m)^{\frac{1}{4}}=v$, & exinde $\frac{v^4-a}{b}=x^m$, & $\sqrt[2]{(c+dx^m)}=\sqrt{\left(c+\frac{dv^4-da}{b}\right)}$ & transformari poteſt data fluxio in fluxionem $\Delta\dot{v}\times\sqrt[2]{\left(\frac{dv^4+bc-da}{b}\right)}$, ubi Δ denotat rationalem functionem literæ v: ſint $\Gamma\times\frac{(a+bx^m)^{\frac{1}{4}}}{(c+dx^m)^{\frac{1}{2}}}\times x^{m-1}\dot{x}$, vel $\Gamma\times\frac{(c+dx^m)^{\frac{1}{2}}}{(a+bx^m)^{\frac{1}{4}}}\times x^{m-1}\dot{x}$, vel $\Gamma\times\frac{x^{m-1}\dot{x}}{(a+bx^m)^{\frac{1}{4}}\times(c+dx^m)^{\frac{1}{2}}}$; & eodem modo pro $(a+bx^m)^{\frac{1}{4}}$ ſcribatur v, & transformentur hæ

fluxiones in fluxiones, quarum variabilis quantitas eſt v, & reſultant fluxiones formularum prius traditarum. Sint fluxiones $\Gamma \times (a+bx^m)^{\frac{3}{4}} \times (c+dx^m)^{\frac{1}{2}} x^{m-1}\dot{x}$, vel $\Gamma \times \frac{(a+bx^m)^{\frac{3}{4}}}{(c+dx^m)^{\frac{1}{2}}} \times x^{m-1}\dot{x}$, vel $\Gamma \times \frac{(c+dx^m)^{\frac{1}{2}}}{(a+bx^m)^{\frac{3}{4}}} \times x^{m-1}\dot{x}$, vel $\frac{\Gamma x^{m-1}\dot{x}}{(a+bx^m)^{\frac{3}{4}} \times (c+dx^m)^{\frac{1}{2}}}$; in his fluxionibus pro $(a+bx^m)^{\frac{3}{4}}$ ſcribatur $\frac{a+bx^m}{(a+bx^m)^{\frac{1}{4}}}$, & transformantur hæ fluxiones in fluxiones præcedentium formularum.

5. Sit data fluxio $\Gamma \times \sqrt[3]{(a+bx^m)} \times \sqrt[2]{(c+dx^m)} \times x^{m-1}\dot{x}$; ſcribatur $v = \sqrt[3]{(a+bx^m)}$, & reſultat $x^m = \frac{v^3-a}{b}$, & exinde $\sqrt{(c+dx^m)}$ $\sqrt[2]{\left(c+d \times \frac{v^3-a}{b}\right)}$, & data fluxio transformatur in fluxionem $\Delta \times \sqrt[4]{\left(\frac{dv^3}{b} + \frac{bc-da}{b}\right)} \times \dot{v}$, (ubi Δ erit rationalis functio literæ v), quæ eſt fluxio formulæ prius traditæ.

Sint fluxiones $\Gamma \times \frac{\sqrt[3]{(a+bx^m)}}{\sqrt{(c+dx^m)}} \times x^{m-1}\dot{x}$, & $\Gamma \times \frac{\sqrt{(c+dx^m)} \times x^{m-1}\dot{x}}{\sqrt[3]{(a+bx^m)}}$ vel $\Gamma \times \frac{x^{m-1}\dot{x}}{\sqrt[3]{(a+bx^m)} \times \sqrt{(c+dx^m)}}$, & per ſubſtitutionem prius traditam hæ fluxiones reduci poſſunt in fluxiones formularum præcedentium.

6. Sit fluxio $\frac{\Delta}{\vartheta} \times \sqrt{(ax^{2m}+b)} \times \sqrt{(cx^{2m}+d)} \times \sqrt{(ex^{2m}+f)} \times x^{m-1}\dot{x}$, ubi Δ ſit quæcunque rationalis functio quantitatis x^m ſine denominatore, & ϑ quæcunque rationalis functio ejus quadrati x^{2m}. Scribatur in datâ fluxione pro x^{2m}, v; & transformetur data fluxio in alteram, cujus variabilis quantitas eſt v; & reſultat fluxio $\phi \times \dot{v} \times \sqrt[2]{(av+b)}$

$\times \sqrt[2]{(cv+d)} \times \sqrt[2]{(ev+f)} + \chi\dot{v}\sqrt{(v)} \times \sqrt{(av+b)} \times \sqrt{(cv+d)} \times \sqrt{(ev+f)}$, ubi φ & χ reſpective denotant rationales functiones quantitatis v; reſultans fluxio habet formulam prius traditam: ſint fluxiones $\frac{\Delta}{\vartheta} \times \sqrt{(ax^{2m}+b)} \times \frac{\sqrt{(cx^{2m}+d)}}{\sqrt{(ex^{2m}+f)}} \times x^{m-1}\dot{x}$, vel $\frac{\Delta}{\vartheta} \times \frac{\sqrt{(ax^{2m}+b)} \times x^{m-1}\dot{x}}{\sqrt{(cx^{2m}+d)} \times \sqrt{(ex^{2m}+f)}}$, vel $\frac{\Delta}{\vartheta} \times \frac{x^{m-1}\dot{x}}{\sqrt{(ax^{2m}+b)} \times \sqrt{(cx^{2m}+b)} \times \sqrt{(ex^{2m}+f)}}$; & per eandem ſubſtitutionem viz. $v = x^{2m}$, reduci poteſt data fluxio in formulas prius traditas.

7. Sit fluxio $\frac{\Gamma \times x^{m-1}\dot{x}}{\Theta + \Delta\sqrt{(ax^{m}+b)} + \varphi\sqrt{(ex^{m}+f)} + \chi\sqrt{(gx^{m}+h)} + \Psi\sqrt{(kx^{m}+l)}}$, ubi Γ, Θ, Δ, φ, X, Ψ reſpective denotant rationales functiones quantitatis x^{m}; inveniatur per prob. 16. quantitas, quæ in denominatorem ducta rationale productum facit, ducatur ea in datam fluxionem, & reſultant fluxiones prædictarum formularum.

Sint fluxiones $\frac{\Gamma \times x^{m-1}\dot{x}}{\Theta + \Delta\sqrt{(ax^{m}+b)} + \varphi\sqrt{(ex^{m}+d)} + \chi\sqrt{(ex^{2m}+fx^{m}+g)}}$, $\frac{\Gamma x^{m-1}\dot{x}}{\Theta + \Delta\sqrt{(ax^{m}+b)} + \varphi\sqrt{(cx^{3m}+dx^{2m}+ex^{m}+f)}}$, $\frac{\Gamma x^{m-1}\dot{x}}{\Theta + \Delta\sqrt{(ax^{2m}+bx^{m}+c)} + \varphi\sqrt{(dx^{2m}+ex^{m}+f)}}$, $\frac{\Gamma x^{m-1}\dot{x} \times \sqrt{(ax^{m}+b)}}{\Theta + \varphi\sqrt{(cx^{m}+d)} + \chi\sqrt{(ex^{m}+f)} + \Psi\sqrt{(gx^{m}+k)}}$, $\frac{\Gamma x^{m-1}\dot{x}\sqrt{(a+bx^{m})}}{\Theta + \varphi\sqrt{(cx^{2m}+dx^{m}+e)} + \Psi\sqrt{(gx^{m}+k)}}$, $\frac{\Gamma x^{m-1}\dot{x}\sqrt{(a+bx^{m})}}{\Theta + \varphi\sqrt{(cx^{3m}+dx^{2m}+ex^{m}+f)}}$, $\frac{\Gamma \times x^{m-1}\dot{x}\sqrt{(ax^{2m}+bx^{m}+c)}}{\Theta + \Delta\sqrt{(dx^{m}+e)} + \varphi\sqrt{(gx^{m}+k)}}$, $\frac{\Gamma \times x^{m-1}\dot{x}\sqrt{(ax^{2m}+bx^{m}+c)}}{\Theta + \Delta\sqrt{(dx^{2m}+ex^{m}+f)}}$, $\frac{\Gamma x^{m-1}\dot{x}\sqrt{(ax^{3m}+bx^{2m}+cx^{m}+d)}}{\Theta + \Delta\sqrt{(ex^{m}+f)}}$, &c. vel $\frac{\Gamma x^{m-1}\dot{x}}{\Theta + \Delta^{3 \text{ vel } \frac{3}{2}}\sqrt{(ax^{2m}+b)} + \sqrt{(dx^{m}+e)}}$,

$$\frac{\Gamma x^{m-1}\dot{x}\times(ax^m+b)^{\frac{1}{3}\text{ vel }\frac{2}{3}}}{\Theta+\sqrt{(dx^m+e)}},\quad \frac{\Gamma x^{m-1}\dot{x}\sqrt[2]{(ax^m+b)}}{\Theta+\sqrt[3]{(dx^m+e)}};\quad \frac{\Gamma x^{m-1}\dot{x}\sqrt{(ax^m+b)}}{\Theta+\sqrt[3]{((dx^m+e)^2)}};$$

&c. vel $\frac{\Gamma\times x^{m-1}\dot{x}}{\Theta+\Delta\sqrt[4]{(ax^m+b)}+\varphi\sqrt[4]{(cx^m+d)}}$, $\frac{\Gamma x^{m-1}\dot{x}\sqrt[4]{(ax^m+b)}}{\Theta+\varphi\sqrt[4]{(cx^m+d)}}$,

$$\frac{\Gamma x^{m-1}\dot{x}}{\Theta+\Delta\sqrt[2]{(ax^m+b)}+\varphi\sqrt[4]{(cx^m+d)}},\quad \frac{\Gamma x^{m-1}\dot{x}\sqrt[2]{(ax^m+b)}}{\Theta+\varphi\sqrt[4]{(ax^m+d)}},$$

$\frac{\Gamma x^{m-1}\dot{x}\sqrt[4]{(ax^m+b)}}{\Theta+\varphi\sqrt[2]{(cx^m+d)}}$, &c. ducantur hæ fluxiones in irrationalem quantitatem deductam, ita ut denominator fiat rationalis quantitas; & reſultant fluxiones formularum prædictarum.

8. Sit data fluxio $\frac{\Gamma x^{m-1}\dot{x}}{\Theta+\varphi\sqrt[2]{(a+\sqrt[2]{(bx^m+c)})}+\mathrm{X}\sqrt{(d+\sqrt{(bx^m+c)})}+\Psi\sqrt{(e+\sqrt{(bx^m+c)})}+\Delta\sqrt{(f+\sqrt{(bx^m+c)})}}$, ducatur denominator in quantitatem acquiſitam, quæ creat denominatorem rationalem; & reſultat fluxio, quæ (ſcribendo $\sqrt{(bx^m+c)}=v$) erit formulæ prius traditæ. Et ſic quamplurimæ hujuſce generis inveniri poſſunt fluxiones, quæ transformari poſſunt in fluxiones formularum præcedentium. 1$^{\text{mo}}$. Aſſumatur fluxio præcedentis formulæ, in cujus denominatore nulli contineantur irrationales termini; deinde per prob. 16. inveniatur quantitas *(H)* quæ ducta in numeratorem *(N)* creat rationale productum *(M)*. In datâ fluxione pro N ſcribatur $\frac{M}{H}$; & invenitur fluxio, quæ facile reduci poteſt in alteram præcedentis formulæ: nonnullæ vero fluxiones in has formulas reduci poſſunt e ſubſtitutionibus in prob. 17, &c. prolatis.

In datâ formulâ $\frac{x^{\pi m-1}\dot{x}}{\sqrt{(ax^{4m}+bx^{3m}+cx^{2m}+dx^{n}+e)}}$ ſcribatur $v^2 =$

$ax^{4m}+bx^{3m}+cx^{2m}+dx^{m}+e$, & transformetur data fluxio in fluxionem, cujus variabilis quantitas eſt v; & invenitur nova formula, quæ reduci poteſt in datam. Et ſic deinceps. Si vero omnes radices æquationis $ax^{4m}+bx^{3m}+cx^{2m}+dx^{m}+e=0$ ſint impoſſibiles, & a ſit negativa quantitas; tum haud datur poſſibilis fluens.

Methodis haud diſſimilibus inveniri poſſunt quamplurimæ fluxiones, quæ reduci poſſunt in fluxiones formularum $\frac{\Gamma x^{m-1}\dot{x}}{\sqrt{(ax^{5m}+bx^{4m}+cx^{3m}+dx^{2m}+ex^{m}+f)}}$ & $\frac{\Gamma x^{m-1}\dot{x}}{\sqrt{(ax^{6m}+bx^{5m}+cx^{4m}+dx^{3m}+ex^{2m}+fx^{m}+g)}}$, &c.

CAP.

C A P. III.

De Fluxionalibus æquationibus.

PROB. XXV.

DATA algebraicâ æquatione relationem inter abſciſſam (x) *& ejus ordinatas* (y) *exprimente, & datâ fluxionali quantitate quæ eſt algebraica functio abſciſſæ* (x) *& ordinatæ* (y) *& earum fluxionum; invenire ſummam ſingulorum hujus functionis valorum; etiamque æquationem, cujus radix eſt data fluxionalis quantitas.*

E datâ æquatione inveniantur prima fluxio ordinatæ *(y)*, & ſic ſecunda, tertia, &c. fluxiones ordinatæ; quibus valoribus in datâ fluxionali quantitate ſubſtitutis, reſultat algebraica quantitas, cujus ſumma ſingulorum valorum erui poteſt e primo capite medit. algebraic. Et ſic e prædicto capite inveniri poſſunt aggregata rectangulorum e quibuſque duobus valoribus datæ functionis, contentorum e quibuſque tribus, quatuor, &c. valoribus datæ functionis, i. e. inveniri poſſunt diverſæ quæſitæ æquationis coefficientes, & conſequenter æquatio ipſa.

Ex. 1. Datâ æquatione $y^2 - Ay + B = o$, in quâ A & B ſunt quæcunquæ abſciſſæ (x) functiones; invenire æquationem, cujus radices ſunt $\frac{\dot{y}}{\dot{x}}$. Sint duæ ordinatæ *(y)* radices reſpective α & β; & ſumma quæſitæ æquationis radicum erit $\frac{\dot{\alpha} + \dot{\beta}}{\dot{x}} = \frac{\dot{A}}{\dot{x}}$, quoniam $A = \alpha + \beta$: & ob $2y\dot{y} - A\dot{y} - \dot{A}y + \dot{B} = o$, & exinde $\dot{y} = \frac{\dot{A}y - \dot{B}}{2y - A}$, &

conſequenter duo valores fluxionis $\dot{y}$ e ſcribendo α & β reſpective pro y, & $\dot{\alpha}$ & $\dot{\beta}$ reſpective pro $\dot{y}$, evadent $\frac{\dot{A}\alpha-\dot{B}}{2\alpha-A}$ & $\frac{\dot{A}\beta-\dot{B}}{2\beta-A}$; unde rectangulum ſub binis valoribus quæſitæ æquationis radicum erit $\frac{\dot{\alpha}\dot{\beta}}{\dot{x}^2}=\frac{\dot{A}\alpha-\dot{B}}{2\alpha-A}\times\frac{\dot{A}\beta-\dot{B}}{(2\beta-A)\dot{x}^2}$, cujus denominator eſt $(4\alpha\beta-2A(\alpha+\beta)+A^2)\dot{x}^2=(4B-A^2)\dot{x}^2$; numerator vero $\dot{A}^2\alpha\beta-\dot{A}\dot{B}(\alpha+\beta)+\dot{B}^2=B\dot{A}^2-A\dot{A}\dot{B}+\dot{B}^2$: & conſequenter, ſi modo v ejus radicem denotet, æquatio quæſita erit $v^2-\frac{\dot{A}}{\dot{x}}v+\frac{B\dot{A}^2-A\dot{A}\dot{B}+\dot{B}^2}{(4B-A^2)\dot{x}^2}$.

Ex. 2. Datâ æquatione $y^3+By-C=o$, in quâ B & C ſunt quæcunque functiones abſciſſæ x; invenire æquationem, cujus radices ſunt $\frac{\dot{y}}{\dot{x}}$.

Sint tres radices ordinatæ (α, β, γ); tum ſumma earum fluxionum per $\dot{x}$ diviſarum erit $\frac{\dot{\alpha}+\dot{\beta}+\dot{\gamma}}{\dot{x}}=o$, quoniam $\alpha+\beta+\gamma=o$: & $\frac{\dot{\alpha}\dot{\beta}+\dot{\alpha}\dot{\gamma}+\dot{\beta}\dot{\gamma}}{\dot{x}^2}=\frac{\dot{C}-\dot{B}\alpha}{(3\alpha^2+B)\dot{x}}\times\frac{\dot{C}-\dot{B}\beta}{(3\beta^2+B)\dot{x}}+\frac{\dot{C}-\dot{B}\alpha}{(3\alpha^2+B)\dot{x}}\times\frac{\dot{C}-\dot{B}\gamma}{(3\gamma^2+B)\dot{x}}+\frac{\dot{C}-\dot{B}\beta}{(3\beta^2+B)\dot{x}}\times\frac{\dot{C}-\dot{B}\gamma}{(3\gamma^2+B)\dot{x}}$; reducantur hæ fractiones in communem denominatorem, & reſultat denominator $(3\alpha^2+B)(3\beta^2+B)(3\gamma^2+B)\dot{x}^3=(27C^2+4B^3)\dot{x}^3$, numerator vero $(3\gamma^2+B)(\dot{C}-\dot{B}\alpha)(\dot{C}-\dot{B}\beta)+(3\beta^2+B)(\dot{C}-\dot{B}\alpha)(\dot{C}-\dot{B}\gamma)+(3\alpha^2+B)(\dot{C}-\dot{B}\beta)(\dot{C}-\dot{B}\gamma)$; ergo ſumma rectangulorum e quibuſque binis in ſeſe ductis erit $\frac{\dot{B}^2B^2-3\dot{C}^2B+9C\dot{B}\dot{C}}{(27C^2+4B^3)\dot{x}^2}$; & contentum ſub tribus radicibus in ſeſe ductis $\left(\frac{\dot{\alpha}\dot{\beta}\dot{\gamma}}{\dot{x}^3}\right)$ erit $\frac{\dot{C}^3-\dot{B}^3C+\dot{B}^2\dot{C}B}{(27C^2+4B^3)\dot{x}^3}$; unde ſi modo radix quæſitæ æquationis ſit v, æquatio quæſita erit $v^3 + \frac{B^2\dot{B}^2 - 3\dot{C}^2B + 9C\dot{B}\dot{C}}{(27C^2 + 4B^3)\dot{x}^2}$. $v - \frac{\dot{C}^3 - \dot{B}^3C + \dot{B}^2\dot{C}B}{(27C^2 + 4B^3)\dot{x}^3} = 0$.

Ex. 3. Datâ eâdem æquatione $y^3 + By - C = 0$, ac in præcedenti exemplo; invenire ſummam ſingularum quantitatum hujuſce generis $\frac{y\dot{x}}{\dot{y}}$. Sint α, β, γ tres incognitæ quantitatis y radices; & tres quantitates, quarum ſumma quæritur, erunt reſpective $\frac{3\alpha^3 + B\alpha}{\dot{C} - \dot{B}\alpha}\dot{x}$, $\frac{3\beta^3 + B\beta}{\dot{C} - \dot{B}\beta}\dot{x}$, $\frac{3\gamma^3 + B\gamma}{\dot{C} - \dot{B}\gamma}\dot{x}$; quibus fractionibus ad communem denominatorem reductis, fit denominator $-\dot{B}^3 \times \alpha\beta\gamma + \dot{B}^2\dot{C}(\alpha\beta + \alpha\gamma + \beta\gamma) - \dot{B}\dot{C}^2 (\alpha + \beta + \gamma) + \dot{C}^3 = \dot{C}^3 - \dot{B}^3C + \dot{B}^2B\dot{C}$; & eodem modo invenitur numerator $(4B^2\dot{C}\dot{B} - 3BC\dot{B}^2 + 9C\dot{C}^2)\dot{x}$; unde ſumma quæſita $\frac{\alpha\dot{x}}{\dot{\alpha}} + \frac{\beta\dot{x}}{\dot{\beta}} + \frac{\gamma\dot{x}}{\dot{\gamma}} = \frac{4\dot{B}\dot{C}B^2 + 9\dot{C}^2C - 3\dot{B}^2CB}{\dot{C}^3 - \dot{B}^3C + \dot{B}^2\dot{C}B}\dot{x}$.

Eodem fere modo inveniri poſſunt æquationes, quarum radices ſunt quæcunque fluxionales functiones datæ algebraicæ vel datarum algebraicarum æquationum radicum.

PROB. XXVI.

Transformare duas æquationes in unam; ita ut variabilis quantitas y, *& ejus fluxiones exterminentur.*

1. Sit altera æquatio algebraica $y^n - py^{n-1} + qy^{n-2} - ry^{n-3} + \&c. = 0$; in quâ p, q, r, &c. reſpective denotant functiones variabilium quantitatum (z, x, &c.) altera vero quæcunque fluxionalis æquatio $Ay^\pi \dot{y}^\rho \ddot{y}^\sigma \dddot{y}^\tau + \&c. = 0$, in quâ continentur variabiles quantitates (x, y, z, &c.), &c.; eas in unam fluxionalem æquationem ita transformare, ut variabilis quantitas y & ejus fluxiones exterminentur.

Supponantur $\alpha, \beta, \gamma, \delta$, &c. diverſi valores variabilis quantitatis y in datâ algebraicâ æquatione; quibus ſubſtitutis in fluxionali, & eorum fluxionibus pro incognitâ quantitate y & ejus fluxionibus; reſultant quantitates $A\alpha^{\pi}\dot{\alpha}^{\rho}\ddot{\alpha}^{\sigma}$ &c. $+$ &c. $A\beta^{\pi}\dot{\beta}^{\rho}\ddot{\beta}^{\sigma}$ &c. $+$ &c. $A\gamma^{\pi}\dot{\gamma}^{\rho}\ddot{\gamma}^{\sigma}$ &c. $+$ &c. &c. ducantur hæ quantitates continuo in ſeſe, & contenti prob. 25. ope inveniatur aggregatum, quod nihilo fiat æquale, & reſultat æquatio quæſita.

2. 1. Duas fluxionales æquationes $A = 0$ & $B = 0$ in unam tranſformare, ita ut variabilis quantitas y & ejus fluxiones exterminentur.

Sit $\overset{\pi}{y}$ fluxio variabilis quantitatis y, cujus ordo π eſt maximus eorum, qui in datis æquationibus continentur; & ſint duæ æquationes $A\overset{\pi}{y}^{m} + B\overset{\pi}{y}^{m-1} + C\overset{\pi}{y}^{m-2} +$ &c. $= 0$, & $a\overset{\pi}{y}^{n} + b\overset{\pi}{y}^{n-1} +$ &c. $= 0$, ubi A, B, C, &c. a, b, c, &c. ſunt quæcunque functiones variabilium (x, z, &c.) & earum fluxionum, & quantitatum $y, \dot{y}, \ddot{y}$, &c. uſque ad $\overset{\pi-1}{y}$; per methodum in algebraicis æquationibus traditam ita reducantur hæ duæ æquationes in unam, ut exterminetur fluxio $\overset{\pi}{y}$; & reſultat æquatio fluxionem $\overset{\pi}{y}$ haud involvens; deinde inveniatur fluxio æquationis reſultantis, quæ contineat in ſe fluxionem $\overset{\pi}{y}$; hujus æquationis ope & alterius, quæ continet in ſe ſimplicem poteſtatem fluxionis $\overset{\pi}{y}$, exterminetur fluxio $\overset{\pi}{y}$; & reſultant duæ æquationes ſolummodo involventes fluxiones $\dot{y}, \ddot{y}$, &c. $\overset{\pi-1}{y}$, &c. in quibus haud continetur fluxio $\overset{\pi}{y}$: deinde eodem modo exterminentur fluxiones $(\overset{\pi-1}{y}, \overset{\pi-2}{y}$, &c.) & tandem exterminabuntur quantitas y & ejus fluxiones.

2. 2. Sint $\overset{\pi}{y}$ & $\overset{\rho}{y}$ fluxiones maximi ordinis quantitatis y prædictarum æquationum $A = 0$ & $B = 0$ reſpective, & ſit ρ minor quam π; inveniatur fluxio ordinis $\pi - \rho$ æquationis $B = 0$; & reſultet fluxio-

nalis æquatio $C = 0$ in quâ continetur prædicta fluxio $\overset{\pi}{y}$ ordinis π; reducantur duæ æquationes $A = 0$ & $C = 0$ in unam, ita ut exterminetur fluxio $\overset{\pi}{y}$; & reſultat æquatio $D = 0$ haud involvens $\overset{\pi}{y}$; cujus ſit fluxio $\overset{\pi'}{y}$ maxime ſuperioris ordinis: ſi π' major ſit quam ϱ, tum inveniatur fluxio ordinis $\pi' - \varrho$ æquationis $B = 0$, & reſultet æquatio $C' = 0$; deinde reducantur duæ æquationes $D = 0$ & $C' = 0$ in unam, ita ut exterminetur fluxio $\overset{\pi'}{y}$; & ſic deinceps: unde per hanc & præcedentem methodum tandem exterminabuntur variabilis quantitas y & ejus fluxiones.

Cor. Sint duæ fluxionales æquationes $A = 0$ & $B = 0$, in quibus fluxiones maximi ordinis quantitatis y ſunt reſpective $\overset{\pi}{y}$ & $\overset{\varrho}{y}$; deinde reducantur hæ duæ æquationes in unam, ita ut exterminentur quantitas y & ejus fluxiones, & reſultet æquatio $C = 0$; tum in æquatione $C = 0$ haud continentur fluxiones ſuperioris ordinis quam fluxio (π) ordinis quantitatum & fluxionum in æquatione $B = 0$ contentarum, & fluxio (ϱ) ordinis quantitatum & fluxionum in æquatione $A = 0$ contentarum.

2. Conſimilia etiam prædicari poſſunt de ordinibus fluxionum in $(n - 1)$ æquationibus reſultantibus ex reductione (n) æquationum, ita ut exterminentur variabilis quantitas y & ejus fluxiones.

3. Et ſic de n æquationibus in unam transformandis, ita ut $(n - 1)$ variabiles quantitates & earum fluxiones exterminentur.

4. Sint n fluxionales æquationes $a = 0$, $b = 0$, $c = 0$, &c.; $m, r, s,$ &c. ordinum reſpective; reducantur hæ fluxionales æquationes in unam, ita ut exterminentur $(n - 1)$ variabiles quantitates & earum fluxiones, & non neceſſario reſultabit fluxionalis æquatio ſuperioris quam $m + r + s +$ &c. ordinis.

Cor. Ex hâc methodo transformandi æquationes, ita ut variabiles quantitates exterminentur, irrepunt in transformatas æquationes valores, qui in datis æquationibus haud inveniuntur; eodem modo, quo in algebraicas æquationes per conſimilem methodum transformatas irrepunt valores, qui in datis haud inveniuntur, ut conſtat e prob. 31. medit. algebraic. e. g. Sint duæ fluxionales æquationes $(x^m+z^r)\dot{y}+\dot{z}=o$, & $(x^b+y^k)\times\dot{z}+\dot{x}=o$; ita transformare has duas fluxionales æquationes in unam, ut exterminetur z & $\dot{z}$: in unâ æquatione $\dot{z}$ ducitur in 1, in alterâ in x^b+y^k; ducatur reſpective ſingula æquatio in coefficientem fluxionis $\dot{z}$, quæ invenitur in alterâ; & reſultant $(x^m+z^r)\times(x^b+y^k)\dot{y}+(x^b+y^k)\times\dot{z}=o$, & $(x^b+y^k)\times\dot{z}+\dot{x}=o$, ſubducatur poſterior æquatio de priori, & reſiduum erit $(x^m+z^r)\times(x^b+y^k)\times\dot{y}-\dot{x}=o$, in quibus duabus æquationibus $(x^m+z^r)\times(x^b+y^k)\times\dot{y}-\dot{x}=o$, & $(x^b+y^k)\times\dot{z}+\dot{x}=o$ inveniuntur valores, qui in datis haud continentur, viz. in his reſultantibus æquationibus inveniuntur omnes valores, qui in datis continentur; etiamque valores, qui in æquationibus $x^b+y^k=o$ & $(x^b+y^k)\times\dot{z}+\dot{x}=o$ inveniuntur. Et ſic inveniatur fluxio æquationis $(x^m+z^r)\times(x^b+y^k)\times\dot{y}-\dot{x}=o$, & reſultat $(x^m+z^r)\times(x^b+y^k)\times\ddot{y}+(mx^{m-1}\dot{x}+rz^{r-1}\dot{z})\times(x^b+y^k)\dot{y}+(bx^{b-1}\dot{x}+ky^{k-1}\dot{y})(x^m+z^r)\dot{y}-\ddot{x}=o$; hæc vero & datæ æquationes $(x^b+y^k)\dot{z}+\dot{x}=o$ habent omnes valores, quos habent æquationes $(x^m+z^r)\times(x^b+y^k)\dot{y}-\dot{x}+A\dot{v}=o$ & $(x^b+y^k)\dot{z}+\dot{x}=o$, ubi litera A denotat invariabilem quantitatem ad libitum aſſumendam; & v quantitatem, quæ fluit uniformiter: e reſultante & datâ æquatione inveniatur æquatio, in quâ haud continetur fluxio $\dot{z}$; & e præcedente methodo inveniri poſſunt valores in duabus reſultantibus æquationibus, in quibus haud invenitur $\dot{z}$, qui in datis haud inveniuntur; & ſic repetitis operationibus tandem ita transformabuntur æquationes, ut exterminetur z; & eâdem methodo argumentandi continuo repetitâ, tandem deduci poſſunt valores, qui in reſultante æquatione inveniuntur, haud vero in datis æquationibus.

Et ſic de pluribus æquationibus in unam transformandis, ita ut exterminentur duæ vel plures incognitæ quantitates.

Hoc problema aliter ſæpe reſolvi poteſt e multiplicatione datarum æquationum in aſſumptas quantitates, quæ reddunt quantitates evaneſcentes, quas exigat problema. Sed de his nimis.

THEOR.

THEOR. XIV.

1. Datâ fluxionali æquatione $\alpha \times \beta \times \gamma \times \delta \times \&c. = o$; ubi $\alpha, \beta, \gamma, \delta,$ &c. quaſcunque fluxionales quantitates reſpective denotant; ea deprimi poteſt in diverſas æquationes $\alpha = o$, $\beta = o$, $\gamma = o$, $\delta = o$. Facile conſtat.

2. Datis duabus fluxionalibus æquationibus vel algebraicâ & fluxionali æquatione $\alpha \times \beta \times \gamma \times \delta \times \&c. = o$ & $\pi \times \varrho \times \sigma \times \&c. = o$, quæ recipiunt diviſores $\alpha, \beta, \gamma,$ &c. & $\pi, \varrho, \sigma,$ &c. hæ æquationes diſtingui poſſunt in ſubſequentes $\alpha = o$ & $\pi = o$; $\alpha = o$ & $\varrho = o$; $\alpha = o$ & $\sigma = o$; &c. $\beta = o$ & $\pi = o$; $\beta = o$ & $\varrho = o$; $\beta = o$ & $\sigma = o$; &c. $\gamma = o$ & $\pi = o$; $\gamma = o$ & $\varrho = o$; &c.

Et ſic de pluribus æquationibus plures diviſores habentibus.

PROB. XXVII.

Datâ fluxionali æquatione; invenire utrum ea reduci poteſt in duas vel plures alias, necne.

Inveniatur per meditat. algebr. annon duos vel plures diviſores recipiat data æquatio, & conſtat problema.

Ex. Sit data æquatio $(x\ddot{y} + \dot{x}^2 + \dot{y}\dot{x}) \times (y\dot{x} + \frac{1}{n}x\dot{y} + ax^m\dot{x}) = o$; ubi x fluit uniformiter. Hæc fluxionalis æquatio continet duos diviſores $x\ddot{y} + \dot{x}^2 + \dot{y}\dot{x}$, & $y\dot{x} + \frac{1}{n}x\dot{y} + ax^m\dot{x}$; unde data æquatio dividi poteſt in duas alias $x\ddot{y} + \dot{x}^2 + \dot{y}\dot{x} = o$ & $y\dot{x} + \frac{1}{n}x\dot{y} + ax^m\dot{x} = o$: fluens prioris fluxionalis æquationis erit $x\dot{y} + x\dot{x} = o$, quæ correcta fit $x\dot{y} + x\dot{x} + b\dot{x} = o$, ubi litera b quamlibet invariabilem quantitem denotat: fluens vero hujus æquationis erit $y + x + b \times \log. x = o$; quæ correcta fit $y + x + b \times \log. x + c = o$, ubi c denotat quamlibet invariabilem quantitatem: fluens vero poſterioris æquationis $(y\dot{x} + \frac{1}{n}x\dot{y} + ax^m\dot{x} = o)$ eſt $\frac{1}{n}yx^n + \frac{ax^{m+n}}{m+n} + A = o$, ubi A quamcunque invariabilem quantitatem denotat; unde data fluxionalis æquatio duas independentes recipit ſolutiones, viz. $y + x + b \times \log. x + c = o$, vel $\frac{1}{n} y x^n + \frac{a x^{m+n}}{m+n} + A = o$: hæc fluxionalis æquatio ad reſolutionem duorum diverſorum problematum reſpicit: in multis caſibus, ſi fluxionales æquationes diverſos habeant ordines, fluxionales æquationes ſuperioris ordinis ſolummodo pro reſolutione problematis habendæ ſunt.

Si fluxio $\dot{y}$ maximi ordinis in datâ fluxionali æquatione plures (n) habeat radices: inveniantur ſinguli (n) valores prædictæ fluxionis $\dot{y}$, & reſultant (n) diverſæ fluxionales æquationes, quarum fluentes requirantur.

Cor. 1. Hinc facile deduci poſſunt infinitæ fluxionales æquationes, quæ dividi poſſunt in alias: aſſumantur enim fluxionales æquationes, quibus in ſeſe ductis, exorietur fluxionalis æquatio, quæ facile reduci poteſt in aſſumptas.

Cor. 2. Datâ fluxionali æquatione involvente variabiles quantitates x & y & earum fluxiones; datâ etiam aliâ fluxionali æquatione habente duas variabiles quantitates z & v & earum fluxiones; invenire annon z eadem eſt functio vel algebraica vel fluxionalis quantitatis v, ac litera x eſt quantitatis y: pro z & v & earum fluxionibus ſcribantur x & y & earum fluxiones reſpective; deinde inveniatur, utrum hæ æquationes reſultantes communem habeant diviſorem, necne; & conſit coroll.

THEOR. XV.

Datis duabus fluxionalibus æquationibus relationem inter z, y, x & earum fluxiones exprimentibus; ex iis deduci poſſunt duæ aliæ infinitis modis, quarum variabiles quantitates eædem ſunt.

Ducantur datæ æquationes in quaſcunque quantitates, deinde addantur vel ſubducantur a ſe invicem, & reſultabunt æquationes quæſitæ.

PROB.

PROB. XXVIII.

Datam æquationem fluentem involventem in fluxionalem reducere, in quâ nulla continetur fluens.

1. Si fluens ducatur in variabilem quantitatem, dividatur æquatio per illam quantitatem, & inveniatur fluxio quantitatis reſultantis, & exterminabitur ea fluens.

1. 2. Si autem majores ſint dimenſiones (n) fluentialis quantitatis exterminandæ, vel altior ſit ordo (m) ejus fluentis; tum opus eſt toties (n) vel (m) ordinis inveniendi fluxionem datæ æquationis.

Ex. 1. Sit data æquatio $x\dot{x}+y\dot{y}+(x+a)\dot{x}$ flu. $\sqrt{(a^2+x^2)}\dot{x}=0$; requiratur hanc æquationem ita reducere, ut exterminetur fluens $\int.\sqrt{(x^2+a^2)}.\dot{x}$.

Hæc fluens ducitur in variabilem quantitatem $(x+a).\dot{x}$, dividatur data æquatio per hanc quantitatem, & reſultat æquatio $\frac{x\dot{x}+y\dot{y}}{(x+a).\dot{x}}$ + fl. $\sqrt{(a^2+x^2)}\dot{x}=0$. Inveniatur fluxio æquationis reſultantis, & exterminabitur fluens prædicta.

Ex. 2. Sit data æquatio $x\dot{x}+y\dot{y}+(x+a)\dot{x}\int.\overset{n}{\overline{\sqrt{(a^2+x^2)}.\dot{x}^n}}$, dividatur hæc æquatio per $(x+a)\times\dot{x}$, & reſultat æquatio $\frac{x\dot{x}+y\dot{y}}{(x+a).\dot{x}}+\int.\overset{n}{\overline{\sqrt{(x^2+x^2)}.\dot{x}^n}}$; hujus æquationis inveniatur fluxio (n) ordinis, & reſultat æquatio a fluentibus libera.

Ex. 3. Sit data æquatio $x^2+y^2+(c+dx)\times\int.\sqrt{(a^2+x^2)}\dot{x}+$ (fluen. $\dot{x}\sqrt{(a^2+x^2)})^2=0$; maximæ dimenſiones fluentis $\sqrt{(a^2+x^2)}\dot{x}$ inveniuntur duæ, viz. (fluen. $\dot{x}\sqrt{(a^2+x^2)})^2$, ſed hæ maximæ dimenſiones in nullam variabilem quantitatem ducuntur, ergo inveniatur fluxio datæ æquationis, quæ erit $2x\dot{x}+2y\dot{y}+(c+dx)\times\sqrt{(a^2+x^2)}\dot{x}+d\dot{x}$ fluen. $\sqrt{(a^2+x^2)}\dot{x}+2$ fluen. $\dot{x}\sqrt{(a^2+x^2)}\times(a^2+x^2)\dot{x}$ $=0$,

$= o$. In hâc æquatione $\int . \surd(a^2 + x^2)\dot{x}$ ducitur in variabilem quantitatem $(2a^2 + 2x^2 + d) \times \dot{x}$; dividatur inventa æquatio per hanc quantitatem, & reſultat $\frac{2x\dot{x} + 2y\dot{y} + (c + dx)\surd(a^2 + x^2)\dot{x}}{(2a^2 + 2x^2 + d)\dot{x}} + \int . \dot{x} \surd(a^2 + x^2) = o$; hujus æquationis inveniatur fluxio, & reſultat fluxionalis æquatio a fluentibus libera.

Cor. 1. Et ſic iteratis operationibus de pluribus fluentibus exterminandis.

Cor. 2. Facile conſtabit ordo, ad quem aſſurget æquatio a fluentialibus quantitatibus libera.

Cor. 3. Si quantitates irrationales utcunque ex his fluentibus compoſitæ in datâ æquatione contineantur, tum per meditationes algebraicas ab irrationalitate ſuâ liberari poteſt æquatio reſultans.

2. Hæc methodus autem transformandi æquationem, in quâ plures inveniuntur dimenſiones (n) fluentialis quantitatis exterminandæ, invenit fluxionalem æquationem ſuperioris ordinis, quam neceſſario exigat problema; ſubſequenti igitur methodo uti oportet; ſit fluens fluxionis $\mathcal{Q}$, quæ dicatur V, quantitas exterminanda; & ſit data fluentialis æquatio $AV^n + BV^{n-1} + CV^{n-2} +$ &c. $= P$, cujus fluxio eſt $\dot{A}V^n + nV^{n-1}A\dot{V} + (n-1)BV^{n-2}\dot{V} + V^{n-1}\dot{B} +$ &c. $= \dot{P}$; per methodum in medit. algeb. traditam ita reducantur hæ duæ æquationes $AV^n + BV^{n-1} + CV^{n-2} +$ &c. $= P$ & $\dot{A}V^n + nV^{n-1}A\dot{V} + (n-1)BV^{n-2}\dot{V} +$ &c. $= \dot{P}$ in unam, ut exterminetur fluentialis quantitas V, & conſtat problema; i. e. generaliter ſit æquatio $q = o$ reducantur duæ æquationes $q = o$ & $\dot{q} = o$, ita ut exterminetur fluens V, & confit prob. Eâdem operatione ſæpius repetitâ tolli poſſunt duæ vel plures fluentiales quantitates e datâ æquatione; etiamque fluentiales quantitates ſuperiorum ordinum.

Si vero irrationales functiones fluentis V contineantur in datâ æquatione $A = o$, tum e methodo in med. algeb. traditâ ita reducantur duæ æquationes $A = o$ & $\dot{A} = o$, ut ex eâ exterminetur incognita quantitas V.

Et ſic tolli poſſunt fluentiales quantitates e duabus vel pluribus æquationibus plures variabiles quantitates habentibus.

PROB. XXIX.

Datâ fluxionali æquatione duas incognitas quantitates x *&* y *& earum fluxiones involvente, in quâ* x *fluit uniformiter; eam ita transformare, ut ex eâ exterminentur incognita quantitas* x *& ejus fluxio.*

Inveniatur fluxio datæ fluxionalis æquationis, & æquatio data & reſultans ita reducantur, ut exterminetur fluxio $\dot{x}$; æquationis reſultantis, in quâ ſolummodo continetur quantitas x ſine ejus fluxionibus, & quantitas y & ejus fluxiones, inveniatur fluxio; & data & poſterior reſultans æquatio ita reducantur in unam, ut exterminetur fluxio $\dot{x}$; unde duæ inveniuntur æquationes, quæ ſolummodo continent algebraicas functiones quantitatis x & nullas ejus fluxiones; ita reducantur hæ duæ æquationes in unam, ut exterminetur incognita quantitas x; & reſultat æquatio, in quâ ſolummodo continentur quantitas y & ejus fluxiones.

Cor. Datâ fluxionali æquatione $m+2$ ordinis unam ſolummodo variabilem quantitatem (y) & ejus fluxiones involvente; eam tranſformare in alteram, terminos variabilis quantitatis y & alterius x, quæ fluit uniformiter; vel cujus fluxio r ordinis fluit uniformiter, ubi r minor ſit quam $m+2$; involventem.

Inveniatur fluentialis æquatio datæ fluxionalis; fluentiali æquationi adjiciatur quantitas vel fluxio generalis, cujus fluens fluit uniformiter, i. e. quæ erit functio fluxionalis quantitatis, cujus fluens fluit uniformiter; & perficitur corollarium.

PROB. XXX.

Datâ æquatione relationem inter duas quantitates x *&* y *& earum fluxiones exprimente; invenire æquationem relationem inter duas alias quantitates* z *&* v *exprimentem, ubi literæ* x *&* y *exprimi poſſunt in terminis quantitatum* z *&* v.

1. Sub-

1. Subſtituantur pro x & y, $\dot{x}$ & $\dot{y}$, &c. in datâ æquatione quæcunque algebraicæ functiones literarum z & v, & earum fluxionum; & reſultant æquationes quæſitæ.

Ex. 1. Datâ æquatione relationem inter prædictas quantitates x & y & earum fluxiones exprimente, pro literis x & y in datâ æquatione ſcribantur reſpective $az+bv+c$ & $pz+qv+r$; & pro fluxionibus $\dot{x}$ & $\dot{y}$ reſpective $a\dot{z}+b\dot{v}$ & $p\dot{z}+q\dot{v}$; & ſic deinceps; & reſultat æquatio relationem inter z & v exprimens, quæ algebraicè e data relatione inter x & y petenda eſt.

1. 2. Si relationes inter quantitates v & z, x & y exprimantur per algebraicas æquationes; tum e reductione plurium æquationum in unam, ita ut exterminentur omnes incognitæ quantitates præter z & v, petenda eſt relatio inter quantitates z & v.

PROB. XXXI.

Datis algebraicis & fluxionalibus æquationibus; invenire æquationem, cujus radices quamcunque habeant aſſignabilem relationem ad radices datarum æquationum.

Si relatio vel exprimatur per algebraicam vel fluxionalem quantitatem, quæ ſupponatur (w); vel per algebraicas vel fluxionales æquationes; tum perfici poteſt problema e reductione plurium (n) æquationum in pauciores ($n-m$) ita ut exterminentur (m) incognitæ quantitates & earum fluxiones.

2. Si vero verſetur relatio inter diverſos ejuſdem incognitæ quantitatis (v) valores & eorum fluxiones.

Inveniantur æquationes, quarum radices ſunt reſpective v, v^2, v^3 .. v^n (ſi modo n ſit maxima dimenſio ad quam aſcendat v in datâ relatione); etiamque æquationes, quarum radices ſunt $\dot{v}$, $\dot{v}^2$, $\dot{v}^3$, $\dot{v}^4$ .. $\dot{v}^m$ (ſi modo m ſit maxima dimenſio, ad quam aſcendat $\dot{v}$ in datâ relatione); & ſic de fluxionibus ſuperiorum ordinum: e reſultantibus æquationibus inveniri poſſunt (per medit. algebr.) æquationes, quarum radices algebraicam vel fluxionalem relationem quæſitam habent.

Cor. Conſtat æquationes hâc methodo derivatas multo plures habere radices, quam neceſſario exigat problema: ſæpe autem reduci poſſunt per methodum diviſores inveniendi.

3. Hoc problema ſæpe recipere poteſt reſolutionem e ſubſtitutione datæ vel datarum æquationum; etiamque ſingulos ex deducendo valores per infinitas ſeries, & exinde quæſitas radices inveſtigando.

PROB. XXXII.

Datâ æquatione algebraicâ relationem inter abſciſſam x & *ejus ordinatas* y *exprimente; invenire quoſdam caſus, in quibus literæ* x & y *per functionem tertiæ* z *exprimi poſſunt.*

1. 1. Sit data æquatio $Ay^n + (a + bx)y^{n-1} + (dx + ex^2)y^{n-2} +$ &c. $= o$, in quâ continentur ſolummodo termini (n & $n-1$) dimenſionum quantitatum (x & y): ſcribatur pro x rectangulum zy, & reſultat æquatio $(A + bz + ez^2 +$ &c.$)y^n + (a + dz +$ &c.$)y^{n-1} = o$, unde $y = -\frac{a + dz + \text{\&c.}}{A + bz + ez^2 + \text{\&c.}}$.

Cor. Area curvæ, cujus æquatio ſit prædictæ formulæ; vel ſolidum ex ejus rotatione circa axim ſuum tanquam baſim generatum; vel fluens fluxionis, quæ eſt rationalis functio quantitatum x & y in $\dot{x}$ vel $\dot{y}$, &c. ducta; exprimi poſſunt ope finitorum terminorum abſciſſæ & ordinatæ, & earum circularium arcuum & logarithmorum.

Et ſic progredi liceat ad æquationes, in quibus ſolummodo continentur termini (n, $n-1$ & $n-2$) dimenſionum; & ſic deinceps.

1. 2. Sit data æquatio $ay^n + by^{n-1}x + cy^{n-2}x^2 +$ &c. $+ \alpha y^m + \beta y^{m-1}x + \gamma y^{m-2}x^2 +$ &c. $= o$, in quâ ſolummodo continentur termini (n & m) dimenſionum. Scribatur $y = xz$, quâ quantitate pro ejus valore y in datâ æquatione ſubſtitutâ, reſultat $(az^n + bz^{n-1}$

$+ cz^{n-2} + \&c.)\,x^n + (\alpha z^m + \beta z^{m-1} + \&c.)\,x^m = 0$; unde $x = \left(-\dfrac{\alpha z^m + \beta z^{m-1} + \&c.}{az^n + bz^{n-1} + \&c.}\right)^{\frac{1}{n-m}}$, & $y = xz$.

Cor. Si in æquatione relationem inter variabiles x & y exprimente tres dimenſiones (n, $n-m$, $n-2m$) occurrant, quarum ſumma tantum ſuperat mediam, quantum hæc media infimam; tum ope quadraticæ æquationis reſolutionis variabiles y & x per novam z exprimi poterunt. Et ſic deinceps.

2. 1. Sit $\Delta(x,y) = \phi(x,y)$; ubi $\Delta(x,y)$ & $\phi(x,y)$ reſpective denotant homogeneas functiones literarum x & y: dimenſiones vero (n) functionis $\phi(x,y)$ ſuperent dimenſiones ($n-1$) functionis $\Delta(x,y)$ per unitatem: & ſi in prædictis functionibus duæ ſolummodo contineantur radicales, viz. $\sqrt{(\alpha x + \beta y)}$ & $\sqrt{(\varepsilon x + \delta y)}$: vel una ſolummodo $\sqrt{(\alpha x^2 + \beta xy + \gamma y^2)}$ vel $(\alpha x + \beta y)^{\frac{1}{m}}$; vel $\left(\dfrac{\alpha x + \beta y}{\varepsilon x + \delta x}\right)^{\frac{r}{m}}$ $= P$; vel $\sqrt[m]{(a + b\sqrt[r]{(c + \&c.\sqrt[s]{\left(\dfrac{e+fx}{h+kx}\right)})})} = \mathcal{Q}$, &c.; tum per prob. 15. fluens cujuſcunque rationalis functionis literarum x & y in $\dot{x}$ vel $\dot{y}$ ductæ reduci poteſt ad fluentem fluxionis, quæ eſt rationalis fractio variabilis quantitatis (u) in fluxionem $\dot{u}$ ductæ: ſcribatur enim zy pro x in datâ æquatione, & inveniatur valor quantitatis (y) in terminis quantitatis (z); in hoc valore ſolummodo continentur radicales quantitates $\sqrt{(\alpha z + \beta)}$ & $\sqrt{(\varepsilon z + \delta)}$; vel $\sqrt{(\alpha z^2 + \beta z + \gamma)}$, &c.; & nullæ aliæ; unde conſtat fluxiones $\dot{x}$ & $\dot{y}$ ſolummodo involvere eaſdem radicales quantitates; & exinde rationalem functionem literarum x vel y in $\dot{x}$ vel $\dot{y}$ ductam eaſdem ſolummodo involvere irrationales quantitates, & conſequenter eam per prob. 15. reduci poſſe ad rationalem functionem variabilis quantitatis (u) in ejus fluxionem ($\dot{u}$) ductam.

2. 2. Sit $\Delta(x,y) = \phi(x,y)$; ubi $\Delta(x,y)$ & $\phi(x,y)$ reſpective denotant homogeneas functiones quantitatum (x & y) abſque denominatore,

natore, quarum dimenſiones reſpective ſunt $n-1$ & n; functio autem $\Delta(x,y)$ nullas in ſe involvat radicales præter eas, quæ formulam $\surd(\alpha x+\beta y)$ habeant; quarum etiam haud duæ vel plures in ſeſe ducantur: functio vero $\phi(x,y)$ contineat nullas radicales quantitates; tum fluens fluxionis $y\dot{x}$ vel $x\dot{y}$ detegi poteſt ex fluentibus fluxionum, quæ conſtant ex rationalibus functionibus quantitatis u in $\dot{u}$, & quantitatis v in $\dot{v}$, &c.

Et ſic æquationes facile deduci poſſunt, quarum fluentes inveniri poſſunt ope fluentium in ſcholio præcedentis capitis traditarum.

Omnia hæc facile conſtant e ſubſtitutione rectanguli zy pro x in datâ æquatione.

3. 1. Si vero dimenſiones functionis $\phi(x,y)$ in datâ æquatione $\phi(x,y)=\Delta(x,y)$ majores ſint quam dimenſiones functionis $\Delta(x,y)$ per duas; tum fluens fluxionis $y\dot{x}$ in æquationibus, quæ eaſdem involvunt radicales quantitates, ac eæ, quæ in tribus præcedentibus caſibus traduntur, inveniri poteſt ope rationalis fractionis: ſcribatur enim zx pro y in datâ æquatione $\Delta(x,y)=\phi(x,y)$ & reſultat $Ax^2=B$; ubi literæ A & B denotant functiones quantitatis z, in quibus nullæ aliæ inveniuntur radicales quantitates præter eas, quæ radicalibus in datâ æquatione contentis correſpondent. e. g. ſit radicalis in datâ æquatione contenta $\surd(\alpha x+\beta y)$, & ejus correſpondens radicalis in reſultante æquatione erit $\surd(\alpha+\beta z)$; ſit etiam radicalis quantitas in datâ æquatione $\surd(\alpha x^2+\beta xy+\gamma y^2)$, & ejus correſpondens radicalis in reſultante æquatione erit $\surd(\alpha+\beta z+\gamma z^2)$, &c. ſed ex æquatione prius tradita $Ax^2=B$ ſequitur $x=\frac{B^{\frac{1}{2}}}{A^{\frac{1}{2}}}$, cujus fluxio $\dot{x}=\frac{1}{2}\frac{B^{-\frac{1}{2}}}{A^{-\frac{1}{2}}}\times\frac{A\dot{B}-B\dot{A}}{A^2}$; ſed per ſubſtitutionem prius traditam $y=zx=\frac{zB^{\frac{1}{2}}}{A^{\frac{1}{2}}}$, & conſequenter $y\dot{x}=\frac{1}{2}z\times\frac{A\dot{B}-B\dot{A}}{A^2}$, quæ continebit nullas radicales quantitates, præter eas, quæ correſpondent irrationalibus datâ æquatione contentis: vel magis generaliter, ſit V

homogenea & rationalis functio quantitatum x & y, cujus numerus dimenſionum ſit impar, tum fluxio $V\dot{x}$ vel $V\dot{y}$ etiam continebit nullas radicales quantitates præter eas, quæ correſpondent irrationalibus datâ æquatione contentis.

3. 2. In genere ſint $\phi(x,y)$ & $\Delta(x,y)$ homogeneæ functiones n & m dimenſionum, quarum n major ſit quam m; & fluens omnis fluxionis $A\dot{x}+B\dot{y}$ ubi literæ A & B denotant homogeneas functiones literarum x & y dimenſiones (r) habentes deſignari poteſt per fluentem functionis variabilis quantitatis (z) ductæ in fluxionem $\dot{z}$ quæ continebit nullas radicales quantitates præter eas, quæ correſpondent irrationalibus datâ æquatione contentis: ſi modo literæ A & B nullas involvant radicales quantitates præter eas, quæ in datâ æquatione continentur, & $\frac{r+1}{n-m}$ ſit integer numerus vel affirmativus vel negativus.

Conſtat e ſubſtitutione prædictâ zy pro x.

Eadem principia etiam applicari poſſunt ad fluentes fluxionalium quantitatum ſuperiorum ordinum. e. g. Sit s ordo fluxionalium quantitatum & r dimenſio homogenearum functionum $(P, Q, R$, &c.) quantitatum (x & y), quæ in fluxiones reſpective ducantur; deinde pro y ſcribatur zx, & pro $\dot{y}$ ejus fluxio; & ſic deinceps; tum in fluxione reſultante nulla involvetur radicalis quantitas; quæ non correſpondet iis, quæ vel in datâ æquatione vel in quantitatibus (P, Q, R, &c.) inveniuntur; ſi modo $\frac{r+s}{n-m}$ ſit integer numerus vel affirmativus vel negativus.

Et ſic facile deduci poſſunt conſimiles æquationes, quarum quædam fluxiones aſſignari poſſunt, & quarum fluentes exprimi poſſunt ope fluentium in ſcholio præcedentis capitis traditarum.

Hæc principia etiam ad plures æquationes plures incognitas quantitates habentes applicari poſſunt.

4. Sit æquatio $x^{\alpha}(e + fx^{n}y^{p} + gx^{2n}y^{2p} + \&c.)^{\lambda} + Hx^{\beta}(l + mx^{n}y^{p} + ox^{2n}y^{2p} + \&c.)^{\mu} = Kx^{\pi}(q + rx^{n}y^{p} + sx^{2n}y^{2p} + \&c.)^{\nu} + \&c.$ ſcribatur $v^{n} = x^{n}y^{p}$, & reſultat æquatio $v^{\alpha}y^{-\frac{p\alpha}{n}}(e + fv^{n} + gv^{2n} + \&c.)^{\lambda} + Hv^{\beta}y^{-\frac{p\beta}{n}}(l + mv^{n} + ov^{2n} + \&c.)^{\mu} = Kv^{\pi}y^{-\frac{p\pi}{n}}(q + rv^{n} + sv^{2n} + \&c.)^{\nu} + \&c.$

Ex. Sit æquatio $x^{\alpha} + ax^{n+\alpha}y^{p} = by^{\beta}$, i. e. $x^{\alpha}(1 + ax^{n}y^{p}) = by^{\beta}$; ſcribatur $v^{n} = x^{n}y^{p}$, & reſultat æquatio $v^{\alpha}y^{-\frac{p\alpha}{n}}(1 + av^{n}) = by^{\beta}$, unde $by^{\frac{\beta n + p\alpha}{n}} = v^{\alpha}(1 + av^{n})$ & $y = b^{\frac{-n}{\beta n + p\alpha}}((v^{\alpha})(1 + \alpha v^{n}))^{\frac{n}{\beta n + p\alpha}}$, & exinde $x = v \times b^{\frac{p}{\beta n + p\alpha}}((v^{\alpha})(1 + av^{n}))^{\frac{-p}{\beta n + p\alpha}}$, &c.

5. Ducatur data æquatio in aſſumptas quantitates, & nonnunquam deduci poteſt æquatio, cujus incognitæ quantitates x & y exprimi poſſunt ope tertiæ z.

Ex. 1. Sit data æquatio $y^{n} + ax^{m}y^{r} = bx^{s}$, ducatur data æquatio in x^{α}, & reſultat $y^{n}x^{\alpha} + ay^{r}x^{m+\alpha} = bx^{s+\alpha}$; ſit $n : \alpha :: r : m + \alpha$; unde $\alpha = \frac{nm}{r - n}$; ſcribatur $y^{n}x^{\alpha} = v^{n}$, & reſultat æquatio $v^{n} + av^{\frac{r}{n}} = bx^{s+\alpha}$, & $y^{n} = \frac{v^{n}}{x^{\alpha}}$.

Cor. Si n vel m vel $s = 0$; vel $r = n$, vel $m = s$; tum ad inveniendos valores quantitatum x & y per novam quantitatem v haud neceſſario ducenda eſt data æquatio in quantitatem x^{α}.

Ex. 2. Sit data æquatio $y^{n} + ax^{m}y^{r} + bx^{p}y^{s} = cx^{q}$ ducatur hæc æquatio in x^{α}, & reſultat $y^{n}x^{\alpha} + ax^{m+\alpha}y^{r} + bx^{p+\alpha}y^{s} = cx^{q+\alpha}$.

Si omnes poteſtates quantitatum y vel x in datâ æquatione contentæ ſint inter ſe æquales; vel duæ ſint diverſæ poteſtates, quarum una dimidium eſt alterius, &c. vel ſi in æquationem horum generum reduci poſſit data æquatio ex ejus multiplicatione in x^{α} vel y^{α}, ubi α ſit index ad libitum aſſumendus; vel ſi modo ducatur data æquatio in x^{α}; & vel pro $y^{n}x^{\alpha}$, vel pro $y^{r}x^{\alpha+m}$, vel pro $y^{s}x^{p+\alpha}$ ſcribatur v, & æquatio data reducatur in terminos quantitatum v & x; & ſi æquationes reſultantes ſint prædictorum generum, &c. tum facile exprimi poſſunt quantitates x & y per novam quantitatem v.

Et ſic de transformandis pluribus æquationibus, ita ut earum incognitæ quantitates exprimantur per novam quantitatem v.

6. 1. Nonnunquam ita transformari poſſunt e ſubſtitutione æquationes, ut earum incognitæ quantitates (x, y, &c.) exprimi poſſint in terminis novæ aſſumptæ v; e. g. datâ algebraicâ æquatione duas incognitas quantitates x & y involvente; invenire utrum ſingula quantitas ex iis ſit rationalis functio tertiæ z, necne; aſſumantur rationales functiones $a + bv + cv^2 + \&c. = x$, vel $\frac{a + bv + cv^2 + \&c.}{p + qv + rv^2 + \&c.} = x$ & $a' + b'v + c'v^2 + \&c. = y$ vel $\frac{a' + b'v + c'v^2 + \&c.}{p' + q'v + r'v^2 + \&c.} = y$; ſubſtituantur hæ quantitates pro ſuis valoribus in datâ æquatione, & ſupponantur correſpondentes termini reſultantis æquationis nihilo æquales; & ex æquationibus reſultantibus deduci poſſunt incognitarum quantitatum valores, ſi modo ulli dentur.

2. Et ſic inveniri poteſt, utrum ſingula variabilis quantitas (x, y, &c.) in datâ vel datis æquationibus contenta ſit rationalis functio vel irrationalis datæ formulæ variabilis v, necne; aſſumantur in genere irrationales functiones datæ formulæ pro incognitis quantitatibus (x, y, &c.); quibus pro ſuis valoribus in datâ vel datis æquationibus ſubſtitutis, & terminis reſultantium æquationum correſpondentibus nihilo æqualibus eſſe ſuppoſitis; ex æquationibus reſultantibus erui poſſunt incognitarum quantitatum valores, ſi modo tales recipiant.

3. E prob. 54. medit. algebr. deduci poſſunt æquationes, quarum incognitæ quantitates x & y exprimi poſſunt ope tertiæ v aſſumendæ: 1. aſſumatur quæcunque functio literæ v pro y, deinde aſſumatur quæcunque functio literarum v & y pro x; ita transformentur hæ duæ æquationes in unam, ut exterminetur litera v; & reſultat æquatio relationem inter literas x & y exprimens, ubi literæ x & y ſunt

datæ functiones tertiæ v. Ex. g. aſſumatur $y = \frac{a + bv + cv^2 + \&c.}{p + qv + rv^2 + \&c.}$ & $xy = v$ vel $\frac{x}{y} = v$, quibus quantitatibus pro ſuo valore v in primâ æquatione ſubſtitutis, reſultant æquationes quæſitæ.

4. Aſſumantur quæcunque functiones quantitatis v pro x & y, deinde aſſumatur æquatio relationem inter x & y exprimens formulæ, quæ recipere poteſt prædictas functiones quantitatis v pro valoribus correſpondentibus quantitatum x & y; ſubſtituantur aſſumptæ functiones quantitatis v pro ſuis valoribus x & y in aſſumptâ æquatione; & fiant correſpondentes termini æquationis reſultantis inter ſe æquales; & ex æquationibus reſultantibus detegi poteſt æquatio quæſita.

Omnia hæc etiam ad fluxionales æquationes applicari poſſunt.

7. Sit æquatio $a(y^n + z^n) + b(y^{n-1}z + z^{n-1}y) + c(y^{n-2}z^2 + z^{n-2}y^2) + \&c. + p(y^{n-1} + z^{n-1}) + q(y^{n-1}z + z^{n-1}y) + \&c. = 0$, in quâ ſimiliter involvuntur quantitates z & y; pro $y + z$ ſcribatur w, & pro zy ſcribatur v; & æquatio reſultans erit $aw^n + (b - na)\,w^{n-2}v + \left(c - (n-2)\,b + n \cdot \frac{n-3}{2}\,a\right) w^{n-4}v^2 + \left(d - (n-4)\,c + (n-2) \times \frac{n-5}{2}\,b - n \cdot \frac{n-4}{2} \cdot \frac{n-5}{3}\,a\right) w^{n-6}v^3 + \left(e - (n-6)\,d + (n-4) \times \frac{n-7}{2}\,c - (n-2) \times \frac{n-6}{2} \times \frac{n-7}{3}\,b + n \times \frac{n-5}{2} \times \frac{n-6}{3} \times \frac{n-7}{4}\,a\right) w^{n-8}v^4 + \&c. + pw^{n-1} + (q - (n-1)\,p)\,w^{n-3}v + \left(r - (n-3)\,q + (n-1) \times \frac{n-4}{2}\,p\right) w^{n-5}v^2 + \&c. = 0$. Lex coefficientium hujuſce ſeriei facile conſtabit ex obſervat. leg. ſeriei in theor. 38. medit. algeb. contentæ.

Quantitas v nunquam aſſurgit ad majores quam $\frac{n}{2}$ dimenſiones, ſi n ſit par numerus; vel majores quam $\frac{n-1}{2}$, ſi n ſit impar: ſi vero detur quantitas v e terminis quantitatis w, tum reſolutione quadraticæ æquationis dantur z & y in terminis quantitatis w.

Cor. Sit $n = 2$ vel 3, & v haud aſſurgit ad majores quam unam; ſit $n = 4$ vel 5, & v haud aſſurgit ad majores quam 2; ſit $n = 8$ vel 9, & haud aſſurget v ad majores quam 4 dimenſiones; & quoniam extractio biquadraticæ æquationis cognoſcitur; ergo fluens cujuſcunque fluxionis, quæ eſt functio quantitatum x & y in $\dot{x}$ vel $\dot{y}$ ducta, per methodos prius traditas deduci poteſt; ſi modo dimenſiones datæ æquationis relationem inter variabiles x & y exprimentis, in quâ x & y ſimiliter involvuntur, haud ſuperent novem dimenſiones.

7. 2. Sit æquatio relationem inter variabiles z & w exprimens ejuſmodi, ut quantitas (z) ſit cognita functio quantitatis w; vel utræque (z & w) ſint cognitæ functiones novæ quantitatis (v); deinde aſſumantur duæ æquationes relationes inter z, w, x & y exprimentes, ita ut ex datis valoribus quantitatum z & w ſemper acquiri poſſunt correſpondentes valores incognitarum quantitatum x & y; deinde reducantur hæ æquationes in unam, ita ut exterminentur incognitæ quantitates z & w, & reſultat æquatio relationem inter x & y exprimens, cujus utræque x & y facile deſignari poſſunt ope tertiæ quantitatis aſſumptæ: & conſequenter fluxio, quæ eſt functio quantitatum x & y & earum fluxionum, ope prædictæ tertiæ quantitatis & ejus fluxionum exprimi poteſt.

Conſimilia etiam ad tres vel plures æquationes applicari poſſunt.

Si vero plures dentur æquationes plures variabiles quantitates habentes, nonnunquam detegi poſſunt reſolutiones ex his vel ex aliis principiis in medit. algeb. traditis.

8. Datis methodis inveniendi continuas approximationes ad fluentes datarum fluxionum, ex iis haud nunquam conſtant fluentes ipſæ, i. e. haud nunquam terminatur ſeries ipſa.

Conſtant ex infinitis ſeriebus infinitæ formulæ harum æquationum.

Ex. Sit data æquatio $y^m + ax^n = by^e x^r$; ſeries, quæ exprimit fluentem fluxionis $y\dot{x}$, poteſt eſſe hujuſce formulæ $Axy + By^{e+1}x^{c+1} + Cy^{2e+1}x^{2c+1} + Dy^{3e+1}x^{3c+1} +$ &c. ubi $c = r - n$: inveniatur fluxio hujuſce ſeriei & reſultat æquatio $(P)\ y\dot{x} = (Ax + (e+1)By^e x^{c+1} + (2e+1)Cy^{2e}x^{2c+1} +$ &c.$)\dot{y} + (Ay + (c+1)By^{e+1}x^c + (2c+1)$

$Cy^{2e+1}x^{2c}+\&c.)\dot{x}$: fluxio autem datæ æquationis erit $(my^{m-1}-eby^{e-1}x^r)\dot{y}=(rby^ex^{r-1}-nax^{n-1})\dot{x}$, in æquatione (P) pro $\dot{y}$ ſcribatur ejus valor $\frac{rby^ex^{r-1}-nax^{n-1}}{my^{m-1}-eby^{e-1}x^r}\dot{x}$ e poſteriori æquatione fluxionali deductus, & reſultat $y\dot{x}=(Ax+(e+1)By^ex^{c+1}+(2e+1)Cy^{2e}x^{2c+1}+\&c.)\left(\frac{rby^ex^{r-1}-nax^{n-1}}{my^{m-1}-eby^{e-1}x^r}\right)\dot{x}+(Ay+(c+1)By^{e+1}x^c+(2c+1)Cy^{2e+1}x^{2c}+\&c.)\dot{x}$; quâ reductâ, fit $(\mathcal{Q})$

$$\left.\begin{array}{r} naAx^n \\ -(mA-m)y^m \end{array}\right\} \begin{array}{l} -eby^ex^r \\ +ebAy^ex^r \\ -rbAy^ex^r \\ +na(e+1)By^ex^{c+n(r)} \\ -m(c+1)By^{e+m}x^c \end{array} +\&c.=0.$$

In terminis $(mA-m)y^m$ & $m(c+1)By^{e+m}x^c$ pro y^m ſcribatur ejus valor $-ax^n+by^ex^r$ e datâ æquatione deductus, & reſultant reſpective $-(mA-m)ax^n+(mA-m)by^ex^r$ & $-m(c+1)Bay^ex^{n+c(r)}+m(c+1)By^{2e}x^{r+c(n+2c)}$; quibus quantitatibus pro ſuis valoribus in æquatione $\mathcal{Q}$ ſubſtitutis, reſultat $(\mathcal{Q})=$

$$\left.\begin{array}{r} nAa \\ +ma(A-1) \end{array}\right\} \left.\begin{array}{l} x^n-eb \\ x^n+ebA \\ -rbA \\ +na(e+1)B \\ +m(c+1)aB \\ -(mA-m)b \end{array}\right\} y^ex^r+\&c.=0.$$

Fiant correſpondentes termini reſultantis æquationis nihilo reſpective æquales, & habebimus $nA+mA-m=0$, (unde $A=\frac{m}{m+n}$); & $(n(e+1)+m(c+1))aB-(e-eA+rA+mA-m)b=0$; pro A & r in hâc æquatione ſcribantur earum valores $\frac{m}{m+n}$ & $n+c$ reſpective, & invenietur $B=\frac{ne+mc}{(n(e+1)+m(c+1))(m+n)}\times\frac{b}{a}$, &

ſic deinceps: ſeries, quæ exprimit $\int. y\dot{x}$ erit $= \frac{m}{m+n} xy + \frac{mc+ne}{(m(c+1)+n(e+1))(m+n)} \times \frac{b}{a} y^{e+1} x^{c+1} + \frac{(m-e)(c+1)+r(e+1)}{m\times(2c+1)+n\times(2e+1)} \times \frac{bB}{a} y^{2e+1} x^{2c+1} + \frac{(m-e)(2c+1)+r(2e+1)}{m(3c+1)+n(3e+1)} \times \frac{bC}{a} y^{3e+1} x^{3c+1} + \frac{(m-e)(3c+1)+r(3e+1)}{m(4c+1)+n(4e+1)} \times \frac{bD}{a} y^{4e+1} x^{4c+1} +$ &c.

Cor. 1. Si $(m-e)(lc+1)+r(le+1)=o$, cum l ſit integer affirmativus numerus, tum terminatur ſeries; aliter vero non.

Cor. 2. Si $e=o$, tum hæc ſeries facile transformari poteſt in binomiale theorema.

Ex. 2. Sit data æquatio $y^m = ax^n + by^e x^{c+n} + dy^{2e} x^{2c+n} + fy^{3e} x^{3c+n} +$ &c. & ſeries, quæ exprimit aream poteſt eſſe hujuſce formulæ $\int. y\dot{x} = Axy + By^{e+1} x^{c+1} + Cy^{2e+1} x^{2c+1} + Dy^{3e+1} x^{3c+1} +$ &c. Ex calculo facile deduci poſſunt coefficientes A, B, C, &c.

Ex. 3. Sit fluxionalis æquatio $y^m\dot{y} = ax^{n-1}\dot{x} + (by^{e-1} x^{c+n} + dy^{2e-1} x^{2c+n} + fy^{3e-1} x^{3c+n} +$ &c.$)\dot{y} + (py^e x^{c+n-1} + qy^{2e} x^{2c+n-1} + ry^{3e} x^{3c+n-1} +$ &c.$)\dot{x}$; & ſeries quæ exprimit $\int. y\dot{x}$ poteſt eſſe hujuſce formulæ Axy $By^{e+1} x^{c+1} + Cy^{2e+1} x^{2c+1} +$ &c.

Facile infinitæ formulæ ſerierum fingi poſſunt, quæ exprimunt fluentes quarumcunque fluxionum, quæ ſunt datæ functiones variabilium quantitatum in datis algebraicis vel fluxionalibus æquationibus contentarum; ſed de his poſtea in capite de infinitis ſeriebus.

PROB. XXXIII.

1. *Datâ algebraicâ æquatione* n *dimenſionum* $y^n + (a+bx)y^{n-1} + (c+dx+ex^2)y^{n-2} +$ &c. $= 0$ *relationem inter abſciſſam* x *&* *ejus correſpondentes ordinatas* y *exprimente; invenire æquationem relationem inter abſciſſam* x *&* *ejus aream* v *deſignantem, ſi modo curva generaliter quadrari poſſit.*

Datâ abſciſsâ x, n ſunt diverſi valores ordinatæ y, qui datæ abſciſſæ correſpondent, & conſequenter n ſunt diverſi areæ valores ei correſpondentes; æquatio relationem inter abſciſſam x & aream v erit hujuſce formulæ $v^n + (A + Bx + Cx^2)v^{n-1} + (D + Ex + Fx^2 + Gx^3 + Hx^4)v^{n-2} + (K + Lx + Mx^2 + \,.\,.\, Px^6)v^{n-3}$ &c. $= 0$; aſſumatur hæc æquatio pro æquatione quæſitâ, & per problema 25. inveniatur æquatio, cujus radix eſt ordinata $y = \frac{\dot{v}}{\dot{x}}$; ſingulis reſultantis & datæ æquationis terminis reſpective inter ſe æquatis, reſultant æquationes, quarum incognitæ quantitates erunt reſpective A, B, C, D, &c. & quarum valores per methodum communes diviſores inveniendi facile erui poſſunt; ſi vero curva haud quadraturam admittat, tum nulli dantur prædicti communes diviſores.

Ex. 1. Sit data æquatio $y^2 + x^2 - 1 = 0$, & ſupponatur æquatio ad aream $v^2 + (A + Bx + Cx^2)v + D + Ex + Fx^2 + Gx^3 + Hx^4 = 0$; per prædictum problema inveniatur æquatio, cujus radix eſt $\frac{\dot{v}}{\dot{x}} = y$; & ſi modo evaneſcat diverſorum areæ valorum ſumma, cum evaneſcat abſciſſa x, i. e. ſit $A = 0$; erunt B & C etiam nihilo æquales; & per problema 25. $\frac{(E + 2Fx + 3Gx^2 + 4Hx^3)^2}{4(D + Ex + Fx^2 + Gx^3 + Hx^4)} = x^2 - 1$, & conſequenter $4H = 16H^2$, $4G = 24HG$, $4F - 4H = 16HF + 9G^2$, $4E - 4G = 8HE + 12FG$, $4D - 4F = 6GE + 4F^2$, $-4E = 4EF$, $-4D = E^2$; ſed e methodo communes diviſores inveniendi, &c. conſtat has æquationes inter ſe contradictorias eſſe, & conſequenter curvam haud generaliter eſſe quadrabilem.

Ex. 2. 1. Sit æquatio $y^2 + (4 + 2x)y + x^2 + 3x + 1 = 0$; aſſumatur æquatio $v^2 + (A + Bx + Cx^2)v + Dx^4 + Ex^3 + Fx^2 + Gx + H = 0$; tum erunt $B = 4$, $C = 1$ & $A = 0$, ſi modo evaneſcat arearum ſumma, cum evaneſcat abſciſſa; per exemplum primum problematis 25. invenietur $\frac{(Dx^4 + Ex^3 + Fx^2 + Gx + H)(2x + 4)^2 - (x^2 + 4x)}{4Dx^4 + 4Ex^3 +}$

(2x +

$\frac{(2x+4)(4Dx^3+3Ex^2+2Fx+G)+(4Dx^3+3Ex^2+2Fx+G)^2}{4Fx^2+4Gx+4H-(4x+x^2)^2} = x^2+3x+1$, reducatur hæc æquatio, & correſpondentes termini inter ſe æquales eſſe ſupponantur; reſultat $16D^2-8D=-1$, unde $D=\frac{1}{4}$; quo valore in reliquis reſultantibus terminis ſubſtituto, exoriuntur æquationes $o=o$; & $9E^2-32E+28=o$, unde E erit vel $\frac{14}{9}$ vel 2; ſubſtituatur $\frac{14}{9}$ pro E in reliquis æquationibus, & inveniuntur $F=o$, $G=-12$, & $H=-12$; & conſequenter æquatio quæſita erit $v^2+(4x+x^2)v+\frac{1}{4}x^4+\frac{14}{9}x^3-12x-12=o$.

2. Sit A quæcunque quantitas, & conſequenter non evaneſcat arearum ſumma, cum evaneſcat abſciſſa; tum erit æquatio ad aream $v^2+(A+4x+x^2)v+\frac{1}{4}x^4+\frac{14}{9}x^3+\frac{1}{2}Ax^2+(2A-12)x+\frac{1}{4}A^2-12=o$.

Hæc æquatio facile deduci poteſt e priore, ſcribendo pro v ejus valorem $v+\frac{A}{2}$: nam per hanc ſubſtitutionem transformatur prior æquatio ad aream, i. e. cujus diverſorum valorum ſumma evaneſcit, cum abſciſſa evadat nihilo æqualis; in æquationem ad aream, cujus ſumma valorum erit A, cum abſciſſa $=o$.

2. 2. Eodem modo ſit æquatio ad aream (v) $v^n+(Bx+Cx^2)v^{n-1}+(Dx^4+Ex^3+\&c.)v^{n-2}+\&c.=o$, cujus valorum ſumma evaneſcit, cum abſciſſa $=o$: in hâc æquatione pro v ſcribatur $v+\frac{A}{n}$, & æquatio reſultans erit æquatio ad aream ejuſdem curvæ, cujus ſumma valorum erit A, cum evaneſcat abſciſſa x.

3. Paulo aliter progredi licet in hoc & multis aliis caſibus; e. g. eâdem æquatione datâ, & iiſdem literis eaſdem quantitates denotantibus; aſſumatur etiam æquatio ad aream $v^n+(A+Bx+Cx^2)$ $v^{n-1}+$

$v^{n-1} + (D + Ex + Fx^2 + Gx^3 + Hx^4)v^{n-2} + \&c. = 0$, & conſequenter pro $\frac{\dot{v}}{\dot{x}}$ ſcribendo y, erit $nyv^{n-1} + (n-1)(A + Bx + Cx^2)v^{n-2}y + (n-2)(D + Ex + Fx^2 + \&c.)v^{n-3}y + \&c. + (B + 2Cx)v^{n-1} + (E + 2Fx + 3Gx^2 + 4Hx^3)v^{n-3} + \&c. = 0$; e quibus æquationibus, ſi methodis notis extermінetur v, habebimus æquationem, quæ exprimit relationem inter x & y; hujus autem æquationis coefficientes æquari debent coefficientibus datæ æquationis $y^n + (a + bx)y^{n-1} + (c + dx + ex^2)y^{n-2} + \&c. = 0$. & ſi quantitates A, B, C, &c. exinde determinari poſſint, curva quadratur; aliter autem quadrari non poteſt.

4. Datâ præcedente algebraicâ æquatione $y^n + (a + bx)y^{n-1} + (c + dx + ex^2)y^{n-2} + \&c. = 0$; invenire utrum fluens v fluxionis $((\alpha + \beta x + \gamma x^2 + \&c. + x^s)y^m + (p + qx + rx^2 + \&c. + x^{s-1})y^{m+1} + (\&c.))\dot{x} = \Delta\dot{x}$ inveniri poteſt, necne; ubi literæ s, m, &c. integros denotant numeros. Per medit. algeb. inveniatur æquatio, cujus radix ſit Δ: aſſumatur æquatio ad fluentem $v^n + (A + Bx + Cx^2 \ldots Px^{m+s+1})v^{n-1} + (D + E\dot{x} + Fx^2 + \&c. + Qx^{2m+2s+2})v^{n-2} + \&c. = 0$, deinde inveniatur æquatio cujus radix ſit $\frac{\dot{v}}{\dot{x}}$; reſultantis & inventæ æquationis, cujus radix ſit Δ, fiant termini inter ſe æquales, & e reſultantibus exinde æquationibus per methodum communes diviſores inveniendi erui poſſunt valores quantitatum A, B, C, &c. P; D, E, F, &c. Q; &c. ſi vero nulli dentur correſpondentes valores harum quantitatum, i. e. æquationes reſultantes inter ſe contradictoriæ ſint; tum finitis terminis haud exprimi poteſt fluens.

Cor. 1. Si pro $\dot{x}$ ſcribatur $\dot{x}^r$, & requiratur fluens (r) ordinis: pro x^{m+s+1} ſcribe x^{m+s+r}; pro $x^{2m+2s+2}$ ſcribe $x^{2m+2s+2r}$, &c. & eodem modo quo prius deduci poteſt reſolutio.

5. Datâ algebraicâ æquatione $Ay^n + By^{n-1} + Cy^{n-2} + \&c. = 0$, (ubi literæ A, B, C, &c. quaſcunque rationales functiones literæ x reſpective denotant) relationem inter duas incognitas quantitates x & y exprimente;

exprimente; datâ etiam quantitate, quæ eſt fluxionalis functio ($\dot{v}$) incognitarum datæ æquationis quantitatum (x & y); invenire utrum fluens (v) prædictæ fluxionalis functionis inveniri poteſt, necne; i. e. relatio inter incognitas quantitates x & y & v per algebraicam æquationem exprimi poteſt, necne.

Ex datâ algebraicâ æquatione inveniatur æquatio, cujus radix eſt data fluxionalis functio variabilium datæ æquationis quantitatum (x & y).

Aſſumatur æquatio, cujus radix eſt fluens quæſita (v); & formula, quæ neceſſario continet radices quæſitæ fluentis v, ſi modo illæ radices finitis algebraicis terminis quantitatum x & y exprimi poſſint, viz. $v^m + (A + Bx + \&c.)\, v^{m-1} + (D + Ex + \&c.)\, v^{m-2} + \&c. = o$, vel $(\alpha + \beta x + \&c.)\, v^m + (A + Bx + \&c.)\, v^{m-1} + \&c. = o$.

Hujus æquationis formula plerumque e datâ æquatione & fluxionali quantitate facile erui poteſt: e numero enim radicum, quas præbent data æquatio & fluxionalis quantitas, conſtat numerus dimenſionum quantitatis v in quæſitâ æquatione contentarum; & ſic de numero dimenſionum, &c. quantitatum x vel y in quæſitâ æquatione contentarum.

Ex aſſumptâ æquatione inveniatur altera, cujus radix eſt $\dot{v}$; reſultantis æquationis, etiamque æquationis, cujus radix eſt fluxionalis quantitas, terminis correſpondentibus inter ſe æqualibus eſſe ſuppoſitis; reſultant æquationes, e quibus methodo communes diviſores inveniendi erui poſſunt incognitæ coefficientes aſſumptæ æquationis; ſi vero hæ æquationes inter ſe contradictoriæ ſunt, tum fluens quæſita haud inveniri poteſt.

Et ſic de duabus vel pluribus æquationibus tres vel plures variabiles quantitates habentibus.

Cor. 1. Hinc inveniri poſſunt quamplurimæ algebraicæ curvæ, quarum relationes inter earum areas & abſciſſas (x) exprimi poſſunt per algebraicas æquationes: aſſumantur enim quæcunque datæ æquationes $Av^n + Bv^{n-1} + Cv^{n-2} + \&c. = o$, ubi A, B, C, &c. quaſcunque

que functiones abſciſſæ x reſpective denotant; & per prob. 25. inveniatur æquatio, cujus radix ſit $\frac{\dot{v}}{\dot{x}} = y$; & invenitur æquatio relationem inter abſciſſam x & ejus ordinatas y exprimens; & cujus æquatio, quæ exprimit relationem inter aream v prædictæ curvæ & ejus abſciſſam x, eſt $Av^n + Bv^{n-1} + \&c. = o$.

Facile conſtat, ſi modo æquatio $Av^n + Bv^{n-1} + \&c. = o$ dividi poſſit in duas alias, æquationem relationem inter abſciſſam x & ejus ordinatas y exprimentem etiam dividi poſſe in duas alias; ſin aliter vero non.

Cor. 2. Hinc eâdem methodo etiam inveniri poſſunt quamplurimæ algebraicæ curvæ, quarum relationes inter abſciſſas x & fluentem v cujuſcunque algebraicæ functionis (P) abſciſſæ x & ordinatæ y in $\dot{x}$ ductæ per algebraicam æquationem deſignari poſſunt: aſſumatur enim æquatio relationem inter y & x exprimens, cujus radix v eſt fluens fluxionalis functionis; deinde inveniatur æquatio, cujus radix eſt $\frac{\dot{v}}{\dot{x}} = P$, pro P ſcribatur prædicta functio quantitatum x & y; & reſultat æquatio quæſita relationem inter x & y exprimens.

Et ſic de conſimilibus ad fluentes ſuperiorum ordinum prædictarum fluxionalium functionum quantitatum x & y & earum fluxionum applicandis.

6. Hi quinque caſus etiam reſolutionem accipere poſſunt e methodo convergentes ſeries inveniendi: e. g. datâ algebraicâ æquatione $y^n + (a + bx)y^{n-1} + \&c. = o$; invenire, annon ejus area exprimi poteſt in algebraicis terminis abſciſſæ x.

Per convergentes ſeries inveniantur n valores ordinatæ y, qui ſint reſpective y, y', y'', &c. & reſultabunt $y = \alpha x + \beta + \frac{\gamma}{x} + \frac{\delta}{x^2} + \&c.$ $y' = \alpha' x + \beta' + \frac{\gamma'}{x} + \frac{\delta'}{x^2} + \&c.$ $y'' = \alpha'' x + \beta'' + \&c.$ ubi $\gamma, \gamma', \gamma''$, &c. nihilo debent eſſe æquales, aliter area haud inveniri poteſt. Sit v area,

& ejus diverſi valores v, v', v'', &c. erunt $v = \frac{\alpha x^2}{2} + \beta x + A - \frac{\delta}{x}$ — &c. $v' = \frac{\alpha' x^2}{2} + \beta' x + A' - \frac{\delta'}{x}$ — &c. $v'' = \alpha'' x^2 + \beta'' x + A'' - \frac{\delta''}{x}$ — &c. &c. = &c.

Ducantur hæ æquationes in ſeſe, & reſultat æquatio $v^n + (\pi x^2 + \rho x + \sigma) v^{n-1} +$ &c. quæ erit æquatio ad aream, ſi modo ea in algebraicis terminis inveniri poſſit; quod conſtabit e terminis reſultantis æquationis haud in infinitum progredientibus, i. e. haud in negativas dimenſiones quantitatis v deſcendentibus.

2. Datâ algebraicâ æquatione $Ay^n + By^{n-1} +$ &c. $= 0$, ubi A, B, C, &c. ſunt algebraicæ functiones quantitatis x, & fluxionali quantitate; per cap. 1. medit. algebr. inveniri poteſt, utrum ejus fluens exprimi poteſt finitis algebraicis terminis abſciſſæ x, necne.

THEOR. XVI.

Si fluens cujuſcunque fluxionis e datâ æquatione deductæ generaliter exprimi poſſit finitis terminis ejus variabilium quantitatum x & y; tum ſumma e ſingulis valoribus prædictæ fluxionis exprimi poteſt finitis terminis prædictarum variabilium quantitatum x & y; ſi igitur ita transformentur variabiles quantitates x & y, ut conſtet ſummam prædictam haud exprimi poſſe generaliter finitis terminis prædictarum quantitatum x & y, concludere licet fluentem ipſam haud exprimi poſſe finitis terminis quantitatum x & y.

Ex. Datâ algebraicâ æquatione haud diviſorem recipiente $Ay^n + (a + bx) y^{n-1} + (c + dx + ex^2) y^{n-2} +$ &c. $= 0$, & relationem inter abſciſſam x & ejus correſpondentes ordinatas y exprimente. In datâ æquatione pro literis x & y ſcribantur reſpective $lv + mz + n$ & $pv + qz + r$, & (ſi modo poſſibile ſit) reſultet æquatio $(Pz + Q) v^{n-1} + (Rz^2 + Sz + T) v^{n-2} +$ &c. $= 0$, cujus primus terminus v^n deeſt; & ſi $Pz + Q$ haud ſit diviſor quantitatis $Rz^2 + Sz + T$, tum curva quadraturam haud recipiet finitis terminis.

Sit vero æquatio reſultans in genere $(Pz^{m} + Qz^{m-1} + \&c.)\, v^{n-m} + (Rz^{m+1} + Sz^{m} + Tz^{m-1} + \&c.)\, v^{n-m-1} + \&c. = 0$, & ſi fluens fluxionis $\frac{Rz^{m+1} + Sz^{m} + Tz^{m-1} + \&c.}{Pz^{m} + Qz^{m-1} + \&c.}\,\dot{z}$ finitis terminis haud exprimi poſſit, tum curva haud finitis terminis quadraturam recipiet.

Eadem principia etiam applicari poſſunt ad contenta ſolidorum, ad fluentes quarumcunque fluxionum, &c.

THEOR. XVII.

1. Datâ algebraicâ æquatione $Ay^{n} + (a + bx)\,y^{n-1} + \&c. = 0$, relationem inter abſciſſam x & ejus correſpondentes ordinatas y exprimente; ſumma e ſingulis valoribus cujuſcunque algebraicæ functionis literarum x & y & earum fluxionum ſemper exprimi poteſt in finitis terminis abſciſſæ & ordinatæ, earum circularium arcuum & logarithmorum: ſumma enim e ſingulis valoribus irrationalis partis nihilo erit æqualis, & ſumma e ſingulis valoribus rationalis functionis ſemper inveniri poteſt ope finitorum terminorum, circularium arcuum & logarithmorum.

2. Datâ fluxionali æquatione, & fluxionali quantitate continente rationalem & irrationalem algebraicam quantitatem, cujus rationalis quantitatis fluens inveniri poteſt; tum ſumma e ſingulis valoribus fluxionalis quantitatis inveniri poteſt; ſumma enim e ſingulis valoribus irrationalis quantitatis erit nihilo æqualis.

PROB. XXXIV.

1. *Ita transformare æquationem fluxionalem, in quâ ſolummodo continentur duæ variabiles quantitates* (x & y), *quarum* x *fluit uniformiter, ut fluat uniformiter* y.

FIG. 1. pag. 95. Sit quæcunque curva (MRS), cujus abſciſſa $(x = AP)$ fluit uniformiter; ſit ejus fluxio (Pp), & fluxio ordinatæ $(y = PM)$ Mh; ſecunda vero fluxio $2mf$; nunc fluat ordinata (PM) uniformiter, ſit ejus fluxio *Mb*, tum prima abſciſſæ (*AP*) fluxio erit *be*, & ſecunda ejus fluxio $2me$: ſed duo triangula *Mbe*, *fme* ſunt ſimilia, ergo $eh(\dot{x}) : hM(\dot{y}) :: 2me(\ddot{x})$ ſecunda abſciſſæ fluxio, cum fluat uniformiter $y : 2mf(-\ddot{y})$ ſecunda ordinatæ fluxio, cum fluat uniformiter x; & conſequenter $\ddot{y} = -\frac{\dot{y}\ddot{x}}{\dot{x}}$, quo valore pro $\ddot{y}$ in datâ fluxionali æquatione ſubſtituto, & ſic deinceps; & ita transformabitur æquatio, ut fluat uniformiter y. Aliter.

2. Sit $y = Ax^n + Bx^{n-r} + Cx^{n-2r} +$ &c. & erit $\dot{y} = nAx^{n-1}\dot{x} + (n-r).Bx^{n-r-1}\dot{x} + (n-2r)Cx^{n-2r-1}\dot{x} +$ &c. &

$$\ddot{y} = n\times(n-1)Ax^{n-2}\dot{x}^2 + (n-r)(n-r-1)Bx^{n-r-2}\dot{x}^2 + (n-2r)(n-2r-1)Cx^{n-2r-2}\dot{x}^2$$
$$+nAx^{n-1}\ddot{x} + (n-r)Bx^{n-r-1}\ddot{x} + (n-2r)Cx^{n-2r-1}\ddot{x} + \text{\&c.}$$

$+$ &c. $= Q\dot{x}^2 + P\ddot{x}$; fluat uniformiter x, & conſequenter evaneſcet $\ddot{x}$, & erit $\ddot{y} = Q\dot{x}^2$; deinde fluat uniformiter y, & erit $\ddot{y} = 0 = Q\dot{x}^2$ ($\ddot{y}$ cum x fluat uniformiter) $+ P\ddot{x}\left(\frac{\dot{y}\ddot{x}}{\dot{x}}\right)$, quoniam $P = \frac{\dot{y}}{\dot{x}}$; utræque enim hæ quantitates P & $\frac{\dot{y}}{\dot{x}}$ ſunt $nAx^{n-1} + (n-r)Bx^{n-r-1} +$ &c. unde $Q\dot{x}^2 + \frac{\dot{y}\ddot{x}}{\dot{x}} = \ddot{y} + \frac{\dot{y}\ddot{x}}{\dot{x}} = 0$, & conſequenter $\ddot{y} = -\frac{\dot{y}\ddot{x}}{\dot{x}}$, &c.

3. Sit $\dot{y} = P\dot{x}$, & 1^mo^. fluat uniformiter $\dot{x}$; tum erunt $\ddot{y} = \dot{P}\dot{x}$, $\dddot{y} = \ddot{P}\dot{x}$, $\ddddot{y} = \dddot{P}\dot{x}$, $\overset{\cdot\cdot\cdot\cdot\cdot}{y} = \ddddot{P}\dot{x}$, &c.; unde $P = \frac{\dot{y}}{\dot{x}}$, $\dot{P} = \frac{\ddot{y}}{\dot{x}}$, $\ddot{P} = \frac{\dddot{y}}{\dot{x}}$, $\dddot{P} = \frac{\ddddot{y}}{\dot{x}}$, &c.: deinde 2^do^. fluat uniformiter y, tum erunt $P = \frac{\dot{y}}{\dot{x}}$, $\dot{P} = -\frac{\dot{y}\ddot{x}}{\dot{x}^2}$, $\ddot{P} = -\frac{\dot{y}\dot{x}\dddot{x} - 2\ddot{x}^2\dot{y}}{\dot{x}^3}$, $\dddot{P} = -\frac{\dot{y}\dot{x}^2\ddddot{x} - 6\dot{y}\dot{x}\ddot{x}\dddot{x} + 6\dot{y}\ddot{x}^3}{\dot{x}^4}$, &c.: & ex æquatis duobus diverſis valoribus quantitatum $\dot{P}$, $\ddot{P}$, $\dddot{P}$, &c.; reſultabunt $\dot{P}$

$= \dot{y}$

$= \frac{\ddot{y}}{\dot{x}} = -\frac{\dot{y}\ddot{x}}{\dot{x}^2}$, unde $\ddot{y} = -\frac{\dot{y}\ddot{x}}{\dot{x}}$; deinde $\ddot{P} = \frac{\dddot{y}}{\dot{x}} = -\frac{\dot{y}\dot{x}\dddot{x} - 2\ddot{x}^2\dot{y}}{\dot{x}^3}$, & exinde $\dddot{y} = -\frac{\dot{y}\dot{x}\dddot{x} - 2\ddot{x}^2\dot{y}}{\dot{x}^2}$; tum $\dddot{P} = \frac{\ddddot{y}}{\dot{x}} = -\frac{\dot{y}\dot{x}^2\ddddot{x} - 6\dot{y}\dot{x}\ddot{x}\dddot{x} + 6\dot{y}\ddot{x}^3}{\dot{x}^4}$; &c.: ſcribantur valores exinde deducti $-\frac{\dot{y}\ddot{x}}{\dot{x}}$, $-\frac{\dot{y}\dot{x}\dddot{x} - 2\ddot{x}^2\dot{y}}{\dot{x}^2}$, $-\frac{\dot{y}\dot{x}^2\ddddot{x} - 6\dot{y}\dot{x}\ddot{x}\dddot{x} + 6\dot{y}\ddot{x}^3}{\dot{x}^3}$, &c., pro $\ddot{y}$, $\dddot{y}$, $\ddddot{y}$, &c. in datâ fluxionali æquatione; & reſultat fluxionalis æquatio quæſita.

2. Datam æquationem involventem x & y & earum fluxiones, in quâ x fluat uniformiter, ita transformare, ut nec x nec y fluat uniformiter: aſſumatur $\dot{y} = p\dot{x}$, & exinde $\ddot{y} = \dot{p}\dot{x} + p\ddot{x} = \dot{p}\dot{x}$ cum x fluat uniformiter; ſed $p = \frac{\dot{y}}{\dot{x}}$ & $\dot{p} = \frac{\ddot{y}\dot{x} - \ddot{x}\dot{y}}{\dot{x}^2}$, ſcribatur hæc quantitas pro ſuo valore $\dot{p}$ in æquatione $\ddot{y} = \dot{p}\dot{x}$, & reſultat æquatio $\ddot{y} = \frac{\dot{x}\ddot{y} - \dot{y}\ddot{x}}{\dot{x}}$; eodem modo $\dddot{y} = \ddot{p}\dot{x} + 2\dot{p}\ddot{x} + p\dddot{x} = \ddot{p}\dot{x}$ cum x fluat uniformiter; in hâc expreſſione ſcribantur $\frac{\dot{y}}{\dot{x}}$ & $\frac{\ddot{y}\dot{x} - \ddot{x}\dot{y}}{\dot{x}^2}$ pro p & $\dot{p}$; & reſultat $\dddot{y} = \ddot{p}\dot{x} + 2\left(\frac{\ddot{y}\dot{x} - \ddot{x}\dot{y}}{\dot{x}^2}\right)\ddot{x} + \frac{\dot{y}}{\dot{x}} \times \dddot{x}$, & conſequenter $\ddot{p} = \frac{\dddot{y}\dot{x}^2 - 2\ddot{y}\dot{x}\ddot{x} + 2\ddot{x}^2\dot{y} - \dot{y}\dot{x}\dddot{x}}{\dot{x}^3} = \frac{\dddot{y}}{\dot{x}}$; &c.: in datâ fluxionali æquatione ſcribendi ſunt igitur valores $\frac{\dot{x}\ddot{y} - \dot{y}\ddot{x}}{\dot{x}}$ & $\frac{\dddot{y}\dot{x}^2 - 2\ddot{y}\dot{x}\ddot{x} + 2\ddot{x}^2\dot{y} - \dot{y}\dot{x}\dddot{x}}{\dot{x}^2}$ pro $\ddot{y}$ & $\dddot{y}$; & ſic deinceps: & reſultat æquatio quæſita.

3. In genere ſit $\dot{y} = pm\dot{x}$, & $m\dot{x}$ conſtans quantitas; & erit $\ddot{y} = m\dot{p}\dot{x}$, ſed $p = \frac{\dot{y}}{m\dot{x}}$ & exinde $\dot{p} = \frac{\ddot{y}m\dot{x} - \dot{m}\dot{x}\dot{y} - m\ddot{x}\dot{y}}{m^2\dot{x}^2}$; ſubſtituatur hic valor pro $\dot{p}$ in æquatione $\ddot{y} = m\dot{x}\dot{p}$, & habebimus æquationem $\ddot{y} = \frac{m\dot{x}\ddot{y} - \dot{m}\dot{x}\dot{y} - m\dot{y}\ddot{x}}{m\dot{x}}$; per eundem modum inveniri poſſunt $\dddot{y}.\ddddot{y}$, &c.

$\dddot{y}$, $\ddddot{y}$, &c.; ſcribantur hi valores pro $\ddot{y}$, $\dddot{y}$, $\ddddot{y}$, &c. in datâ æquatione, & reſultat quæſita.

Si vero requiratur, ut $q\dot{x} + r\dot{y}$ fluat uniformiter, eodem prorſus modo progrediendum eſt.

4. Datam fluxionalem æquationem $\alpha = o$ relationem inter x & y & earum fluxiones exprimentem, in quâ $p = \phi(x \,\&\, y)$ data functio quantitatum x & y fluit uniformiter, ita transformare, ut reſultet æquatio relationem inter x & p & earum fluxiones exprimens.

Reducantur duæ æquationes $\alpha = o$ & $p = \phi(x \,\&\, y, \&c.)$, ita ut exterminetur quantitas y & ejus fluxiones, & reſultat æquatio quæſita.

PROB. XXXV.

Duas vel plures (n) *fluxionales æquationes* (n+1) *vel plures variabiles quantitates habentes, in quibus una* x *fluit uniformiter, ita transformare, ut fluat uniformiter* y.

Pro $\ddot{y}$ ſcribatur in datis æquationibus ejus valor $-\frac{\dot{y}\ddot{x}}{\dot{x}}$, & pro $\dddot{y}$ fluxio prius inventa, & ſic deinceps; & ita transformabuntur æquationes ut fluat uniformiter y.

Et ſic transformari poſſunt duæ vel plures (n) æquationes ($n+1$) variabiles quantitates habentes; in quibus fluit uniformiter $m\dot{x}$, ita ut fluat uniformiter $p\dot{x}$ vel $p\dot{x} + q\dot{y}$, &c.

THEOR. XVIII.

1. Sit algebraica æquatio $P = Q$, e quâ deducatur fluxionalis $\dot{P} = \dot{Q}$: ſi modo utriſque hujuſce æquationis partibus addantur vel ſubducantur eædem quantitates; vel utræque partes fluxionalis æquationis in eaſdem quantitates ductæ; tum eadem manet fluxionalis æquatio, (forſan vero etiam adjiciantur novæ), cujus fluens erit $P = Q + a$, ubi a denotat invariabilem quantitatem ad

libitum aſſumendam: ſi vero hæ duæ partes æquationis haud ducantur in eaſdem ſed in æquales quantitates utcunque e datâ algebraicâ ($P = Q$) æquatione deductas, tum particularis fluentialis æquatio deducta e fluxionali æquatione reſultante etiam erit $P = Q$; quodſi generaliter corrigatur, i. e. in genere inveniatur æquatio fluentialis ad fluxionalem æquationem reſultantem, haud erit $P = Q + a$. e. g. ducatur utraque pars fluxionalis æquationis reſultantis in P & Q, & reſultat $P\dot{P} = Q\dot{Q}$, cujus fluentialis æquatio erit in genere $P^2 = Q^2 + a$, haud $P = Q + a$; dividatur autem utraque pars fluxionalis æquationis ($\dot{P} = \dot{Q}$) per P & Q reſpective, & reſultat $\frac{\dot{P}}{P} = \frac{\dot{Q}}{Q}$, cujus fluentialis æquatio in genere erit $P = aQ$, & haud $P = Q + a$.

Cor. 1. Hinc, ſi modo detur æquatio, cujus inveniatur fluxio; & in fluxionali æquatione reſultante ſubſtituantur quæcunque quantitates e datâ æquatione deductæ pro ſuis valoribus; reſultabunt diverſæ fluxionales æquationes, quarum eædem dantur particulares fluentiales, quamvis forſan hæ generaliter correctæ haud erunt eædem.

Cor. 2. Eadem etiam affirmari poſſunt de fluxionalibus æquationibus ſuperiorum ordinum, & de pluribus æquationibus plures variabiles quantitates habentibus.

2. E contra æquationes fluxionales diverſæ generaliter eaſdem præbeant fluentiales æquationes: aſſumatur enim fluentialis æquatio, in quâ unus terminus ſit invariabilis & ad libitum aſſumendus: ita transformetur hæc æquatio, ut fiat alter terminus invariabilis & ad libitum aſſumendus: harum duarum æquationum inveniantur fluxiones, & duarum fluxionalium æquationum reſultantium fluentes invenientur eædem æquationes.

Ex. Sit æquatio algebraica aſſumpta $ay^n + bx^r y^s + cx^t + d = o$, ubi termini d & a ſunt invariabiles quantitates ad libitum aſſumendæ: dividatur data æquatio per y^n, & reſultat $a + bx^r y^{s-n} + cx^t y^{-n} + dy^{-n} = o$; inveniantur fluxiones ex utriſque his æquationibus, & reſultant duæ fluxionales æquationes diverſæ, viz. $nay^{n-1}\dot{y} + sbx^r y^{s-1}\dot{y}$
$+ rb$

$+ rby^{s}x^{r-1}\dot{x} + tcx^{t-1}\dot{x} = 0$ & $rbx^{r-1}y^{s-n}\dot{x} + (s-n)bxy^{s-n-1}\dot{y} + tcx^{t-1}y^{-n}\dot{x} - ncx^{t}y^{-n-1}\dot{y} - ndy^{-n-1}\dot{y} = 0$; quæ duæ fluxionales æquationes eaſdem præbent fluentiales.

In hâc æquatione duæ vel plures ſunt invariabiles quantitates ad libitum aſſumendæ.

Et ſic de fluxionalibus æquationibus ſuperiorum ordinum, & pluribus æquationibus.

PROB. XXXVI.

Corrigere fluentes in genere.

1. Datis quibuſcunque fluxionalibus æquationibus; & quantitate, quæ eſt fluxionalis functio literarum in datis fluxionalibus æquationibus contentarum; datam quantitatem generaliter corrigere.

Sit data quantitas fluxio (n) ordinis; & $\overset{n-1}{\dot{v}}$ fluxio $n-1$ ordinis, quæ fluit uniformiter; tum correctio primæ fluentis erit $a \times \overset{n-1}{\dot{v}}$, ubi litera a invariabilem quantitatem denotat; ſi vero V ſit prima fluens datæ quantitatis, tum erit prima fluens correcta $V + a\overset{n-1}{\dot{v}}$: ſit $\overset{n-2}{\dot{w}}$ fluxio $n-2$ ordinis, quæ fluit uniformiter, & correctio ſecundæ fluentis erit ſ. $a\overset{n-1}{\dot{v}} + b\overset{n-2}{\dot{w}}$, ubi b denotat invariabilem coefficientem; ſit W fluens fluxionis $(V + a\overset{n-1}{\dot{v}})$; & ſecunda fluens correcta erit $W + b\overset{n-2}{\dot{w}}$; & ſic deinceps.

2. Fluentialem æquationem in genere corrigere; 1^mo^. ducatur utraque datæ æquationis pars in eandem quantitatem; inveniatur fluens æquationis reſultantis (ſi modo fieri poſſit), quæ ſit fluxionalis æquatio $(n-1)$ ordinis; alteri parti æquationis fluentis inventæ addatur vel ſubtrahatur quantitas $a\overset{n-1}{\dot{v}}$, ubi a denotat invariabilem quantitatem, & $\overset{n-1}{\dot{v}}$ denotat fluxionem $n-1$ ordinis, quæ fluit uniformiter; & vere corrigitur prædicta æquatio, & vera etiam eſt fluentialis æquatio,

quam deſignat data fluxionalis: eodem modo inveniri poteſt correctio æquationis reſultantis; & ſic deinceps.

Ex. 1. Sit data fluxio $2y\dot{y} = 3x^2\dot{x}$, cujus fluens eſt $y^2 = x^3$; alteri æquationis parti addatur vel ſubtrahatur invariabilis quantitas a ad libitum aſſumenda, & erit fluens correcta $y^2 = x^3 + a$, quæ eſt fluentialis æquatio prædictæ fluxionalis.

Cave vero, ne ita ratiocineris, quoniam $y^2 = x^3$, erit $y = x^{\frac{3}{2}}$, & exinde correcta fluens $y = x^{\frac{3}{2}} + a$: hæc æquatio minime erit generaliter fluentialis æquatio datæ fluxionalis $2y\dot{y} = 3x^2\dot{x}$; ſubſtituatur enim pro y in datâ fluxionali æquatione ejus valor aſſumptus $x^{\frac{3}{2}} + a$, & haud reſultant utræque æquationis partes inter ſe æquales.

Ex. 2. Sit data fluxionalis æquatio $\frac{\dot{x}}{x} = \frac{2\dot{y}}{y}$; & erit æquatio, cujus fluxio eſt prædicta æquatio log. $x =$ log. y^2; addatur invariabilis quantitas ad alteram hujuſce æquationis partem, & reſultat æquatio vere correcta log. $x = 2$ log. $y + a$, unde $x = ay^2$ eſt æquatio vere correcta, ſed non $x = y^2 + a$.

Cor. 1. Hinc cavendum eſt in reſolutionibus problematum vel arithmeticorum vel geometricorum, ne in corrigendis fluentibus errores in ſolutiones irrepant, & reſultans fluentialis æquatio haud ſit æquatio, quam deſignat problema: data quantitas ad libitum aſſumenda fluenti datæ fluxionalis æquationis ſemper adjungenda eſt, vel fluenti æquationis e datâ (ducendo utraſque æquationis partes in eandem & haud diverſas quantitates) deductæ; & haud quantitati e prædictâ fluente quocunque modo deductæ.

3. Si data fluxionalis æquatio $P = 0$ plures recipiat diviſores A, B, C, &c. i. e. $P = A \times B \times C \times$ &c. $= 0$; & inveniantur quantitates α, β, γ, &c, quæ in datas A, B, C, &c. ductæ præbent quantitates, quarum fluentes detegi poſſunt, & quæ ſint reſpective W, X, Y, &c. tum erunt diverſi valores datæ fluxionalis æquationis $W + a = 0$, $X + b = 0$, $Y + c = 0$, &c. ubi literæ a, b, c, &c. denotant invariabiles quantitates ad libitum aſſumendas.

Cor. 3. Eadem etiam applicari poſſunt ad plures fluxionales æquationes plures variabiles quantitates involventes; ſingula enim fluxionalis æquatio corrigenda eſt prædictâ methodo. Et ſic de æquationibus, in quibus continentur fluxiones ſuperiorum ordinum.

4. Cum nulla inveniatur quantitas; quæ in datam fluxionalem æquationem ducta, creat fluxionalem, cujus fluens inveniri poteſt, tum e datâ fluxionali æquatione exprimente relationem inter quantitates x & y & earum fluxiones inveniatur particularis valor (Y) quantitatis y terminis vero variabilis quantitatis x; ut corrigatur valor quantitatis y inventus, ſcribatur pro y ejus valor $Y+X$ (correc. quæſit.); & e datâ & reſultante æquationibus inveniatur X terminis quantitatis x, & erit correctio quæſita.

Cor. In generali correctione fluxionalis æquationis primi ordinis continetur invariabilis quantitas ad libitum aſſumenda, ſecundi vero ordinis duæ continentur invariabiles quantitates ad libitum aſſumendæ, & ſic deinceps.

Eadem etiam applicari poſſunt ad plures (n) æquationes fluxionales plures $(n+1)$ variabiles quantitates quorumcunque ordinum involventes.

5. Si vero generalis fluens haud inveniri poſſit finitis terminis, confugiendum eſt ad infinitas ſeries, e. g. datâ fluxionali æquatione n ordinis duas involvente variabiles quantitates x & y & earum fluxiones; e methodo inveniendi ſeries progredientes ſecundum dimenſiones cujuſcunque literæ x inveniatur ſeries, in quâ continentur (n) literæ invariabiles ad libitum aſſumendæ, & erit ſeries vere correcta.

THEOR. XIX.

1. Datâ generali fluente fluxionalis æquationis (m) ordinis, quæ continet (m) invariabiles quantitates $(A, B, C, D$, &c.) ad libitum aſſumendas, etiamque (r) variabiles quantitates $(x, y, z$, &c.): datis etiam (m) diverſis valoribus e ſingulis (r) variabilibus quantitatibus $(x, y, z$, &c.); qui ſint reſpective a, a', a'', &c.; b, b', b'', &c.; c, c', c'', &c.; &c.;

&c.; ſcribantur hi valores a, b, c, &c.; a', b', c', &c.; a'', b'', c'', &c.; &c.; reſpective pro ſuis valoribus in datâ generali fluente, & reſultabunt (m) æquationes totidem (m) incognitas quantitates habentes (A, B, C, D, &c.) e quibus deduci poſſunt quantitates A, B, C, D, &c.; quæ præbent correctiones datæ fluentis datis (m) valoribus e ſingulis variabilibus reſpondentes.

2. Sint duæ vel plures (n) fluxionales æquationes (m, r, s, &c.) ordinum reſpective, tres vel plures ($n+1$) variabiles quantitates x, y, w, &c. & earum fluxiones habentes, in quibus $\dot{x}$ eſt conſtans; inveniantur generales fluentes variabilium y, w, &c.; quæ ſint $y = \phi : (x)$, $w = \phi' : (x)$, &c. vel quantitates quâcunque aliâ methodo deſignatæ: & ſi modo ſint $m + r + s +$ &c. invariabiles quantitates ad libitum aſſumendæ in hiſce functionibus ϕ, ϕ', &c. tum ex $m + r + s +$ &c. invariabilibus quantitatibus ad libitum aſſumendis in unâ functione ϕ, deduci poſſunt $m + r + s +$ &c. invariabiles quantitates ad libitum aſſumendæ in ſingulis reliquis; vel quod idem eſt, prædictæ invariabiles quantitates in ſingulis diverſis functionibus erunt eædem.

PROB. XXXVII.

Datâ æquatione vel algebraicâ vel fluxionali, cujus radix y *exprimit ſummam ſeriei* $a x^{h} + b x^{h \pm m} + c x^{h \pm l} +$ &c. *ſecundum dimenſiones literæ* x *progredientis; invenire quantitatem alternis ſeriei terminis æqualem; & denique ſeriei terminis, quorum diſtantia a ſemetipſis ſit* $n+1$.

Si modo ſeries requiratur alternis ſeriei terminis æqualis; pro x, $\dot{x}$, $\ddot{x}$, &c. in datâ æquatione ſcribantur reſpective x, $\dot{x}$, $\ddot{x}$, &c. & $-x$, $-\dot{x}$, $-\ddot{x}$, &c. & exoriuntur duæ æquationes, quarum radices ſint y & y'; & erit ſumma quæſita $\frac{y + y'}{2}$: ſi vero requiratur ſumma ſecundi, quarti, ſexti, &c. terminorum, erit $\frac{y - y'}{2}$ ſumma quæſita. Si requiratur ſumma terminorum, quorum diſtantia a ſe invicem ſit $n+1$; ſint α, β, γ, δ, &c. reſpective radices æquationis $x^{n+1} - 1 = 0$, & in datâ æquatione pro x, $\dot{x}$, $\ddot{x}$, &c. ſcribatur reſpective αx, $\alpha\dot{x}$, $\alpha\ddot{x}$, &c. βx, $\beta\dot{x}$, $\beta\ddot{x}$, &c. γx, $\gamma\dot{x}$, $\gamma\ddot{x}$, &c. &c. & reſultant $n+1$ æquationes, quarum correſpondentes radices ſint y, y', y'', &c. & erit ſumma quæſita $\frac{y+y'+y''+\&c.}{n+1}$: & ſic e medit. algebr. inveniri poſſunt ſumma e ſecundo, $n+3$, $2n+4$, $3n+5$, &c. terminis; ſumma e tertio, $n+4$, $2n+5$, $3n+6$, &c. terminis; & ſic deinceps.

Cor. Si requiratur æquatio, cujus radix (z) eſt ſumma alternorum ſeriei terminorum, cujus ſumma eſt y: per prob. 25. inveniantur æquationes, quarum radices ſunt y & y': ergo e duabus reſultantibus æquationibus & tertiâ $\frac{y+y'}{2}=z$ inveniatur æquatio, cujus radix eſt z; & conſit coroll.

Et ſic inveniri poteſt æquatio, cujus radix eſt $\frac{y+y'+y''+\&c.}{n+1}$.

Cor. Eodem modo ratiocinari liceat de pluribus æquationibus plures incognitas quantitates habentibus: ſed haud tam facilis vel elegans dici poteſt hujuſce problematis ſolutio in æquationibus ac in quantitatibus datis: in æquationibus enim plurimæ dantur radices, quæ minime ad problema reſolvendum attinent.

PROB. XXXVIII.

Invenire, annon data æquatio $W=0$ *ſit generalis fluens datæ fluxionalis æquationis* ($\alpha=0$) *primi ordinis.*

1. Sit a quantitas invariabilis ad libitum aſſumenda, quæ in datâ fluxionali æquatione haud invenitur, tum ex æquatione $W=0$ inveniatur $a=V$, ubi V eſt quantitas in quâ non continetur litera (a); tum, ſi $\frac{\alpha}{\dot{V}}$ ad minimos terminos reducta ſit quantitas, quæ nullas continet fluxiones, erit $W=0$ fluens generalis datæ fluxionalis æquationis; ſin aliter non.

2. Si non deduci poſſit $a = V$; reducantur duæ æquationes $W = o$ & $\dot{W} = o$ in unam, ita ut exterminetur quantitas a, & reſultet æquatio fluxionalis $(\beta = o)$; tum, ſi quotiens $\left(\frac{\alpha}{\beta}\right)$ ad minimos terminos reducta nullas contineat fluxiones, $W = o$ erit fluens prædicta.

2. 2. Si quotiens $\frac{\alpha}{\beta}$ vel $\frac{\alpha}{\dot{V}}$ contineat fluxiones, reducantur utræque fluxionales æquationes $(\alpha = o$ & $\beta = o)$, ita ut fluxiones $(\dot{x}, \dot{y}$, &c.) in iis contentæ ſolummodo habeant unam dimenſionem, & reſultent quantitates $\pi = o$ & $\varrho = o$, &c.; tum ſi quotiens $\left(\frac{\pi}{\varrho}\right)$ ad minimos terminos reducta nullas contineat fluxiones, fluens quæſita erit $W = o$. Si autem non ita reduci poſſint prædictæ fluxionales æquationes $\alpha = o$ & $\beta = o$, ut ſolummodo habeant unam dimenſionem fluxiones contentæ; tum per medit. algebr. exterminentur irrationales functiones fluxionum $(\dot{x}, \dot{y}$, &c.) ex prædictis æquationibus $\alpha = o$ & $\beta = o$, & reſultent æquationes $\gamma = o$ & $\delta = o$; deinde inveſtigetur, annon æquationes $\gamma = o$ & $\delta = o$ communem habeant diviſorem, qui involvit fluxiones $\dot{x}$ & $\dot{y}$; ſi communem habeant diviſorem, tum $W = o$ erit generalis fluens fluxionalis æquationis $\alpha = o$; vel in æquatione $W = o$ continetur diviſor, qui erit fluens fluxionalis æquationis $\alpha = o$.

3. Si $W = o$ ſit generalis fluens fluxionalis æquationis $(\alpha = o)$ ordinis (n), & in eâ $(W = o)$ contineantur (n) invariabiles quantitates $(a, b, c$, &c.) ad libitum aſſumendæ; tum ex æquatione $W = o$ inveniatur æquatio $a = V$, cujus fluxio $\dot{V} = o$ haud involvat quantitatem a, deinde ex æquatione $\dot{V} = o$ inveniatur $b = V'$, cujus fluxio $\dot{V}'$ haud continet vel quantitatem a vel b, & ſic deinceps; uſque donec exterminantur omnes prædictæ (n) invariabiles quantitates ad libitum aſſumendæ, quæ in datâ fluxionali æquatione non continentur; & reſultet fluxionalis æquatio $(\varrho = o)$ ordinis (n); tum, ſi fractio $\frac{\alpha}{\varrho}$ ad minimos terminos reducta, nullas contineat fluxiones ordinis (n), erit $W = o$ generalis fluens fluxionalis æquationis $\alpha = o$.

4. Si non deduci poſſit $a = V$, &c.; tum reducantur duæ æquationes $W = o$ & $\dot{W} = o$ in unam, ita ut exterminetur incognita quantitas a, & ſic de reliquis $b = V'$, &c.; & poſt ſingulas has reductiones reſultet fluxionalis æquatio $\varrho' = o$ ordinis (n), in quâ nec continetur litera a, nec b, nec c, &c.: tum, ſi fractio $\frac{\alpha}{\varrho'}$ ad minimos terminos reducta nullas contineat fluxiones ordinis (n), erit $W = o$ generalis fluens fluxionalis æquationis $\alpha = o$.

4. 2. Si quotientes $\frac{\alpha}{\varrho}$ vel $\frac{\alpha}{\varrho'}$ contineant fluxiones ordinis (n), reducantur utræque fluxionales æquationes $\alpha = o$ & $\varrho' = o$, ita ut in iis fluxio ordinis (n) variabilis quantitatis unam ſolummodo habeat dimenſionem, & reſultent æquationes $\sigma = o$ & $\gamma = o$; tum, ſi fractiones $\frac{\sigma}{\varrho}$ vel $\frac{\sigma}{\gamma}$ ad minimos terminos reductæ contineant nullas fluxiones ordinis (n), erit $W = o$ generalis fluens datæ fluxionalis æquationis $\alpha = o$, ſin aliter non.

4. 3. Si vero non ita reduci poſſint prædictæ æquationes, ut fluxio ordinis (n) variabilis quantitatis ſolummodo habeat unam dimenſionem; tum per medit. algeb. ita reducantur duæ æquationes $\alpha = o$ & $\varrho' = o$, ut exterminentur irrationales functiones fluxionum in iis contentarum, & reſultent æquationes $\beta' = o$ & $\sigma' = o$; tum, ſi $\beta' = o$ & $\sigma' = o$ habeant communem diviſorem, qui in ſe continet fluxionem ordinis (n) prædictæ variabilis quantitatis, fluentialis æquatio $W = o$ continet in ſe generalem fluentem fluxionalis æquationis in datâ fluxionali æquatione $\alpha = o$ contentæ.

5. Si vero dentur plures æquationes fluxionales $\alpha = o$, $\beta = o$, $\gamma = o$, &c.; & fluentiales plures variabiles quantitates habentes; tum per eandem methodum ita reducantur fluentiales æquationes, ut exterminentur (n) invariabiles quantitates generaliter aſſumptæ, quæ in datis fluxionalibus æquationibus ($\alpha = o$, $\beta = o$, &c.) non continentur,

tur, & reſultent fluxionales æquationes $\lambda = o$, $\mu = o$, $\nu = o$, &c.; deinde inveniatur, annon datæ fluxionales æquationes $\alpha = o$, $\beta = o$, $\gamma = o$, &c. & reſultantes $\lambda = o$, $\mu = o$, $\nu = o$, &c. habeant communem diviſorem, qui continet fluxionem (n) ordinis, quod per medit. algebr. perfici poteſt ex reducendo fluxionales æquationes, ita ut exterminentur irrationales functiones fluxionum maximi ordinis in ſingulis æquationibus contentæ, & exinde per methodum communes diviſores inveniendi diviſores quæſiti deduci poſſunt, ſi modo quidam ſint.

5. Hoc problema nonnunquam reſolutionem recipere poteſt ex ſubſtitutione valorum nonnullarum incognitarum quantitatum & earum fluxionum ex datis fluentialibus æquationibus deductarum pro ſuis valoribus in fluxionalibus æquationibus, & ſi nihilo evadant æquales prædictæ fluxionales æquationes; tum fluentiales erunt reſolutiones fluxionalium æquationum, ſin aliter non.

Quamvis quantitates ex datâ fluentiali æquatione acquiſitæ, & in datâ fluxionali ſubſtitutæ, eam reddent nihilo æqualem; attamen nonnunquam data fluentialis æquatio non vere dici poteſt fluens datæ fluxionalis, ex eo quod in generali fluente non continetur; & quamvis nonnunquam etiam in generali fluente continetur, attamen variabiles quantitates ex aſſumptis evadant invariabiles, tum non proprie dici poteſt fluens datæ fluxionalis æquationis, nam omnino contradicit hypotheſi fluxionalis æquationis; hoc ſemper evadet, ex eo quod aliqua quantitas in reſolutione aſſumitur invariabilis vel nihilo æqualis vel penitus ex æquatione ejicitur, quæ in æquatione variabilis ſupponitur. e. g. Sit data fluxionalis æquatio $\alpha\dot{P} + \beta P\dot{q} = o$; tum $P = o$ erit fluens fluxionalis æquationis $\alpha\dot{P} = o$, & non fluens fluxionalis æquationis $\beta P\dot{q} = o$, ni P ſit functio quantitatis q, & conſequenter non fluens datæ fluxionalis æquationis; attamen, ſi modo ſcribantur $P = o$ & $\dot{P} = o$ pro earum valoribus in datâ fluxionali æquatione $\alpha\dot{P} + \beta P\dot{q} = o$, tum proprie evaneſcet terminus $\alpha\dot{P} = o$, ex eo quod $\dot{P} = o$; etiamque evaneſcet terminus $\beta P\dot{q}$, ex eo quod $P = o$, at non proprie; nam in datâ fluxionali æquatione q ſupponitur variabilis;

ſed in aſſumptâ æquatione pro particulari valore penitus ex æquatione ejicitur q. e. 2. Sit $x\dot{x}+y\dot{y}=\dot{y}\sqrt{(x^2+y^2-a^2)}$; aſſumatur $\sqrt{(x^2+y^2-a^2)}=o$ pro particulari fluente datæ fluxionalis æquationis; ſi $x^2+y^2-a^2=o$; tum $x\dot{x}+y\dot{y}=o$, etiamque $\dot{y}\sqrt{(x^2+y^2-a^2)}=o$ & conſequenter $x\dot{x}+y\dot{y}=\dot{y}\sqrt{(x^2+y^2-a^2)}$; ſed non recte $\dot{y}\sqrt{(x^2+y^2-a^2)}=o$, nam $x^2+y^2-a^2$ non eſt functio fluentis (y) fluxionis $(\dot{y})$, & conſequenter non recte $x\dot{x}+y\dot{y}=\dot{y}\sqrt{(x^2+y^2-a^2)}$: etiamque hæç particularis fluens in generali non continetur, nam generalis fluens fluxionalis æquationis $x\dot{x}+y\dot{y}=\dot{y}\sqrt{(x^2+y^2-a^2)}$ (unde $\dot{y}=\dfrac{x\dot{x}+y\dot{y}}{\sqrt{(x^2+y^2-a^2)}}$) erit $y=\sqrt{(x^2+y^2-a^2)}+A$, ubi A eſt quæcunque invariabilis quantitas; & non ita aſſumi poteſt A, ut præcedens $x^2+y^2-a^2=o$ in hâc $y=\sqrt{(x^2+y^2-a^2)}+A$ contineatur: hæc autem fluens $x^2+y^2-a^2=o$ potius dici poteſt reſolutio fluxionalis æquationis $x\dot{x}+y\dot{y}=o$ & algebraicæ $\sqrt{(x^2+y^2-a^2)}=o$.

Cor. Hinc etiam in fluxionalibus æquationibus ſuperiorum ordinum, ſi quæcunque æquatio ſit vere fluens prioris partis fluxionalis æquationis; ſed non vere fluens reliquæ æquationis partis, attamen diviſor vel fluens fluxionalis quantitatis inferioris ordinis (m), quæ eſt diviſor reliquæ æquationis partis, cujus partis fluxionum in eâ contentarum ordo eſt major quam m; tum æquatio prædicta non vere erit fluens datæ fluxionalis æquationis.

6. Datâ fluxionali æquatione $\alpha=o$ vel $\alpha\dot{P}+\beta P\dot{q}=o$; invenire, annon data æquatio $\pi=o$ vel $P=o$ ſit fluens datæ fluxionalis æquationis: in datis æquationibus $\alpha=o$ & $\pi=o$ pro variabili y in iis contentâ, ejus fluxione $\dot{y}$, &c. ſcribantur $v-o$, $\dot{v}-\dot{o}$, &c. ubi o eſt quam minima quantitas; tum ita reducantur duæ reſultantes æquationes, ut exterminentur variabilis v & ejus fluxiones, & reſultet æquatio $\varrho=o$ relationem inter x & o & earum fluxiones exprimens: & ſit o^λ minima poteſtas quantitatis o in æquatione reſultante $\varrho=o$, unde, ſi modo ſit fluxionalis æquatio primi ordinis, erit $\frac{\dot{o}}{o^\lambda}=\int.X\dot{x}$,

ubi X eſt functio quantitatis x, & exinde $\frac{-1}{(\lambda-1)o^{\lambda-1}} = \int. X\dot{x} + A$, ubi A eſt quæcunque invariabilis quantitas; unde $(1-\lambda) \times o^{\lambda-1} = \frac{1}{\int. X\dot{x} + A}$; quodſi y in prædictâ æquatione $\alpha = o$ ſit vere fluens, tum ſaltem $y \pm o$, ubi o eſt quam minima quantitas, erit etiam fluens; ut vero $\frac{1}{\int. X\dot{x} + A}$ fiat nihilo æqualis, neceſſe eſt ut A evadat infinita quantitas, & conſequenter $\lambda - 1$ ſit affirmativa quantitas.

Si $\lambda = 1$, tum erit log. $o = \int. X\dot{x} + \log. A$, unde $o = A \times e^{\int. X\dot{x}}$, & ſi A, tum etiam o evadet nihil.

PROB. XXXIX.

Invenire, annon data algebraica æquatio ſit particularis vel generalis fluens datæ fluxionalis æquationis n *ordinis.*

1^mo^. Reducantur data algebraica & fluxionalis æquatio ad minimos terminos; i. e. auferatur ſingula quantitas, quæ ducitur in totam æquationem vel fluxionalem vel algebraicam; etiamque reducantur omnes fractiones ad minimos terminos ex dividendo earum numeratores & denominatores per maximos eorum communes diviſores.

Datæ algebraicæ æquationis inveniantur fluxiones primi, ſecundi, &c. n ordinis; & exinde prima, ſecunda, tertia, &c. fluxiones alterius variabilis quantitatis y in datâ fluxionali & algebraicâ æquatione contentæ; quibus fluxionibus pro ſuis valoribus in datâ fluxionali æquatione ſubſtitutis; æquatio reſultans vel nihilo æqualis erit, vel reſultans & data algebraica æquatio communem habent diviſorem, ſi modo ſit fluens: aliter; reducantur datæ algebraicæ & fluxionales æquationes in unam, ita ut exterminentur omnes præter unam variabiles quantitates & earum fluxiones; & ex æquatione reſultante & principiis prius traditis facile conſtabit; annon data algebraica æquatio ſit fluens datæ fluxionalis.

Ex. 1. Sit $\alpha\dot{P}=o$ data fluxionalis æquatio, & $P\times Q=o$ data algebraica; ſint $\dot{P}=p\dot{x}+p'\dot{y}$ & $\dot{Q}=q\dot{x}+q'\dot{y}$; tum fluxio algebraicæ æquationis $(PQ=o)$ erit $Qp\dot{x}+Qp'\dot{y}+Pq\dot{x}+Pq'\dot{y}=(Qp+Pq)\dot{x}+(Qp'+Pq')\dot{y}=o$; in datâ fluxionali æquatione $\alpha\dot{P}=\alpha p\dot{x}+\alpha p'\dot{y}=o$ pro $\dot{x}$ ſcribatur ejus valor $-\frac{(Qp'+Pq')}{Qp+Pq}\times\dot{y}$ ex præcedente æquatione deductus, & reſultat $\alpha p\times\frac{Qp'+Pq'}{Qp+Pq}-\alpha p'=P\times\alpha\times\frac{pq'-qp'}{Qp+Pq}=o$; ſed P erit communis diviſor æquationum $Q\times P=o$ & reſultantis æquationis $P\times\alpha\times\frac{pq'-qp'}{Qp+Pq}=o$, & conſequenter $P=o$ erit fluens.

Ex. 2. Invenire, annon data æquatio $(A)\ y^m+x^n+y^rx^s=o$ ſit fluens fluxionalis æquationis $(B)\ m\times(m-1)y^{m-2}\dot{y}^2+my^{m-1}\ddot{y}+n\times(n-1)x^{n-2}\dot{x}^2+r\times(r-1)y^{r-2}x^s\dot{y}^2+ry^{r-1}x^s\ddot{y}+2rsy^{r-1}x^{s-1}\dot{y}\dot{x}+s(s-1)y^rx^{s-2}\dot{x}^2+may^m\dot{y}\dot{x}+nax^{n-1}y\dot{x}^2+ray^rx^s\dot{y}\dot{x}+say^{r+1}x^{s-1}\dot{x}^2+b(y^m+x^n+y^rx^s)\dot{y}^2=o$; datæ æquationis A inveniatur ſecunda fluxio, &c. & pro $\ddot{y}$ in æquatione B ſcribatur ejus valor e ſecundâ fluxione æquationis A deductus; in æquatione exinde reſultante ſcribatur pro $\dot{y}$ ejus valor e primâ fluxione prædictæ æquationis A deductus, & data æquatio erit diviſor æquationis reſultantis; e principiis prius traditis conſtat datam algebraicam æquationem non proprie pro fluente fluxionalis æquationis habendam eſſe.

2. Ex iiſdem principiis etiam inveniri poteſt, utrum data fluxionalis æquatio ſit fluens alterius datæ fluxionalis æquationis, necne.

Si in datâ æquatione $\pi=o$, quæ eſt fluens (r) ordinis datæ fluxionalis æquationis $\alpha=o$, contineantur (r) invariabiles quantitates a, b, c, &c. ad libitum aſſumendæ, quæ in datâ fluxionali æquatione haud continentur; & quæ in diverſas functiones variabilium quantitatum in datâ æquatione contentarum ducuntur; i. e. quæ inter ſe ſunt independentes:

pendentes: tum, ſi data æquatio $\pi = 0$ ſit fluens datæ fluxionalis æquationis, i. e. ſi in æquatione $\alpha = 0$ ſcribantur quæcunque quantitates pro ſuis valoribus ex æquatione $\pi = 0$ deductis; & evaneſcat æquatio reſultans, erit $\pi = 0$ generalis fluens (r) ordinis fluxionalis æquationis $\alpha = 0$: ſin autem non contineantur (r) prædicti generis invariabiles quantitates, exceptis excipiendis erit particularis fluens: particularis ſemper continetur in fluente generali.

3. Sint $\alpha = 0$, $\beta = 0$, $\gamma = 0$, &c. fluxionales æquationes ordinum $(n, m, r,$ &c.) reſpective; & $\lambda = 0$, $\mu = 0$, $\nu = 0$, &c. fluentiales æquationes, in quibus continentur $(n + m + r +$ &c.) invariabiles & independentes ad libitum aſſumendæ quantitates, quæ in datis æquationibus non inveniuntur: in æquationibus $\alpha = 0$, $\beta = 0$, $\gamma = 0$, &c. ſcribantur quæcunque quantitates pro ſuis valoribus ex æquationibus $\lambda = 0$, $\mu = 0$, $\nu = 0$, &c. deductis; & ſi evaneſcant æquationes reſultantes; tum erunt $\lambda = 0$, $\mu = 0$, $\nu = 0$, &c. generalis fluens datarum fluxionalium æquationum $\alpha = 0$, $\beta = 0$, $\gamma = 0$, &c.

Cor. Sit $\pi = 0$ æquatio, quæ continet (m) invariabiles quantitates a, b, c, d, &c. ad libitum aſſumendas & inter ſe independentes, quæ in æquatione $\varrho = 0$ non inveniuntur: in æquatione $\varrho = 0$ ſcribantur quæcunque quantitates ex æquationibus $\pi = 0$, $\dot{\pi} = 0$, $\ddot{\pi} = 0$, $\dddot{\pi} = 0$, $\ldots \overset{r}{\pi} = 0$ utcunque deductæ pro ſuis valoribus; & in æquatione reſultante non plures quam (r) e prædictis invariabilibus quantitatibus $(a, b, c, d,$ &c.) evaneſcent.

THEOR. XX.

Sit fluxionalis æquatio r ordinis; & ſi modo ejus fluens (r ordinis) inveniri poſſit, tum a fortiori ejus fluentes $r - 1$, $r - 2$, &c. ordinum detegi poſſunt. Inveniatur enim ejus fluens (r ordinis), cujus prima fluxio erit fluxionalis æquatio primi ordinis, & fluens $r - 1$ ordinis datæ fluxionalis æquationis; & ſic deinceps; unde conſtat theor.

PROB. XL.

Datâ fluxionali æquatione tres (x, y, z,) *variabiles quantitates & earum fluxiones habente; invenire utrum una* (x) *ex iis exprimi poteſt in terminis duarum reliquarum* (y, z), *necne.*

1. Aſſumatur x tanquam invariabilis quantitas, deinde inveniatur quantitas (p), quæ in reſultantem æquationem ducta, creat æquationem, cujus fluens (P) inveniri poteſt; deinde aſſumatur y tanquam invariabilis, & inveniatur correſpondens quantitas q, quæ in æquationem exinde reſultantem ducta, creat fluxionem, cujus fluens $(\mathcal{Q})$ inveniri poteſt; 3^{tio}. aſſumatur z invariabilis, & inveniatur correſpondens quantitas r, quæ in æquationem reſultantem ducta præbet fluxionem, cujus fluens (R) inveniri poteſt.

Si vero quantitates p & q habeant omnes eoſdem factores, in quibus invenitur z; & quantitates p & r eoſdem involvant factores, in quibus continetur y; & tertio quantitates q & r eoſdem habeant factores, in quibus invenitur x; i. e. ſint correſpondentes; & ſi omnes termini, in quibus continentur literæ (x & y & z) iidem ſint in tribus fluentibus P, $\mathcal{Q}$ & R; & omnes termini, in quibus inveniuntur z, y, x ſint iidem in duabus fluentibus P & $\mathcal{Q}$, P & R, $\mathcal{Q}$ & R reſpective; tum inveniri poteſt fluens datæ fluxionalis æquationis, ſin aliter vero non.

Ex. 1. Datâ fluxionali æquatione $((x^2y+c)\dot{z}+(x^2z+d)\dot{y}+2xzy\dot{x})\surd(z^2+cx^2)+z\dot{z}+cx\dot{x}=o$, invenire utrum ejus fluens exprimi poteſt in algebraicis terminis quantitatum (x, y, z), necne. Supponatur x eſſe invariabilis quantitas, & reſultat $(x^2z+d)\surd(z^2+cx^2)\dot{y}+((x^2y+c)(\surd(z^2+cx^2)+z)\dot{z}=o$; ducatur hæc æquatio in $\frac{1}{\surd(z^2+cx^2)}$, & reſultat $(x^2z+d)\dot{y}+\left(x^2y+c+\frac{z}{\surd(z^2+cx^2)}\right)\dot{z}=o$, cujus fluens (P) erit $x^2zy+dy+cz+\surd(z^2+cx^2)+A$: ubi A eſt functio quantitatis x: eodem modo ſupponatur z invariabilis, & ducatur æquatio reſultans in prædictam quantitatem $\frac{1}{\surd(z^2+cx^2)}$, & invenietur fluens ($R$) reſultantis fluxionis $x^2yz+dy+\surd(z^2+cx^2)+B$, ubi B eſt functio quantitatis z: & ultimo ſupponatur y invariabilis, & ducatur æquatio reſultans in $\frac{1}{\surd(z^2+cx^2)}$, & fluxionis reſultantis invenietur fluens (Q) $x^2yz+cz+\surd(z^2+cx^2)+C$, ubi C eſt functio quantitatis y: terminus vero x^2yz, in quo continentur tres variabiles quantitates x, y, z, idem eſt in tribus fluentibus P, Q & R; & termini, in quibus continetur z, iidem ſunt in fluentibus P & Q; termini, in quibus habetur y, iidem ſunt in fluentibus P & R, &c.; unde fluens datæ fluxionis invenitur $x^2yz+dy+cz+\surd(z^2+cx^2)+A'=0$, ubi A' denotat invariabilem quantitatem ad libitum aſſumendam.

2. 1. Sæpe vero ex hâc methodo haud conſtat reſolutio problematis, quoniam haud cognoſcuntur methodi, e quibus deduci poſſunt prædicti cognati multiplicatores; in his caſibus confugiendum eſt ad infinitas ſeries, i. e. inveniatur variabilis quantitas (x) in terminis progredientibus ſecundum dimenſiones reliquarum (y,z), ſi modo fieri poſſit, & confit problema.

2. 2. Forſan vero in fluxionalibus æquationibus ſuperiorum (n) ordinum haud inveniri poteſt variabilis quantitas (x) in terminis reliquarum (y, z); ſed forſan inveniri poteſt fluxio ($\overset{m}{\dot{x}}$) ordinis m, ubi m minor eſt quam n, in terminis ſecundum dimenſiones reliquarum (y, z) & earum fluxiones ordinum haud majorum quam $n-m$ progredientibus, i. e. reduci poteſt data fluxionalis æquatio in alteram minoris ordinis quam n.

2. 3. Eadem principia etiam applicari poſſunt ad æquationem quatuor vel plures variabiles quantitates & earum fluxiones habentem; & ſic ad duas vel plures æquationes tres vel plures variabiles quantitates & earum fluxiones involventes.

3. Sit æquatio $P\dot{x}+Q\dot{y}+R\dot{z}=0$; & M quantitas, quæ in datam æquationem ducta, præbet fluxionem, cujus fluens inveniri poteſt,

teſt, i. e. ſit quantitas $MP\dot{x} + MQ\dot{y} + MR\dot{z}$ integrabilis; & ſint $\dot{M} = m\dot{x} + m'\dot{y} + m''\dot{z}$, $\dot{P} = p\dot{x} + p'\dot{y} + p''\dot{z}$, $\dot{Q} = q\dot{x} + q'\dot{y} + q''\dot{z}$ & $\dot{R} = r\dot{x} + r'\dot{y} + r''\dot{z}$; tum ex theor. 2. erit $Mp' + Pm' = Mq + Qm$, $Mp'' + Pm'' = Mr + Rm$ & $Mq'' + Qm'' = Mr' + Rm'$; ducantur æquationes exinde reſultantes $Mp' + Pm' - Mq - Qm = o$, $Mp'' + Pm'' - Mr - Rm = o$ & $Mq'' + Qm'' - Mr' - Rm' = o$ reſpective in R, Q & P; & ſumma trium quantitatum reſultantium erit $M(R \times (p' - q) - Q(p'' - r) + P(q'' - r') = o$; unde æquatio erit integrabilis, ſi modo $R(p' - q) - Q(p'' - r) + P(q'' - r) = o$; ſin aliter non.

3. 2. Sit æquatio $P\dot{x} + Q\dot{y} + R\dot{z} + S\dot{v} + T\dot{w} +$ &c. $= o$: 1^{mo}. aſſumantur omnes quantitates præter quaſcunque tres (x, y & z) tanquam invariabiles, & reſultat æquatio $P\dot{x} + Q\dot{y} + R\dot{z} = o$; per præcedentem methodum inveniatur, annon hæc æquatio ſit integrabilis; ſi modo ſit integrabilis, tum aſſumantur omnes quantitates præter tres alias (x, y & v) tanquam invariabiles; & inveniatur, annon æquatio ex hâc hypotheſi reſultans ſit integrabilis; ſi modo ſit integrabilis, tum aſſumantur omnes quantitates præter quaſcunque tres alias tanquam invariabiles, & ſi omnes æquationes ex hujuſmodi aſſumptis reſultantes, ſint integrabiles; tum integrari poteſt reſultans æquatio; ſin aliter vero non.

Fluxiones $\left(\frac{\ddot{M}P}{\dot{y}\dot{z}}\right) = \left(\frac{\ddot{M}Q}{\dot{x}\dot{z}}\right) = \left(\frac{\ddot{M}R}{\dot{x}\dot{y}}\right)$; & $\left(\frac{\ddot{M}P}{\dot{y}\dot{v}}\right) = \left(\frac{\ddot{M}Q}{\dot{x}\dot{v}}\right) = \left(\frac{\ddot{M}S}{\dot{y}\dot{x}}\right)$, &c.; $\left(\frac{\dddot{M}P}{\dot{y}\dot{z}\dot{v}}\right) = \left(\frac{\dddot{M}Q}{\dot{x}\dot{z}\dot{v}}\right) = \left(\frac{\dddot{M}R}{\dot{x}\dot{y}\dot{v}}\right) = \left(\frac{\dddot{M}S}{\dot{x}\dot{y}\dot{z}}\right)$, &c., ubi quantitas $\left(\frac{\dddot{V}}{\dot{x}\dot{y}\dot{z}}\right) = Z$ denotat quantitatem ex hac methodo inventam; nempe ex deducendo fluxionem $W\dot{x}$ quantitatis V ex hypotheſi quod x ſolummodo ſit variabilis; deinde fluxionem $Y\dot{y}$ quantitatis W, ex hypotheſi quod y ſolummodo ſit variabilis; & tertio fluxionem $Z\dot{z}$ quantitatis Y ex hypotheſi quod z ſolummodo ſit variabilis.

Cor. Sit æquatio fluxionalis (n) variabiles quantitates & earum fluxiones habens; tum reſultant $n\times\frac{n-1}{2}\times\frac{n-2}{3}$ æquationes, e quibus dici poteſt, annon data fluxionalis æquatio ſit integrabilis.

3. 3. Si modo fluxiones variabilium ſuperiorum (n, $n-1$, $n-2$, &c.) ordinum in datâ æquatione ($\alpha = o$) contineantur: ducatur data æquatio in M, functionem variabilium (x, y, z, &c.) in datâ æquatione contentarum, & earum fluxionum ($n-1$, $n-2$, $n-3$, . 3, 2, 1) ordinum; aſſumatur N functio incognita prædictarum (x, y, z, &c.) variabilium; deinde inveniatur fluxio (n) ordinis functionis N; & ex æquatis correſpondentibus terminis reſultantis & quantitatis $M\alpha$ fluxionis n ordinis deduci poteſt, annon data æquatio ſit integrabilis.

PROB. XLI.

Datâ æquatione relationem inter quantitates x *&* y *& earum fluxiones deſignante; & datâ quantitate, quæ eſt fluxionalis functio* (P) *literarum* x *&* y; *invenire æquationem duas involventem incognitas quantitates* z *&* v, *ita ut fluens quantitatis (quæ eſt eadem fluxionalis functio* (P) *literarum* z *&* v, *ac prædicta fuit functio literarum* x *&* y) *algebraicè e fluente datæ quantitatis, quæ fuit data fluxionalis functio literarum* x *&* y, *deduci poteſt.*

Aſſumantur duæ æquationes, quæ exprimunt algebraicas relationes inter x, y & z, v; ita ut fluens functionis P literarum z & v deduci poteſt algebraicè e fluente functionis P, i. e. e datâ fluxionali functione literarum x & y; ita reducantur æquationes reſultantes, ut exterminentur incognitæ quantitates x & y, & reſultat quæſita æquatio relationem inter z & v exprimens.

Ex. Datâ æquatione relationem inter x & y & earum fluxiones exprimente; ſit data quantitas functio ($P = y\dot{x}$) quantitatum x & y: aſſumantur duæ æquationes $x = az + bv + c$ & $y = pz + qv + r$; &

& conſequenter $a'x + b'y + c' = z$ & $p'x + q'y + r' = v$; fluens functionis $P = v\dot{z}$ inveniri poteſt, ſi modo fluens fluxionis $y\dot{x}$ inveniri poſſit; nam $y\dot{x} = (pz + qv + r) \times (a\dot{z} + b\dot{v})$, cujus fluens $\int. y\dot{x} = \frac{1}{2}apz^2 + raz + \frac{1}{2}bqv^2 + rbv + bpzv + \int.(aq - bp)v\dot{z}$; unde e fluente $\int. y\dot{x}$ deduci poteſt $\int. v\dot{z}$; ſcribantur hæ quantitates in datâ æquatione pro ſuis valoribus, & reſultat æquatio quæſita relationem inter z & v exprimens.

Cor. 1. Datâ fluente fluxionis $y\dot{x}$, ex eâ ſemper inveniri poteſt fluens fluxionis $v\dot{z}$ vel $z\dot{v}$: datis fluentibus trium fluxionum $y\dot{x}$, $y^2\dot{x}$, $yx\dot{x}$; & ex iis ſemper inveniri poſſunt fluentes fluxionum $v\dot{z}$, $v^2\dot{z}$, $vz\dot{z}$; $z\dot{v}$, $z^2\dot{v}$, $zv\dot{v}$: datis fluentibus ſex fluxionum $y\dot{x}$, $y^2\dot{x}$, $yx\dot{x}$, $y^3\dot{x}$, $y^2x\dot{x}$, $yx^2\dot{x}$, & ſemper ex iis algebraicè exprimi poſſunt fluentes fluxionum $v\dot{z}$, $v^2\dot{z}$, $vz\dot{z}$; $v^3\dot{z}$, $vz^2\dot{z}$, &c. $z\dot{v}$, $z^2\dot{v}$, &c. Datis vero fluentibus $\left(n . \frac{n-1}{2}\right)$ inter ſe independentibus fluxionum hujuſce formulæ $y^{n-1}\dot{x}$, $y^{n-2}x\dot{x}$, $y^{n-3}x^2\dot{x}$, &c. $y^{n-2}\dot{x}$, $y^{n-3}x\dot{x}$, $y^{n-4}x^2\dot{x}$, &c. $y^{n-3}\dot{x}$, $y^{n-4}x\dot{x}$, &c. . . . $y^2\dot{x}$, $yx\dot{x}$; $y\dot{x}$; & ex iis ſemper deduci poſſunt fluentes omnium conſimilium quantitatum $v^{n-1}\dot{z}$, $v^{n-2}z\dot{z}$, $v^{n-3}z^2\dot{z}$, &c. $v^{n-2}\dot{z}$, $v^{n-3}z\dot{z}$, &c. . . . $v^2\dot{z}$, $vz\dot{z}$; $v\dot{z}$; $z\dot{v}$; $z^2\dot{v}$, $zv\dot{v}$, &c. . .; $z^{n-1}\dot{v}$, $z^{n-2}v\dot{v}$, &c.

Cor. 2. Datis $n . \frac{n-1}{2}$ diverſis, i. e. inter ſe independentibus æquationibus hujuſce generis $a\int. y\dot{x} + b\int. y^2\dot{x} + c\int. yx\dot{x} + d\int. y^3\dot{x} +$ &c. . . . $k\int. y^{n-1}\dot{x} = l$; ubi literæ a, b, c, d, &c. k denotant datas invariabiles quantitates; tum facile conſtabit omnis valor cujuſcunque quantitatis hujuſce generis $p\int. v\dot{z} + q\int. v^2\dot{z} + r\int. vz\dot{z} +$ &c. $t\int. v^{n-1}\dot{z} + p'\int. z\dot{v} + q'\int. z^2\dot{v} +$ &c.; ubi p, q, r, &c.; t, p', q', r', &c. datas etiam denotant invariabiles quantitates.

Et ſic de datis conſimilibus quantitatibus.

THEOR.

THEOR. XXI.

1. Si area inter quoſcunque duos valores abſciſſæ (x) curvæ ſemper detegi poſſit, tum area inter quoſcunque datos valores cujuſcunque abſciſſæ (z) ejuſdem curvæ ſemper detegi poteſt.

Transformetur enim data æquatio relationem inter abſciſſam x & ejus ordinatam y exprimens in alteram relationem inter novam abſciſſam z & ejus correſpondentes ordinatas (v) deſignantem; per trigonometriam cognitum eſt $ax + by + c = z$ & $px + qy + r = v$; ubi a, b, c; $p, q,$ & r invariabiles denotant coefficientes inveniendas, unde fluxio datæ curvæ ad novam abſciſſam erit ut $v\dot{z} = (px+qy+r)$ $(a\dot{x} + b\dot{y})$; & conſequenter $\int . v\dot{z} = \int . (px + qy + r)(a\dot{x} + b\dot{y}) =$ $pa\frac{x^2}{2} + \frac{qby^2}{2} + rax + rby + qa\int . y\dot{x} + pbxy - pb\int . y\dot{x}$; & exinde e fluente $\int . y\dot{x}$ datâ ſemper acquiri poteſt $\int . v\dot{z}$.

2. Datis generaliter centris gravitatis ad duas abſciſſas, tum plerumque dabuntur centra gravitatis ad omnes abſciſſas: detur enim generaliter $\frac{\int . yx\dot{x}}{\int . y\dot{x}}$, dabuntur plerumque $\int . yx\dot{x}$ & $\int . y\dot{x}$: detur etiam centrum gravitatis generaliter ad alteram abſciſſam, quod e præcedente non pendet; tum dabitur etiam generaliter $\int . y^2\dot{x}$; ſed datis generaliter $\int . yx\dot{x}$, $\int . y\dot{x}$ & $\int . y^2\dot{x}$ per præcedentem propoſitionem ſemper acquiri poſſunt generaliter $\int . v^2\dot{z}$, $\int . vz\dot{z}$ & $\int . v\dot{z}$, & conſequenter generaliter centra gravitatis ad omnes abſciſſas.

Ex iiſdem principiis, ſi dentur generaliter centra percuſſionum, viz. $\frac{\int . yx^2\dot{x}}{\int . yx\dot{x}}$ ad quinque abſciſſas; tum plerumque ea dabuntur ad omnes abſciſſas.

3. E datis generaliter contentis ſolidorum a rotatione curvæ circa tres diverſos axes generatorum, quorum unus ſit $\int$. 3, 1456 &c. $\times$ $y^2\dot{x}$; dabuntur contenta ſolidorum a rotatione ejuſdem curvæ circa quoſcunque alios axes generatorum: viz. e tribus independentibus valoribus generalibus fluentium fluxionum $\alpha y^2\dot{x} + \beta yx\dot{x} + \gamma y\dot{x}$ erui poteſt generaliter $\int . v^2\dot{z}$.

4. Circa tres axes ſecum quoſcunque datos angulos facientes gyretur quæcunque curva, cujus abſciſſæ & ordinatæ ſunt reſpective x & y; transformetur data curva in alteram ſubſtituendo pro $x = d'w + b'u + c'$ & pro $y = p'w + q'u + r'$; ubi w & u reſpective denotant abſciſſam & ejus correſpondentes ordinatas novæ curvæ; & d', b', c', &c.; p', q' & r' invariabiles coefficientes: gyretur hæc curva reſultans circa ſuum axem, & e datis contentis ſolidorum datæ curvæ circa tres axes rotatione generatorum inter ſe independentibus deduci poteſt ſolidum generatum e rotatione poſterioris curvæ circa ſuum axem.

Conſtat e principiis prius traditis.

Ea, quæ prius tradita fuere de areis, centris gravitatis & percuſſionum, & contentis ſolidorum a rotatione datæ curvæ circa ejus axes; mutatis mutandis, æque ad areas, prædicta centra & ſolida cujuſcunque curvæ ex hâc ſubſtitutione exortæ applicari poſſunt.

Cor. 4. Si pro x & y in datâ æquatione ſcribantur reſpective $av^2 + bzv + cz^2 + dz + ev + A$ & $fv^2 + gvz + hz^2 + lz + mv + B$ vel quæcunque rationales vel irrationales quantitates; facile deduci poſſunt hujuſmodi propoſitiones.

Et ſic de fluxionalibus functionibus quantitatum z & v pro datis fluxionalibus functionibus quantitatum x & y in datâ æquatione vel datis æquationibus ſcriptis; exinde enim conſimilia deduci poſſunt.

THEOR. XXII.

In fluxione $y\dot{x}$ pro x & y ſcribantur reſpective $az^m + (b + cv)z^{m-1} + (d + ev + fv^2)z^{m-2} +$ &c. & $Az^r + (B + Cv)z^{r-1} + (D + Ev + Fv^2)z^{r-2} +$ &c. ubi $a, b, c, d, e, f,$ &c. $A, B, C, D, E, F,$ &c. ſunt quæcunque invariabiles coefficientes, m vero & r integri numeri; & ex fluentibus independentibus $(m + r - 1) . \frac{m+r}{2}$ fluxionum ex prædictis ſubſtitutionibus reſultantium datis, per ſimplices æquationes erui poſſunt fluentes omnium fluxionum exinde reſultantium; etiamque e prædictis fluentibus datis erui poſſunt fluentes quarumcunque fluxionum hujuſce formulæ $z^{\alpha} v^{\beta} \dot{z}$, ſi modo $\alpha + \beta$ haud major ſit quam $m + r - 1$, & α & β ſint integri numeri.

Totidem $(m + r - 1) \times \frac{m + r}{2}$ enim in datâ fluxione $y\dot{x}$ continentur diverſæ fluxiones, quarum fluentes ſunt independentes, viz. $\int. v\dot{z}$, $\int. v^2 \dot{z}$, $\int. vz\dot{z}$, $\int. v^2 \dot{z}$, $\int. v^2 z\dot{z}$, $\int. vz^2 \dot{z}$, &c.

Et ſic ratiocinari liceat de aliis ſubſtitutionibus in plurimis aliis fluxionibus; viz. exponentialibus, fluentialibus, &c.

THEOR. XXIII.

1. Sit data æquatio $(ay + bx + c)\dot{x} + (Ay + Bx + C)\dot{y} = o$; & ex datâ fluente fluxionis $y\dot{x}$ (quæ plerumque acquiri poteſt e datâ æquatione) erui poteſt fluens cujuſcunque formulæ $(hy^m + (k + lx) y^{m-1} + \&c.)\dot{x} + (Hy^m + (K + Lx) y^{m-1} + \&c.)\dot{y}$; ubi literæ h, k, l, &c. H, K, L, &c. quaſcunque invariabiles coefficientes reſpective denotant; & m eſt integer numerus.

Si enim detur fluens fluxionis $y\dot{x} = \alpha$, tum dabitur fluens omnium fluxionum formularum $y\dot{y}$, $y\dot{x}$, $x\dot{y}$, $x\dot{x}$, $\dot{x}$, $\dot{y}$; ducatur data æquatio in x & y reſpective, & reſultant $(ayx + bx^2 + cx)\dot{x} + (Ayx + Bx^2 + Cx)\dot{y} = \pi = o$ & $(ay^2 + bxy + cy)\dot{x} + (Ay^2 + Bxy + Cy)\dot{y} = \varrho = o$; ſed per hypotheſin dantur fluentes omnium fluxionum hiſce æquationibus contentarum præter duas fluentes (P & Q) fluxionum $yx\dot{x}$ & $yx\dot{y}$; vel fluxionum $x^2\dot{y}$ & $y^2\dot{x}$, quæ ex iis pendent; erunt enim $x^2 y - 2P = \int. x^2\dot{y}$ & $y^2 x - 2Q = \int. y^2\dot{x}$; ſcribantur hæ duæ quantitates pro ſuis valoribus in æquationibus $\pi = o$ & $\varrho = o$ & reſultant $Bx^2y + (a - 2B)P + \frac{bx^3}{3} + \frac{cx^2}{2} + AQ + C(xy - \alpha) + F = o$ & $ay^2x + (B - 2a)Q + bP + c\alpha + \frac{Ay^3}{3} + \frac{Cy^2}{2} + G = o$, (ubi F & G ſunt invariabiles quantitates) quæ ſunt fluentes duarum reſultantium æquationum $\pi = o$ & $\varrho = o$; i.e. exoriuntur duæ æquationes, quarum ſingulæ fluentes præter P & Q deduci poſſunt, ergo facile acquiri poſſunt fluentes P & $\mathfrak{Q}$; & exinde dabitur fluens omnis fluxionis hujuſce formulæ $(\alpha' y^2 + \beta y x + \gamma x^2 + \delta y + \varepsilon x + \zeta)\dot{x} + (\pi' y^2 + \varrho' y x + \sigma x^2 + \tau y + \lambda x + \mu)\dot{y}$, ubi α', β, γ, &c. π', ϱ', σ, &c. ſunt invariabiles quantitates; & ſic deinceps.

2. Sit data æquatio $(a y^n + (b + c x) y^{n-1} + (d + e x + f x^2) y^{n-2} +$ &c.$)\dot{x} + (A y^n + (B + C x) y^{n-1} +$ &c.$)\dot{y} = 0$, & dentur fluentes ſingularum quantitatum hujuſce generis *(V)* $(h y^m + (k + l x) y^{m-1} +$ &c.$)\dot{x} + (H y^m + (K + L x) y^{m-1} +$ &c.$)\dot{y}$, ubi literæ h, k, l, &c. H, K, L, &c. quaſcunque invariabiles quantitates reſpective denotant; etiamque dentur $(n - 1)$ fluentes fluxionum hujuſce generis *(W)* $(\alpha y^{m+1} + (\beta + \gamma x) y^m +$ &c.$)\dot{x} + (\pi y^{m+1} + \varrho y^m x +$ &c.$)\dot{y}$, (ubi n & m integri ſunt numeri affirmativi, & m major quam n; & α, β, γ, &c. π, ϱ, &c. ſunt invariabiles quantitates) quæ nec a fluentibus quibuſcunque in V, nec a ſe invicem detegi poſſunt; tum ex omnibus fluentibus generis *(V)* & prædictis $n - 1$ fluentibus detegi poteſt quæcunque fluens generis W.

Ducatur data æquatio reſpective in $(m - n + 2)$ quantitates y^{m-n+1}, $y^{m-n} x$, $y^{m-n-1} x^2$, $y^{m-n-2} x^3$, $y^{m-n-3} x^4$, &c.; & reſultant $m - n + 2$ diverſæ æquationes: ſed in æquatione W continentur $(m + 1)$ fluxiones, viz. $y^{m+1}\dot{x}$, $y^m x \dot{x}$, $y^{m-1} x^2 \dot{x}$, &c. . . $y x^m \dot{x}$; quarum fluentes forſan nec a ſe invicem, nec a fluentibus fluxionum in V contentarum deduci poſſunt: unde ex $(m + 1) - (m - n + 2) = n - 1$ independentibus fluentibus fluxionum formularum $y^{m+1}\dot{x}$, $y^m x \dot{x}$, $y^{m-1} x^2 \dot{x}$, $y^{m-2} x^3 \dot{x}$, &c. & ex omnibus fluentibus generis V acquiri poſſunt fluentes omnium fluxionum generis W.

In quamplurimis caſibus fluens W e paucioribus fluentibus quam $n - 1$ prædictis deduci poteſt.

In nonnullis caſibus denominator nihilo evadat æqualis, in quo caſu fluens quæſita e prædictis non detegi poteſt.

Eadem etiam ad algebraicas, fluxionales & incrementiales æquationes; & fluxionales, &c. quantitates applicari poſſunt; in pleriſque caſibus facile deduci poteſt numerus datarum fluentium vel integralium, quas exigit data æquatio.

PROB.

PROB. XLII.

Datâ algebraicâ vel fluxionali æquatione $\alpha = 0$, *& fluxionali quantitate* π; *in quibuſdam caſibus invènire, annon fluens quantitatis* π *detegi poteſt ope datarum fluentium.*

Inveniatur, annon data quantitas π ſit ſumma quarumcunque directarum functionum quantitatis α in finitas quantitates vel variabiles vel invariabiles ductarum; & fluxionum, quarum fluentes detegi poſſunt ope datarum fluentium.

Hoc vero ſæpe perfici poteſt, 1^{mo}. ex auferendo vel pro nihilo habendo quaſcunque quantitates in quantitate π contentas, quæ e datis fluentibus detegi poſſunt: 2^{do}. ex obſervando & inter ſe comparando irrationales quantitates, & dimenſiones & rationalium & irrationalium quantitatum in α & π contentarum; exinde enim conſtabunt functiones quantitatum in α contentarum; etiamque quantitates, in quas ducendæ ſunt prædictæ functiones, ita ut præbeant quantitates in π contentas, quæ haud detegi poſſunt ope datarum fluentium: 3^{tio}. e reducendo per methodum prius traditam datam fluxionalem quantitatem in fluxionalem quantitatem, cujus ordo ſit minor quam ordo datæ fluxionalis æquationis; & deinde ex inveniendo, utrum fluens fluxionalis quantitatis reſultantis ope datarum fluentium deduci poteſt, necne.

Per hanc methodum, viz. reductionem fluxionalis quantitatis ita ut ejus ordo minor ſit quam datæ fluxionalis æquationis ordo, ſæpe ex reſultante fluxione detegi poteſt, utrum fluens datæ fluxionalis quantitatis ſic exprimi poteſt, necne; cum in eâ plures involvantur variabiles quantitates & earum fluxiones.

PROB. XLIII.

Data fluxionali æquatione duas variabiles quantitates x *&* y, *& earum fluxiones involvente; invenire functiones prædictarum quantitatum* x *&* y *& earum fluxionum, quarum fluentes dantur.*

1. Dividatur data æquatio in duas partes ſibi invicem æquales, ita ut fluens unius partis inveniri poteſt; tum etiam erit fluens alterius partis.

2. Ducatur data æquatio in quamcunque quantitatem, & ita dividatur fluxionalis æquatio reſultans in duas inter ſe æquales partes, ut fluens unius partis inveniatur; & erit etiam fluens alterius.

Ex. 1. Sit fluxionalis æquatio $y\dot{x} = nax^{n-1}y^{m}\dot{x} + may^{m-1}x^{m}\dot{y} + bx^{r}\dot{x} + cy^{s}\dot{y}$, & erit fluens fluxionis $y\dot{x}$, i. e. area curvæ, cujus relatio inter abſciſſam & ejus correſpondentes ordinatas per datam fluxionalem æquationem exprimitur $= ax^{n}y^{m} + \frac{bx^{r+1}}{r+1} + \frac{cy^{s+1}}{s+1} + A$, ubi A ſit invariabilis quantitas.

Ex. 2. Sit $y^{2}\dot{x} = P\dot{Q} + Q\dot{P} + A\dot{x} + B\dot{y}$, ubi A & B ſunt quæcunque functiones incognitarum quantitatum x & y reſpective; tum erit fluens fluxionis $3,1416\,y^{2}\dot{x}$ (i. e. contentum ſolidi generati a rotatione curvæ, cujus relatio inter abſciſſas & ejus correſpondentes ordinatas deſignatur per datam æquationem) $= 3,1416\,(QP + \int. A\dot{x} + \int. B\dot{y} + a)$, ubi a eſt invariabilis quantitas.

Ex. 3. Sit æquatio fluxionalis $y\dot{x} + nx\dot{y} = b\dot{y}\sqrt{(x^{2n} + cx^{n} + d)}$, ducatur data æquatio in quamcunque quantitatem, e. g. y^{n-1}, & reſultat æquatio $y^{n}\dot{x} + ny^{n-1}x\dot{y} = by^{n-1}\dot{y}\sqrt{(x^{2n} + cx^{n} + d)}$, unde $y^{n}x = b\int. y^{n-1}\dot{y}\sqrt{(x^{2n} + cx^{n} + d)} + a$.

3. Si modo e fluentibus prædictâ methodo inveſtigatis deduci poſſint fluentes plurium fluxionum, tum harum fluxionum conſequentur fluentes.

Ex. Sit fluxionalis æquatio $y^{2}\dot{x} = ax^{r}\dot{x} + by^{s}\dot{y}$; & ſi modo detur fluens fluxionis $y^{2}\dot{x}\ (A) = \frac{ax^{r+1}}{r+1} + \frac{by^{s+1}}{s+1} + c$, dabitur etiam fluens fluxionis $yx\dot{y}\ (\dot{B})$; nam $\frac{y^{2}x - A}{2} = B = \frac{y^{2}x}{2} - \frac{ax^{r+1}}{2(r+1)} - \frac{bx^{s+1}}{2(s+1)} - \frac{c}{2}$.

Et ſic de pluribus fluxionalibus æquationibus datis.

4. Sit æquatio vel algebraica vel fluxionalis $P=o$, dentur fluentes fluxionum L, M, &c. & ſint etiam R, S, T, &c. fluxiones, quarum fluentes e fluentibus datis fluxionum L, M,&c. vel ſeparatim vel conjunctim deduci poſſunt: tum fluens cujuſcunque fluxionis $\alpha\phi:(P) + aR + bS + cT +$ &c. $+ \dot{\sigma}$ (ubi a, b, c, &c. ſunt invariabiles quantitates; & α quæcunque fluxio vel quantitas, quæ pro ejus denominatore haud habet functionem quantitatis P vel ejus diviſorem; & $\phi:(P)$ ſit quæcunque functio quantitatis $P=o$, in quâ nulla continetur poteſtas vel nihilo æqualis vel negativa) e prædictis fluentibus L, M, &c. deduci poteſt.

Cor. Datâ quâcunque fluxione ($\mathfrak{Q}$), facile deduci poſſunt infinitæ fluxionales æquationes, quarum fluens fluxionis ($\mathfrak{Q}$) inveniri poteſt. Dividatur fluxio $\mathfrak{Q}$ in quaſcunque duas partes α & β, i. e. $\mathfrak{Q}=\alpha+\beta$; quarum una ($\alpha$) inveniri poteſt ope datarum fluentium; tum vel aſſumatur $\beta=o$, vel quæcunque functio (π) quantitatis β nihilo æqualis, quæ talis ſit, ut functio (π) quantitatis β haud in ſe poteſtatem contineat nihilo æqualem vel negativam; & deducitur æquatio quæſita.

PROB. XLIV.

Datâ fluxionali æquatione $P\dot{x}+Q\dot{y}=0$, *ubi* P & Q *ſunt functiones quantitatum* x & y; *invenire fluentem fluxionis* $W\dot{x}+R\dot{y}$, *ubi* W & R *ſunt functiones quantitatum* x & y.

1$^{\text{mo}}$. Si $\dot{W}=\alpha\dot{x}+\beta\dot{y}$, & $\dot{R}=\pi\dot{x}+\varrho\dot{y}$, & $\beta=\pi$; tum integrari poteſt data fluxio.

2. Sit $(AP+W)\dot{x}+(A\mathfrak{Q}+R)\dot{y}$, ubi A eſt functio quantitatum x & y, fluxio integrabilis; tum inveniatur ejus fluens, quæ etiam erit fluens fluxionis $W\dot{x}+R\dot{y}$.

3. Si modo fluens ($\alpha=o$) fluxionalis æquationis $P\dot{x}+\mathfrak{Q}\dot{y}=o$ detegi poſſit, tum in datâ fluxione ($W\dot{x}+R\dot{y}$) pro $\dot{y}$ ſcribatur $-\dfrac{P\dot{x}}{\mathfrak{Q}}$,

&

& reſultat $\frac{WQ - RP}{Q}\dot{x}$; in hâc fluxione pro y ſcribatur ejus valor, qui erit functio quantitatis x ex datâ æquatione $\alpha = o$ deducta, unde reſultat data fluxio $= \dot{x}\varphi : (x)$, cujus fluens (M) inveniri poteſt.

Sit $H =$ conſt. fluens fluxionalis æquationis $P\dot{x} + Q\dot{y} = o$, tum erit $H -$ conſt. $+ M$ fluens datæ fluxionis $W\dot{x} + R\dot{y}$.

4. Aſſumatur $W\dot{x} + R\dot{y} = \dot{V}$; tum ita reducantur duæ æquationes $W\dot{x} + R\dot{y} = \dot{V}$ & $P\dot{x} + Q\dot{y} = o$ in unam, ut exterminentur vel y & $\dot{y}$; vel x & $\dot{x}$, & reſultat fluxionalis æquatio, cujus fluens eſt quæſita.

Et ſic de pluribus æquationibus plures variabiles quantitates & fluxiones ſuperiorum ordinum habentibus.

5. Datâ fluxionali æquatione r ordinis, cujus variabiles quantitates ſint x & y, & $\dot{x}$ ſit conſtans; datâ etiam fluxionali quantitate, quæ ſit functio variabilium x, y & fluxionum $\dot{x}, \dot{y}, \ddot{y}, \ldots \overset{r+m}{\dot{y}}$ uſque ad ordinem $r + m$; invenire quantitatem fluxionalem, quæ haud implicat fluxiones quantitatis y, quarum ordo ſuperior ſit ordini $r - 1$, i. e. ſuperior ordini fluxionis $\overset{r-1}{\dot{y}}$, datæ fluxionali quantitati æqualem.

Inveniatur fluxio datæ æquationis ordinis m; ex æquatione reſultante deducatur valor fluxionis $\overset{r+m}{\dot{y}}$; ſcribatur valor inventus pro $\overset{r+m}{\dot{y}}$ in datâ fluxionali quantitate, & exterminabitur $\overset{r+m}{\dot{y}}$; ex eodem proceſſu repetito exterminari poſſunt fluxiones $\overset{r+m-1}{\dot{y}}$, $\overset{r+m-2}{\dot{y}}$, &c. ad $\overset{r-1}{\dot{y}}$; & tandem reſultabit quantitas quæſita.

Eadem principia etiam applicari poſſunt ad plures fluxionales æquationes & fluxionalem quantitatem datam plures variabiles quantitates & earum fluxiones habentes.

THEOR. XXIV.

Datis fluxionalibus æquationibus involventibus x & y, &c. & earum fluxiones, & datâ functione P quantitatum x & y, &c. & earum fluxio-

fluxionum. Scribantur pro x & y, &c. & earum fluxionibus in datis æquationibus & datâ functione P quæcunque functiones literarum z & v, &c. & earum fluxiones; & reſultabunt æquationes relationem inter z & v, &c. & earum fluxiones exprimentes, & functio $\mathcal{Q}$ quantitatum z & v, &c. & earum fluxionum, cujus fluens e fluente functionis P conſequitur ſcribendo in eâ pro x & y, &c. & earum fluxionibus prædictas functiones quantitatum z, v, &c. & earum fluxiones.

PROB. XLV.

1. *Invenire, quando duo valores quantitatis* y *in fluxionali æquatione duas variabiles quantitates* x *&* y *& earum fluxiones* $\dot{x}$ *&* $\dot{y}$ *habente ſint inter ſe æquales, i. e. in æquatione* $\alpha\dot{x} = \beta\dot{y}$, *ubi* α *&* β *ſunt quæcunque algebraicæ functiones literarum* x *&* y.

Inveniatur fluens hujuſce æquationis, quæ ſit $P = \mathcal{Q}$, ubi P & $\mathcal{Q}$ ſunt etiam quæcunque functiones literarum x & y, & æquatio vere correcta erit $P = \mathcal{Q} + a$, ubi a eſt invariabilis quantitas ad libitum aſſumenda; ſed duo valores quantitatis y fiunt inter ſe æquales, cum $\beta\dot{y} = o$ & exinde $\alpha\dot{x} = o$, & $\beta = o$; in quâ æquatione exprimitur relatio inter x & y, & in æquatione correctâ $P = \mathcal{Q} + a$ exprimitur relatio inter tres incognitas quantitates x, y & a; unde duæ reſultant æquationes tres incognitas quantitates x, y & a habentes, & conſequenter haud detegi poteſt, quando duæ radices quantitatis y fiant inter ſe æquales, ni detur quantitas corrigens a.

Dato autem uno valore (π) quantitatum x vel y, quando fiant duo valores quantitatis y inter ſe æquales, exinde deduci poſſunt reliqui: dato enim valore quantitatis x vel y in æquatione $\beta = o$, exinde ſequitur ejus correſpondens valor quantitatis (y vel x), quibus valoribus pro y & x in æquatione $P = \mathcal{Q} + a$ ſubſtitutis, reſultat valor quantitatis a, & exinde duæ æquationes $\beta = o$ & $P = \mathcal{Q} + a$, quæ duas incognitas quantitates x & y habent, e quibus deduci poſſunt reliqui valores quantitatum x & y, quando duo valores quantitatis y fiant inter ſe æquales.

Si vero æquatio $P = Q$ haud exprimi poſſit in finitis terminis, tum confugiendum eſt ad infinitas ſeries.

2. Datâ æquatione fluxionali n ordinis, i. e. in quâ continetur fluxio $\overset{n}{\dot{y}}$, & x fluit uniformiter; invenire generaliter quando duo valores quantitatum (x vel y) fiant inter ſe æquales. Neceſſe eſt in hoc caſu, ut dentur n correſpondentes valores ſingularum fluxionum in datâ æquatione contentarum, e. g. $\overset{n}{\dot{y}}$, $\overset{n-1}{\dot{y}}$, &c. $\dot{x}$, etiamque quantitatum x & y, quando duo valores quantitatum x vel y fiant inter ſe æquales; & his datis generaliter deduci poteſt, quando duo valores prædictarum quantitatum fiant inter ſe æquales.

Hoc conſtat ex eo, quod n incognitas & invariabiles coefficientes habet æquatio vere correcta.

Eadem etiam applicari poſſunt ad duas vel plures æquationes tres vel plures variabiles quantitates habentes.

Hæc ratiocinandi methodus haud ſolummodo ad problemata reſolvenda, in quibus duo valores quantitatum x vel y inter ſe æquales eſſe ſupponuntur, ſed etiam in quamplurima alia extendi poteſt.

3. Datâ fluxionali æquatione duas variabiles quantitates & earum fluxiones habente, invenire coefficientium conſtitutionem.

Habeat æquatio n terminos, & conſequenter horum terminorum ſunt n coefficientes: ſcribantur pro n correſpondentibus valoribus incognitarum quantitatum (x & y) α, β, γ, δ, &c. π, ρ, σ, τ, &c. pro correſpondentibus valoribus primarum fluxionum reſpective $\dot{\alpha}$, $\dot{\beta}$, $\dot{\gamma}$, $\dot{\delta}$, &c. $\dot{\pi}$, $\dot{\rho}$, $\dot{\sigma}$, $\dot{\tau}$, &c. & ſic de deſignandis ſecundis, tertiis, &c. fluxionibus incognitarum quantitatum (x & y): ſubſtituantur hæ literæ pro ſuis quantitatibus in datâ æquatione, & reſultant n æquationes totidem incognitas coefficientes habentes, e quibus inveniri poſſunt coefficientium conſtitutiones.

Multa de impoſſibilibus radicibus, &c. deduci poſſunt fere ex eâdem ratiocinandi methodo in fluxionalibus ac in algebraicis æquationibus uſitatâ.

PROB. XLVI.

Datis formulis fluxionum; invenire fluxiones prædictarum formularum, quarum fluentes exprimi poſſunt in finitis terminis, vel in terminis fluentium, quarum dantur fluxiones.

1^{mo}. Ex datis formulis fluxionum inveniantur formulæ quantitatum in prædictis formulis contentarum, ita ut fluentes fluxionum reſultantium datarum formularum reſpective exprimi poſſunt vel per finitos terminos vel per fluentes prædictas, quarum fluxiones dantur: deinde ex comparandis quantitatibus prædictarum formularum inter ſe ſæpe inveniri poteſt formula quantitatis, quæ ſatisfacit omnibus conditionibus quæſtionis prædictæ.

Ex. 1. Sit data fluxio $= P\dot{x}$, cujus fluens acquiri poteſt, ubi P eſt functio variabilium x & y; aſſumatur quæcunque functio ($\mathcal{Q}$) quantitatis x pro fluente; tum erit $\frac{\dot{\mathcal{Q}}}{\dot{x}} = P$, quæ eſt algebraica æquatio exprimens relationem inter x & y, unde ex variabili x detegi poteſt variabilis (y), ita ut fluens datæ fluxionis inveniri poteſt.

Ex. 2. Sint duæ formulæ $y\dot{x}$ & $y^2\dot{x}$, quarum fluentes requiruntur in finitis terminis: aſſumatur pro fluente fluxionis $y\dot{x}$ quantitas formulæ $\frac{(a+bx+cx^2+dx^3+\&c.)^{1\frac{1}{2}} \times (A+Bx+Cx^2+Dx^3+\&c.)}{(h+kx)^{\frac{\lambda}{\mu}}}$; cujus fluxio ſit $P\dot{x}$; tum fiat $P = y$, & $P^2\dot{x} = y^2\dot{x}$ (exceptis excipiendis) etiam erit fluxio, cujus fluens in finitis terminis exprimi poteſt: & ſic detegi poſſunt plures hujuſmodi formulæ.

Ex. 3. Sint duæ fluxiones $y\dot{x}$ & $\surd(\dot{x}^2 + \dot{y}^2)$; pro $\dot{y}$ ſcribatur $z\dot{x}$, & evadent $\int.y\dot{x} = yx - \int.x\dot{y} = yx - \int.zx\dot{x}$ & $\surd(1+z^2)\dot{x} = \surd(\dot{x}^2 + \dot{y}^2)$; invenire ($z$) functionem quantitatis x, quæ reddit fluxiones $zx\dot{x}$ & $\surd(1+z^2)\dot{x}$ formularum, quarum fluentes exprimi poſſunt in finitis terminis quantitatis (x): 1^{mo}. aſſumatur $x = z^{\lambda}$, & evadent duæ fluxiones $z x \dot{x} = \lambda z^{2\lambda} \dot{z}$ & $\sqrt{(1+z^2)}\,\dot{x} = \lambda (1+z^2)^{\frac{1}{2}} \times z^{\lambda-1}\dot{z}$; unde, ſi λ ſit par numerus, fluentes duarum prædictarum fluxionum ſemper inveniri poſſunt.

Ex. 4. Aſſumatur $\dot{x} = \frac{(1-R^2)^{\frac{3}{2}}\dot{P}}{-\dot{R}}$, $y = \frac{P\dot{R} - R\dot{P}(1-R^2)}{\dot{R}}$, ubi R fluat uniformiter; tum erit $\sqrt{(\dot{x}^2+\dot{y}^2)} = \sqrt{\left(\frac{(1-R^2)^3\ddot{P}^2 - 6R(1-R^2)_2 R\dot{R}\dot{P}\ddot{P} + 9R^2\dot{R}^2\dot{P}^2 \times (1-R^2)}{\dot{R}^2} + \frac{R^2(1-R^2)^2\ddot{P}^2 - 6R^3(1-R^2)\dot{R}\dot{P}\ddot{P} + 9R^4\dot{R}^2\dot{P}^2}{\dot{R}^2}\right)} = \frac{(1-R^2)\ddot{P} - 3R\dot{R}\dot{P}}{R}$, cujus fluens eſt $-\int.\ R\dot{P} + \frac{(1-R^2)\dot{P}}{\dot{R}}$.

Cor. Hæc methodus in detegendis curvis algebraicis, quæ rectificari poſſunt, haud multum uſui inſervit.

THEOR. XXV.

Sit quæcunque fluxionalis æquatio duas involvens incognitas quantitates x & y & earum fluxiones, in quâ x fluit uniformiter; ſemper erit quantitas, quæ in datam æquationem ducta, creat fluxionem, cujus fluens inveniri poteſt.

Ex. g. Sit data fluxio $\dot{A} = a\dot{x} + b\dot{y} = o$, ſint etiam $\dot{a} = \alpha\dot{x} + \beta\dot{y}$, $\dot{b} = \pi\dot{x} + \varrho\dot{y}$, & $\pi = \beta$; tum ipſius æquationis fluens per theor. 2. inveniri poteſt.

Si vero fluens haud inveniri poſſit; ſemper datur quantitas L, quæ in datam æquationem $a\dot{x} + b\dot{y} = o$ ducta, creat fluxionem $La\dot{x} + Lb\dot{y}$, cujus fluens inveniri poteſt; ſit $\dot{L} = r\dot{x} + s\dot{y}$, & per prædictum theorema erit $as + L\beta = L\pi + br$.

Conſimiles propoſitiones etiam de fluxionalibus æquationibus ſuperiorum ordinum e theor. 2. facile deduci poſſunt.

Cor. 1. Datâ unâ quantitate L, quæ in datam fluxionalem æquationem ducta, præbet fluxionem, cujus fluens inveniri poteſt; ex eâ L facile deduci poſſunt infinitæ aliæ: ſit enim data æquatio $a\dot{x} + b\dot{y} = o$, quæ ducta in L dat fluxionem $La\dot{x} + Lb\dot{y}$, cujus fluens inveniri poteſt, quæ ſit z: i. e. ſit $\dot{z} = La\dot{x} + Lb\dot{y}$: ſit Z quæcunque functio quantitatis z, & fluens fluxionis $Z\dot{z} = ZLa\dot{x} + ZLb\dot{y}$ ſemper deduci poteſt; i. e. fluxio eſt integrabilis: & conſequenter plures deducuntur quantitates ZL, quæ in datam fluxionem $a\dot{x} + b\dot{y}$ ductæ, præbent fluxiones integrabiles.

Cor. 2. Inveniantur duæ quantitates P & R, quæ in $a\dot{x} + b\dot{y}$ ductæ dabunt fluxiones integrabiles $Pa\dot{x} + Pb\dot{y} = \dot{V}$; $Ra\dot{x} + Rb\dot{y} = \dot{W}$, tum V & $\frac{P}{R}$ erunt functiones quantitatis W.

Cor. 3. Sit M particularis valor fluxionalis æquationis vel fluxionalium æquationum; tum erit quæcunque functio quantitatis M, etiam particularis valor prædictæ fluxionalis æquationis, &c. nam particularis ſemper in generali continetur.

Cor. 4. Sit æquatio $P\dot{x} + Q\dot{y} = o$; ducatur hæc æquatio in $M = H \times L$ & reſultat $M(P\dot{x} + Q\dot{y}) = o$, quæ ſit integrabilis æquatio, i. e. ſit $MP\dot{x} + MQ\dot{y}$ fluxio, cujus fluens inveniri poteſt: ſi vero & P & Q nullum habeat diviſorem, qui reciprocat cum quantitate H; tum per prob. 4. & 5, &c. H erit diviſor particularis fluentis fluxionis $M(P\dot{x} + Q\dot{y})$, & conſequenter $H = o$ erit quodam modo particularis fluens fluxionalis æquationis $P\dot{x} + Q\dot{y} = o$.

Ducatur data æquatio in $\frac{1}{M} = \frac{1}{H \times L}$, & reſultat $\frac{1}{M}(P\dot{x} + Q\dot{y})$; ſi vero & P & Q nullum habeat diviſorem H, tum per prædicta problemata H erit diviſor vel numeratoris vel denominatoris fluentis fluxionis $M(P\dot{x} + Q\dot{y})$, & conſequenter $H = o$ vel $\frac{1}{H} = o$ dici poteſt quodammodo particularis valor fluxionalis æquationis $P\dot{x} + Q\dot{y} = o$.

Et ſic de fluxionalibus æquationibus ſuperiorum ordinum; multiplicator autem ſuperioris ordinis poteſt eſſe quæcunque fluxio cujuſcunque inferioris ordinis.

THEOR. XXVI.

1. Sit quæcunque fluxionalis æquatio $P\dot{x} + Q\dot{y} = o$, ducatur hæc æquatio in q, & ſit fluens fluxionis reſultantis generalis $= S$; i. e. $\dot{S} = qP\dot{x} + qQ\dot{y}$; tum fluens datæ æquationis erit univerſaliter quæcunque functio fluentis S.

Sit enim quæcunque functio quantitatis $(S) = \phi : (S) = o$ fluens quæſita, cujus fluxio erit $\Delta : (S) \times \dot{S} = o$, ubi $\Delta : (S)$ deſignat functionem quantitatis S; dividatur hæc æquatio per $\Delta : S$ vel $\Delta : (S) \times q$, & reſultat $\dot{S} = o$ vel $\frac{\dot{S}}{q} = P\dot{x} + Q\dot{y} = o$, quæ eſt data æquatio; ergo quæcunque functio quantitatis S erit reſolutio datæ æquationis.

Hic animadvertendum eſt, quamvis $\phi : S = o$ eſt fluens datæ fluxionalis æquationis; attamen $V \times \phi : S = o$, ubi V non eſt functio quantitatis S, non proprie dici poteſt fluens prædictæ fluxionalis æquationis, quamvis in eâ fluentem continet.

2. Sit æquatio $A\dot{x} + B\dot{y} = o$, ducatur hæc æquatio in q, ita ut fluens quantitatis $qA\dot{x} + qB\dot{y}$ inveniri poſſit, quæ ſit P; pro P vel quâvis functione quantitatis P ſcribatur z, & erit $\dot{z} = o$.

Et ſic de æquationibus fluxionalibus ſuperiorum ordinum.

3. Sit æquatio $Q\dot{z} = o$, & erit ejus fluens $z = a$ vel quæcunque functio quantitatis $(z) = o$; minime vero $Q = o$ proprie dici poteſt reſolutio datæ æquationis; quamvis in caſibus pro hâc re deſignatis ejus fluens erit valor datæ æquationis.

THEOR. XXVII.

1. Sint $\alpha = o$ & $\pi = o$ duæ fluxionales æquationes m & r ordinum reſpective, quorum m minor eſt quam r, & K & L duo multiplicatores

ita conſtituti ut $K\alpha + L\pi = \dot{V}$, ubi V eſt fluxionalis quantitas ordinis $(r - 1)$; duæ datæ fluxionales æquationes reducuntur in duas fluxionales $V =$ conſt. & $\alpha = o$: & ſimiliter deprimi poſſunt æquationes reductæ, &c.

Et ſic de pluribus fluxionalibus æquationibus plures variabiles quantitates habentibus.

2. Sint datæ fluxionales æquationes $a = o$, $b = o$, $c = o$, &c.; & ſint $\alpha = o$, $\beta = o$, &c. generales fluentes quorumcunque ordinum fluxionalium æquationum $a = o$, $b = o$, &c.; tum erit generalis fluens fluxionalium æquationum $\alpha = o$, $\beta = o$, &c., $c = o$, &c. eadem ac generalis fluens fluxionalium æquationum datarum $a = o$, $b = o$, $c = o$, &c.

THEOR. XXVIII.

Si detur fluens generalis $(W = o)$ fluxionalis æquationis $(P = o)$, facile exinde deduci poteſt multiplicator M, qui in datam æquationem ductus, reddit eam integrabilem.

In datâ generali fluente ſit C quæcunque invariabilis quantitas generalis in datâ fluxionali æquatione haud contenta, & ita reducatur per vulgarem algebram æquatio W, ut fiat $C \times V = Z$, ubi V & Z ſunt quantitates in quibus haud continetur C; erit $\frac{\dot{Z}V - Z\dot{V}}{V^2 P} = M$ multiplicator quæſitus.

Et ſic de fluxionalibus æquationibus ſuperiorum ordinum.

E. g. Detur generalis fluens $W = o$ fluxionalis æquationis, quæ continet duas C & D invariabiles coefficientes ad libitum aſſumendas; ex æquatione $W = o$ inveniatur vel $C = \alpha$ vel $D = \beta$, in quibus (α & β) haud continentur reſpective C & D, quarum fluxiones erunt $\dot{\alpha} = o$ & $\dot{\beta} = o$; deinde ex æquationibus $\dot{\alpha} = o$ & $\dot{\beta} = o$ reſpective deducantur $D = \pi$ & $C = \varrho$, & evadunt duæ æquationes $\dot{\pi} = o$ & $\dot{\varrho} = o$

eadem æquatio, cujus generalis fluens ſecundi ordinis eſt data æquatio; duo vero inveniuntur ejus primi multiplicatores vel $\frac{\dot{\alpha}}{\pi}$ vel $\frac{\dot{\beta}}{\varrho}$.

Et ſic deinceps.

THEOR. XXIX.

1. Sit quæcunque fluxionalis æquatio n ordinis $P = o$, cujus generalis fluens ſit $W = o$, quæ continet quaſcunque n diverſas invariabiles quantitates a, b, c, d, e, &c. ad libitum aſſumendas & in datâ fluxionali æquatione non contentas; ex æquatione $W = o$ inveniantur $a = \alpha$, $b = \beta$, $c = \gamma$, $d = \delta$, &c. tum erunt $\dot{\alpha} = o$, $\dot{\beta} = o$, $\dot{\gamma} = o$, $\dot{\delta} = o$, &c. (n) diverſæ æquationes, quæ ſunt fluentes $(n - 1)$ ordinis datæ æquationis $P = o$, in quibus continentur ſolummodo $n - 1$ invariabiles quantitates prædicti generis; deinde ex æquatione $\dot{\alpha} = o$ inveniatur vel $b = \beta'$ vel $c = \gamma'$, &c. & ſic ex æquationibus $\dot{\beta} = o$, $\dot{\gamma} = o$, $\dot{\delta} = o$, &c. inveniantur $a = \alpha''$, $c = \gamma''$, &c, $a = \alpha'''$, $b = \beta'''$, &c. &c. reſpective, tum erunt $\dot{\beta}' = o$, $\dot{\gamma}' = o$, &c. $\dot{\alpha}'' = o$, $\dot{\gamma}'' = o$, &c. $\dot{\alpha}''' = o$, &c. &c. $\left(n \times \frac{n-1}{2}\right)$ diverſæ fluxionales æquationes, quæ ſunt fluentes $(n - 2)$ ordinis datæ æquationis $P = o$; & ſic inveniri poſſunt $n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3} \ldots \frac{n-m+1}{m}$ diverſæ fluxionales æquationes, quæ ſunt fluentes m ordinis fluxionalis æquationis $P = o$; & quæ continent (m) diverſas invariabiles quantitates ad libitum aſſumendas, quæ in datâ æquatione haud continentur; ſi $m = n$, tum ex ſingulis hiſce ſubſtitutionibus eadem reſultabit æquatio.

Cor. Sit quæcunque algebraica $(\pi = o)$ æquatio, cujus variabiles ſint x, y, z, &c.; & n invariabiles $(a, b, c, d, e$, &c.) tum ex eâ inveniantur (n) fluxionales æquationes $\alpha = o$, $\beta = o$, $\gamma = o$, &c.; ex quibus exterminatur una invariabilis quantitas, viz. a, b, c, &c. reſpective; deinde detegantur $n \cdot \frac{n-1}{2}$ fluxionales æquationes ſecundi ordinis, viz. $\alpha' = o$, $\beta' = o$, $\gamma' = o$, &c. ex quibus ſingulis exterminantur quæcunque duæ invariabiles a & b, a & c, b & c, &c. reſpective; & ſic inveniantur $n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3}$ fluxionales æquationes tertii ordinis, ex quibus exterminantur quæcunque tres invariabiles quantitates reſpective; & tandem invenientur fluxionales æquationes n ordinis, ex quibus exterminantur (n) invariabiles quantitates; & omnes hæ fluxionales æquationes reſultantes erunt eædem.

2. Si vero haud ita reduci poſſint prædictæ æquationes, ut inveniantur $a = \alpha$, $b = \beta$, $c = \gamma$, &c.; tum ita reducantur duæ æquationes $W = o$ & $\dot{W} = o$ in unam; ut exterminetur vel a vel b vel c, &c.: & reſultabunt prædictæ æquationes $h\dot{\alpha} = o$, $k\dot{\beta} = o$, $l\ddot{\gamma} = o$, &c. ubi h, k & l ſunt quæcunque algebraicæ quantitates; deinde eodem modo reducantur duæ æquationes $h\dot{\alpha} = o$ & $h\ddot{\alpha} + \dot{h}\dot{\alpha} = o$; $k\dot{\beta} = o$ & $k\ddot{\beta} + \dot{k}\dot{\beta} = o$; $l\dot{\gamma} = o$ & $l\ddot{\gamma} + \dot{l}\dot{\gamma} = o$; &c. reſpective in unam, ita ut exterminetur ſecunda invariabilis quantitas vel b vel c, &c.; & reſultant $n \cdot \frac{n-1}{2}$ diverſæ fluxionales æquationes $b'\ddot{\beta}' = o$, $k'\ddot{\gamma}' = o$, &c., $b''\ddot{\alpha}'' = o$, &c.; ubi b', k', &c.; b'', &c. reſpective denotant functiones algebraicarum quantitatum & fluxionum primi ordinis; & ſic deinceps.

Cor. 3. Ex datâ generali fluente datæ æquationis methodo prius traditâ deduci poteſt multiplicator, qui eam reddit integrabilem; ſed conſtat e problemate, quod in æquatione fluxionali n ordinis (n) dantur diverſæ fluxionales æquationes ($n-1$) ordinis, quæ ſunt fluentes generales datæ æquationis; & conſequenter (n) ſunt proprii multiplicatores diverſorum generum, qui ducti in datam æquationem præbent fluxiones, quarum fluentes primi ordinis inveniri poſſunt; & ſic de fluxionalibus æquationibus $n-2$, $n-3$, $n-4$, &c. ordinum inveniendis.

Hic autem obſervandum eſt, quod ſi æquatio fluxionalis ſit $P = o$ & M ſit multiplicator eam reddens integrabilem, i. e. MP ſit fluxio, cujus fluens inveniri poteſt; per $M\phi : (\int . MP)$ volo multiplicatorem ejuſdem haud vero diverſi generis.

THEOR. XXX.

Sint $A=o$, $B=o$, $C=o$, $D=o$, &c. (n) generales fluentes primi ordinis fluxionalis æquationis $\alpha=o$, &c.; ſit $\alpha=o$ fluxionalis æquatio n ordinis, variabiles quantitates x, y, & earum fluxiones $\dot{x}, \dot{y}, \ddot{y}, \dddot{y}, \ldots \overset{n}{\dot{y}}$ involvens, & $\dot{x}$ ſit conſtans; & in $A=o$ continetur a invariabilis quantitas ad libitum aſſumenda, quæ in fluxionali æquatione $\alpha=o$ non continetur; & ſimiliter in $B=o$ continetur invariabilis b, quæ in prædictâ æquatione $\alpha=o$ non invenitur; deinde in $C=o$, $D=o$, &c. continentur reſpective invariabiles quantitates c & d, &c. independentes & ad libitum aſſumendæ, quæ in $\alpha=o$ non inveniuntur, & in ſingulis his æquationibus $A=o$, $B=o$, &c. haud continetur $\overset{n}{\dot{y}}$; tum ita reducantur æquationes $A=o$, $B=o$, $C=o$, $D=o$, &c. ut exterminetur fluxio $\overset{n-1}{\dot{y}}$ & reſultant $n . \frac{n-1}{2}$ æquationes, quæ erunt fluentes ſecundi ordinis datæ fluxionalis æquationis $\alpha=o$; & ſic deinceps; ita reducantur hæ æquationes $A=o$, $B=o$, $C=o$,&c. in unam, ut exterminentur fluxiones $\overset{n-1}{\dot{y}}, \overset{n-2}{\dot{y}}, \overset{n-3}{\dot{y}}$, &c. & tandem reſultabit æquatio, quæ nullas implicat fluxiones, & quæ generalis erit fluens datæ fluxionalis æquationis.

Cor. 1. Fluens generalis $n-1$ ordinis e ſingulis (n) æquationibus $A=o$, $B=o$, $C=o$, $D=o$, &c. eadem evadet.

PROB. XLVII.

Sint x, y, w *tres variabiles quantitates,* & *ſint* $\phi=0$ & $\phi'=0$ *duæ functiones quantitatum* x, y & z, *in quibus functionibus contineantur* $m+r$ *invariabiles quantitates* (a, b, c, d, e, &c.) *ad libitum aſſumendæ*; *invenire*

nire duas æquationes fluxionales m & r *ordinum, quarum generales fluentes ſunt* $\phi = o$ & $\phi' = o$.

1. Ex utriſque æquationibus $\phi = o$ & $\phi' = o$ inveniantur $a = \pi$, & $a = \beta$ reſpective, unde $\beta = \pi$, $\dot{\pi} = o$, & $\dot{\beta} = o$ tres æquationes, quarum una eſt algebraica, duæ vero reliquæ fluxionales primi ordinis, in quibus haud continetur a; deinde ex æquatione $\pi = \beta$ inveniatur $b = \pi'$, etiamque ex æquationibus $\dot{\pi} = o$, & $\dot{\beta} = o$, inveniantur reſpective $b = \varrho$ & $b = \gamma$, unde reſultant duæ fluxionales primi ordinis æquationes $\dot{\pi}' = o$, & $\varrho = \gamma$, in quibus nec continetur a nec b; tum ex æquationibus $\dot{\pi}' = o$, & $\varrho = \gamma$, inveniantur $c = \sigma$, & $c = \delta$, & exinde reſultant duæ fluxionales æquationes $\sigma = \delta$, & $\dot{\sigma} = o$, quarum prior erit fluxionalis æquatio primi ordinis, poſterior vero ſecundi ordinis, in quibus continetur nec a nec b nec c: & ſic ex hâc methodo ratiocinandi continuo repetitâ, erui poſſunt duæ æquationes quæſitæ.

2. Si vero ex datâ æquatione $\lambda = o$ non inveniri poſſit $a = \lambda'$, & conſequenter $\dot{\lambda}' = o$ æquatio, quæ non involvit quantitatem a; tum reducantur duæ æquationes $\lambda = o$ & $\dot{\lambda} = o$ in unam, ita ut exterminetur quantitas (a), & reſultabit æquatio prædicta $\dot{\lambda}' = o$; &c. aliter: reducantur duæ æquationes $\phi = o$ & $\phi' = o$ in unam, ita ut exterminetur (a), & reſultet æquatio $\alpha = o$; deinde inveniantur duæ æquationes $\dot{\phi} = o$ & $\dot{\phi}' = o$, & ita reducantur hæ duæ æquationes in unam, ut exterminetur a; & reſultet æquatio $\pi = o$; unde reſultant duæ æquationes, una algebraica $\alpha = o$, altera vero fluxionalis primi ordinis $\pi = o$, in quibus non continetur invariabilis (a); deinde ita reducantur tres æquationes $\alpha = o$, $\dot{\alpha} = o$ & $\pi = o$ in duas fluxionales æquationes ($\mu = o$ & $\nu = o$) primi ordinis, ita ut exterminentur invariabiles quantitates (a & b): tum ita reducantur æquationes $\phi = o$ & $\phi' = o$, ut exterminetur invariabilis quantitas b, & reſultet æquatio $\alpha' = o$; & ita reducantur æquationes $\dot{\phi} = o$ & $\dot{\phi}' = o$ in unam, ut exterminetur quantitas b, & reſultet æquatio

$\dot{\pi} = o$; unde reſultant duæ æquationes, una algebraica $\dot{\alpha} = o$, altera vero fluxionalis primi ordinis $\pi' = o$, in quibus non continetur invariabilis b; deinde ita reducantur tres æquationes $\dot{\alpha} = o$, $\dot{\alpha}' = o$ & $\pi' = o$ in duas fluxionales æquationes ($\mu' = o$ & $\nu' = o$) primi ordinis, ita ut exterminentur invariabiles quantitates a & b; tum inveniantur fluxionales æquationes ſecundi ordinis $\dot{\mu} = o$, $\dot{\nu} = o$; $\dot{\mu}' = o$, & $\dot{\nu}' = o$, in quibus non inveniuntur variabiles quantitates a & b: ita reducantur prædictæ æquationes, ut exterminentur fluxiones $\dot{x}$, $\dot{y}$, $\dot{z}$, &c.; a & b; & reſultabit algebraica æquatio, in quâ non inveniuntur invariabiles a & b; unde reſultant duæ æquationes una algebraica, & altera fluxionalis ſecundi ordinis, in quibus non inveniuntur a & b; & ſic deinceps ad exterminationem quantitatum (a, b, &c. &c.) progredi liceat.

Conſimilia etiam applicari poſſunt ad tres vel plures datas æquationes quatuor vel plures variabiles quantitates & h invariabiles involventes.

Ea, quæ in hoc prob. tradita fuere, ſimiliter applicari poſſunt ad plures (n) fluxionales æquationes m, r, r, s, &c. ordinum detegendas, quarum variabilium quantitatum dantur n algebraicæ æquationes generales relationes exprimentes.

THEOR. XXXI.

1. Sit fluxionalis æquatio $\lambda = o$ ordinis (n), cujus (n) generales fluentes ſint $\alpha = o$, $\beta = o$, $\gamma = o$, $\delta = o$, &c.; & cujus (n) generales multiplicatores ſint $\pi = o$, $\varrho = o$, $\sigma = o$, $\tau = o$, &c.; erunt $\dot{\alpha} = \pi\lambda$, $\dot{\beta} = \varrho\lambda$, $\dot{\gamma} = \sigma\lambda$, &c.; & $\frac{\dot{\alpha}}{\dot{\beta}} = \frac{\pi}{\varrho}$, $\frac{\dot{\alpha}}{\dot{\gamma}} = \frac{\pi}{\sigma}$, $\frac{\dot{\beta}}{\dot{\gamma}} = \frac{\varrho}{\sigma}$, &c.: hic α, β, γ, δ, &c.; π, ϱ, σ, τ, &c. ſunt algebraicæ vel fluxionales quantitates minorum ordinum quam (n).

2. Generalis fluens fluxionalis æquationis erit quæcunque functio fluentium α, β, γ, δ, &c. i. e. erit $\varphi : (\alpha, \beta, \gamma, \delta, \&c.)$: inveniatur enim ejus fluxio, & reſultat $\varphi' : (\alpha, \beta, \gamma, \delta, \&c.) \times \dot{\alpha} + \varphi'' : (\alpha, \beta, \gamma, \delta, \&c.)\, \dot{\beta}$ $+ \varphi'''$

$$+\varphi''':(\alpha,\beta,\gamma,\delta,\&c.)\times\dot{\gamma}+\&c.=(\varphi':(\alpha,\beta,\gamma,\delta,\&c.)+\varphi'':(\alpha,\beta,\gamma,\delta,\&c.)\frac{\varrho}{\pi}+\varphi''':(\alpha,\beta,\gamma,\delta,\&c.)\frac{\sigma}{\pi}+\&c.)\times\dot{\alpha}=(\varphi':(\alpha,\beta,\gamma,\delta,\&c.)\,\pi+\varphi'':(\alpha,\beta,\gamma,\delta,\&c.)\,\varrho+\varphi''':(\alpha,\beta,\gamma,\delta,\&c.)\times\sigma+\&c.)\times\lambda=o.$$

THEOR. XXXII.

Datâ fluxionali æquatione $P-p=o$, ubi P ſit quæcunque functio literarum x & y & earum fluxionum, p functio literæ x & ejus primarum fluxionum, formulæ vero ita ut multiplicator (M) ſit functio quantitatis x, qui ductus in P præbet fluxionem, cujus fluens, i. e. $\int.MP$ inveniri poteſt; tum etiam erit $\int.M\,(P-p)$ fluxio, cujus fluens innoteſcet; ſed datâ generali fluente æquationis $P=o$ inveniri poſſunt omnes ejus multiplicatores, & conſequenter inveniri poſſunt omnes multiplicatores æquationis $P-p=o$, iidem enim invenientur; & exinde deduci poteſt fluens æquationis prædictæ $P-p=o$.

E. g. Sit æquatio $A\overset{n}{\dot{y}}+B\overset{n-1}{\dot{y}}\ldots Py\dot{x}^n+\mathcal{Q}\dot{x}^n=o$, ubi $A, B\,..\,P, \mathcal{Q}$ ſunt functiones quantitatis x; tum, ſi modo detur generalis valor quantitatis y in æquatione $A\overset{n}{\dot{y}}+B\overset{n-1}{\dot{y}}\,..\,Py\dot{x}^n=o$, in quâ deficit ultimus terminus, & conſequenter omnes multiplicatores eam reddentes integrabilem, qui vero multiplicatores erunt functiones quantitatis x; conſequenter dabuntur etiam multiplicatores, qui reddunt æquationem $A\overset{n}{\dot{y}}+B\overset{n-1}{\dot{y}}\,..\,Py\dot{x}^n+\mathcal{Q}\dot{x}^n=o$ integrabilem.

Ex. 2. Fluens fluxionalis æquationis $x^2\ddot{y}+(m+n+1)\,x\dot{x}\dot{y}+mny\dot{x}^2=o$ erit vel $x^{m+1}\dot{y}+nyx^m\dot{x}=a\dot{x}$, vel $x^{n+1}\dot{y}+myx^n\dot{x}=d'\dot{x}$; ubi x fluit uniformiter: fluentes autem harum duarum fluxionalium æquationum erunt $x^ny=\frac{a\,x^{n-m}}{n-m}+b$ vel $x^my=\frac{d'x^{m-n}}{m-n}+b'$; ubi a, b, d' & b' ſunt invariabiles quantitates ad libitum aſſumendæ.

Si vero aſſumantur $\frac{a}{n-m} = b'$ & $b = \frac{-d'}{n-m}$; tum hæ duæ fluentes erunt eædem.

PROB. XLVIII.

Invenire fluxionalem æquationem, cujus particularis valor ſit data æquatio, viz. $y = \phi : (x)$, *vel quæcunque æquatio* ($\pi = 0$) *relationem inter* x & y *exprimens.*

Aſſumatur quæcunque fluxionalis quantitas $P\dot{x} + Q\dot{y}$, ubi P & Q ſunt functiones quantitatum x & y: in hâc quantitate nonnunquam pro y ſcribatur $\phi : x$ vel pro quibuſdam quantitatibus ſcribantur earum valores ex æquatione $\pi = 0$ deducti; & pro $\dot{y}$ ſcribatur ejus valor ex æquatione $y = \phi : (x)$ vel ex æquatione $\pi = 0$ deductus; & reſultabit fluxionalis æquatio, cujus particularis fluens erit $y = \phi : (x)$ vel $\pi = 0$.

2. Invenire æquationem fluxionalem, cujus particulares valores ſunt quæcunque datæ quantitates $\alpha = 0$, $\beta = 0$, $\gamma = 0$, &c.

Inveniatur fluxionalis æquatio, cujus partiularis fluens erit $\phi : (\alpha) \times \phi : (\beta) \times \phi : (\gamma) \times$ &c. $= 0$: vel inveniatur æquatio, cujus particularis fluens erit $p + a\pi + b\rho + c\sigma +$ &c. $= 0$, ubi $p + a\pi = \alpha$, $p + b\rho = \beta$, $p + c\sigma = \gamma$, &c. & a, b, c, &c. ſunt coefficientes ad libitum aſſumendæ; & perficitur problema.

Infinitis modis facile deduci poteſt hujuſce problematis reſolutio.

PROB. XLIX.

Datis duabus quantitatibus p & q, *quæ ſunt functiones vel algebraicæ vel fluxionales quantitatum* x & y; *invenire utrum* p *ſit functio quantitatis* q, *necne.*

Sint P & Q functiones quantitatum x & y, quibus æquales ſunt p & q reſpective, i. e. ſint $P = p$, $Q = q$; ita reducantur hæ duæ æquationes

nes in unam, ut exterminentur x & ejus fluxiones, & ſi etiam evaneſcat quantitas y & ejus fluxiones, tum p erit functio quantitatis q.

Et ſic de pluribus æquationibus plures variabiles quantitates habentibus.

Cor. Datâ fluente (p) fluxionalis æquationis, & p & q functionibus quantitatum (x & y): per hoc problema detegi poteſt, annon q eſt etiam fluens datæ æquationis.

PROB. L.

1. *Invenire fluxionales æquationes, quarum fluentiales innoteſcunt. Aſſumatur quæcunque æquatio pro fluentiali, inveniatur ejus fluxio, & dividatur per quamcunque quantitatem* N; *tum erit reſultans æquatio fluxionalis, cujus fluentialis innoteſcit.*

2. *Ex iiſdem principiis deduci poſſunt duæ vel tres vel plures* (n) *fluxionales æquationes, tres vel quatuor & denique quemlibet numerum variabilium & earum fluxionum involventes, quarum fluentiales innoteſcunt.*

Aſſumantur quæcunque (n) æquationes involventes ($n+1$ vel plures) variabiles quantitates & earum fluxiones, &c. pro fluentialibus; inveniantur fluxiones cujuſcunque ordinis prædictarum (n) aſſumptarum æquationum, & ducantur æquationes reſultantes in quaſcunque invariabiles quantitates; addantur ſimul hæc producta, ita ut evadant (n) diverſæ independentes æquationes; tum dividantur hæ (n) æquationes per quaſlibet quantitates; & reſultant (n) fluxionales æquationes, quarum (n) fluentiales innoteſcunt.

Aſſumendæ ſunt æquationes pro fluentialibus hujuſce formulæ $P+a=0$, &c. ubi P eſt quælibet functio variabilium & earum fluxionum, & a eſt quæcunque invariabilis quantitas ad libitum aſſumenda.

THEOR.

THEOR. XXXIII.

De methodis inveniendi fluentes fluxionalium æquationum.

1. Sit æquatio, in quâ continentur variabiles quantitates x & y & earum fluxiones r ordinum, & in quâ nulla quantitas fluit uniformiter; & ſi ſupponantur omnes termini, in quibus haud continentur $\overset{r}{\dot{x}}$ vel $\overset{r}{\dot{y}}$ nihilo æquales, & ſit æquatio reſultans $P\overset{r}{\dot{x}} + \mathcal{Q}\overset{r}{\dot{y}} = o$, & aſſumatur æquatio $P\dot{x} + \mathcal{Q}\dot{y} = o$, & ducatur ea in quantitatem q, ita ut fiat integrabilis, & inveniatur fluens æquationis reſultantis $qP\dot{x} + q\mathcal{Q}\dot{y} = o$, quæ dicatur t; fluens datæ æquationis erit functio quantitatis t, quâ inventâ plerumque e principiis in theor. 2. datis deduci poteſt fluens quæſita.

Demonſtrari poteſt hoc theorema e methodo inveniendi fluxiones fluentium.

Et ſic inveniri poteſt fluens m ordinis datæ æquationis, ſi modo fluentem recipiat data æquatio.

2. Si $\overset{r}{\dot{y}}{}^{s}$ ſit maxima dimenſio fluxionis $\overset{r}{\dot{y}}$, cujus ordo eſt maximus datâ æquatione contentus; tum 1^{mo}. ita reducenda eſt data æquatio, ut haud plures quam una dimenſio quantitatis $\overset{r}{\dot{y}}$ in eâ contineantur: & reſultabunt plures fluxionales æquationes $P = o$, $\mathcal{Q} = o$, $R = o$, &c., quarum diverſæ ſunt reſolutiones, diverſique etiam multiplicatores; e. g. ſit fluxionalis æquatio $\overset{r}{\dot{y}}{}^{2} - (A + B)\overset{r}{\dot{y}} + AB = o$, ubi literæ A & B reſpective denotant functiones quantitatum $y, \dot{y}, \ddot{y}, \dddot{y}, \ldots \overset{r-1}{\dot{y}}$; reliquarum variabilium & earum fluxionum; tum reduci poteſt data fluxionalis æquatio ad duas alias $\overset{r}{\dot{y}} - A = o$ & $\overset{r}{\dot{y}} - B = o$, quarum fluentes & multiplicatores poſſunt eſſe diverſi.

In reſolvendis fluxionalibus æquationibus, in quibus involvuntur ſolummodo x & y, $\dot{x}$ & $\dot{y}$: 1^{mo}. ita reducendæ ſunt datæ æquationes, ut haud plures quam una dimenſio fluxionum $\dot{x}$ & $\dot{y}$ in datis æquationibus contineantur.

Ex. g. Sit æquatio fluxionalis $x^2\dot{x}^2 + xy\dot{x}\dot{y} = a^2\dot{y}^2$, erit $x^2\dot{x}^2 + xy\dot{x}\dot{y} + \frac{1}{4}y^2\dot{y}^2 = (a^2 + \frac{1}{4}y^2)\dot{y}^2$, unde $x\dot{x} + \frac{1}{2}y\dot{y} = \pm\surd(a^2 + \frac{1}{4}y^2)\dot{y}$.

3. Transformetur data fluxionalis æquatio in eandem æquationis partem, i. e. nihilo fiat æqualis; & quantitatis reſultantis inveniatur per methodos in præcedente capite traditas fluens, ſi modo fieri poſſit, & invenitur fluens quæſita.

4. Si fluens haud præcedente methodo deduci poſſit; inveſtiganda eſt quantitas, quæ in datam æquationem ducta, creat æquationem, cujus fluens inveniri poteſt.

In hiſce quantitatibus deducendis, quæ in datam æquationem ductæ, præbent æquationes, quarum fluentes inveniri poſſunt; haud inutiles forſan erunt ſubſequentes obſervationes.

1$^{\text{ma}}$. Nullam compoſitam irrationalem quantitatem, i. e. irrationalem quantitatem duobus vel pluribus terminis conſtantem involvi neceſſe eſt plerumque in quæſitâ reſultante æquatione, quæ in datâ haud invenitur, ni exponentiales in quæſitâ fluente contineantur quantitates; e. g. ſit data fluxionalis æquatio $x\dot{x} + 2a(x\dot{x} + y\dot{y})(a^2 + x^2)^{\frac{1}{2}}(x^2 + y^2)^{\frac{1}{2}} = y\dot{y}(a^2 + x^2)^{\frac{1}{2}}$; in hâc æquatione duæ continentur irrationales compoſitæ quantitates, viz. $(a^2 + x^2)^{\frac{1}{2}}$ & $(x^2 + y^2)^{\frac{1}{2}}$, ergo haud improbabile erit finitam æquationem, quæ exprimit fluentem $(a^2 + x^2)^{\frac{1}{2}} + \frac{2a}{3}(x^2 + y^2)^{1\frac{1}{2}} = \frac{1}{2}y^2 + b$, nullas alias compoſitas irrationales quantitates involvere, ni in eâ contineantur exponentiales vel fluentiales quantitates: in genere igitur aſſumatur quantitas prædicti generis, quæ eſt generalis rationalis functio quantitatum x^n & irrationalium quantitatum in datâ æquatione contentarum, ad poteſtatem vel radicem (m) evecta; (hìc forſan haud indignum eſt obſervatu generalem rationalem functionem quantitatum x^n & v^m eſſe fractionem

$$\frac{(ax^{rn} + bx^{(r+s)n} + cx^{(r+2s)n} + \&c.)\,v^{em} + (a'x^{r'n} + bx^{(r'+s')n} + \&c.)\,v^{(e+h)m} + \&c.}{(Ax^{\alpha n} + Bx^{(\alpha+\beta)n} + Cx^{(\alpha+2\beta)n} + \&c.)\,v^{\tau m} + (A'x^{\alpha' n} + B'x^{(\alpha'+\beta')n} + \&c.)\,v^{(\tau'+h')m} + \&c.}\Bigg);$$

& exinde deduci poteſt fluens, ni in eâ contineantur exponentiales vel fluentiales quantitates.

2^{da}. In datis diverſis terminis datæ æquationis contineantur diverſæ irrationales quantitates; ſupponantur omnes termini, in quibus continentur eædem irrationales quantitates, vel eædem duæ vel tres diverſæ irrationales vel rationales quantitates nihilo æquales; tum ex æquatione reſultante ſæpe detegi poteſt quantitas, quæ in datam æquationem ducta, facit æquationem, cujus fluens inveniri poteſt.

3^{tia}. Dimenſiones quantitatis y in fluxionem $\dot{x}$ ductæ debent eſſe majores per unitatem, quam dimenſiones ejuſdem quantitatis in $\dot{y}$ ductæ, in termino, in quo involvuntur ſimul literæ x & y; & ſic de literâ x.

Ex. Sit æquatio $p\dot{y}+qy\dot{x}=r\dot{x}$, ubi literæ p, q & r reſpective denotant quaſcunque functiones literæ x; ducatur data æquatio in P functionem quantitatis x, & reſultat $Pp\dot{y}+Pqy\dot{x}=Pr\dot{x}$; ſupponatur $+Ppy$ fluens fluxionis $Pp\dot{y}+Pqy\dot{x}$, & conſequenter $Pp\dot{y}+y(P\dot{p}+\dot{P}p)=Pp\dot{y}+Pqy\dot{x}$, unde $P\dot{p}+p\dot{P}=Pq\dot{x}$, & $P(q\dot{x}-\dot{p})=p\dot{P}$, & exinde $\frac{q\dot{x}-\dot{p}}{p}=\frac{\dot{P}}{P}$, & $\int.\frac{q\dot{x}}{p}=\log. Pp$, & $P=e^{\int.\frac{q\dot{x}-\dot{p}}{p}}$, & $Ppy=e^{\int.\frac{q\dot{x}}{p}}y=e\int.\frac{q\dot{x}-\dot{p}}{p}\times r\dot{x}$.

Cor. 1. Si $\int.\frac{q\dot{x}}{q}$ ſit logarithmus quantitatis $\mathcal{Q}$, tum erit $P=\frac{\mathcal{Q}}{p}$.

Cor. 2. Sit æquatio $pv^{n-1}\dot{v}+qv^{n}\dot{x}=r\dot{x}$; in eâ pro v^{n} & $v^{n-1}\dot{v}$ ſcribantur reſpective y & $\frac{\dot{y}}{n}$, reſultat æquatio præcedentis formulæ $\frac{p}{n}\dot{y}+qy\dot{x}=r\dot{x}$.

Cor. 3. Sit $p\dot{v}+v\dot{x}=rv^{-n+1}\dot{x}$; ducatur hæc æquatio in v^{n-1}, & reſultat æquatio $pv^{n-1}\dot{v}+qv^{n}\dot{x}=r\dot{x}$ præcedentis formulæ.

5. Sit fluxio $A\dot{x}+B\dot{y}+C\dot{x}+D\dot{y}=o$, (ubi literæ A & B denotant functiones quantitatum x & y reſpective, & C & D ſunt quæcunque functiones quantitatum x & y conjunctim) cujus inveniri poteſt fluens: ſint vero dimenſiones quantitatis y in functione B

majores quam dimenſiones ejuſdem quantitatis y in functione D; dividatur æquatio $A\dot{x} + B\dot{y} + C\dot{x} + D\dot{y} = 0$ per functionem P quantitatis x, & reſultat æquatio in quâ dimenſiones quantitatis y in fluxionem $\dot{x}$ ductæ minores erunt quam dimenſiones ejuſdem quantitatis y in termino $\frac{B}{P}$ (in quo continentur literæ x & y conjunctim) in $\dot{y}$ ductæ per quantitatem majorem quam unitatem; ſed facile reduci poteſt æquatio $\frac{A+C}{P}\dot{x} + \frac{B+D}{P}\dot{y} = 0$ ad æquationem integrabilem $A\dot{x}+B\dot{y}+C\dot{x}+D\dot{y} = 0$ ex ejus multiplicatione in P: omnes igitur datæ fluxionales æquationes, (ubi dimenſiones quantitatis y in fluxionem $\dot{x}$ ductæ haud ſuperant dimenſiones ejuſdem quantitatis y in termino, in quo involvuntur quantitates x & y conjunctim in $\dot{y}$ ductæ per unitatem) reducendæ ſunt ad fluxionales æquationes formulæ prædictæ; quod ſæpe perficietur, ſi modo auferantur quantitates in quibus continetur x e terminis, in quibus dimenſiones quantitatis y ſunt nimis magnæ.

Et ſic de reducendis terminis, in quibus dimenſiones quantitatis x ſunt nimis magnæ.

Ex. Sit fluxionalis æquatio $8y^7x^{-6}\dot{y} + 7ay^5\dot{x} + 5axy^4\dot{y} + 6bx^{-1}\dot{x} = 0$: in hac æquatione dimenſiones (5) quantitatis y in $\dot{x}$ ductæ haud ſuperant prædictas dimenſiones (7) ejuſdem quantitatis y in $\dot{y}$ ductas per unitatem; ergo auferatur quantitas (x^{-6}), in quâ continetur x, e termino prædicto $(8y^7x^{-6}\dot{y})$, i. e. ducatur data æquatio in x^6; & reſultat æquatio quæſita $8y^7\dot{y} + 7ay^5x^6\dot{x} + 5ax^7y^4\dot{y} + 6bx^5\dot{x} = 0$, cujus fluens erit $y^8 + ay^5x^7 + bx^6 =$ conſt.

6. Si vero multiplicator conſtet e pluribus terminis; aſſumatur pro multiplicatore primus terminus e prædicta methodo inventus $+p$, & e methodo prius traditâ inveniatur proximus valor quantitatis p, qui erit ſecundus terminus quæſitus; & ſic deinceps. Hic vero animadvertendum eſt, multiplicatorem conſtare poſſe e terminis & in numeratore & denominatore contentis; etiamque, quodſi nulla exponentialis & fluentialis quantitas in datâ æquatione contineatur,

nullam exponentialem niſi formulæ (e^{v}) vel in multiplicatore vel in fluente neceſſario contineri; & generaliter ſi modo ſint π, ϱ, σ, &c. exponentiales quantitates in datâ fluxionali æquatione contentæ, nullas eſſe exponentiales quantitates in ejus multiplicatore vel fluente neceſſario contentas, niſi formularum π, ϱ, σ, &c. e^{v}, e^{π}, e^{ϱ}, e^{σ}, &c. exponentiales.

7. Eadem etiam principia applicari poſſunt ad detegendas fluentes fluxionum, in quibus continentur fluxiones ſuperiorum ordinum; ſed aſſumi debent indices, &c. in terminis magis generalibus quam in fluentibus fluxionum primi ordinis detegendis.

In hoc loco animadvertendum eſt.

1. Sit data æquatio, in quâ continentur quæcunque functiones quantitatum x & y in $\dot{x}$ & $\dot{y}$ ductarum; æquationis multiplicator, qui eam integrabilem reddit, quantitas erit algebraica, itemque functio quantitatum x & y.

2. Si in datâ æquatione contineantur functiones quantitatum $x, y, \dot{x}, \dot{y}, \ddot{x}$ & $\ddot{y}$; tum poteſt eſſe ejus multiplicator quæcunque functio quantitatum x, y, $\dot{x}$ & $\dot{y}$.

3. Si in datâ æquatione contineantur functiones quantitatum $x, y, \dot{x}, \dot{y}, \ddot{x}, \ddot{y}, \dddot{x}$ & $\dddot{y}$; tum poteſt eſſe ejus multiplicator quæcunque functio quantitatum $x, y, \dot{x}, \dot{y}, \ddot{x}$ & $\ddot{y}$: & ſic deinceps.

Ex. 1. Sit $2ay\ddot{y} - 4a\dot{y}^2 - y^{n+5}\dot{x}^2(1+x^2)^{\frac{n-1}{2}} = 0$; ducatur hæc æquatio in $\frac{x\dot{x}}{y^4} + \frac{(1+x^2)\dot{y}}{y^5}$, & reſultat $\frac{2ax\dot{x}\ddot{y}}{y^3} + \frac{2a(1+x^2)\dot{y}\ddot{y}}{y^4} - \frac{4ax\dot{x}\dot{y}^2}{y^4} - \frac{4a(1+x^2)\dot{y}^3}{y^5} - (y^{n+1}\dot{x}^3x + (1+x^2)y^n\dot{x}^2\dot{y})(1+x^2)^{\frac{n-2}{2}} = 0$, cujus fluens reperietur $\frac{a\dot{x}^2}{y^2} + \frac{2ax\dot{x}\dot{y}}{y^3} + \frac{a(1+x^2)\dot{y}^2}{y^4} = \frac{1}{n+1}y^{n+1}\dot{x}^2(1+x^2)^{\frac{n+1}{2}} + C\dot{x}^2$, ubi C & $\dot{x}$ ſunt invariabiles quantitates.

Ex. 2. Sit $by\dot{x}\dot{y} + bsx\dot{y}^2 + bxy\ddot{y} + \frac{c\dot{y}^{t-1}\ddot{y}}{y^{s-1}(a\dot{x} + bxy^s\dot{y})^{t-1}} = 0$, ducatur hæc æquatio in multiplicatorem $ty^{s-1}(a\dot{x} + bxy^s\dot{y})^{t-1}$; & reſultat æquatio, cujus fluens erit $(a\dot{x} + bxy^s\dot{y})^t + c\dot{y}^t =$ conſt.

Ex. 3. Sit $z^{m+1}\dot{x}^m\ddot{x} + \dot{y}^{m+1}\frac{\dot{z}}{z} = \dot{y}^m\ddot{y}$; dividatur hæc æquatio per z^{m+1}, & reſultat $\dot{x}^m\ddot{x} + \dot{y}^{m+1}\frac{\dot{z}}{z^{m+2}} = \frac{\dot{y}^m\ddot{y}}{z^{m+1}}$, cujus fluens erit $\dot{x}^{m+1} = \frac{\dot{y}^{m+1}}{z^{m+1}} + C\dot{v}^{m+1}$, ubi v eſt quantitas, quæ fluit uniformiter.

Ex. 4. Sit $\ddot{y} = 4x^3\ddot{y} + 6x^2\dot{x}\dot{y} - 2xy\dot{x}^2$; ducatur hæc æquatio in $2x\dot{y} - y\dot{x}$, & æquationis reſultantis fluens erit $x\dot{y}^2 - y\dot{x}\dot{y} = 4x^4\dot{y}^2 - 4x^3y\dot{y}\dot{x} + x^2y^2\dot{x}^2 + C\dot{x}^2$, ubi $C =$ conſt.

8. Sæpe e quibuſdam datæ fluxionalis æquationis terminis nihilo æqualibus eſſe ſuppoſitis detegi poteſt multiplicator, qui in datam fluxionalem æquationem multiplicatus, præbet fluxionem integrabilem. Inveniantur enim multiplicatores, qui in hos & reliquos terminos multiplicati, præbent fluxiones integrabiles; ex his vero ſæpe conſtabit multiplicator quæſitus.

1. Sit fluxionalis æquatio $A + B = 0$; inveniatur quantitas (a), quæ in A ducta, reddit fluxionem aA integrabilem, cujus fluens ſit A'; etiamque inveniatur quantitas b, quæ in B ducta, præbet fluxionem bB integrabilem, cujus fluens ſit B'; inveniantur tales functiones quantitatum A' & B', viz. $\phi A'$ & $\Delta B'$, ut ſit $\phi A' : \Delta B' :: b : a$; & quantitas $a\phi A'$ in datam æquationem $A + B = 0$ ducta, eam reddit integrabilem.

2. Sit æquatio $A + B + C + D +$ &c. $= 0$, & ſint fluentes fluxionum aA, bB, cC, dD, &c. reſpective A', B', C', D', &c.: deinde ſit $a\phi : A' = b\phi' : B' = c\phi'' : C' = d\phi''' : D$; & ex his æquationibus inveniri poteſt valor quantitatis $a\phi A$, qui ductus in datam fluxionalem æquationem, eam reddit integrabilem.

Cor. Hinc, ſi detegi poſſit generaliter reſolutio prædictæ algebraicæ æquationis $a\phi : A' = b\phi' : B'$, detegi poſſunt generaliter multiplicatores; nam, ſi generaliter detegi poſſint reſolutiones æquationum $a\phi : A' = b\phi' : B'$, quæ ſit $\Delta : \alpha$; detegi poteſt generaliter reſolutio æquationis $\Delta : \alpha = c\phi'' : C'$; & ſic deinceps.

Cor. 2. Hinc inveniri poſſunt fluxionales æquationes, quarum fluentes dantur: inveniantur enim duæ diverſæ fluxiones A & B, quæ in datam quantitatem vel fluxionem ductæ, præbent fluxiones, quarum fluentes cognoſcuntur; & erit $A + B = o$ fluxionalis æquatio, cujus fluens inveniri poteſt.

Ex. 1. $\ddot{y} - \frac{m\dot{y}^2}{y}$, & $y^n \dot{x}^2 (\alpha + 2\beta x + \gamma x^2)^{\frac{n-4m+3}{2m-2}}$ ſunt fluxiones, quæ in (P) factorem $\frac{(\beta + \gamma x)\dot{x}}{(m-1)y^{2m-1}} + \frac{(\alpha + 2\beta x + \gamma x^2)\dot{y}}{y^{2m}}$ ductæ, præbent fluxiones, quarum fluentes ſunt reſpective $\frac{(\beta + \gamma x)\dot{x}\dot{y}}{(m-1)y^{2m-1}} + \frac{(\alpha + 2\beta x + \gamma x^2)\dot{y}^2}{2y^{2m}} + \frac{\gamma\dot{x}^2}{2(m-1)^2 y^{2m-2}}$, & $\frac{1}{n-2m+1} y^{n-2m+1} \dot{x}^2 (\alpha + 2\beta x + \gamma x^2)^{\frac{n-2m+1}{2m-2}}$; & conſequenter fluens fluxionalis æquationis $\ddot{y} = \frac{m\dot{y}^2}{y} + y^n \dot{x}^2 (\alpha + 2\beta x + \gamma x^n)^{\frac{n-4m+3}{2m-2}} = o$ acquiri poteſt.

Cor. 1. In æquatione $\ddot{y} - \frac{m\dot{y}^2}{y} + y^n \dot{x}^2 (\alpha + 2\beta x + \gamma x^2)^{\frac{n-4m+3}{2m-2}} = o$ ſcribatur pro y ejus valor aſſumptus $z^{\frac{1}{1-m}}$, & reſultat $\frac{\ddot{z}}{1-m} + z^{\frac{n-m}{1-m}} \dot{x}^2 (\alpha + 2\beta x + \gamma x^2)^{\frac{n-4m+3}{2m-2}} = o$.

Ex. 2. Sint $\ddot{y}$ & $\frac{a y^{n+1} \dot{x}^2}{(\alpha + 2\beta x + \gamma x^2 + c y^2)^{\frac{n+4}{2}}}$ quantitates, quæ in eandem fluxionem $\dot{y}(\alpha + 2\beta x + \gamma x^2) - y\dot{x}(\beta + \gamma x)$ ductæ, præbent fluxiones integrabiles, quarum fluentes ſunt $\frac{1}{2}\dot{y}^2(\alpha + 2\beta x + \gamma x^2) - y\dot{x}\dot{y}$

$-y\dot{x}\dot{y}(\beta+\gamma x)+\frac{1}{2}\gamma\dot{x}^2y^2$, & $\dfrac{ay^{n+2}\dot{x}^2}{(n+2)(\alpha+2\beta x+\gamma x^2+cy^2)^{\frac{n+2}{2}}}$, unde fluens fluxionalis æquationis $\ddot{y}+\dfrac{ay^{n+1}\dot{x}^2}{(\alpha+2\beta x+\gamma x^2+cy^2)^{\frac{n+4}{2}}}=0$ detegi poteſt.

Cor. Sit æquatio $\ddot{y}+\dfrac{ay^{n+1}\dot{x}^2}{(x^2+y^2)^{\frac{n+4}{2}}}=0$, cujus fluens erit $\frac{1}{2}(y\dot{x}-x\dot{y})^2+\dfrac{ay^{n+2}\dot{x}^2}{(n+2)(x^2+y^2)^{\frac{n+2}{2}}}=C\dot{x}$: in hâc æquatione ſcribatur ux pro y, & erit $-x^2\dot{u}=y\dot{x}-x\dot{y}$, & fluens $\frac{1}{2}(y\dot{x}-x\dot{y})^2+\dfrac{ay^{n+2}\dot{x}^2}{(n+2)(x^2+y^2)^{\frac{n+2}{2}}}=\frac{1}{2}x^4\dot{u}^2+\dfrac{au^{n+2}\dot{x}^2}{(n+2)(1+u^2)^{\frac{n+2}{2}}}=C\dot{x}$, cujus fluxionis fluens inveniri poteſt.

Ex· 3. Propoſitâ æquatione fluxionali $\alpha y\dot{x}+\beta x\dot{y}=x^my^n(\gamma y\dot{x}+\delta x\dot{y})$; omnes vero multiplicatores, qui ducti in $\alpha y\dot{x}+\beta x\dot{y}$ præbent fluxiones, quarum fluentes detegi poſſunt, erunt functiones quantitatis $x^\alpha y^\beta$ in fractionem $\dfrac{1}{xy}$ ductæ: eodem modo omnes multiplicatores, qui ducti in $x^my^n(\gamma y\dot{x}+\delta x\dot{y})$ præbent fluxiones, quarum fluentes detegi poſſunt, erunt functiones quantitatis $x^\gamma y^\delta$ in fractionem $\dfrac{1}{x^{m+1}y^{n+1}}$ ductæ; ſed hi multiplicatores poſſunt eſſe hujuſmodi $x^{\mu\alpha-1}y^{\mu\beta-1}$ & $x^{\nu\gamma-m-1}y^{\nu\delta-n-1}$; unde, ſi modo $\mu\alpha-1=\nu\gamma-m-1$, & $\mu\beta-1=\nu\delta-n-1$, tum multiplicator inventus in propoſitam æquationem ductus eam reddit integrabilem; ſed in hoc caſu $\mu=\dfrac{\gamma n-\delta m}{\alpha\delta-\beta\gamma}$ & $\nu=\dfrac{\alpha n-\beta m}{\alpha\delta-\beta\gamma}$.

Cor. Si $\alpha\delta-\beta\gamma=0$, tum propoſita æquatio evadet $\alpha y\dot{x}+\beta x\dot{y}$

$=\gamma$

$= \frac{\gamma}{\alpha} x^m y^n (\alpha y \dot{x} + \beta x \dot{y})$, unde erit $(\alpha y \dot{x} + \beta x \dot{y})(1 - \frac{\gamma}{\alpha} x^m y^n) = o$, & exinde $\alpha y \dot{x} + \beta x \dot{y} = o$.

Multiplicator nonnunquam eſt algebraica functio variabilium quantitatum & earum fluxionum in datâ æquatione contentarum; nonnunquam eſt etiam algebraica, fluentialis & exponentialis functio prædictarum quantitatum; & nonnunquam per infinitas ſeries prædictarum quantitatum ſolummodo exprimi poteſt.

PROB. LI.

Datâ formulâ æquationis fluxionalis, & quantitatis in prædictam æquationem ducendæ; invenire caſus, in quibus quantitas reſultans evadit integrabilis.

Aſſumantur generaliter datæ formulæ fluxionalis æquatio & ejus multiplicator; ducantur hæ duæ quantitates in ſeſe, & inveniantur per methodos in præcedente capite traditas caſus, in quibus datur fluens æquationis reſultantis; & perficitur problema.

Si innoteſcant formulæ multiplicatorum, tum etiam innoteſcent formulæ fluentium, & conſequenter cum detur fluens ex aſſumptis multiplicatoribus, tum etiam dabitur fluens ex aſſumptis fluentibus: aſſumatur enim generaliter fluens datæ formulæ, cujus inveniatur fluxio, dividatur hæc fluxio per datam fluxionalem æquationem, & ſi quotiens ſit quantitas vel algebraica vel fluxionalis minoris quam ordinis datæ fluxionalis æquationis; tum aſſumpta erit fluens datæ fluxionalis, ſin aliter non.

Hic forſan haud indignum eſt obſervatu; cum in datâ fluxionali æquatione dimenſiones quantitatis y in x non ſuperent ejus dimenſiones in $\dot{y}$ ductas per unitatem; tum pro ejus multiplicatore vel fluente aſſumenda eſt quantitas, quæ continet poteſtatem vel radicem quantitatis y, quæ in nullam functionem quantitatis x ducitur. e. g. Sit quantitas $y\dot{y} + Py\dot{x} + Q\dot{x} = o$, ubi P & Q ſunt functiones ipſius x; tum multiplicatores poſſunt eſſe $(y+M)^n \times (y+N)^m \times$ &c.: vel etiam ex datis fluxionibus reſultantibus a datâ fluxione in multiplicatorem ductâ erui poteſt formula fluentis; vel facile aſſumi poteſt formula fluentis, & exinde erui poſſunt caſus, in quibus fluens datæ fluxionis habet formulam aſſumptam.

Ex. Sit data æquatio $y\dot{y} + Py\dot{x} + \mathcal{Q}\dot{x} = 0$, ducatur hæc æquatio in $(y+M)^n$, ubi P, $\mathcal{Q}$ & M ſunt functiones ipſius x, & reſultat $(y+M)^n y\dot{y} + P(y+M)^n y\dot{x} + \mathcal{Q}(y+M)^n \dot{x} = 0$; ſed per præcedens caput $\int. (y+M)^n y\dot{y} = \frac{(y+M)^{n+2}}{n+2} - \frac{M(y+M)^{n+1}}{n+1}$, ſi modo M ſit invariabilis; nunc autem ſit M variabilis, & fluxio quantitatis $\frac{(y+M)^{n+2}}{n+2} - \frac{M(y+M)^{n+1}}{n+1}$ erit $(y+M)^{n+1}(\dot{y}+M) - M(y+M)^n (\dot{y}+\dot{M}) - \frac{\dot{M}(y+M)^{n+1}}{n+1}$; unde $(y+M)^{n+1}(\dot{y}+\dot{M}) - M(y+M)^n (\dot{y}+\dot{M}) - \frac{\dot{M}(y+M)^{n+1}}{n+1} = (y+M)^n y\dot{y} + P(y+M)^n y\dot{x} + \mathcal{Q}(y+M)^n \dot{x}$, & exinde $\left(\frac{n\dot{M}}{n+1} - P\dot{x}\right)y - \left(\frac{M\dot{M}}{n+1} + \mathcal{Q}\dot{x}\right) = 0$; fingantur $\frac{n\dot{M}}{n+1} - P\dot{x} = 0$ & $\frac{M\dot{M}}{n+1} = -\mathcal{Q}\dot{x}$, & conſequenter erunt $\frac{n\dot{M}}{(n+1)\dot{x}} = P$, & $-\frac{M\dot{M}}{(n+1)\dot{x}} = \mathcal{Q}$, & data æquatio $y\dot{y} + \frac{n\dot{M}}{(n+1)} y - \frac{M\dot{M}}{(n+1)} = 0$.

2. Hoc problema etiam e principiis in theor. 2. traditis reſolvi poteſt. E. g. Sit fluxio $\alpha\dot{x} + \beta\dot{y}$, cujus fluens inveniri poteſt; tum erit $\left(\frac{\dot{\alpha}}{\dot{y}}\right) = \left(\frac{\dot{\beta}}{\dot{x}}\right)$.

Ex. 1. Sit æquatio $Py\dot{x} + (y+\mathcal{Q})\dot{y} = 0$ & multiplicator $\frac{1}{y^3 + My^2 + Ny}$; ubi P, $\mathcal{Q}$, M & N ſunt functiones quantitatis x:

ducatur æquatio in multiplicatorem, & reſultat $\frac{Py\dot{x}}{y^3+My^2+Ny}+\frac{(y+Q)\dot{y}}{y^3+My^2+Ny}=0$; ut vero fluens hujuſce fluxionis inveniatur, neceſſe eſt fluxionem factoris $\frac{Py}{y^3+My^2+Ny}$, in quo y ſolummodo ſumitur variabilis, per $\dot{y}$ diviſam, æqualem eſſe fluxioni quantitatis $\frac{y+Q}{y^3+My^2+Ny}$, in quâ ſolummodo ſumitur x variabilis, per $\dot{x}$ diviſæ; i.e. $-2Py^3-PMy^2=(y^3+My^2+Ny)\frac{\dot{Q}}{\dot{x}}-(y+Q)\frac{(y^2\dot{M}+y\dot{N})}{\dot{x}}$, quæ reducta ad terminos ſecundum poteſtates quantitatis y progredientes, evadet $0=(2P\dot{x}+\dot{Q}-\dot{M})y^3+(PM\dot{x}+M\dot{Q}-\dot{N}-Q\dot{M})y^2+(N\dot{Q}-Q\dot{N})y$; unde ſingulis poteſtatibus ſeorſim ad nihilum productis, exorietur $N\dot{Q}-Q\dot{N}=0$, & conſequenter $N=aQ$, ubi a erit conſtans quantitas; quo valore pro N in duabus reliquis quantitatibus ſubſtituto, reſultant $2P\dot{x}+\dot{Q}-\dot{M}=0$ & $PM\dot{x}+M\dot{Q}-a\dot{Q}-Q\dot{M}=0$; ita reducantur hæ æquationes, ut exterminetur $\dot{x}$, & reſultat æquatio $-M\dot{Q}-M\dot{M}+2a\dot{Q}+2Q\dot{M}=0$, quâ multiplicatâ in $\frac{1}{(2a-M)^3}$; fluxionis reſultantis fluens inventa dat $\frac{Q}{(2a-M)^2}=\int.\frac{M\dot{M}}{(2a-M)^3}=\frac{-1}{2a-M}+\frac{a}{(2a-M)^2}+b$. Sed erit $2P\dot{x}=\dot{M}-\dot{Q}$, unde pro M aſſumatur quæcunque functio quantitatis variabilis x, & exinde deduci poſſunt reliquæ quantitates Q, P & N.

Ex. 2. Sit præcedens æquatio $Py\dot{x}+(y+Q)\dot{y}=0$, & multiplicator $\frac{y^{n-1}}{y^2+My+N}$: ducatur æquatio in prædictum multiplicatorem & reſultat $\frac{Py^n\dot{x}+y^{n-1}(y+Q)\dot{y}}{y^2+My+N}=0$; ut vero fluens hujuſce fluxionis

nis inveniatur, neceſſe eſt fluxionem factoris $\frac{Py^{n}}{y^{2}+My+N}$, in quo y ſolummodo ſumitur variabilis, per $\dot{y}$ diviſam, æqualem eſſe fluxioni quantitatis $\frac{(y+Q)y^{n-1}}{y^{2}+My+N}$, in quâ ſolummodo ſumitur x variabilis, per $\dot{x}$ diviſæ; unde reſultans æquatio ad terminos ſecundum poteſtates quantitatis y progredientes reducta, evadet $((n-2)P\dot{x}-\dot{Q}+\dot{M})y^{n+1}+((n-1)PM\dot{x}-M\dot{Q}+\dot{N}+Q\dot{M})y^{n}+(nPN\dot{x}-N\dot{Q}+Q\dot{N})y^{n-1}=0$; & e ſingulis poteſtatibus ſeorſim nihil evadere ſuppoſitis, reſultabunt æquationes $(n-2)P\dot{x}=\dot{Q}-\dot{M}$; $(n-1)MP\dot{x}=M\dot{Q}-Q\dot{M}-\dot{N}$; & $nNP\dot{x}=N\dot{Q}-Q\dot{N}$; ex his æquationibus inveniri poſſunt in quibuſdam caſibus correſpondentes valores quantitatum P, Q, M & N.

Ex. 3. Sit data æquatio $yP\dot{x}+(Qy+R)\dot{y}=0$, in quâ P, Q & R ſint functiones ipſius x, & multiplicator $\frac{y^{m}}{(1+Sy)^{n}}$, ubi S eſt etiam functio ipſius x; ex hâc methodo inveniri poſſunt correſpondentes valores quantitatum P, Q, R & S, quando data æquatio in multiplicatorem ducta, fiat integrabilis; etiamque correſpondentes valores earundem quantitatum, cum multiplicator ſit $\frac{y^{m}}{(1+Sy+Ty^{2})^{n}}$; &c. & ſic inveniri poſſunt caſus, in quibus æquatio $(Py+Q)\dot{x}+y\dot{y}=0$ redditur integrabilis per multiplicatorem $(y+R)^{m}\times(y+S)^{n}$, ubi P, Q, R & S ſunt etiam functiones quantitatis x; & ſimiliter detegi poſſunt caſus, in quibus æquatio $\dot{y}+y^{2}\dot{x}+X\dot{x}=0$ redditur integrabilis per multiplicatorem $\frac{1}{Py^{2}+Qy+R}$, &c.

3. 1. Sit æquatio $(Px^{m}+Qx^{m-1}+Rx^{m-2}+\ldots+Tx^{m-b})\dot{v}+(px^{m-1}+qx^{m-2}+rx^{m-3}+\ldots+tx^{m-b-1})\dot{x}=0$; ducatur data æquatio in quantitatem $ax^{n}+bx^{n-1}+cx^{n-2}+\ldots+ex^{n-l}$, & reſultat $(aPx^{m+n}+(aQ+bP)x^{m+n-1}+(aR+bQ+cP)x^{m+n-2}+\&c.)$

$\dot{v} + (apx^{m+n-1} + (aq+bp)x^{m+n-2} + (ar+bq+cp)x^{m+n-3} + \&c.)$ $\dot{x} = 0$; & ſi modo reſultans quantitas ſit integrabilis, tum fluxio quantitatis ap, quæ eſt $a\dot{p} + p\dot{a} = (m+n)aP\dot{v}$, & ſimiliter fluxio quantitatis $aq + bp$, quæ erit $a\dot{q} + q\dot{a} + b\dot{p} + p\dot{b} = (m+n-1)$ $(aQ + bP)\dot{v}$, etiamque $a\dot{r} + r\dot{a} + b\dot{q} + q\dot{b} + c\dot{p} + p\dot{c} = (m$ $+ n - 2)(aR + bQ + cP)\dot{v}$; & ſic deinceps: unde reſultant $(k + h + 1)$ æquationes involventes $(k + 2h + 3)$ variabiles quantitates; deinde ita aſſumantur hæ $(k + 2h + 3)$ variabiles, ut ſatisfaciant prædictis $(k + h + 1)$ fluxionalibus æquationibus, & conſequenter aſſumi poſſunt ad libitum $h + 1$ variabiles quantitates; at in pleriſque caſibus relationes inter variabiles quantitates aſſumere præſtat, ita ut ſolutio magis facilis evadat; & ſequitur fluens deductæ fluxionalis æquationis.

2. Et ſimiliter ducatur data æquatio in fractionalem quantitatem $\frac{ax^n + bx^{n-1} + cx^{n-2} + \&c.}{d'x^{n'} + b'x^{n'-1} + c'x^{n'-2} + \&c.}$; vel in $\frac{(ax^n + bx^{n-1} + cx^{n-2} + \&c.)^\pi}{(a'x^{n'} + b'x^{n'-1} + c'x^{n'-2} + \&c.)^\rho}$;

$$\text{vel in } \frac{(ax^n + bx^{n\pm\frac{1}{l}} + cx^{n\pm\frac{2}{l}} + dx^{n\pm\frac{3}{l}} \ldots x^{n-1} + dx^{n-1\pm\frac{1}{l}} + \&c.)^\pi \times}{(ex^{n'} + fx^{n'\pm\frac{1}{l}} + gx^{n'\pm\frac{2}{l}} + \&c.)^\rho \times}$$

$\frac{(Ax^i + Bx^{i\pm k'} + \&c.)}{(A'x^{i'} + B'x^{i'\pm k''} + \&c.)}$; vel denique in quamcunque functionem quantitatis x & aliarum quantitatum, quæ ſunt functiones variabilis v, ita vero aſſumendarum, ut quantitas reſultans evadat integrabilis.

Quantitates P, Q, R, &c.; p, q, r, &c.; a, b, c, &c.; a', b', c', &c.; e, f, g, &c.; A, B, &c.; A', B', &c.; denotant functiones quantitatis (v); & m, n, n', i, i', k' & k'', π & ρ ſunt invariabiles; & h, k, l & l' ſunt integri numeri.

4. Sit æquatio $(ax^m + bx^{m+h} + cx^{m+2h} \ldots + Mx^{n-h-1} + Px^{n-1}$ $+ Qx^{n+h-1} + Rx^{n+2h-1} + \&c.)\,\dot{x} + (px^n + qx^{n+h} + rx^{n+2h} + \&c.)\,\dot{v}$ $= 0$; ducatur ea in quantitatem $Ax^l + Bx^{l+h} + Cx^{l+2h} \ldots Dx^{l+n-m-h-1}$ $+ Ex^{l+n-m-1} + Fx^{l+n-m+h-1} + \&c.$; & exorietur $(aAx^{m+l} + (aB + bA)$

x^{m+l+h}

$x^{m+l+b} + \ldots + (AM + \ldots + aD)\,x^{l+n-b-1})\dot{x} + (Apx^{l+n} + \&c.)\,\dot{v} = o$; tum aſſumantur $aA = \alpha$, $aB + bA = \beta$, &c., uſque ad æquationem $aD + \ldots + AM = \theta$; deinde hoc modo aſſumantur reliquæ $A\dot{P} + P\dot{A} + \ldots\ldots + E\dot{a} + a\dot{E} = (l+n)\,Ap\dot{v}$, $A\dot{Q} + Q\dot{A} + \ldots\ldots + a\dot{F} + F\dot{a} = (l+n+b)\,(Aq + Bp)\,\dot{v}$, & ſic deinceps. In hoc caſu literæ a, b, c, &c., M, P, Q, R, &c.; p, q, r, &c.; A, B, C, &c., E, F, &c. functiones literæ v denotent, & α, β, &c. θ ſunt invariabiles quantitates; deinde ita deducendæ ſunt prædictæ quantitates, ut ſatisfaciant æquationibus aſſumptis, & inveſtigantur fluxionales æquationes, quarum fluentes innoteſcunt.

2. Et ſimiliter ducatur data æquatio in quantitatem $\dfrac{(Ax^{l} + Bx^{l+b} + \&c.)^{\pi} \times (Hx^{\sigma} + Ix^{\sigma+h} + \&c.)}{(A'x^{l'} + B'x^{l'+b} + \&c.)^{\rho} \times (H'x^{\sigma'} + I'x^{\sigma'+h} + \&c.)}$; ubi A, B, &c.; A', B', &c.; H, I, &c.; H', I', &c.; ſunt functiones quantitatis v, & π, ρ, σ, &c. ſunt invariabiles quantitates; vel in quamcunque functionem literarum x & v, ita aſſumendam, ut quantitas reſultans evadat integrabilis.

3. Sit æquatio $(ax^{m} + bx^{m+b} + cx^{m+2b} + \ldots + kx^{n-b} + px^{n} + qx^{n+b} + \&c.)\,\dot{v} + (Px^{n-1} + Qx^{n+b-1} + \&c.)\,\dot{x} = o$; ducatur data æquatio in $Ax^{-m} + Bx^{-m+b} + Cx^{-m+2b} + \&c. \ldots Dx^{n-2m-b} + Ex^{n-2m} + Fx^{n-2m+b} + \&c.$; & aſſumantur $Ab + Ba = o$, $Ac + Bb + Ca = o$, &c. uſque ad terminum $Ak + \&c. = o$, deinde hoc modo inveniantur reliqui $A\dot{P} + P\dot{A} = (n-m) \times (Ap + \ldots + aE)\,\dot{v}$, &c.: vel ejus multiplicator poteſt eſſe functio variabilis x & quantitatum, quæ ſunt functiones variabilis v; ſi modo ſupponantur evaneſcentes termini æquationis reſultantis, in quibus dimenſiones quantitatis x in $\dot{v}$ ductæ majores ſint quam dimenſiones quorumcunque terminorum quantitatis x in $\dot{x}$ ductorum per quantitatem majorem quam unitatem: hìc a, b, &c.; P, Q, &c.; A, B, &c. ſunt functiones quantitatis v.

Ex his methodis inveniri poteſt infinita ſeries, quæ reddit datam æquationem integrabilem.

Eadem principia etiam ad omnes fluxionales æquationes quorumcunque ordinum applicari poſſunt.

PROB. LII.

Datâ fluxionali æquatione ſuperioris ordinis; invenire quantitatem, quæ in datam æquationem ducta, tandem haud raro reddet integrabilem.

1. Aſſumatur multiplicator (P) primi generis, qui ducatur in datam æquationem, & in multis caſibus per notas regulas fluentes inveniendi ita aſſumi poteſt P, ut inveniatur fluens æquationis reſultantis: ſæpe quoque e præcedentibus principiis conſtabit formula multiplicatoris P, & exinde hoc problema in præcedens tranſibit.

2. Si vero in fluxionalibus æquationibus ſuperiorum ordinum haud inveniatur multiplicator primi generis P, aſſumatur multiplicator ſecundi generis $P\dot{x} + Q\dot{y}$, &c. & per prædictas regulas in multis caſibus inveniri poteſt fluens fluxionalis æquationis; & ſic deinceps.

3. Invenire fluentem fluxionis $P\overset{r}{\dot{y}} + Q\overset{r-1}{\dot{y}} + R\overset{r-2}{\dot{y}} +$ &c. $+ T$; ubi P eſt functio quantitatum $\overset{r-1}{\dot{y}}, \overset{r-2}{\dot{y}}, \overset{r-3}{\dot{y}}, \ldots \overset{2}{\dot{y}}\, \dot{y}$, & $\dot{x}$, y & x; Q vero functio quantitatum $\overset{r-2}{\dot{y}}, \overset{r-3}{\dot{y}}, \ldots \overset{2}{\dot{y}}, \dot{y}$ & $\dot{x}$, y & x; R autem functio quantitatum $\overset{r-3}{\dot{y}}, \overset{r-4}{\dot{y}}, \ldots \overset{2}{\dot{y}}, \dot{y}$ & $\dot{x}$, y & x, &c.; T functio vel rationalis vel irrationalis nonnullarum prædictarum fluxionum & quantitatum: in fluxione $P\overset{r}{\dot{y}}$, aſſumantur omnes quantitates præter $\overset{r-1}{\dot{y}}$ tanquam invariabiles, & ex hâc hypotheſi inveniatur fluens fluxionis $P\overset{r}{\dot{y}}$, quæ ſit π; deinde inveniatur $\dot{\pi}$ ex hypotheſi quod $\overset{r-1}{\dot{y}}, \overset{r-2}{\dot{y}}, \overset{r-3}{\dot{y}}, \ldots \ddot{y}, \dot{y}, y, \dot{x}$ & x ſint variabiles, quæ ſit $P\overset{r}{\dot{y}} + H\overset{r-1}{\dot{y}} + b\overset{r-2}{\dot{y}} +$ &c.: tum inveniatur fluens fluxionis $(Q - H)\overset{r-1}{\dot{y}}$, ex hypotheſi quod omnes quantitates præter $\overset{r-2}{\dot{y}}$ ſint

invariabiles; quæ ſit ϱ; deinde inveniatur $\dot{\varrho}$ ex hypotheſi quod $\overset{r-2}{\dot{y}}$, $\overset{r-3}{\dot{y}}$, . . . $\ddot{y}$, $\dot{y}$, y & x ſint variabiles, quæ ſit $(Q - H)\overset{r-1}{\dot{y}} + I\overset{r-2}{\dot{y}} +$ &c.; tum inveniatur fluens fluxionis $(R - h - I)\overset{r-2}{\dot{y}}$ ex hypotheſi quod omnes quantitates præter $\overset{r-3}{\dot{y}}$ ſint variabiles; & ſic deinceps: & ultimo perveniendum eſt ad functionem quantitatis x in $\dot{x}^n$ ductam, cujus inveniatur fluens, & generaliter corrigatur; & fluens datæ fluxionis, ſi modo ſit integrabilis, reperietur.

3. 2. Sit fluxionalis quantitas (A) $P\overset{n}{\dot{y}} + Q\overset{n}{\dot{z}} + R\overset{n}{\dot{v}} +$ &c. $+ F +$ &c., in quâ plures continentur variabiles quantitates $(y, z, v,$ &c.$)$ & earum fluxiones; tum primo aſſumantur una (y) & ejus fluxiones ſolummodo variabiles, & ex hâc hypotheſi inveniatur fluens (π), cujus (π) inveniatur fluxio ex hypotheſi quod omnes prædictæ quantitates $(y, z, v,$ &c.$)$ & earum fluxiones ſint variabiles, quæ ſupponatur α; ſubtrahatur hæc fluxio de datâ, & reſultabit fluxio $A - \alpha = B$, in quâ nec continetur variabilis y, nec ejus fluxiones, ſi modo fluens datæ fluxionis exprimi poſſit: ſi evaneſcant prædictæ quantitates, tum per eundem modum in quantitate B aſſumantur (z) & ejus fluxiones ſolummodo variabiles, & ex hâc hypotheſi inveniatur fluens, quæ ſit ϱ, cujus (ϱ) inveniatur fluxio ex hypotheſi quod prædictæ quantitates $(z, v,$ &c.$)$ & earum fluxiones ſint variabiles, quæ ſupponatur β; ſubtrahatur hæc fluxio de B, & reſultabit fluxio $B - \beta$, in quâ nec continetur variabilis z, nec ejus fluxiones, ſi modo fluens datæ fluxionis exprimi poſſit: & ſic de reliquis variabilibus quantitatibus.

Ex. 1. Sit $\ddot{y} = y^n X\dot{x}^2$; ubi X ſit functio quantitatis x; aſſumatur P multiplicator primi generis, ducatur data æquatio in P, & reſultat $P\ddot{y} = Py^n X\dot{x}^2$; per regulam hìc traditam pro fluente aſſumenda eſt $P\dot{y} - \dot{A} = \int.(P\ddot{y} - Py^n X\dot{x}^2)$, ſed hujuſce æquationis fluxio erit $P\ddot{y} + \dot{y}\dot{P} - \ddot{A} = P\ddot{y} - Py^n X\dot{x}$, unde $\dot{y}\dot{P} - \ddot{A} = -Py^n X\dot{x}$, cujus pro fluente per eandem regulam aſſumenda eſt $y\dot{P} + \dot{B} = \int.(\dot{y}\dot{P} - \ddot{A} + Py^n X\dot{x}^2)$;

$P y^n X \dot{x}^2$), cujus fluxio erit $\dot{y} \dot{P} + y \ddot{P} + \ddot{B} = \dot{y} \dot{P} - \ddot{A} + P y^n X \dot{x}^2$, unde $y \ddot{P} + \ddot{B} = - \ddot{A} + P y^n X \dot{x}^2$; ſed ut correſpondentes termini hujuſce æquationis inter ſe conveniant, neceſſe eſt, ut $\ddot{B} = - \ddot{A}$ & $y \ddot{P} = P y^n X \dot{x}^2$, & exinde $y^n = y$ & $n = 1$ & $\frac{\ddot{P}}{P \dot{x}^2} = X$.

Cor. Sit data æquatio $X \dot{x}^2 = m v^\mu \dot{v}^2 + n v^{\mu+1} \ddot{v}$, ſcribatur $v = y^{\frac{n}{m+n}}$ & reſultant $\dot{v} = \frac{n}{m+n} y^{\frac{-m}{m+n}} \dot{y}$ & $\ddot{v} = - \frac{mn}{(m+n)^2} y^{\frac{-2m-n}{m+n}} \dot{y}^2 + \frac{n}{m+n} y^{\frac{-m}{m+n}} \ddot{y}$; & exinde $m v^\mu \dot{v}^2 + n v^{\mu+1} \ddot{v} = \frac{n^2}{m+n} y^{\frac{\mu n - m + n}{m+n}} \ddot{y} = X \dot{x}^2$, hæc autem æquatio eſt ejuſdem formulæ ac æquatio $y^n X \dot{x}^2 = \ddot{y}$.

Ex. 2. Sit $\ddot{y} + \frac{A y^n \dot{x}^2}{(\alpha + 2 \beta x + \gamma x^2)^{\frac{n+3}{2}}} = 0$, ducatur hæc æquatio in $p \dot{x} + q \dot{y}$ & reſultat $p \dot{x} \ddot{y} + q \dot{y} \ddot{y} + \frac{p A y^n \dot{x}^3 + q A y^n \dot{y} \dot{x}^2}{(\alpha + 2 \beta x + \gamma x^2)^{\frac{n+3}{2}}} = 0$; per prædictam regulam pro fluente fluxionis $p \dot{x} \ddot{y} + q \dot{y} \ddot{y}$ ($\dot{\pi}$) aſſumenda eſt $p \dot{x} \dot{y} + \frac{q \dot{y}^2}{2} + L \dot{x}^2$, cujus fluxio erit $p \dot{x} \ddot{y} + q \dot{y} \ddot{y}$ ($\dot{\pi}$) $+ \dot{p} \dot{x} \dot{y} + \frac{\dot{q} \dot{y}^2}{2} + \dot{L} \dot{x}^2$; ſit igitur $p = P y$ & erit $\dot{p} = \dot{P} y + P \dot{y}$, unde $\dot{p} \dot{x} \dot{y} = P \dot{x} \dot{y}^2 + y \dot{P} \dot{x} \dot{y}$; fingatur $P \dot{x} \dot{y}^2 = - \frac{\dot{q} \dot{y}^2}{2}$, & exinde $P \dot{x} = - \frac{\dot{q}}{2}$, & multiplicator $p \dot{x} + q \dot{y}$ erit $- \frac{\dot{q}}{2} y + q \dot{y}$; ducatur data æquatio $\ddot{y} + \frac{A y^n \dot{x}^2}{(\alpha + 2 \beta x + \gamma x^2)^{\frac{n+3}{2}}} = 0$ in $- \dot{q} y + 2 q \dot{y}$ & reſultat æquatio $- \dot{q} y \ddot{y} + 2 q \dot{y} \ddot{y} - \frac{A y^{n+1} \dot{q} \dot{x}^2 - 2 q A y^n \dot{y} \dot{x}^2}{(\alpha + 2 \beta x + \gamma x^2)^{\frac{n+3}{2}}}$, cujus fluens erit $- y \dot{q} \dot{y} + q \dot{y}^2 + \int . y \dot{y} \ddot{q} - \int . \frac{A y^{n+1} \dot{q} \dot{x}^2 - 2 q A y^n \dot{y} \dot{x}^2}{(\alpha + 2 \beta x + \gamma x^2)^{\frac{n+3}{2}}}$ (H); ſi vero fluens H inveniri poſſit, erit $\frac{-2qAy^{n+1}\dot{x}^2}{(n+1)(\alpha+2\beta x+\gamma x^2)^{\frac{n+3}{2}}}$, cujus fluxio erit $\frac{-2qAy^n\dot{x}^2\dot{y}}{(\alpha+2\beta x+\gamma x^2)^{\frac{n+3}{2}}}$ $+\frac{(-2\dot{q}(\alpha+2\beta x+\gamma x^2)+2(n+3)q(\beta+\gamma x)\dot{x})Ay^{n+1}\dot{x}^2}{(n+1)(\alpha+2\beta x+\gamma x^2)^{\frac{n+5}{2}}}$, unde $Ay^{n+1}\dot{x}^2\dot{q}=-Ay^{n+1}\dot{x}^2\frac{(2\dot{q}(\alpha+2\beta x+\gamma x^2)-2(n+3)q(\beta+\gamma x)\dot{x})}{(n+1)(\alpha+2\beta x+\gamma x^2)}$, & exinde $(n+1)(\alpha+2\beta x+\gamma x^2)\dot{q}=-2(\alpha+2\beta x+\gamma x^2)\dot{q}+2\dot{x}(n+3)q(\beta+\gamma x)$, & conſequenter $(n+3)(\alpha+2\beta x+\gamma x^2)\dot{q}=2(n+3)q(\beta+\gamma x)\dot{x}$, unde $\frac{\dot{q}}{q}=\frac{2(\beta+\gamma x)\dot{x}}{\alpha+2\beta x+\gamma x^2}$, & $q=\alpha+2\beta x+\gamma x^2$: multiplicator igitur quæſitus erit $-2(\beta+\gamma x)y\dot{x}+2\dot{y}(\alpha+2\beta x+\gamma x^2)$, qui ductus in datam æquationem $\ddot{y}+\frac{Ay^n\dot{x}^2}{(\alpha+2\beta x+\gamma x^2)^{\frac{n+3}{2}}}=o$, præbet fluxionem, cujus fluens erit $-2y\dot{x}\dot{y}(\beta+\gamma x)+\dot{y}^2(\alpha+2\beta x+\gamma x^2)\pm\frac{2}{n+1}\frac{Ay^{n+1}\dot{x}^2}{(\alpha+2\beta x+\gamma x^2)^{\frac{n+1}{2}}}+\gamma y^2\dot{x}^2=C\dot{x}^2$, ubi C ſit invariabilis quantitas ad libitum aſſumenda.

Ex. 3. Sit $\ddot{y}+yX\dot{x}^2=o$, ubi X ſit functio quantitatis x: ejus multiplicator erit $-y\ddot{q}+2q\dot{y}$.

Ex. 4. Sit $\ddot{y}+\frac{Ay^2\dot{x}^2}{x^{\frac{15}{7}}}=o$, & ejus fluens invenietur $\frac{24\dot{x}\dot{y}}{243A}+\frac{6y\dot{x}\dot{y}}{7x^{\frac{1}{7}}}+x^{\frac{6}{7}}\dot{y}^2+\frac{2Ay^3\dot{x}^2}{3x^{\frac{9}{7}}}-\frac{3y^2\dot{x}^2}{49x^{\frac{8}{7}}}=C\dot{x}^2$.

Ex. 5. Sit $\ddot{y}+y^{-\frac{5}{3}}\dot{x}^2(\alpha+2\beta x+\gamma x^2)=o$, multiplicetur in quantitatem $36y^{\frac{1}{3}}(\beta+\gamma x)\dot{x}^3-12y^{-\frac{2}{3}}(\alpha+2\beta x+\gamma x^2)\dot{x}^2\dot{y}+4\dot{y}^3$ formulæ $P\dot{x}^3+2Q\dot{x}^2\dot{y}+3R\dot{x}\dot{y}^2+4S\dot{y}^3$, & reſultat æquatio, cujus fluens erit $36y^{\frac{1}{3}}(\beta+\gamma x)\dot{x}^3\dot{y}-6y^{-\frac{2}{3}}(\alpha+2\beta x+\gamma x^2)\dot{x}^2\dot{y}^2+\dot{y}^4+9y^{-\frac{4}{3}}(\alpha+2\beta x+\gamma x^2)^2\dot{x}^4-27\gamma y^{\frac{4}{3}}\dot{x}^4=C\dot{x}^4$.

Cor. In his æquationibus, quarum multiplicatores inveniuntur, pro y & ejus fluxionibus $\dot{y}$, $\ddot{y}$, &c. ſcribantur fz^n, vel quæcunque alia quantitas, & ejus fluxiones; & reſultant æquationes, quarum dabuntur etiam multiplicatores.

Ex. 6. Sit $y^2\ddot{y} + y\dot{y}^2 + Ax\dot{x}^2 = 0$, ducatur ea in multiplicatorem formulæ $3L\dot{y}^2 + 2M\dot{x}\dot{y} + N\dot{x}^2$, ubi A ſit conſtans, & L, M & N functiones quantitatis x; & æquationis reſultantis inveniatur fluens per notas regulas; invenietur multiplicator $3y\dot{y}^2 + 3Ax\dot{x}^2$, qui ductus in datam æquationem, dat fluxionem, cujus fluens erit $y^3\dot{y}^3 + 3Axy^2\dot{x}^2\dot{y} - Ay^3\dot{x}^3 + A^2x^3\dot{x}^3 = C\dot{x}^3$.

Cor. Sit $y = u\dot{x}$ & $\dot{y} = p\dot{x}$, unde $\frac{\dot{x}}{x} = \frac{\dot{u}}{p-u}$; ita reducatur data æquatio, ut exprimat relationem inter p & u, quæ evadit $A\dot{u} + up^2\dot{u} + pu^2\dot{p} - u^3\dot{p} = 0$; pro p & $\dot{p}$ ſcribantur $\frac{q}{u}$ & $\frac{\dot{q}u - \dot{u}q}{u^2}$, & reſultat $A\dot{u} + q\dot{q} + qu\dot{q} - u^2\dot{q} = 0$.

7. Sit æquatio $y^2\ddot{y} - \frac{1}{2}y\dot{y}^2 = a\dot{x}\dot{x}^2$ vel $\ddot{y} - \frac{1}{2}\frac{\dot{y}^2}{y} = \frac{ax\dot{x}^2}{y^2}$, ducatur hæc æquatio in $-2\dot{x}^2 + \frac{2x\dot{x}\dot{y}}{y}$, & fluens quæſita erit $-2\dot{x}^2\dot{y} + \frac{x\dot{x}\dot{y}^2}{y} + \frac{ax^2\dot{x}^3}{y^2} = C\dot{x}^3$.

8. Sit $\ddot{y} - \frac{2\dot{y}^2}{5y} - \frac{ax\dot{x}^2}{y^2} = 0$, ducatur hæc æquatio in $-\frac{10}{3}xy^{\frac{1}{5}}\dot{x}^2 + 2x^2y^{-\frac{4}{5}}\dot{x}\dot{y}$, & fluxionis reſultantis fluens erit $-\frac{10}{3}xy^{\frac{1}{5}}\dot{x}^2\dot{y} + x^2y^{-\frac{4}{5}}\dot{x}\dot{y}^2 + \frac{10}{9}ax^3y^{-\frac{9}{5}}\dot{x}^3 + \frac{25}{9}y^{\frac{6}{5}}\dot{x}^3 = C\dot{x}^3$.

9. Sit $\ddot{y} + \frac{2\dot{y}^2}{y} - \frac{ax\dot{x}^2}{y^2} = 0$, ducatur ea in $y^2\dot{x}^2$, & fluens fluxionis reſultantis erit $y^2\dot{x}^2\dot{y} - \frac{1}{2}ax^2\dot{x}^3 = C\dot{x}^3$.

10. Sit æquatio $\ddot{y} + \frac{\dot{y}^2}{y} - \frac{ax\dot{x}^2}{y^2} = 0$; multiplicetur ea per $3y^3\dot{y}^2$

$- 3ay^2x\dot{x}^2$, & reſultat fluxio, cujus fluens erit $y^3\dot{y}^3 - 3axy^2\dot{x}^2\dot{y} + ay^3\dot{x}^3 + a^2x^3\dot{x}^3 = C\dot{x}^3$.

11. Sit $\ddot{y} + y^2X\dot{x}^2 = 0$, ubi $X = -\frac{24\beta^3}{343(\alpha+\beta x)^{\frac{20}{7}}}$; & invenietur ejus multiplicator $(\alpha + \beta x - \frac{8}{7}\beta y(\alpha+\beta x)^{\frac{1}{7}})\dot{x} + 2(\alpha+\beta x)^{\frac{8}{7}}\dot{y}$, qui ductus in datam æquationem præbet fluxionem, cujus fluens invenietur $(\alpha + \beta x - \frac{8}{7}\beta y(\alpha+\beta x)^{\frac{1}{7}})\dot{x}\dot{y} + (\alpha+\beta x)^{\frac{8}{7}}\dot{y}^2 - \frac{16\beta^3y^3\dot{x}^2}{343(\alpha+\beta x)^{\frac{12}{7}}} - \beta y\dot{x}^2 + \frac{4\beta^2y^2\dot{x}^2}{49(\alpha+\beta x)^{\frac{6}{7}}} = C\dot{x}^2$.

12. Sit $\ddot{y} + \frac{Ay^2\dot{x}^2}{x^{\frac{15}{7}}} = 0$, ducatur hæc æquatio in $\frac{24\dot{x}}{343A} - \frac{6y\dot{x}}{7x^{\frac{1}{7}}} + 2x^{\frac{6}{7}}\dot{y}$, & reſultat fluxio, cujus fluens erit $\frac{24\dot{x}\dot{y}}{343A} - \frac{6y\dot{x}\dot{y}}{7x^{\frac{1}{7}}} + x^{\frac{6}{7}}\dot{y}^2 + \frac{2Ay^3\dot{x}^2}{3x^{\frac{9}{7}}} - \frac{3y^2\dot{x}^2}{49x^{\frac{8}{7}}} = C\dot{x}^2$.

13. Sit $2y^3\ddot{y} + y^2\dot{y}^2 + X\dot{x}^2 = 0$, ubi $X = \alpha + \beta x + \gamma x^2$; ducatur ea in $\frac{2\dot{y}^3}{y} - \frac{2(\alpha+\beta x+\gamma x^2)\dot{x}^2\dot{y}}{y^3} + \frac{2\dot{x}^3(\beta + 2\gamma x)}{y^2}$, & reſultat fluxio, cujus fluens erit $(y^2\dot{y}^2 - (\alpha+\beta x+\gamma x^2)\dot{x}^2)^2 + 4y^3\dot{x}^3\dot{y}(\beta + 2\gamma x) - 4\gamma y^4\dot{x}^4 = Cy^2\dot{x}^4$.

14. Si in æquatione $y^2\ddot{y} + y\dot{y}^2 + X\dot{x}^2 = 0$ ſcribatur $y\dot{y} = \frac{1}{2}\dot{z}$ ſeu $y^2 = z$; reſultat $\frac{1}{2}\ddot{z}\sqrt{(z)} + X\dot{x}^2 = 0$, quæ ope multiplicatoris $\frac{3\dot{z}^2}{4\sqrt{(z)}} + 3X\dot{x}^2$ redditur integrabilis, ſi modo $X = Ax$ vel $\alpha + \beta x$: eodem modo ſit æquatio $y^2(2y\ddot{y} + \dot{y}^2) + X\dot{x}^2 = 0$, ubi $X = \alpha + \beta x + \gamma x^2$; ponatur $y = z^{\frac{2}{3}}$ & prodit æquatio $\frac{4}{3}z^{\frac{5}{3}}\ddot{z} + X\dot{x}^2 = 0$, per eundem modum integrabilis.

15. Sit $\ddot{y} + \frac{Ay\dot{x}^2}{(By^2 + C + 2Dx + Ex^2)^2} = 0$; ubi A, B, C, D, E

ſunt conſtantes, & ejus multiplicator invenietur $2\dot y(C+2Dx+Ex^2)-2y\dot x(D+Ex)$, cujus producti fluens erit $\frac{\dot y^2}{\dot x^2}(C+2Dx+Ex^2)-\frac{2y\dot y}{\dot x}(D+Ex)+\frac{Ay^2}{By^2+C+2Dx+Ex^2}+Ey^2=$ const.

16. Sit æquatio $\ddot y+\frac{\dot X}{2X}\dot y+\frac{Y}{X}\dot x^2=0$; ubi Y & X ſunt functiones quantitatum y & x reſpective; ducatur hæc æquatio in $X\dot y$, & reſultat $X\dot y\ddot y+\frac{1}{2}\dot X\dot y^2+Y\dot y\dot x^2=0$, cujus fluens erit $\frac{1}{2}X\dot y^2=-\dot x^2\int.Y\dot y$.

PROB. LIII.

1. *Sit* $\mathrm{p}\dot{\mathrm{x}}+\mathrm{q}\dot{\mathrm{y}}=0$, *ducatur hæc æquatio in* P, *ubi* P *ſit functio quantitatis* x; *invenire, annon ita aſſumi poteſt* P, *ut fluens fluxionis* $\mathrm{Pp}\dot{\mathrm{x}}+\mathrm{Pq}\dot{\mathrm{y}}$ *inveniri poteſt.*

Sint $\dot p=\alpha\dot x+\beta\dot y$ & $\dot q=\pi\dot x+\varrho\dot y$, tum erit $P\beta=P\pi+\frac{\dot P}{\dot x}\times q$; unde $\frac{\beta-\pi}{q}\times\dot x=\frac{\dot P}{P}$; & exinde, ſi differentia $\beta-\pi$ per q diviſa ſit functio quantitatis x, ita aſſumi poteſt $P=e^{\int.\frac{\beta-\pi}{q}\dot x}$, ut fluens fluxionis $Pp\dot x+Pq\dot x$ inveniri poteſt; ſin aliter vero non.

1.2. Fluens terminorum datæ æquationis, in quibus continetur y, erit $P\int.q\dot y$; ubi per $\int.q\dot y$ intelligo fluentem (V) fluxionis $q\dot y$, cum x fingatur invariabilis; & fluens fluxionalis æquationis erit $PV+\int.\mathcal{Q}\dot x=A=0$, ubi $\mathcal{Q}\dot x=(Pp-W)\dot x$, & $W\dot x$ eſt fluxio rectanguli $P\times V$ ex hypotheſi quod x ſolummodo ſit variabilis.

Aliter: inveniatur fluens fluxionis $(q\dot y)$ ex hypotheſi quod x ſit invariabilis quantitas, quæ ſit V; tum fiat $V\dot P+P\dot V=P\mathcal{Q}'\dot x$, ubi $\mathcal{Q}'$ eſt ea pars quantitatis p, in quâ continetur y vel ejus functio & non ſolummodo quantitas x & ejus functiones; tum ex hypotheſi quod in quantitatibus (V & Q') ſolummodo habetur variabilis x, quæratur multiplicator P ex fluxionali æquatione $P\dot{V} + V\dot{P} = PQ'\dot{x}$, unde $\frac{\dot{P}}{P} = \frac{Q'\dot{x} - \dot{V}}{V}$; ergo, ſi $\frac{Q\dot{x} - \dot{V}}{V}$ ſit functio unica quantitatis x in $\dot{x}$, P erit etiam functio quantitatis x.

2. Sit æquatio $P\overset{n}{\dot{y}} - Q\overset{n-1}{\dot{y}}\dot{x} + R\overset{n-2}{\dot{y}}\dot{x}^2 - S\overset{n-3}{\dot{y}}\dot{x}^3 + \&c. + \pi\dddot{y}\dot{x}^{n-3} + \varrho\ddot{y}\dot{x}^{n-2} + \sigma\dot{y}\dot{x}^{n-1} + \tau y\dot{x}^n = X\dot{x}^n$, ubi x fluit uniformiter; & P, Q, R, S, &c. X ſunt functiones quantitatis x; tum ejus (n) multiplicatores (M) erunt functiones quantitatis x, quæ erunt (n) fluentes fluxionalis æquationis $(\overset{n}{\dot{M}}P) + (\overset{n-1}{\dot{M}}Q)\dot{x} + (\overset{n-2}{\dot{M}}R)\dot{x}^2 + (\overset{n-3}{\dot{M}}S)\dot{x}^3 + \ldots = \tau\dot{x}^n = \overset{n}{\dot{M}}P + (n\dot{P} + Q\dot{x})\overset{n-1}{\dot{M}} + (n.\frac{n-1}{2}\ddot{P} + (n-1)\dot{Q}\dot{x} + R\dot{x}^2)\overset{n-2}{\dot{M}} + (n.\frac{n-1}{2}.\frac{n-2}{3}\dddot{P} + (n-1).\frac{n-2}{2}\ddot{Q}\dot{x} + (n-2)\dot{R}\dot{x}^2 + S\dot{x}^3) \times \overset{n-3}{\dot{M}} + \&c. = 0$.

Ejus fluens erit $MP\overset{n-1}{\dot{y}} - (\dot{M}P + M\dot{P} + MQ\dot{x})\overset{n-2}{\dot{y}} + (\ddot{M}P + (2\dot{P} + Q\dot{x})\dot{M} + (\ddot{P} + \dot{Q}\dot{x} + R\dot{x}^2)M)\overset{n-3}{\dot{y}} + (\dddot{M}P + (3\dot{P} + Q\dot{x})\ddot{M} + (3\ddot{P} + 2\dot{Q}\dot{x} + R\dot{x}^2)\dot{M} + (\dddot{P} + \ddot{Q}\dot{x} + \dot{R}\dot{x}^2 + S\dot{x}^3)M)\overset{n-4}{\dot{y}}$ &c. $= \alpha = \dot{x}^{n-1}\int. MX\dot{x}$, &c.

Cor. Sit data prædicta æquatio $P\overset{n}{\dot{y}} - Q\overset{n-1}{\dot{y}}\dot{x} + R\overset{n-2}{\dot{y}}\dot{x}^2 \ldots + \pi\dddot{y}\dot{x}^{n-3} + \varrho\ddot{y}\dot{x}^{n-2} + \sigma\dot{y}\dot{x}^{n-1} + \tau y\dot{x}^n = X\dot{x}^n$; & ſit $\tau\dot{x}^n = \frac{1}{M}((\sigma\dot{M} + M\dot{\sigma})\dot{x}^{n-1} - (\ddot{\varrho}M + 2\dot{\varrho}\dot{M} + \varrho\ddot{M})\dot{x}^{n-2} + \&c.)$; tum ſemper detegi poteſt fluens; vel, ſi $\sigma\dot{x}^{n-1} = \frac{1}{M}\int.(\tau M\dot{x}^n + (\ddot{\varrho}M + 2\dot{\varrho}\dot{M} + \varrho\ddot{M})\dot{x}^{n-2} - (\dddot{\pi}M + 3\ddot{\pi}\dot{M} + 3\dot{\pi}\ddot{M} + \pi\dddot{M})\dot{x}^{n-3} + \&c.)$; vel $\varrho\dot{x}^{n-2} = \frac{-1}{M}\int.\int.(\tau\dot{M}\dot{x}^n - (\sigma\dot{M} + M\dot{\sigma})\dot{x}^{n-1} - (\dddot{\pi}M + 3\ddot{\pi}\dot{M}$

$+ 3\ddot{\pi}\ddot{M}$

$+ 3 \dot{\pi} \ddot{M} + \pi \dddot{M}) \dot{x}^{n-3} +$ &c.), &c.; tum fluens datæ fluxionalis æquationis ſemper detegi poteſt.

Cor. Sit fluens datæ fluxionis $(P \overset{n}{\dot{y}} - Q \overset{n-1}{\dot{y}} +$ &c.$) = \Gamma = o$; tum, ut e præcedentibus conſtat, erit multiplicator $M\phi : \Gamma$.

3. Sit fluxionalis æquatio ordinis (n), viz. $P \overset{n}{\dot{y}} + Q \overset{n-1}{\dot{y}} \dot{x} + R \overset{n-2}{\dot{y}} \dot{x}^2 +$ &c. $= o$; ubi P, Q, R, &c. ſunt functiones quantitatum x & y & earum fluxionum ordinum haud majorum quam $n-1$, $n-2$, $n-3$, &c.; & x fluit uniformiter: ſit multiplicator (M), qui reddit æquationem integrabilem, functio quantitatis x; tum inveniatur fluens quantitatis $P \overset{n}{\dot{y}}$ ex hypotheſi, quod $\overset{n-1}{\dot{y}}$ ſolummodo ſit variabilis; quæ ſit α; & exinde inveniatur fluxio quantitatis αM, viz. $\alpha \dot{M} + M \dot{\alpha}$, in quâ ſolummodo ſupponitur ($\dot{x}$) invariabilis; deinde inveniatur fluens fluxionis $M Q \overset{n-1}{\dot{y}} \dot{x} - \alpha \dot{M} - M \dot{\alpha} + M P \overset{n}{\dot{y}}$ ex hypotheſi, quod $\overset{n-2}{\dot{y}}$ ſolummodo ſit variabilis, quæ ſit $- \dot{M} \int . \alpha + M \int . (Q \overset{n-1}{\dot{y}} \dot{x} - \dot{\alpha} + P \overset{n}{\dot{y}}) = - \dot{M} \beta' + M \beta$: tum inveniatur fluxio quantitatis $\beta M - \beta' \dot{M}$ ex hypotheſi quod $\dot{x}$ ſolummodo ſit invariabilis, quæ ſit $M \gamma + \dot{M} \gamma' + \ddot{M} \gamma''$: deinde inveniatur fluens fluxionis $M R \overset{n-2}{\dot{y}} \dot{x}^2 - M \gamma - \dot{M} \gamma' - \ddot{M} \gamma'' - M \dot{\alpha} - \alpha \dot{M} + M P \overset{n}{\dot{y}} + M Q \overset{n-1}{\dot{y}} \dot{x}$ ex hypotheſi quod $\overset{n-3}{\dot{y}}$ ſolummodo ſit variabilis; quæ ſit $M \delta + \dot{M} \delta' + \ddot{M} \delta''$; cujus inveniatur fluxio ex hypotheſi quod $\dot{x}$ ſolummodo ſit invariabilis; & ſic deinceps: & ſi ultimo evaneſcant quantitas y & ejus fluxiones; tum integrari poteſt data æquatio per multiplicatorem, quæ eſt functio quantitatis x; ſin aliter vero non.

Cor. Si ex aſſumptis valoribus prædictarum quantitatum ultimo evaneſcant quantitas y & ejus fluxiones, tum dabuntur fluxionales æquationes, quarum fluentes innoteſcunt.

4. Datâ fluxionali æquatione n ordinis ($A = o$), quæ ſit data functio quantitatis y, ejus fluxionum; x & $\dot{x}$; ubi $\dot{x}$ eſt invariabilis; &

multiplicator eandem habeat formulam ac data æquatio, i.e. in eo involvantur datæ functiones quantitatis y & ejus fluxionum $\dot{y}, \ddot{y}, \dddot{y}$, &c. ad $\overset{n-1}{\dot{y}}$; per quamcunque datam methodum cum incognitis & cognitis functionibus quantitatum x & $\dot{x}$; invenire ejus fluentem, ſi modo data æquatio ſit integrabilis.

1^mo^. Reducenda eſt data fluxionalis æquatio, ita ut fluxio maximi ordinis in eâ contenta unam ſolummodo habeat dimenſionem, i. e. ſit $P\overset{n}{\dot{y}} + \&c. = A = 0$; deinde ducatur data æquatio in multiplicatorem M; & inveniatur fluens fluxionis $MP\overset{n}{\dot{y}}$ ex hypotheſi quod $\overset{n-1}{\dot{y}}$ ſolummodo ſit variabilis, quæ ſit α; & exinde inveniatur fluxio quantitatis α ex hypotheſi, quod quantitates $y, \dot{y}, \ddot{y}, \ldots \overset{n-1}{\dot{y}}$ & x; ſint variabiles, & reſultet fluxionalis quantitas B; tum inveniatur fluens fluxionalis quantitatis $MA - B$ ex hypotheſi quod $\overset{n-2}{\dot{y}}$ ſolummodo ſit variabilis, ſi $\overset{n-1}{\dot{y}}$ in quantitate $MA - B$ unam ſolummodo habeat dimenſionem, quæ ſit γ; ſi autem in quibuſdam terminis quantitatis $MA - B$ inveniantur aliæ dimenſiones præter unam fluxionis $\overset{n-1}{\dot{y}}$; tum ita aſſumantur functiones F, G, H, &c., ut evaneſcant illæ dimenſiones fluxionis $\overset{n-1}{\dot{y}}$; deinde inveniatur fluxio quantitatis γ ex hypotheſi quod $y, \dot{y}, \ddot{y}, \ldots, \overset{n-2}{\dot{y}}$ & x & prædictæ incognitæ functiones (F, G, &c.) & earum fluxiones ſint variabiles, & reſultet fluxionalis quantitas C: tum inveniatur fluens fluxionalis quantitatis $MA - B - C$ ex hypotheſi quod $\overset{n-3}{\dot{y}}$ ſolummodo ſit variabilis, ſi $\overset{n-2}{\dot{y}}$ ſolummodo habeat unam dimenſionem in quantitate $MA - B - C$; quæ ſit δ; ſi autem in quibuſdam terminis quantitatis prædictæ $MA - B - C$ inveniantur aliæ dimenſiones præter unam fluxionis $\overset{n-2}{\dot{y}}$, tum ita aſſumantur quantitates & earum fluxiones ut evaneſcant illæ dimenſiones fluxionis $\overset{n-2}{\dot{y}}$:

deinde inveniatur fluxio quantitatis δ ex hypotheſi quod $y, \dot{y}, \ddot{y}, \ldots, \overset{n-3}{\dot{y}}$ & x & prædictæ quantitates ($F, G,$ &c. &c.) & earum fluxiones ſint variabiles, & reſultet quantitas D; tum inveniatur fluens quantitatis $MA - B - C - D$ ex hypotheſi quod $\overset{n-4}{\dot{y}}$ ſolummodo ſit variabilis, & ſic deinceps; uſque donec evaneſcant quantitas y & ejus fluxiones ex fluxione deductâ: & ſi ex hâc reductione reſultent (l) independentes æquationes totidem incognitas functiones quantitatum x & $\dot{x}$ &c. involventes; tum ex fluentibus prædictarum fluxionalium æquationum detegi poteſt data fluxionalis æquatio & ejus multiplicator: ſi plures (m) ſint incognitæ functiones prædictæ quam æquationes, tum forſan pro $m - l$ functionibus aſſumi poſſunt quæcunque functiones quantitatis x & ejus fluxionum, &c.

Cor. Datâ æquatione $p\overset{n}{\dot{y}} + q\overset{n-1}{\dot{y}} + r\overset{n-2}{\dot{y}} + s\overset{n-3}{\dot{y}} \ldots t\overset{n-m}{\dot{y}} = 0$; ubi $2m$ minor eſt quam n, & p eſt data functio quantitatum $x, \dot{x}, y, \dot{y}, \ddot{y}$, uſque ad $\overset{n-2m-1}{\dot{y}}$; & q data functio quantitatum $x, \dot{x}, y, \dot{y}, \ddot{y}$, &c. uſque ad $\overset{n-2m}{\dot{y}}$; & r eſt functio quantitatum $x, \dot{x}, y, \dot{y}, \ddot{y}, \ldots \overset{n-2m+1}{\dot{y}}$; & s functio quantitatum $x, \dot{x}, y, \dot{y}, \ldots \overset{n-2m+2}{\dot{y}}$; & ſic deinceps: tum erit fluens primi ordinis datæ æquationis $Pp\overset{n-1}{\dot{y}} + (Pq - P\dot{p} - p\dot{P})\overset{n-2}{\dot{y}} + (Pr - P\dot{q} - q\dot{P} + P\ddot{p} + 2\dot{P}\dot{p} + p\ddot{P})\overset{n-3}{\dot{y}} + (Pv - \overline{\dot{P}r} + \overline{\ddot{P}q} - \overline{\dddot{P}p})\overset{n-4}{\dot{y}} +$ &c. $\overset{n-m+1}{\dot{y}} = 0$ $(A) = C\dot{x}^{n-1}$, ubi C eſt invariabilis; ſi modo $\overline{\overset{m}{\dot{P}}p} - \overline{\overset{m-1}{\dot{P}}q} + \overline{\overset{m-2}{\dot{P}}r} - \overline{\overset{m-3}{\dot{P}}s} +$ &c. $= 0$, tum fluens datæ fluxionis (n ordinis) detegi poteſt e fluente fluxionalis æquationis (B) $\overline{\overset{m}{\dot{P}}p} - \overline{\overset{m-1}{\dot{P}}q} + \overline{\overset{m-2}{\dot{P}}r} -$ &c. $= 0$: ſi P ſit fluxionalis quantitas π ordinis; & p, q, r, s, &c. fluxionales quantitates $\alpha, \beta, \gamma, \delta, \epsilon$, &c. ordinum reſpective; tum erit prædicta (B) fluxionalis æquatio ordinis, qui inventus fuerit maximus inter numeros $\alpha + m$, $\beta + m - 1$, $\gamma + m - 2$, $\delta + m - 3$, &c.

Æquatio reſultans per conſimilem methodum reduci poteſt, ſi $2m$ minor ſit quam $n - 1$; & ſic deinceps.

THEOR. XXXIV.

1. Sit fluxionalis æquatio $\alpha = 0$, quæ continet fluxionalem quantitatem $\overset{n}{\dot{y}}$ ordinis n; & ſi ducatur æquatio $\alpha = 0$ in P fluxionalem quantitatem $\overset{m}{\dot{y}}$ ordinis m, ita ut æquatio reſultans evadat integrabilis; tum plures erunt fluxionales æquationes ordinis (m), quæ ſunt fluentiales æquationes ordinis $(n - m)$ datæ æquationis $\alpha = 0$, i. e. in fluentiali æquatione ordinis $(n - m)$ datæ æquationis fluxio $\overset{m}{\dot{y}}$ aſcendet ad majores quam unam dimenſiones.

2. Si vero dentur (n) algebraici multiplicatores, viz. $a, b, c, d,$&c; tum una ſolummodo datur generalis fluens, viz. $\int . a \int . b . \int . c \int . d$ &c. $\times \alpha = 0$; &, ſi una ſolummodo detur generalis fluens, tum dantur (n) algebraici multiplicatores; etiamque (n) diverſi algebraici multiplicatores, qui reddent datam æquationem integrabilem.

3. Sit fluxio $M\dot{x}^{n} + N\dot{x}^{n-1}\dot{y} + P\dot{x}^{n-2}\ddot{y} + \mathcal{Q}\dot{x}^{n-3}\dddot{y} + R\dot{x}^{n-4}\ddddot{y} +$ &c.; & $\dot{\pi} = M\dot{x} + M'\dot{y}$; tum erit ejus fluens, (ſi modo exprimi poſſit), $= \pi\dot{x}^{n-1} + \rho\dot{x}^{n-2}\dot{y} + \sigma\dot{x}^{n-3}\ddot{y} + \tau\dot{x}^{n-4}\dddot{y} +$ &c. $= A$; ubi ρ, σ, τ, &c. ſunt functiones quantitatis x; & exinde $N = M' + \varphi : (x)$; & P, $\mathcal{Q}$, R, &c. ſunt etiam functiones quantitatis x.

Fluens fluxionis A, (i. e. fluens ſecundi ordinis datæ fluxionis), ſi modo integrabilis ſit, erit $\pi'\dot{x}^{n-2} + \rho'\dot{x}^{n-3}\dot{y} + \sigma'\dot{x}^{n-4}\ddot{y} + \tau'\dot{x}^{n-5}\dddot{y} +$ &c.; ubi π' eſt functio quantitatis x in $ay + b$; & ρ', σ', τ', &c. ſunt functiones quantitatis x: & fluens omnis ſuperioris ordinis $(m + 1)$ datæ fluxionis erit $\overset{m}{\pi}{}'\dot{x}^{n-m} + \overset{m}{\rho}{}'\dot{x}^{n-m-1}\dot{y} + \overset{m}{\sigma}{}'\dot{x}^{n-m-2}\ddot{y} + \overset{m}{\tau}{}'\dot{x}^{n-m-3}\dddot{y} +$ &c.; ubi $\overset{m}{\pi}{}'$ fuerit functio quantitatis x in $a'y + b'$; & $\overset{m}{\rho}{}'$, $\overset{m}{\sigma}{}'$, $\overset{m}{\tau}{}'$, &c. functiones quantitatis x.

4. Sit fluxio $M\dot{x}^n + N\dot{x}^{n-1}\dot{y} + P\dot{x}^{n-2}\ddot{y} + \mathcal{Q}\dot{x}^{n-3}\dddot{y} + R\dot{x}^{n-4}\ddddot{y} + \&c.$
$$+ P'\dot{x}^{n-2}\dot{y}^2$$
ejus fluens erit (ſi modo integrari poſſit,) $\pi\dot{x}^{n-1} + \varrho\dot{x}^{n-2}\dot{y} + \sigma\dot{x}^{n-3}\ddot{y} + \tau\dot{x}^{n-4}\dddot{y} + \&c.$; ubi M, N, P & P', π & ϱ ſunt functiones quantitatum x & y; & $\mathcal{Q}$, R, &c. σ, τ, &c. ſunt functiones quantitatis x; hæc fluxio eandem habet formulam ac fluxio in priori caſu data.

5. Sit fluxio $M\dot{x}^n + N\dot{x}^{n-1}\dot{y} + P\dot{x}^{n-2}\ddot{y} + \mathcal{Q}\dot{x}^{n-3}\dddot{y} + R\dot{x}^{n-4}\ddddot{y} + \&c.$
$$+ P'\dot{x}^{n-2}\dot{y}^2 + \mathcal{Q}'\dot{x}^{n-3}\dot{y}\ddot{y}$$
$$+ \mathcal{Q}''\dot{x}^{n-3}\dot{y}^3$$
ejus fluens erit (ſi modo integrari poſſit) $\pi\dot{x}^{n-1} + \varrho\dot{x}^{n-2}\dot{y} + \sigma\dot{x}^{n-3}\ddot{y} + \sigma'\dot{x}^{n-3}\dot{y}^2 + \tau\dot{x}^{n-4}\dddot{y} + \&c.$; ubi literæ M, N, P, P', $\mathcal{Q}$, $\mathcal{Q}'$, $\mathcal{Q}''$; π, ϱ, σ & σ' ſunt functiones quantitatum x & y; & R, &c., τ, &c. ſunt functiones quantitatis x; hæc fluxio eandem habet formulam ac data fluxio in præcedente caſu; & ſic deinceps: unde, ſi modo datæ fluxiones hujuſce generis integrari poſſint, continuo ex inveniendis earum fluentibus deprimentur datæ fluxiones in alias, quarum formula eſt magis ſimplex.

PROB. LIV.

Per infinitas ſeries invenire multiplicatores datarum fluxionalium æquationum.

1. Ex datis fluxionalibus æquationibus ($\alpha = 0$) inveniatur ſeries, quæ exprimit valorem unius variabilis in terminis reliquarum & earum fluxionum; & ſi data fluxionalis æquatio ſit primi ordinis, tum in ſerie generali continetur una ſolummodo invariabilis quantitas (A) ad libitum aſſumenda; ex hâc ſerie per methodum infinitarum ſerierum poſtea traditam inveniatur quantitas $A = P$; tum $A = P$ erit generalis fluens æquationis $\alpha = 0$ & $\frac{\dot{P}}{\alpha}$ multiplicator quæſitus.

2. Si data ſit fluxionalis æquatio ſuperioris (m) ordinis; tum in ſerie generali prædictâ inventâ continebuntur (m) invariabiles & inter ſe

non dependentes (A, B, C, D, &c.): ex hâc ſerie inveniatur quæcunque prædicta invariabilis quantitas, e. g. $A = P$; tum erit $A = P$ generalis fluens & $\frac{\dot{P}}{\alpha}$ multiplicator quæſitus.

THEOR. XXXV.

1. Sit æquatio fluxionalis $\alpha = 0$, quæ ducta in P fiat fluxio, cujus fluens n ordinis inveniri poteſt: ſit β fluens n ordinis fluxionalis æquationis $\alpha = 0$, tum $\frac{\overset{\overline{n-1}}{\dot{\beta}}}{\alpha}$ erit quantitas, quæ ducta in α creat fluxionem, cujus fluens $(n - 1)$ ordinis inveniri poteſt; & ſic $\frac{\overset{\overline{n-2}}{\dot{\beta}}}{\alpha}$, $\frac{\overset{\overline{n-3}}{\dot{\beta}}}{\alpha}$, &c. erunt multiplicatores, qui ducti in datam fluxionalem æquationem, creant fluxiones, quarum fluentes $n-2$, $n-3$, &c. ordinum inveniri poſſunt.

2. Et ſic ſit $\alpha = 0$ data æquatio, & $P\alpha = \dot{\beta}$ & $\mathcal{Q}\beta = \dot{\gamma}$, $R\gamma = \dot{\delta}$, $S\delta = \dot{\varepsilon}$, &c. tum erunt $\mathcal{Q}\int.P\alpha = \dot{\gamma}$, $R\int.\mathcal{Q}\int.P\alpha = \dot{\delta}$, & ſic $S\int.R\int.\mathcal{Q}\int.P\alpha = \dot{\varepsilon}$, &c. & multiplicatores, qui ducti in datam æquationem $\alpha = 0$ præbent æquationes fluxionales, quarum fluentes primi, ſecundi, tertii, &c. ordinis inveniri poſſunt, erunt reſpective P, $\frac{\mathcal{Q}\int.P\alpha}{\alpha}$, $\frac{R\int.\mathcal{Q}\int.P\alpha}{\alpha}$, $\frac{S\int.R\int.\mathcal{Q}\int.P\alpha}{\alpha}$, &c.

THEOR. XXXVI.

1. Sæpe e ſubſtitutione ſeparari poſſunt variabiles quantitates: e methodis prius traditis deleri poſſunt quædam irrationales quantitates, e quibus nonnunquam reſultabit ſeparatio; vel ſæpe ſubſtitutio ex æquatione datâ ſatis manifeſta erit, quæ ſeparationem variabilium quantitatum inducet; ſæpe etiam diſtingui poteſt data æquatio in

diverſas partes, quarum ſingula eſt functio fluentis in ejus fluxionem, quod e ſubſequentibus exemplis conſtabit.

Ex. 1. Sit $(x\dot{y} + y\dot{x})\sqrt{(a^4 - x^2y^2)} = \frac{x\dot{x} + y\dot{y}}{(x^2+y^2)^{\frac{3}{4}}}$; facilis eſt obſervatio, ut $f.\,(x\dot{y} + y\dot{x}) = xy$, cujus quadratum x^2y^2 ſolummodo continetur in quantitate $a^4 - x^2y^2$; fluens vero fluxionis $x\dot{x} + y\dot{y}$ erit $\frac{1}{2}(x^2+y^2)$; ſed quantitas x^2+y^2 ſolummodo continetur in quantitate $(x^2+y^2)^{\frac{3}{4}}$: ſcribantur igitur $xy = z$ & $x^2+y^2 = v$; & exinde reſultabit æquatio $\dot{z}\sqrt{(a^4 - z^2)} = \frac{\dot{v}}{2v^{\frac{3}{4}}}$, in quâ variabiles z & v ſeparantur.

Ex. 2. Sit $a\dot{x} = (x+y)^n \times \dot{y}$, ſcribatur $x+y = z$, & erit $\dot{x} + \dot{y} = \dot{z}$, unde exorietur (ſi modo exterminentur x & $\dot{x}$) æquatio $a\dot{z} - a\dot{y} = z^n\dot{y}$, & conſequenter $\dot{y} = \frac{a\dot{z}}{a + z^n}$.

Ex. 3. Sit fluxio $\frac{2x^2\dot{x} + xy\dot{y} + y^2\dot{x}}{x^4 + x^2y^2 + a^4} = \frac{x\dot{x} + y\dot{y}}{\sqrt{(x^2+y^2)}}$, nunc ſupponatur irrationalis quantitas $\sqrt{(x^2+y^2)} = z$, quâ quantitate & ejus fluxionibus pro ſuis valoribus in datâ æquatione ſubſtitutis reſultat $\frac{z}{x^2z^2 + a^4} \times (x\dot{z} + z\dot{x}) = \dot{z}$; ſcribatur $xz = p$, & ex hoc aſſumpto valore deduci poteſt $\frac{\dot{p}}{p^2 + a^4} = \frac{\dot{z}}{z}$.

Ex. 4. Sit æquatio $y^m(x\dot{x} + y\dot{y}) = x^n(y\dot{x} - x\dot{y})$, quæ hoc modo ſcribi poteſt $\frac{y^{m-2}}{x^n}(x\dot{x} + y\dot{y}) = \frac{y\dot{x} - x\dot{y}}{y^2}$; aſſumatur $\frac{y\dot{x} - x\dot{y}}{y^2} = \dot{p}$ & $x\dot{x} + y\dot{y} = q\dot{q}$, unde $\frac{x}{y} = p$, & $x^2 + y^2 = q^2$; his poſitis, deduci poteſt æquatio $q^{m-n-1} \times \dot{q} = p^n \times (p^2+1)^{\frac{m-n-2}{2}}\dot{p}$.

Ex. 5. Sit æquatio $\frac{x^{mt-1}y\dot{x}}{(bx^t + ay^nx^r)^m} = \dot{y}$, ſupponatur $(bx^t + ay^nx^r)^m = zx^{mt}$;

$= z x^{mt}$; e datâ æquatione exterminentur y & ejus fluxio, & reſultat

$$\frac{z^{\frac{1}{m}}\dot{z}}{mnz^{\frac{1}{m}} + (r - t)\,mz^{\frac{1}{m}+1} - (mr - mt)(bz) - mnb} = \frac{\dot{x}}{x}.$$

Ex. 6. Sit $\frac{y^u \dot{x}}{(bx^t + ay^n x^r)^m} = cx^{\frac{ut - mnt - t + r + n - ur}{n}}\dot{y}$, & per præcedentem methodum ſeparari poſſunt indeterminatæ. Et ſic ex eâdem ſubſtitutione ſeparari poſſunt variabiles quantitates in æquationibus $\frac{y^u \dot{y}}{(bx^t + ay^n x^r)^m} = cx^{\frac{tu - n - tmn - ru + t - r}{n}}\dot{x}$. & $\frac{(bx^t + fy^n x^r)^v y^u \dot{y}}{(bx^t + ay^n x^r)^m} = cx^{\frac{ut - n - tmn - ru + t - r + ntv}{n}}\dot{x}$.

Ex. 7. Sit æquatio $\frac{y^{n-1}\dot{y}}{(ax^t + bx^r + cy^n x^r)^m} = fx^{t-r-1-mt}\dot{x}$; vel $\frac{(gx^t + hx^r + ky^n x^r)^e y^{n-1}\dot{y}}{(ax^t + bx^r + cy^n x^r)^m} = x^{t-r-1+te-mt}\dot{x}$, cum $ch = bk$; ex ſubſtitutione $ax^t + bx^r + cy^n x^r = x^t z^{\frac{1}{m}}$ facile conſtat reſolutio, &c.

Ex. 8. Sit æquatio $\frac{a\dot{y}}{b + (cy^n + fx)^u} = gy^{1-n}\dot{x}$ & $\frac{ay^{n-1}\dot{y}}{b + (cy^n + fx^m)^u} = gx^{m-1}\dot{x}$; ſcribatur pro priori $(cy^n + fx)^u = z$, & pro ſecundâ $(cy^n + fx^m)^u = x$, & reſultabunt duæ æquationes $\frac{az^{\frac{1-u}{u}}\dot{z}}{nubcg + nucgz + auf} = \dot{x}$, & $x^{m-1}\dot{x} = \frac{az^{\frac{1-u}{u}}\dot{z}}{bcgnu + cgnuz + mafu}$.

Ex. 9. Sit $\frac{ay^{n-1}\dot{y}}{b + (cy^n + p)^u} = gq\dot{x}$, ubi p & q ſunt algebraicæ functiones literæ x; ſi $q = \frac{\dot{p}}{\dot{x}}$, tum ſeparari poſſunt indeterminatæ a ſcribendo $(cy^n + p)^u = z$, reſultat enim æquatio $\frac{az^{\frac{1-u}{u}}\dot{z}}{nbcgu + ncguz + au} = q\dot{x} = \dot{p}$.

Ex. 10. Sit æquatio $a^2\dot{y} - x^2\dot{y} + xy\dot{x} = a\dot{x}\sqrt{(x^2 + y^2 - a^2)}$

ſcribe $y = u\sqrt{(a^2 - x^2)}$, & reſultat $\frac{\dot u}{\sqrt{(u^2-1)}} = \frac{a\dot x}{a^2 - x^2}$, unde $u + \sqrt{(u^2 - 1)} = n\sqrt{\left(\frac{a+x}{a-x}\right)}$; ubi n ſit quæcunque conſtans quantitas.

11. Sit $x^m\dot x + (by + y^2)\frac{c\dot x}{x} = y\dot y$, quæ ita ſcribi poteſt $x^m\dot x + \frac{bcy\dot x}{x} = y^2 \times \left(\frac{\dot y}{y} - \frac{c\dot x}{x}\right)$; fingatur $\frac{\dot y}{y} - \frac{c\dot x}{x} = \frac{\dot p}{p}$, unde $y = px^c$, quâ quantitate pro ſuo valore y in datâ æquatione ſubſtitutâ reſultat $x^m\dot x + bcpx^{c-1}\dot x = p\dot p\, x^{2c}$; dividatur ea per x^{2c}, & erit reſultans æquatio $x^{m-2c}\dot x + bcpx^{-c-1}\dot x = p\dot p$, fiat $m - 2c = -c - 1$, & ſeparantur invariabiles.

Hæc methodus etiam nonnunquam applicari poteſt ad ſubſequentes æquationes $x^m\dot x + \frac{ay^n\dot x}{x} + \frac{by^s\dot x}{x} = y^t\dot y$; poſitis enim $n - 1$ vel $s - 1 = t$; abbreviationem recipiet formula.

2. Sit æquatio $ax^m\dot x + by^q\dot x = \dot y$; ſcribatur $y = z^{\frac{1}{1-q}}$, & reſultat æquatio $ax^m\dot x + bz^{\frac{q}{1-q}}\dot x = \frac{1}{1-q}z^{\frac{q}{1-q}}\dot z$; unde, ſi vel $m = 0$ vel $= \frac{q}{1-q}$, reſultantis æquationis fluens inveniri poteſt.

3. Sit $x\dot y + gy\dot x + ax^n\dot x + by^n\dot y = 0$; ſcribatur $x = y^r u$, & exinde deduci poteſt æquatio $y^r u\dot y + gy(ry^{r-1}u\dot y + y^r\dot u) + ay^{rn}u^n(ry^{r-1}u\dot y + y^r\dot u) + by^n\dot y = 0$, cujus fluens inveniri poteſt, ſi modo $gr + 1 = 0$ & $rn + r - 1 = n$.

Ex. 12. Sit $\alpha y\dot x + \beta x\dot y + x^m y^n(\gamma y\dot x + \delta x\dot y) = 0$, dividatur hæc æquatio per xy, & reſultat $\frac{\alpha\dot x}{x} + \frac{\beta\dot y}{y} + x^m y^n\left(\frac{\gamma\dot x}{x} + \frac{\delta\dot y}{y}\right) = 0$; fingantur $x^\alpha y^\beta = t$ & $x^\gamma y^\delta = u$, & inveniatur fluxionalis æquatio, cujus variabiles quantitates ſunt t & u, & evadit $t^{\frac{\gamma n - \delta m}{\alpha\delta - \beta\gamma} - 1}\dot t + u^{\frac{\alpha n - \beta m}{\alpha\delta - \beta\gamma} - 1}\dot u = 0$.

Ex. 13. Sit $\dot{y}y + \dot{y}\,(a + bx + nx^2) = y\dot{x}\,(c + nx)$; ſubſtituatur $\frac{y\,(c + nx)}{y + a + bx + nx^2} = u$; & inveniatur fluxionalis æquatio, cujus variabiles quantitates ſunt x & u, quæ erit $\frac{\dot{x}}{(a + bx + nx^2)\,(c + nx)}$ $= \frac{\dot{u}}{u\,(na + c^2 - bc + (b - 2c)\,u + u^2)}$.

Ex. 14. Sit $(y - x)\,\dot{y} = \frac{n\dot{x}\,(1 + y^2)^{\frac{3}{2}}}{\surd(1 + x^2)}$, ſcribatur $y = \frac{x - u}{1 + xu}$, deinde inveniatur fluxionalis æquatio, cujus variabiles quantitates ſunt x & u, & reſultabit $\frac{\dot{x}}{1 + x^2} = \frac{u\dot{u}}{(1 + u^2)\,(n\surd(1 + u^2) + u)}$.

Ex. 15. Sit $(x^n\dot{x} + ay^{\frac{-n-1-c}{c}}\,\dot{y})\,p = (x\dot{y} + by\dot{x})\,q$, ſubſtituantur $y = t^s z^{\frac{r}{n+1}}$ & $x = t^{-\frac{s}{c}}\,(a \pm abz^{-\frac{r}{c}})^{\frac{1}{n+1}}$, ubi literæ s & r quantitates invariabiles reſpective denotant; & p & q ſunt algebraicæ functiones literarum x & y, tales vero ut in ſingulis terminis in functione p vel q contentis exponens quantitatis y in c ducta ſemper ſuperet exponentem quantitatis x per datam quantitatem; e. g. ſit terminus $ex^m y^n$ in functione p, & neceſſe eſt ut $cn - m$ ſemper in omnibus terminis functione p contentis eandem præbeat ſummam, & reſultat æquatio

$$-\frac{s}{c}\,t^{\frac{-sn-c-cs}{c}}\,\dot{t} = \frac{rz^{\frac{r-n-1}{n+1}}\,\dot{z} \times \frac{q}{p}}{(n+1)\,(a \pm abz^{-\frac{r}{c}})^{\frac{n}{n+1}}};$$

in quâ ſeparantur variabiles t & z.

Ex. 16. Sit æquatio $(x^n\dot{x} \pm ay^{\frac{-nf-c-f}{c}}\,\dot{y})\,p = (fx\dot{y} + cy\dot{x})\,q$: ſcribantur $y = t^{\frac{s}{f}}\,z^{\frac{r}{f(n+1)}}$ & $x = t^{-\frac{s}{c}}\,\left(a \pm \frac{acz^{-\frac{r}{c}}}{f}\right)^{\frac{s}{n+1}}$, ubi exponens quantitatis y ducta in c ſuperat exponentem quantitatis x ductam in f in ſingulis terminis quantitatum p vel q per eandem quantitatem,

&

& exorietur $\frac{s}{c}\,t^{\frac{-fc-fsn-sc}{cf}}\,\dot{t} = \dfrac{\frac{r}{n+1}\,z^{\frac{r-nf-f}{fn+f}} \times \dot{z} \times \frac{q}{p}}{\left(a \pm \frac{ac}{f}\,z^{-\frac{r}{c}}\right)^{\frac{n}{n+c}}}$, in quâ ſeparantur variabiles t & z.

Ex. 17. Sit $\dot{y} + Py\dot{x} = Q\dot{x}$, ſcribatur $y = Xu$, & oritur $X\dot{u} + u\dot{X} = Q\dot{x} - PXu\dot{x}$; ut fiat æquatio, cujus fluens detegi poteſt, fingatur $\dot{X} + PX\dot{x} = 0$, unde $\frac{\dot{X}}{X} = -P\dot{x}$ & log. $X = -\int. P\dot{x}$ & $X = e^{-\int. P\dot{x}}$; æquatio vero transformata erit $X\dot{u} = Q\dot{x}$ & $u = \int. \frac{Q\dot{x}}{X} = \int. e^{\int. P\dot{x}}\,Q\dot{x} = \frac{y}{X}$, unde $y = e^{-\int. Px}\int. e^{\int. P\dot{x}}\,Q\dot{x}$; & quantitates y & x ſeparabiles ſunt, ſi modo P & Q ſint functiones quantitatis x.

Ex. 18. Sit $x = \frac{y\dot{x}}{\dot{y}} + \frac{ay^2\dot{x}^2}{\dot{y}^2} + \frac{by^3\dot{x}^3}{\dot{y}^3} +$ &c., ſcribatur $t\dot{y} = \dot{x}$, & æquatio reſultans erit $x = yt + ay^2t^2 + by^3t^3 +$ &c.; in hâc æquatione pro ty ſcribatur z, & transformetur reſultans æquatio in alteram exprimentem relationem inter x & z, & evadet $x = z + az^2 + bz^3 +$ &c., unde $\dot{x} = t\dot{y} = \frac{z\dot{y}}{y} = \dot{z} + 2az\dot{z} + 3bz^2\dot{z} +$ &c.; & exinde log. $y =$ log. $z + 2az + \frac{3bz^2}{2} +$ &c. $+$ conſt.

Cor. 1. Hinc conſtat in nonnullis caſibus quantitates y & x in datâ fluxionali æquatione contentas poſſe exprimi per tertiam aſſumptam z, at minime per ſe ipſas.

2. Sit $\dot{y}^n$ maxima dimenſio fluxionis $(\dot{y})$ in datâ æquatione contenta, tum ea reduci poteſt in (n) diverſas fluxionales æquationes, quarum ſingulis fluentibus correſpondent prædicti valores quantitatum x & y.

Ex. 19. Datâ fluxionali æquatione $P\dot{x} = Q\dot{y}$, quæ ſit homogenea, i. e. ubi P & Q ſint homogeneæ functiones ejuſdem dimenſionum numeri ipſarum x & y, invenire ejus fluentem.

In æquatione $\frac{P\dot{x}}{Q} = \dot{y}$ pro y ſcribatur ux, & quoniam P & Q ſunt quantitates quæ eundem habet numerum dimenſionum, erit $\frac{P}{Q}$ functio quantitatis u, quæ dicatur U; ergo $U\dot{x} = \dot{y} = u\dot{x} + x\dot{u}$, unde $\frac{\dot{x}}{x} = \frac{\dot{u}}{U - u}$ & log. $x = \int . \frac{\dot{u}}{U - u}$.

Cor. 1. Quod ſi $\int . \frac{\dot{u}}{U - u}$ per logarithmos exprimi poſſit, habebitur algebraica æquatio inter x & u, & exinde inter u & y, etiamque inter x & y.

Cor. 2. Si in æquatione $P\dot{x} = Q\dot{y}$, ubi in literis P & Q haud ſolummodo continentur homogeneæ functiones ejuſdem dimenſionum numeri, ſed inſint etiam quantitates hujuſmodi formulæ; log. $\frac{\sqrt{(x^2 + y^2)}}{x}$; $e^{y:x}$; ang. ſin. $\frac{x}{\sqrt{(x^2 + y^2)}}$; coſ. $\frac{nx}{y}$; &c. in quibus tranſcendentes functiones afficiant functiones nullius dimenſionis ipſarum (x & y); pro y ſcribatur ux, & reſultabit æquatio hujuſce formulæ $U\dot{x} = \dot{y}$, ubi U erit functio quantitatis u, unde $U\dot{x} = u\dot{x} + x\dot{u}$ & $\frac{\dot{x}}{x} = \frac{\dot{u}}{U - u}$ & log. $x = \int . \frac{\dot{u}}{U - u}$.

Cor. 3. Sit $p\dot{x} + q\dot{y} = o$, ubi literæ p & q reſpective denotant homogeneas functiones literarum x & y, quarum dimenſiones ſunt m; ſit quantitas L, quæ in datam æquationem ducta, creat fluxionem, cujus fluens inveniri poteſt; & erit $Lp\dot{x} + Lq\dot{y} = P\dot{x} + Q\dot{y} = \dot{V}$, ubi P & Q ſunt homogeneæ n dimenſionum functiones literarum x & y, & $nV = Px + Qy$: ſcribatur enim $y = xz$, & erit $V = x^n Z$, ubi Z functio eſt quantitatis z, unde $\dot{V} = nx^{n-1}Z\dot{x} + x^n\dot{Z}$; at quoniam $\dot{V} = P\dot{x} + Q\dot{y}$ & $\dot{y} = x\dot{z} + z\dot{x}$, erit $\dot{V} = (P + Qz)\dot{x} + Qx\dot{z} = nx^{n-1}Z\dot{x} + x^n\dot{Z}$, & exinde $P + Qz = nx^{n-1}Z$; ducatur

catur hæc æquatio in x, & reſultat $Px + Qzx = Px + Qy = nx^nZ = nV$.

Hinc etiam erit $\left(\frac{\dot{V}}{\dot{x}}\right) = P = \frac{(P\dot{x} + Q\dot{y})y - (Px + Qy)\dot{y}}{y\dot{x} - x\dot{y}} = \frac{y\dot{V} - nV\dot{y}}{y\dot{x} - x\dot{y}}$, ſi modo x ſolummodo fluat in quantitate $\left(\frac{\dot{V}}{\dot{x}}\right)$; &c.

Eodem modo quoniam p & q ſunt homogeneæ functiones quantitatum x & y, erunt etiam $\left(\frac{\dot{p}}{\dot{y}}\right) = \frac{np\dot{x} - x\dot{p}}{y\dot{x} - x\dot{y}}$ & $\left(\frac{\dot{q}}{\dot{x}}\right) = \frac{y\dot{q} - nq\dot{y}}{y\dot{x} - x\dot{y}}$; &c.

Cor. 4. Sit $\dot{L} = a\dot{x} + b\dot{y}$; e præcedentibus principiis invenientur $a = \frac{y\dot{L} - mL\dot{y}}{y\dot{x} - x\dot{y}}$, & $b = \frac{mL\dot{x} - x\dot{L}}{y\dot{x} - x\dot{y}}$; L enim erit homogenea functio quantitatum x & y.

Quoniam $\dot{V} = Lp\dot{x} + Lq\dot{y}$, & $V = n(Lpx + Lqy)$, erit $\frac{\dot{V}}{V} = \frac{p\dot{x} + q\dot{y}}{n(px + qy)}$, unde fluens V datæ æquationis $p\dot{x} + q\dot{y} = 0$, facile deduci poteſt; & conſtat unum valorem quantitatis L eſſe $\frac{1}{px + qy}$.

Aliter: ſit $p\dot{x} + q\dot{y} = 0$ æquatio, in quâ literæ p & q denotant homogeneas n dimenſionum functiones literarum x & y; ſcribatur $y = ux$, & evadent $p = x^nU$ & $q = x^nW$, ubi literæ U & W denotant functiones literæ u; & conſequenter $x^nU\dot{x} + x^nW\dot{y} = p\dot{x} + q\dot{y} = 0$; dividatur hæc æquatio per x^n, & reſultat $U\dot{x} + W\dot{y} = 0$; pro $\dot{y}$ ſcribatur ejus valor $u\dot{x} + x\dot{u}$, & reſultat æquatio $(U + Wu)\dot{x} + Wx\dot{u} = 0$; dividatur hæc æquatio per $x(U + Wu)$, & fit $\frac{\dot{x}}{x} + \frac{W\dot{u}}{U + Wu} = 0$ integrabilis æquatio.

Ex. 20. Sit $ax^n\dot{x} + by^n\dot{y} + (x\dot{y} - y\dot{x})\left(\frac{p}{q} + \frac{\pi}{w}\right) = 0$, ubi p & q

ſunt homogeneæ functiones quantitatum x & y, π & w ſunt homogeneæ functiones earundem quantitatum, dimenſionum, quarum differentia eſt $n-1$; ſcribatur $x=yz$ & deduci poteſt æquatio $ay^n z^n (y\dot{z}+z\dot{y})+by^n\dot{y}-y^2\dot{z}(y^k\varphi:(z)+y^{n-1}\varphi:(z))=0$.

Ex. 21. Sit fluxionalis æquatio $a\dot{x}+b\dot{y}=0$, cujus ſinguli termini quantitatum a & b habeant m vel $m-1$ dimenſiones; in hâc æquatione pro y & $\dot{y}$ ſcribantur xv & $x\dot{v}+v\dot{x}$ reſpective, & reſultat æquatio formulæ $(Px^2+\mathcal{Q}x)\dot{v}+(Rx+S)\dot{x}=0$; ubi P, $\mathcal{Q}$, R & S ſunt functiones quantitatis v; unde, ſi modo $\frac{\mathcal{Q}\dot{v}-\dot{S}}{S}=\frac{2P\dot{v}-\dot{R}}{R}$, tum ſemper fluens æquationis reſultantis detegi poteſt: ducatur enim reſultans æquatio in $e^{\int.\frac{(\mathcal{Q}\dot{v}-\dot{S})}{S}}$, & exorietur æquatio, cujus fluens erit $e^{\int.\frac{(\mathcal{Q}\dot{v}-\dot{S})}{S}}\left(\frac{Rx^2+2Sx}{2}\right)=$ conſt.

2. Scribantur in æquatione $(Px^2+\mathcal{Q}x)\dot{v}+(Rx+S)\dot{x}=0$, z^{-n} & $-nz^{-n-1}\dot{z}$ pro x & $\dot{x}$, & reſultat æquatio $(Pz+\mathcal{Q}z^{n+1})\dot{v}-(nR+nSz^n)\dot{z}=0$; & exinde conſtabit fluentem prædictæ æquationis inveniri poſſe, ſi $\frac{P\dot{v}+n\dot{R}}{R}=\frac{(n+1)\mathcal{Q}\dot{v}+n\dot{S}}{S}$; erit enim $e^{\int.-\frac{P\dot{v}+n\dot{R}}{nR}}(nRz+\frac{nS}{n+1}z^{n+1})=$ conſt.

3. Si vero termini ſinguli quantitatum a & b habeant n, $n-1$ & $n-2$ dimenſiones, tum e ſubſtitutione prius traditâ reſultabit æquatio formulæ $(Px^3+\mathcal{Q}x^2+Rx)\dot{v}+(px^2+qx+r)\dot{x}=0$; & ſic deinceps; ubi P, $\mathcal{Q}$, R, p, q & r ſunt functiones quantitatis v; cujus fluens inveniri poteſt, ſi vel $\frac{3P\dot{v}-\dot{p}}{p}=\frac{2\mathcal{Q}\dot{v}-\dot{q}}{q}=\frac{R\dot{v}-\dot{r}}{r}$; vel $\frac{P\dot{v}+n\dot{p}}{p}=\frac{(n+1)\mathcal{Q}\dot{v}+n\dot{q}}{q}=\frac{(2n+1)R\dot{v}+n\dot{r}}{r}$; fluens enim ejus in priori caſu erit $e^{\int.\frac{R\dot{v}-\dot{r}}{r}}\left(\frac{px^3}{3}+\frac{qx^2}{2}+rx\right)=$ conſt.; in po-

ſteriori vero invenietur $e^{\int . -\frac{P\dot{v}+n\dot{p}}{np}}\left(npz+\frac{nq}{n+1}z^{n+1}+\frac{nr}{2n+1}z^{2n+1}\right)$ $=$ conſt. ſi $z=x^{-n}$.

4. Si vero terminos n, $n-1$, $n-2\,\ldots\,n-m$ dimenſionum habeant quantitates a & b in fluxionali æquatione $a\dot{x}+b\dot{y}=o$, ſcribantur xv & $x\dot{v}+v\dot{x}$ pro y & $\dot{y}$ in datâ æquatione, & reſultat æquatio formulæ $(Px^{m+1}+Qx^{m}+Rx^{m-1}+Sx^{m-2}+\&c.)\,\dot{v}+(px^{m}+qx^{m-1}$ $+rx^{m-2}+\&c.)\,\dot{x}=o$, ubi P, Q, R, &c. p, q, r, &c. ſunt functiones ipſius v; cujus fluens inveniri poteſt, ſi modo vel $\frac{(m+1)P\dot{v}-\dot{p}}{p}=$ $\frac{mQ\dot{v}-\dot{q}}{q}=\frac{(m-1)R\dot{v}-\dot{r}}{r}=\frac{(m-2)S\dot{v}-\dot{s}}{s}=$ &c. vel $\frac{P\dot{v}+n\dot{p}}{p}$ $=\frac{(n+1)Q\dot{v}+n\dot{q}}{q}=\frac{(2n+1)R\dot{v}+n\dot{r}}{r}=\frac{(3n+1)S\dot{v}+n\dot{s}}{s}=$ &c. fluens enim in priori caſu erit $e^{\int . \frac{(m+1)P\dot{v}-\dot{p}}{p}}\left(\frac{px^{m+1}}{m+1}+\frac{qx^{m}}{m}+\right.$ $\left.\frac{rx^{m-1}}{m-1}+\&c.\right)=$ conſt.; in poſteriori vero erit $e^{\int . -\frac{P\dot{v}+n\dot{p}}{np}}\left(npz+\frac{nqz^{n+1}}{n+1}\right.$ $\left.+\frac{nrz^{2n+1}}{2n+1}+\&c.\right)=$ conſt. ſi $z=x^{-n}$.

Ex. 22. Sit data algebraica æquatio homogenea relationem inter x, y & z exprimens; ubi $\dot{z}=P\dot{x}+Q\dot{y}$, in quâ literæ P & Q reſpective denotant homogeneas functiones quantitatum x & y.

In datâ algebraicâ æquatione pro y & z ſcribantur reſpective vx & wx; & reſultat algebraica æquatio relationem inter v & w exprimens; in poſteriori æquatione pro y & z ſubſtituantur prædicti valores, & reſultat $w\dot{x}+x\dot{w}=x^{m}\times(P'\dot{x}+Q'(v\dot{x}+x\dot{v}))$, ubi P' & Q' ſunt functiones quantitatum v & w: ex priori æquatione deduci poteſt quantitas v in terminis quantitatis w, i. e. $v=\phi:(w)$: ſubſtituatur $\phi:(w)$ & ejus fluxio pro v & $\dot{v}$ in ſecunda æquatione, & reſultat æquatio $w\dot{x}+x\dot{w}=x^{m}\,(P''\dot{x}+Q''(\phi:(w)\times\dot{x}+x\times\dot{\phi}:(w))$.

Hujuſce æquationis fluens facile detegi poteſt, cum $m = o$; & in multis aliis caſibus.

Et ſic progredi liceat ad fluxionales æquationes ſuperiorum ordinum; etiamque ad inveſtigandos plures caſus prædictarum fluxionalium æquationum, quarum fluentes inveniri poſſunt.

Hinc facile deduci poſſunt infinitæ æquationes formularum prædictarum, quarum fluentes inveniri poſſunt.

Cor. Deducere quam plurimas fluxionales æquationes, quæ reduci poſſunt in alias, quarum fluentes deduci poſſunt, vel quarum variabiles ſeparantur.

Aſſumantur æquationes fluxionales, in quibus ſeparantur variabiles, vel quarum fluentes innoteſcunt; pro variabilibus (x & y) & earum fluxionibus in his æquationibus ſcribantur functiones quantitatum (z & v) & earum fluxionum; & reſultant æquationes, quæ reduci poſſunt ad aſſumptas.

THEOR. XXXVII.

Fluxionalis æquatio, cujus termini haud videntur eaſdem habere dimenſiones, ſæpe transformari poteſt in alteram homogeneam, i. e. cujus termini eaſdem habent dimenſiones. Hoc plerumque perfici poteſt, ſi modo ita transformetur æquatio, ut ii termini, qui haud eaſdem habent dimenſiones in datâ eaſdem præbeant in reſultanti æquatione: vel ut diruat transformatio omnes reſultantis æquationis terminos, in quibus haud eædem inveniuntur dimenſiones.

Ex. 1. Sit æquatio $\dot{x}\sqrt{(ax^2 + bz^3)} = z^2\dot{z}$; ſupponantur quantitates z^3 & x^2 eaſdem habere dimenſiones, i. e. ſit $z^3 = y^2$ quantitati aſſumptæ, quæ eaſdem habet dimenſiones ac x^2; & exinde $z^2\dot{z} = \frac{2}{3}y\dot{y}$; ſubſtituantur pro z^3 & $z^2\dot{z}$ in datâ æquatione $\dot{x}\sqrt{(ax^2 + bz^3)} = z^2\dot{z}$ earum valores y^2 & $\frac{2}{3}y\dot{y}$, & reſultabit æquatio $\dot{x}\sqrt{(ax^2 + by^2)} = \frac{2}{3}y\dot{y}$, cujus termini manifeſto eaſdem habent dimenſiones.

Ex. 2. Sit æquatio $x^3\dot{x} + \frac{x^2\dot{y}}{\sqrt{(a+y)}} = \dot{y}$; ut liberetur data æquatio a termino, cujus dimenſiones haud ſunt æquales, fingatur $\surd(a+y)=v$, & reſultat $v^2-a=y$, & exinde $2v\dot{v}=\dot{y}$, unde deducitur æquatio $x^3\dot{x}+2x^2\dot{v}=2v\dot{v}$; nunc ſupponantur $x^3\dot{x}$ & $2x^2\dot{v}$ eaſdem habere dimenſiones, & conſequenter v duplas habere dimenſiones quantitatis x; aſſumatur igitur pro v quantitas u^2 duarum dimenſionum, quâ ſubſtitutâ pro v in datâ æquatione, reſultabit homogenea æquatio $x^3\dot{x}+4x^2u\dot{u}=8u^3\dot{u}$.

Ex. 3. Sit æquatio $ay^nx^m\dot{x}+by^qx^p\dot{x}+ex^ry^s\dot{y}=0$, ſi haud ſint $n+m=q+p=r+s$; ſcribatur $y=z^t$, unde reſultabit $az^{nt}x^m\dot{x}+bz^{qt}x^p\dot{x}+tez^{(s+1)t-1}x^r\dot{z}=0$; fingatur, ſi modo poſſibile ſit, $nt+m=qt+p=(s+1)t+r-1$, & reſultabit homogenea æquatio.

Ex. 4. Sit data æquatio $ay\dot{y}+by\dot{x}+cx\dot{y}+dx\dot{x}+e\dot{y}+f\dot{x}=0$; ſi vero $b=c$, tum fluens datæ æquationis erit $\frac{ay^2}{2}+cyx+\frac{dx^2}{2}+ey+fx=A$ conſt. quant. Si vero b haud ſit $=c$; ſcribantur $v+h$ & $z+k$ reſpective pro y & x, & earum fluxiones $\dot{v}$ & $\dot{z}$ pro $\dot{y}$ & $\dot{x}$ in datâ æquatione; & reſultat $av\dot{v}+bv\dot{z}+cz\dot{v}+dz\dot{z}+(ah+ck+e)\dot{v}+(bh+dk+f)\dot{z}=0$; fiant coefficientes $ah+ck+e$ & $bh+dk+f$ terminorum ($\dot{v}$ & $\dot{z}$) nihilo reſpective æquales, & reſultant $ah+ck+e=0$ & $bh+dk+f=0$, unde $h=\frac{de-cf}{cb-da}$ & $k=\frac{fa-be}{cb-da}$, & æquatio reſultans $av\dot{v}+bv\dot{z}+cz\dot{v}+dz\dot{z}=0$ erit homogenea.

Si vero $cb-da=0$, tum per hanc methodum haud reduci poteſt data æquatio in homogeneam; in hoc caſu erit $f\dot{x}+(by+dx)\dot{x}+n(by+dx)\dot{y}+e\dot{y}=0$, ponatur $by+dx=z$, & erunt $\frac{\dot{y}}{\dot{x}}=-\frac{f+z}{e+nz}$ & $\dot{y}=\frac{\dot{z}-d\dot{x}}{b}$, unde $\dot{x}=\frac{\dot{z}(e+nz)}{-bf+ed+(-b+nd)z}$.

Ex. 5. Sit $(a+bxy+cx^2y^2+dx^3y^3+\&c.)\dot{x}+x^2(l+mxy+nx^2y^2+\&c.)\dot{y}=0$, vel $(ay+bxy^2+cx^2y^3+\&c.)\dot{x}+(lx+$

$mx^2y + nx^3y^2 +$ &c.$)\dot{y} = o$; ſcribatur z^{-1} in his æquationibus pro y, & exponentes omnium terminorum evadent eædem.

Si vero generaliter $(ax^{\pi} + bx^{\pi-1}y^{\tau} + cx^{\pi-2}y^{2\tau} +$ &c.$)\dot{x} + (l'x^{\pi}y^{\tau-1} + mx^{\pi-1}y^{2\tau-1} + nx^{\pi-2}y^{3\tau-1} +$ &c.$)\dot{y} = o$; ſcribatur z pro y^{τ}, & reſultat æquatio homogenea.

Ex. 6. Sit $\dot{y} + y^2\dot{x} = \frac{a\dot{x}}{x^2}$, ſcribatur $\frac{1}{z}$ pro y in data æquatione, & reſultat æquatio homogenea $x^2\dot{z} = \dot{x}(xz - az^2)$.

Ex. 7. Sit æquatio $(ax - bcy + by^2)\dot{x} + (dcx - 2dxy + c^2y - 3cy^2 + 2y^3)\dot{y} = o$; ſubſtituatur pro y ejus valor aſſumptus $\frac{c \pm \surd(c^2 + az)}{2}$, & reſultabit æquatio homogenea hujuſce formulæ $\alpha x\dot{x} + \beta z\dot{x} + \gamma x\dot{z} + \delta z\dot{z} = o$.

Et ſimiliter progredi liceat in reductione datarum æquationum in alias, quarum ſinguli termini vel ſint quantitates n, $n-1$, $n-2$, &c. dimenſionum.

Si tres vel plures (x, y, z, &c.) variabiles quantitates in datâ homogeneâ æquatione contineantur; pro literis y, z, &c. ſcribantur reſpective px, qx, &c. deinde per methodum, quæ de æquationibus duas variabiles quantitates habentibus fuit tradita, progrediendum eſt.

PROB. LV.

Invenire æquationes, quas reducere liceat ad homogeneas æquationes.

Aſſumantur æquationes homogeneæ, ſubſtituantur quæcunque functiones novarum quantitatum pro variabilibus quantitatibus in aſſumptis æquationibus contentis; & reſultant æquationes, quæ reduci poſſunt ad homogeneas æquationes.

Cor. Datâ homogeneâ æquatione duas variabiles quantitates x & y & earum fluxiones habente; pro x vel y ſcribatur $\frac{r}{z^s}$ in datâ æquatione, & reſultat æquatio; quæ facile reduci poteſt in priorem ex æquatis terminis reſultantis æquationis, quorum dimenſiones haud videntur eſſe eædem.

Et ſic de infinitis aliis hujuſcemodi æquationibus deducendis, quæ vel in homogeneas æquationes vel in alias cujuſcunque datæ formulæ reduci poſſunt.

2. Sit data fluxionalis homogenea æquatio (n) ordinis, quantitates x & y, & earum fluxiones $\dot x, \dot y, \ddot y, \dddot y, \ldots, \overset{n-1}{\dot y}$ & $\overset{n}{\dot y}$ involvens, ubi x fluit uniformiter: in eâ pro $y, \dot y, \ddot y, \ldots, \overset{n-1}{\dot y}$ & $\overset{n}{\dot y}$ ſcribantur reſpective vx, $v'\dot x, \frac{v''\dot x^2}{x}, \frac{v'''\dot x^3}{x^2}, \frac{v''''\dot x^4}{x^3}, \ldots, \frac{\overset{n-1}{v'}\dot x^{n-1}}{x^{n-2}}$ & $\frac{\overset{n}{v'}\dot x^n}{x^{n-1}}$, & reſultat æquatio algebraica ($A = o$) relationem inter quantitates $v, v', v'', v''', \ldots \overset{n-1}{v'}$ & $\overset{n}{v'}$ exprimens: deindc, ob $y = vx$, erit $v\dot x + x\dot v = \dot y = v'\dot x$, & exinde $\frac{\dot x}{x} = \frac{\dot v}{v'-v}$; & ſimiliter ob $\dot y = v'\dot x$, erit $\ddot y = \dot v'\dot x = \frac{v''\dot x^2}{x}$, unde $\frac{\dot x}{x} = \frac{\dot v'}{v''}$; & denique $\frac{\dot x}{x} = \frac{\dot v}{v'-v} = \frac{\dot v'}{v''} = \frac{\dot v''}{v'''+v''} = \frac{\dot v'''}{v''''+2v'''} = \frac{\dot v''''}{v'''''+3v''''} = \text{\&c.} = \frac{\overset{n-1}{\dot v'}}{\overset{n}{v'}+(n-2)\overset{n-1}{v'}}$: hæ autem ſunt ($n-1$) diverſæ æquationes, viz. $\frac{\dot v}{v'-v} = \frac{\dot v'}{v''} = \text{\&c.}$, in quibus nec continetur x nec $\dot x$: continentur autem ſolummodo $v, v', v'', v''', \ldots, \overset{n-1}{v'}$ & $\overset{n}{v'}$; & fluxiones $\dot v, \dot v', \dot v'', \ldots \overset{n-2}{\dot v'}$ & $\overset{n-1}{\dot v'}$; deinde ex datâ algebraicâ æquatione $A = o$ inveniatur $\overset{n}{v'}$ in terminis reliquarum $v, v', v'', v''', \overset{n-2}{v'}$ & $\overset{n-1}{v'}$; & ſubſtituatur hæc quantitas in ultimâ æquatione $\frac{\overset{n-2}{\dot v'}}{\overset{n-1}{v'}+(n-3)\overset{n-2}{v'}} = \frac{\overset{n-1}{\dot v'}}{\overset{n}{v'}+(n-2)\overset{n-2}{v'}}$ pro ejus valore $\overset{n}{v'}$; & reſultant ($n-1$) fluxionales æquationes primi ordinis

involventes (n) quantitates v, v', v'', . . ., $\overset{n-1}{v'}$ & earum fluxiones primi ordinis: & exinde, ſi modo reducantur hæ ($n-1$) æquationes in unam, ita ut exterminentur omnes præter quaſcunque duas variabiles & earum fluxiones, per prob. 26. reſultabit fluxionalis æquatio involvens duas prædictas variabiles quantitates & earum fluxiones ordinis, qui non major eſt quam ($n-1$).

3. Sint duæ homogeneæ fluxionales æquationes primi ordinis involventes variabiles (x, y & z) & earum primas fluxiones: in his duabus æquationibus pro y & z ſcribantur reſpective vx & wx, & pro $\dot{y}$ & $\dot{z}$ ſubſtituantur $v'\dot{x}$ & $w'\dot{x}$, & reſultabunt duæ algebraicæ æquationes $A=0$ & $B=0$ quatuor variabiles v & w, v' & w' involventes: ex præcedente methodo deduci poteſt $\frac{\dot{x}}{x}=\frac{\dot{v}}{v'-v}=\frac{\dot{w}}{w'-w}$; deinde in æquatione $\frac{\dot{v}}{v'-v}=\frac{\dot{w}}{w'-w}$ pro v' & w' ſcribantur reſpective functiones quantitatum v & w ex æquationibus $A=0$ & $B=0$ petitæ; & reſultabit fluxionalis æquatio primi ordinis relationem inter v & w & earum fluxiones exprimens.

4. Sint duæ, tres vel m homogeneæ fluxionales æquationes ordinis (n), tres vel quatuor vel ($m+1$) variabiles quantitates (x, y, z, &c.) involventes; pro y, $\dot{y}$, $\ddot{y}$, . . ., $\overset{n-1}{\dot{y}}$ & $\overset{n}{\dot{y}}$ ſcribantur vx, $v'\dot{x}$, $\frac{v''\dot{x}^2}{x}$, . . . $\frac{\overset{n-1}{v'}\dot{x}^{n-1}}{x^{n-2}}$ & $\frac{\overset{n}{v'}\dot{x}^n}{x^{n-1}}$; & pro z, $\dot{z}$, $\ddot{z}$, . . ., $\overset{n}{\dot{z}}$ ſubſtituantur wx, $w'\dot{x}$, $\frac{w''\dot{x}^2}{x}$, . . . $\frac{\overset{n}{w'}\dot{x}^n}{x^{n-1}}$; &c. &c. & reſultabunt (m) algebraicæ æquationes (m) ($n+1$) variabiles quantitates habentes; & per methodum prius traditam deduci poſſunt æquationes $\frac{\dot{x}}{x}=\frac{\dot{v}}{v'-v}=\frac{\dot{v}'}{v''}=\frac{\dot{v}''}{v'''+v''}=$ &c. $=$

$$\frac{\overset{n-1}{\dot{v}'}}{\overset{n}{v'}+(n-2)\overset{n-1}{v'}}=\frac{\dot{w}}{w'-w}=\frac{\dot{w}'}{w''}=\frac{\dot{w}''}{w'''+w''}=\&c.=\frac{\overset{n-1}{\dot{w}'}}{\overset{n}{w'}+(n-2)\overset{n-1}{w'}}$$

$=$ &c.: in his æquationibus pro $\overset{n}{v'}$, $\overset{n}{w'}$, &c. ſcribantur earum valores; & reſultabunt $m \times n-1$ diverſæ fluxionales æquationes primi ordinis involventes $m \times n$ variabiles quantitates; ſi hæ æquationes in unam reducantur, ut exterminentur omnes præter duas variabiles quantitates & earum fluxiones; tum reſultabit fluxionalis æquatio, cujus ordo non major eſt quam $mn-1$.

5. Ex iiſdem principiis & reductione æquationum facile conſtat (ſi modo dentur (m) homogeneæ fluxionales æquationes ordinum $(r, s, t,$ &c.); & reducantur per methodum prius traditam æquationes deductæ in unam, ita ut exterminentur $(m-1)$ variabiles quantitates & earum fluxiones) æquationem reſultantem habere ordinem, qui non major eſt quam $r+s+t+$ &c.-1.

THEOR. XXXVIII.

Defin. Æquatio relationem inter x & y & earum fluxiones exprimens dicitur homogenea, ſi non ſolum variabilibus x & y, ſed etiam ſingulis earum fluxionibus $\dot{x}$ & $\dot{y}$, itemque $\ddot{y}, \dddot{y}$ &c. unam dimenſionem obtinentibus, omnes æquationis termini eundem dimenſionum numerum contineant.

Hinc; ſi $\dot{y}=p\dot{x}$ & $\frac{\dot{p}}{\dot{x}}=q$, litera p nullam habet dimenſionem, litera q unam habet dimenſionem negativam, &c.

Ex iiſdem principiis, viz. ſubſtitutionibus, fluxionales æquationes ſuperiorum ordinum reduci poſſunt ad fluxionales inferiorum ordinum æquationes.

Ex. 1. Datam æquationem fluxionalem ſecundi ordinis, quæ ſit homogenea, ad æquationem fluxionalem primi ordinis reducere.

Sint $\dot{y}=p\dot{x}$ & $\dot{p}=q\dot{x}$, & habeatur æquatio inter quatuor quantitates

titates x, y, p & q relationem exprimens: fingatur $y = ux$ & $q = \frac{v}{x}$, quibus valoribus pro y & q in datâ æquatione ſubſtitutis, reſultat æquatio relationem inter u, v & p exprimens, ex quâ unam per duas reliquas definire liceat: cum autem ſit $\dot{y} = p\dot{x}$, erit $u\dot{x} + x\dot{u} = p\dot{x}$ & $\frac{\dot{x}}{x} = \frac{\dot{u}}{p - u}$; deinde $\dot{p} = q\dot{x} = \frac{v\dot{x}}{x}$, erit $\frac{\dot{x}}{x} = \frac{\dot{p}}{v}$, unde $\frac{\dot{p}}{v} = \frac{\dot{u}}{p - u}$ & $p\dot{p} - u\dot{p} = v\dot{u}$: ſed e præcedente æquatione inveniatur v in terminis quantitatum p & u, ſubſtituatur valor inventus pro v in prædictâ æquatione $p\dot{p} - u\dot{p} = v\dot{u}$, & reſultabit æquatio relationem inter variabiles p & u & earum primas fluxiones exprimens.

Ex. 2. Si æquatio fluxionalis evadat homogenea, cum alteri variabili y tribuantur n dimenſiones, tum ejus reſolutio eadem erit ac æquationis in ex. præced. contentæ; ſcribatur enim w^n pro y & reſultabit æquatio homogenea; vel quod ad idem redit, fingantur $y = x^n u$, $\dot{y} = x^{n-1} t\dot{x}$ & $\ddot{y} = x^{n-2} v\dot{x}^2$; & ob $\dot{y} = p\dot{x}$ & $\dot{p} = q\dot{x}$, erit $x\dot{u} + nu\dot{x} = t\dot{x}$, & $x\dot{t} + (n - 1)t\dot{x} = v\dot{x}$, unde $\frac{\dot{x}}{x} = \frac{\dot{u}}{t - nu} = \frac{\dot{t}}{v - (n - 1)t}$: ſubſtituantur etiam in datâ æquatione pro y, p & q prædicti valores, & reſultabit æquatio relationem exprimens inter u, t & v; ex quâ inveniri poteſt v in terminis quantitatum t & u; ſcribatur hic valor inventus pro v in æquatione $\frac{\dot{u}}{t - nu} = \frac{\dot{t}}{v - (n - 1)t}$, & reſultabit æquatio quæſita.

Sit $n = 0$, i. e. ſi quantitati y & ejus fluxionibus nullæ tribuantur dimenſiones, data æquatio evadet homogenea; hoc ſcilicet caſu variabilis x cum ſuis fluxionibus in ſingulis terminis eaſdem conſtituunt dimenſiones.

Sit n infinitus, & dimenſiones e variabili y & ejus fluxionibus in ſingulis terminis evadent eædem.

Ex. gr. Sit $\ddot{y} = cx^{\alpha} y^{\beta} \dot{x}^{2-\gamma} \dot{y}^{\gamma}$, ejus fluentem invenire.

Quoniam $\dot{y} = p\dot{x}$, & $\ddot{y} = q\dot{x}^2$, erit $\ddot{y} = cx^{\alpha} y^{\beta} p^{\gamma} \dot{x}^2$; fingantur $y = x^n u$,

$\dot{y} = x^{n-1} t \dot{x}$ & $\ddot{y} = x^{n-2} v \dot{x}^2$, & evadet $x^{n-2} v = c x^{\alpha+n\beta+\gamma n-\gamma} u^{\beta} t^{\gamma}$, unde $n = \frac{\gamma - \alpha - 2}{\beta + \gamma - 1}$, & exinde $v = c u^{\beta} t^{\gamma}$; & $\frac{\dot{u}}{t - nu} = \frac{\dot{t}}{c u^{\beta} t^{\gamma} - (n-1) t}$ æquat. quæſit. primi ordinis.

Si n ſit infinitus, ſcribatur $\dot{y} = p \dot{x}$ & $\ddot{y} = q \dot{x}^2$, & æquatio ita erit comparata, ut in eâ tres variabiles y, p & q ubique eundem dimenſionum numerum obtineant: ſtatuantur $p = uy$ & $q = vy$, quibus quantitatibus pro ſuis valoribus in datâ æquatione ſubſtitutis, reſultabit æquatio relationem inter variabiles x, u & v exprimens, ex quâ inveniatur v in terminis duarum reliquarum u & x: ſed quoniam $p = uy$ erit $\dot{y} = p \dot{x} = u y \dot{x}$, & quoniam $\ddot{y} = q \dot{x}^2$, erit $\ddot{y} = v y \dot{x}^2 = \dot{x} (u \dot{y} + y \dot{u})$; ex his æquationibus ſequitur $\frac{\dot{y}}{y} = u \dot{x} = \frac{v \dot{x} - \dot{u}}{u}$, unde $u^2 \dot{x} + \dot{u} = v \dot{x}$, ſed invenitur valor quantitatis v in terminis quantitatum u & x, ſcribatur hic valor pro v in æquatione $u^2 \dot{x} + \dot{u} = v \dot{x}$, & reſultat æquatio quæſita.

Si in æquatione $u^2 \dot{x} + \dot{u} = v \dot{x}$ ſubſtituantur $\frac{\dot{y}}{y \dot{x}}$ & $\frac{\ddot{y}}{y \dot{x}} - \frac{\dot{y}^2}{y^2 \dot{x}}$ (ubi $\dot{x}$ ſit conſtans) pro ſuis valoribus u & $\dot{u}$, reſultabit æquatio $\frac{\ddot{y}}{y \dot{x}} = v \dot{x}$.

Facile conſtant caſus, in quibus fluens fluxionalis æquationis inveniri poteſt ex æquatione reſultante, &c.

Ex. 3. Si in datâ æquatione relationem inter x & y & earum fluxiones deſignante, in quâ fluit uniformiter x, deſit altera incognita quantitas y; ſcribatur in datâ æquatione pro $\dot{y}$ ejus valor aſſumptus $p \dot{x}$, & pro fluxione $\ddot{y} = \dot{p} \dot{x}$, pro $\dddot{y} = \ddot{p} \dot{x}$ & ſic deinceps; & reſultabit æquatio, in quâ ordo fluxionum minor unitate quam in datâ, i. e. ſi data contineat ſecundas fluxiones, tum reſultans continebit ſolummodo primas; &c.

In genere ſi A ſit quantitas, quæ fluit uniformiter, ſupponatur $\dot{x} = p \dot{A}$, unde $\ddot{x} = \dot{p} \dot{A}$, $\dddot{x} = \ddot{p} \dot{A}$ & ſic deinceps; quibus quantitatibus pro ſuis valoribus in datâ æquatione ſubſtitutis reſultabit æquatio, in quâ ordo fluxionis quantitatis p minor erit unitate quam ordo quantitatis x in datâ æquatione inventus. Si nullus terminus in datâ æquatione aſſumatur tanquam conſtans, tum aſſumi poteſt quæcunque quantitas, cujus fluxio ſit conſtans; ſed præſtat aſſumere talem quantitatem fluentem uniformiter, quâ ſcilicet aſſumptâ deleantur plurimæ quantitates e datâ æquatione; vel ita transformentur quantitates, ut exinde ejus fluens inveſtigari poſſit; vel reducatur æquatio in formulas prædictas, quarum reſolutiones docentur.

Æquationes, quæ haud apparent ſub hâc formulâ, e ſubſtitutione nonnunquam in eam reduci poſſunt, e.g. ſit æquatio $x^m\ddot{x} = y\ddot{y} + \dot{y}^2 + \dot{y}^2y^2$, ſcribatur $y\dot{y} = z$, & reſultat $x^m\ddot{x} = \ddot{z} + \dot{z}^2$.

Ex. 4. Si duæ variabiles (x & y) quantitates in datâ æquatione contentæ ſimul cum earum fluxionibus ſemper in ſingulis terminis eaſdem conficiant dimenſiones; & $\dot{x}$ ſit conſtans quantitas; aſſumantur $x = e^u$ & $y = e^u t$, ubi $e =$ num. cujus log. eſt 1; tum $\dot{x} = e^u\dot{u}$ & $\ddot{x} = e^u(\dot{u}^2 + \ddot{u}) = 0$, quoniam $\dot{x}$ eſt conſtans, erit $\ddot{u} = -\dot{u}^2$; $\dot{y} = e^u(t\dot{u} + \dot{t})$, & $\ddot{y} = e^u(\ddot{t} + 2\dot{u}\dot{t} + t\ddot{u} + t\dot{u}^2) = e^u(\ddot{t} + 2\dot{u}\dot{t})$, quibus quantitatibus pro ſuis valoribus in datâ æquatione ſubſtitutis reſultabit æquatio, in quâ haud invenitur u.

Hoc in loco obſervandum eſt, nec quantitatem t nec u in æquatione reſultante fluere uniformiter, ſed e^u.

Ex. 5. Si variabilis y cum ſuis fluxionibus $\dot{y}$ & $\ddot{y}$ ubique eundem dimenſionum numerum adimpleat in datâ fluxionali æquatione, eam in fluxionalem æquationem primi ordinis reducere.

Sint $\dot{y} = p\dot{x}$ & $\dot{p} = q\dot{x}$, unde variabiles y, p & q ubique eundem dimenſionum numerum obtineant: ſtatuantur $p = uy$ & $q = vy$ & ſubſtituantur hæ quantitates in datâ æquatione pro ſuis valoribus, & reſultabit æquatio relationem inter x, u & v exprimens, e quâ inveniatur v functio quantitatum x & u, ob $p = uy$, erit $\dot{y} = uy\dot{x}$, & exinde $u\dot{x} = \frac{\dot{y}}{y} = \frac{v\dot{x} - \dot{u}}{u}$, unde $\dot{u} + u^2\dot{x} = v\dot{x}$ æquatio quæſita.

Cor. . Ex formulis integralium æquationum prius traditis conſtant formulæ hujuſce generis integrabiles.

Ex. 6. Si variabilis y cum ſuis fluxionibus $\dot{y}$, $\ddot{y}$, &c. $\overset{n}{\dot{y}}$ ubique eundem dimenſionum numerum adimpleat in datâ æquatione, eam in fluxionalem æquationem $(n - 1)$ ordinis reducere. Subſtituantur pro, y, $\dot{y}$, $\ddot{y}$, $\dddot{y}$, &c. in datâ æquatione reſpective $e^{\int . u\dot{x}}$, $u\dot{x}e^{\int . u\dot{x}}$, $(u^2\dot{x}^2 + \dot{u}\dot{x})e^{\int . u\dot{x}}$, $(u^3\dot{x}^3 + 3u\dot{u}\dot{x}^2 + \ddot{u}\dot{x})e^{\int . u\dot{x}}$, &c. ubi $\dot{x}$ eſt conſtans, & reſultabit fluxionalis æquatio $(n - 1)$ ordinis. Hoc etiam perfici poteſt e præcedentibus principiis, quæ omnino eadem ſunt ac ea hoc in loco data.

E. g. Sit æquatio $\ddot{y} + P\dot{x}\dot{y} + Qy\dot{x}^2 = o$, ſcribatur $y = e^{\int . u\dot{x}}$, unde $\dot{y} = e^{\int . u\dot{x}}u\dot{x}$ & $\ddot{y} = e^{\int . u\dot{x}}(\dot{u}\dot{x} + u^2\dot{x}^2)$; ſubſtituantur hæ quantitates pro ſuis valoribus in datâ æquatione, & reſultabit $\dot{u} + u^2\dot{x} + Pu\dot{x} + Q\dot{x} = o$.

Cor. 1. Scribatur in æquatione $(\dot{u} + u^2\dot{x} + Pu\dot{x} + Q\dot{x} = o)$ Mz pro u & reſultabit æquatio $M\dot{z} + z(\dot{M} + PM\dot{x}) + M^2z^2\dot{x} + Q\dot{x} = o$, aſſumatur $\dot{M} + PM\dot{x} = o$, & exinde $M = ae^{-\int . P\dot{x}}$, ubi a eſt conſtans quantitas; & æquatio reſultans $ae^{-\int . P\dot{x}}\dot{z} + a^2e^{-2\int . P\dot{x}}z^2\dot{x} + Q\dot{x} = o$.

Eodem modo ſcribantur pro u; vel $\frac{M}{z}$, vel $K + Mz$, vel $\frac{K + Mz}{L + Nz}$, &e. in datâ æquatione, & reſultantis æquationis fiant quicunque termini nihilo æquales, ita vero ut æquatio exinde reſultans evadat maxime ſimplex; &c.

Paulo aliter in quibuſdam caſibus progredi licet: e. g. ſit æquatio $\ddot{y} - \frac{n\dot{y}\dot{x}}{x} + Ax^n\dot{y}\dot{x} + Bx^{2n}y\dot{x}^2 = o$; ponatur $\dot{y} = x^n yu\dot{x}$, ſcribatur $e^{\int . x^n u\dot{x}}$ pro y, & ejus fluxio pro $\dot{y}$, & ſic deinceps in datâ æquatione; & reſultabit $\dot{u} + x^n u^2\dot{x} + Ax^n u\dot{x} + Bx^n\dot{x} = o$, unde $x^n\dot{x} = \frac{-\dot{u}}{u^2 + Au + B}$; ſint radices æquationis $u^2 + Au + B = o$ reſpective α & β, tum erit

$y =$

$y = ae^{\alpha \int . x^n \dot{x}} + be^{\beta \int . x^n \dot{x}}$, ubi a & b ſunt quæcunque invariabiles quantitates. Conſtat e ſubſtitutione.

Facile deduci poſſunt formulæ æquationum, quarum ex his ſubſtitutionibus $y = e^{\int . u \dot{x}}$, vel $y = e^u t$ & $x = e^{ru}$, &c. fluentes deduci poſſunt: aſſumantur enim formulæ æquationum, quarum fluentes dentur, & exinde inveſtigentur æquationes, quæ prædictis ſubſtitutionibus præbent æquationes formularum aſſumptarum: e. g. fluentes fluxionalium æquationum primi ordinis, in quibus deeſt una variabilis quantitas, vel quæ in hanc formulam reducantur, inveniri poſſunt; &c. & e priori ſubſtitutione patet omnem æquationem, cujus ſinguli termini habeant formulam $Ay^n \dot{y}^m \ddot{y}^p \dot{x}^q$ (ubi in ſingulis A, n, m, & p ſunt invariabiles & $n + m + p$ eadem quantitas) tranſire in æquationem fluxionalem primi ordinis, in quâ deeſt x: deinde e ſubſtitutione poſteriori conſtat omnem æquationem, cujus primus terminus ſit $\ddot{y}$, cæteri vero habeant formulam $Ay^n \dot{y}^m \dot{x}^{2-m}$, in quâ A, n & m ſunt invariabiles, & $n + m + r(2 - m) = 1$, (ubi r & $2 - m$ in ſingulis terminis eadem manet), tranſire in fluxionalem æquationem ſecundi ordinis, in cujus ſingulis terminis variabilis t cum ejus fluxionibus eaſdem conficit dimenſiones, & deeſt variabilis u.

Idem etiam affirmari poteſt, ſi modo pro $\dot{x}$ in ſingulis terminis prædictis ſcribatur $\frac{\dot{z}}{z}$, vel fluxio cujuſcunque functionis quantitatis x. Et ſic de inveniendis æquationibus, quæ ſubſtitutionibus prædictis reduci poſſint ad homogeneas æquationes primi ordinis, cæteraſve æquationes, quarum fluentes dentur.

Ex. 7. Sit $ax^m \dot{x}^p = y^n \dot{y}^{p-2} \ddot{y}$, ubi $\dot{x}$ eſt invariabilis quantitas; fingatur $x = e^{hu}$ & $y = e^u t$, ubi e ſit numerus, cujus logarith. $= 1$; tum $\dot{x} = he^{hu} \dot{u}$, $\dot{y} = e^u \dot{t} + e^u t \dot{u}$, $\ddot{y} = e^u (\ddot{t} + 2\dot{u}\dot{t} + t\dot{u}^2 + t\ddot{u})$; quibus valoribus pro x, $\dot{x}$, $\dot{y}$ & $\ddot{y}$ in datâ æquatione ſubſtitutis, reſultat $e^{hu(m+p)} h^p \dot{u}^p = e^{u(n+p-1)} t^n (\dot{t} + t\dot{u})^{p-2} (\ddot{t} + 2\dot{t}\dot{u} + t\ddot{u} + t\dot{u}^2)$; ut vero deſtruatur exponentialis e^u, ſcribatur $hu(m+p) = u(n+p-1)$, unde $h =$

$n + p$

$\frac{n+p-1}{m+p}$; quo valore ſubſtituto reſultat æquatio, in quâ nec continetur quantitas u, nec ejus functiones, viz. $h^p\dot{u}^p = t^n(\dot{t} + t\dot{u})^{p-2}(\ddot{t} + 2\dot{t}\dot{u} + t\ddot{u} + t\dot{u}^2)$: ſed quoniam per hypotheſin $\ddot{x} = o$, erit $\ddot{u} = -h\dot{u}^2 = -\frac{n+p-1}{m+p}\dot{u}^2$, quâ quantitate pro ſuo valore ſubſtitutâ, reſultat $\left(\frac{n+p-1}{m+p}\right)^p \dot{u}^p = t^n(\dot{t} + t\dot{u})^{p-2}(\ddot{t} + 2\dot{t}\dot{u} + \frac{m-n+1}{m+p}t\dot{u}^2) = o$.

E. g. Sit æquatio $ax^{-1}\dot{x} = \dot{y}^{-1}\ddot{y}$, ejus fluens haud inveniri poteſt e præcedente exemplo, nam $m = -1$ & $p = 1$, unde $m + p = o$, & $\left(\frac{n+p-1}{m+p}\right)^p$ hâc methodo haud innoteſcit.

In hoc exemplo animadvertendum eſt nec t nec u fluere uniformiter, ſed e^{bu}; unde reſultans æquatio hâc methodo haud generalem reſolutionem recipiet.

Ex. 8. Sit æquatio $P\ddot{y} + Q\dot{y}\dot{x} + R\dot{x}^2 = o$; ubi P, Q & R ſunt homogeneæ functiones n, $n-1$ & $n-1$ dimenſionum literarum x & y reſpective; ſcribatur $y = xz$, & reſultabunt $\dot{y} = x\dot{z} + z\dot{x}$ & $\ddot{y} = x\ddot{z} + 2\dot{z}\dot{x}$, ſi modo x fluat uniformiter; quibus quantitatibus pro ſuis valoribus in datâ æquatione ſubſtitutis, reſultat æquatio $px^2\ddot{z} + qx\dot{z}\dot{x} + r\dot{x}^2 = o$, ubi literæ p, q & r reſpective denotant functiones literæ z.

9. Sit æquatio $P\overset{n}{\dot{y}} + Q\overset{n-1}{\dot{y}} + R\overset{n-2}{\dot{y}} + \&c. = o$, ubi literæ P, Q, R, &c. ſunt homogeneæ functiones m, $m-1$, $m-2$, ... $m-n+1$, dimenſionum literarum x & y reſpective; fingatur $y = xz$, & exinde $\dot{y} = x\dot{z} + z\dot{x}$ & $\ddot{y} = x\ddot{z} + 2\dot{z}\dot{x}$, $\dddot{y} = x\dddot{z} + 3\dot{x}\ddot{z}$, & ſic deinceps; quibus quantitatibus pro ſuis valoribus in datâ æquatione ſubſtitutis, reſultat æquatio $px^n\overset{n}{\dot{z}} + qx^{n-1}\overset{n-1}{\dot{z}}\dot{x} + rx^{n-2}\overset{n-2}{\dot{z}}\dot{x}^2 + \&c. = o$, ubi literæ p, q, r, &c. reſpective ſunt functiones literæ z, &c.

Sint x & y variabiles quantitates in datâ æquatione contentæ;

cavendum eſt, ne in ſubſtitutionibus pro x & y ſcribantur tales functiones quantitatum aſſumptarum z & v, quales reddant quantitatem x exinde deductam, pendentem e quantitate y; aliter in errores ut nos inducamur probabile eſt: e. g. aſſumantur pro x & y quæcunque homogeneæ diverſæ functiones nullius dimenſionis quantitatum aſſumptarum z & v, ſcribantur hæ functiones pro ſuis valoribus in datâ æquatione, & reſultat homogenea æquatio, unde reducitur omnis æquatio ad homogeneam; ſed in hoc obſervandum eſt, quod aſſumptio duarum diverſarum homogenearum functionum nullius dimenſionis præſupponit quandam relationem inter quantitates x & y exiſtentem.

PROB. LVI.

Propoſitâ æquatione fluxionali, cujus fluens ex introductione novarum variabilium pro iis in datâ æquatione contentis detegi poteſt; invenire multiplicatorem, qui ductus in propoſitam æquationem, præbeat fluxionem, cujus fluens inveniri poteſt.

Sit $P\dot{x} + Q\dot{y} = 0$ propoſita æquatio, quæ per ſubſtitutionem reducatur ad æquationem $P\dot{x} + Q\dot{y} = R\dot{t} + S\dot{u}$, ubi duæ variabiles t & u introducuntur loco ipſarum x & y; dividatur æquatio $R\dot{t} + S\dot{u} = 0$ per V, cum igitur fluens fluxionis $\frac{R\dot{t} + S\dot{u}}{V}$ inveniri poſſit, tum etiam fluens fluxionis $\frac{P\dot{x} + Q\dot{y}}{V} = \frac{R\dot{t} + S\dot{u}}{V}$ inveniri poteſt, unde $\frac{1}{V}$ erit multiplicator, qui in datam æquationem ductus præbet fluxionem, cujus fluens inveniri poteſt: in multiplicatore $\frac{1}{V}$ pro variabilibus u & t reſtituantur variabiles x & y, & reſultat multiplicator quæſitus.

Cor. Hinc multæ æquationes inveniri poſſunt, quarum multiplicatores

catores innoteſcunt; in æquationibus, quarum multiplicatores dantur, pro variabilibus, &c. ſcribantur quæcunque functiones aliarum variabilium quantitatum & earum fluxiones reſpective; & exinde deduci poſſunt fluxionales æquationes, quarum multiplicatores detegi poſſunt.

THEOR. XXXIX.

Aſſumptâ generali relatione inter variabiles datarum æquationum quantitates & novas; & his novis, pro earum valoribus in datis æquationibus generaliter expreſſis, ſubſtitutis; ſæpe ita aſſumi poſſunt coefficientes vel indices in generali relatione vel æquationibus, ut evaneſcant quidam termini, & exinde reſultet æquatio, cujus integratio datur.

Ex. 1. Sit $\dot{x} = ax^m y^n \dot{y} + bx^e y^b \dot{y} + cx^g y^l \dot{y} +$ &c. fingatur $u = y^\alpha x^\beta$; ubi α & β ſunt indeterminatæ quantitates; & erit integrabilis data æquatio, cum $-\frac{\alpha}{\beta} - 1 = \frac{-m\alpha}{\beta} + n = \frac{-e\alpha}{\beta} + b = \frac{-g\alpha}{\beta} + l$, &c. vel $\frac{n+1}{m-1} = \frac{b+1}{e-1} = \frac{l+1}{g-1}$, &c. &c.

Ex. 2. Sit æquatio $ay^l x^\beta \dot{x} + by^n x^\gamma \dot{x} = \dot{y}$; ſcribatur $y = ex^\pi$ & reſultat $ae^l x^{\pi l+\beta} \dot{x} + be^n x^{\pi n+\gamma} \dot{x} = \pi e x^{\pi-1} \dot{x}$; ut hi termini ſeſe mutuo deſtruant, neceſſe eſt ſupponere $\pi l + \beta = \pi n + \gamma = \pi - 1$, e quibus æquationibus conſtabit $\pi = \frac{\gamma-\beta}{l-n} = \frac{\gamma+1}{1-n}$, & exinde $\beta = \frac{l\gamma + l - n - \gamma}{n-1}$; & $ae^{l-1} + be^{n-1} = \frac{\gamma-\beta}{l-n}$; & ſi igitur $\beta = \frac{l\gamma + l - n - \gamma}{n-1}$, & $ae^{l-1} + be^{n-1} = \frac{\gamma-\beta}{l-n}$ in datâ æquatione, erit etiam $y = ex^{\frac{\gamma+1}{1-n}}$, ſed non erit ejus y generalis valor.

Ex. 3. Æquationem $ax^r \dot{x} + bx^s y^2 \dot{x} = \dot{y}$ in æquationem formulæ

$a x^m \dot{x} + b y^2 \dot{x} = \dot{y}$ reducere; ſcribatur $x^{s+1} = z$, & exinde $x = z^{\frac{1}{s+1}}$; qui valor in datâ æquatione ſubſtituatur, & reſultat $\frac{a}{s+1} z^{\frac{r+1}{s+1}-1} \dot{z} + \frac{b}{s+1} y^2 \dot{z} = \dot{y}$ æquatio formulæ quæſitæ.

Si $s + 1 = 0$, fallit hæc methodus; in quo caſu pro x ſcribatur e^z, & reſultat æquatio prædictæ formulæ.

Ex. 4. Sit data æquatio (P), $(a + b x^n) x^2 \dot{z} + (c + f x^n) x z \dot{x} + (a + b x^n) x^2 z^2 \dot{x} + (g + h x^n) \dot{x} = 0$; ſcribantur $T y + S$ & $T \dot{y} + y \dot{T} + \dot{S}$ pro z & $\dot{z}$ reſpective in datâ æquatione, & reſultat $(a + b x^n) x^2 T \dot{y} + (a + b x^n) x^2 y \dot{T} + (a + b x^n) x^2 \dot{S} + (c + f x^n) x T y \dot{x} + (c + f x^n) x S \dot{x} + (a + b x^n)(T y + S)^2 x^2 \dot{x} + (g + h x^n) \dot{z} = 0$, (R); ſupponatur omnium terminorum ſumma, in quibus continetur y, nihilo æqualis, i. e. $(a + b x^n) x \dot{T} + 2 (a + b x^n) x T S \dot{x} + (c + f x^n) T \dot{x} = 0$, unde $\frac{\dot{T}}{T} + 2 S \dot{x} + \frac{(c + f x^n) \dot{x}}{(a + b x^n) x} = 0$: fingatur $T = x^p$, & exinde ſequitur $\frac{p \dot{x}}{x} + 2 S \dot{x} + \frac{(c + f x^n) \dot{x}}{(a + b x^n) x} = 0$; ex hâc æquatione conſtat $S = \frac{-p a - p b x^n - c - f x^n}{2 x (a + b x^n)}$; ſcribantur hæc quantitas & ejus fluxio pro ſuis valoribus S & $\dot{S}$ in æquatione R, & reſultat fluxionalis æquatio, cujus conſtant caſus, qui integrationem admittunt, e caſibus æquationis P, qui integrationem recipiunt.

Cor. Ex his generalibus fluxionalibus æquationibus facile deduci poſſunt æquationes magis particulares, quæ majorem habeant concinnitatem.

E. g. Sit æquatio (P), $(a + b x^n) x^2 \dot{z} + (a + b x^n) x^2 z^2 \dot{x} + (c + f x^n) x z \dot{x} + (g + h x^n) \dot{x} = 0$, in eâ pro z & $\dot{z}$ ſcribantur reſpective $T y$ & $T \dot{y} + y \dot{T}$, ubi T ſit incognita functio quantitatis x; & reſultat æquatio (Q) $(a + b x^n) x^2 T \dot{y} + (a + b x^n) y x^2 \dot{T} + (a + b x^n) x^2 T^2 y^2 \dot{x} + (c + f x^n) x T y \dot{x} + (g + h x^n) \dot{x} = 0$; ſupponantur termini, in quibus invenitur y, nihilo æquales, i. e. $(a+bx^n)yx^2\dot{T}+(c+fx^n)xTy\dot{x}=0$, & conſequenter $\frac{\dot{T}}{T}=-\frac{c+fx^n}{(a+bx^n)x}\dot{x}$, cujus fluens erit $\log.\ T+\frac{c}{a}\log.\ x+\frac{fa-bc}{nba}\log.\ (a+bx^n)=A$ invariabili quantitati ad libitum aſſumendæ, unde $T=\frac{(a+bx^n)^{\frac{bc-af}{nba}}}{x^{\frac{c}{a}}}\times A$; ſcribatur $\frac{(a+bx^n)^{\frac{bc-af}{nba}}}{x^{\frac{c}{a}}}$ pro T in reſultante æquatione, & transformatur reſultans æquatio in ſubſequentem $\dot{y}+\frac{(a+bx^n)^{\frac{bc-af}{nba}}}{x^{\frac{c}{a}}}Ay^2\dot{x}+\frac{(g+hx^n)x^{\frac{c}{a}-2}\dot{x}}{A(a+bx^n)^{\frac{bc-af}{nba}+1}}=0$, ($\mathfrak{Q}$).

Cor. . Hujus æquationis facile conſtant caſus, in quibus detegi poteſt ejus fluens: inveniantur enim caſus in quibus integrari poteſt altera æquatio vel P vel $\mathfrak{Q}$, & exinde conſtant caſus alterius æquationis $\mathfrak{Q}$ vel P, qui integrationem admittunt.

Ex. 1. Sit $bc=af$, & $A=1$, & reſultans æquatio erit $\dot{y}+x^{-\frac{c}{a}}y^2\dot{x}+\frac{(g+hx^n)x^{\frac{c}{a}-2}\dot{x}}{a+bx^n}=0$; pro x & ejus fluxione ſcribatur $t^{\frac{a}{a-c}}$ & $\frac{a}{a-c}t^{\frac{c}{a-c}}\dot{t}$, & exorietur $\dot{y}+\frac{ay^2\dot{t}}{a-c}+\frac{a\left(g+ht^{\frac{na}{a-c}}\right)\times t^{-2}\dot{t}}{(a-c)\left(a+bt^{\frac{na}{a-c}}\right)}=0$.

PROB.

PROB. LVII.

Invenire fluentem datæ fluxionalis æquationis ex aſſumptâ æquationis formulâ, quæ continet fluentem ipſam.

Inveniatur fluxio æquationis pro fluente aſſumptâ, &c. & ita reducatur æquatio reſultans, ut evadant termini datæ & reſultantis æquationis inter ſe æquales ; & id quod requiritur, fit.

Ex. 1. Aſſumantur quantitates $\frac{a x^n y^r + b x^m y^s}{(a' x^{n'} y^{r'} + b' x^{m'} y^{s'})^p} = A$, vel $\frac{a x^n y^r + b x^m y^s + c y^t + d y^l + \&c.}{(a' x^{n'} y^{r'} + b' x^{m'} y^{s'} + c' y^{t'} + d' y^{l'} + \&c.)^b} = A$, &c. pro fluente; & inveniatur ejus fluxio; rejiciantur earum denominatores; & reſultant fluxionales æquationes, quarum fluentes innoteſcunt.

Ex. 2. Sit æquatio $(a x^{2m-1} + b x^{2m-2} y + c x^{2m-3} y^2 + \&c. + d x^{2m-2} + e x^{2m-3} y + \&c. + f x^{2m-3} + g x^{2m-4} y + \&c. + \&c.\ k x + l y + m') \dot{y} = (A x^{2m-1} + B x^{2m-2} y + \&c. + D x^{2m-2} + E x^{2m-3} y + \&c.) \dot{x}$.

Fingatur $(P x^m + Q x^{m-1} y + R x^{m-2} y^2 + \&c. + S x^{m-1} + T x^{m-2} y + \&c.)^{\alpha - \beta} = M (p x^m + q x^{m-1} y + r x^{m-2} y^2 + \&c. + s x^{m-1} + t x^{m-2} y + \&c.)^{\alpha + \beta}$, cujus fluxio per fluentem ipſam dividatur, & æquationis reſultantis terminis correſpondentibus inter ſe æqualibus eſſe ſuppoſitis, nonnunquam ſequitur fluens particularis datæ æquationis.

Ex. 2. Æquatio $(a x + b + c y) \dot{y} = (f x + g x + h y) \dot{x}$ ſemper reduci poteſt in homogeneam, cum $a h$ haud æquat $f c$, ſcribendo in datâ æquatione pro x & y earum aſſumptos valores $z + \alpha$ & $v + \beta$; & in æquatione reſultante $(a z + (a \alpha + b + c \beta) + c v) \dot{v} = (f z + (f \alpha + g + h \beta) + h v) \dot{z}$ ſupponendo terminos $a \alpha + b + c \beta = o$ & $f \alpha + g + h \beta = o$, unde $\beta = \frac{b f - a g}{a h - c f}$ & $\alpha = \frac{c g - h b}{a h - c f}$, & data æquatio transformatur in homogeneam $(a z + c v) \dot{v} = (f z + h v) \dot{z}$.

Sit $ab = fc$, & conſequenter $(b + (ax + cy))\dot{y} = (g + l(ax + cy))\dot{x}$; ſcribe $\dot{y} = \frac{\dot{z} - a\dot{x}}{c}$, unde $(b + z) \times \frac{\dot{z} - a\dot{x}}{c} = (g + lz)\dot{x}$, & conſequenter $\frac{(b + z)\dot{z}}{gc + ba + az + clz} = \dot{x}$, cujus fluens innoteſcit.

Ex. 3. Sit æquatio $ax^m\dot{x} + by^2\dot{x} = \dot{y}$: 1$^{\text{mo}}$. fingatur $y = cx^n + dx^p v$, ſcribatur hæc quantitas pro ejus valore in datâ æquatione, & reſultat $(ax^m + bc^2x^{2n} + 2bcdx^{n+p}v + bd^2x^{2p}v^2)\dot{x} = ncx^{n-1}\dot{x} + pdx^{p-1}v\dot{x} + dx^p\dot{v}$: fiant termini hujuſce æquationis $2bcdx^{n+p}v\dot{x} = pdx^{p-1}v\dot{x}$, & $bc^2x^{2n}\dot{x} = ncx^{n-1}\dot{x}$: colligi poſſunt e priori reſultante æquatione $n + p = p - 1$, unde $n = -1$; & $2bcd = pd$ & exinde $2bc = p$: e poſteriori vero $bc^2 = nc$, unde $bc = -1$ & $c = -\frac{1}{b}$, & $p = 2bc = -2$; & exinde $y = -\frac{1}{b}x^{-1} + x^{-2}v$: æquatio vero ex hâc ſubſtitutione reſultans erit $(ax^m + d^2bx^{-4}v^2)\dot{x} = dx^{-2}\dot{v}$, & conſequenter $\frac{a}{d}x^{m+2}\dot{x} + dbx^{-2}v^2\dot{x} = \dot{v}$; pro x ſcribatur $-\frac{1}{z}$ in hâc æquatione & reſultat $-\frac{a}{d}z^{-m-4}\dot{z} - dbv^2\dot{z} = \dot{v}$.

Sint $m = -2$, & $2bc = p$; & fingatur $a + bc^2 = nc = -c$, & conſequenter $c = \frac{\sqrt{(1 - 4ab)} - 1}{2b}$, & reſultabit æquatio transformata $d^2bx^{4bc}v^2\dot{x} = dx^{2bc}\dot{v}$.

2$^{\text{do}}$. Fingatur $y = cx^n + dx^p v^{-1}$, ſcribatur hæc quantitas pro ejus valore in æquatione datâ $ax^m + by^2\dot{x} = \dot{y}$, & reſultat $(ax^m + bc^2x^{2n} + 2bcdx^{p+n}v^{-1} + bd^2v^{2p}v^{-2})\dot{x} = ncx^{n-1}\dot{x} + pdx^{p-1}v^{-1}\dot{x} - dx^p v^{-2}\dot{v}$: fiant $2bcdx^{p+n}v^{-1}\dot{x} = pdx^{p-1}v^{-1}\dot{x}$ & $ncx^{n-1} = bc^2x^{2n}$, unde $p + n = p - 1$ & $n = -1$, $2bcd = pd$ & $2bc = p$, $nc = -c = bc^2$ & $bc = -1$; & exinde $p = 2bc = -2$: & æquatio reſultans erit $ax^m\dot{x} + bd^2x^{-4}v^{-2}\dot{x} = -dx^{-2}v^{-2}\dot{v}$, & conſequenter $ax^{m+2}v^2\dot{x}$

$+ bd^2$

$+ b d^2 x^{-2} \dot{x} = - d\dot{v}$; ſcribatur $\frac{x^{m+3}}{m+3} = z$ in hâc æquatione, & invenietur $a v^2 \dot{z} + b d^2 (m+3)^{-\frac{m+4}{m+3}} z^{-\frac{m+4}{m+3}} \dot{z} = - d\dot{v}$; & conſequenter ſi indeterminatæ quantitates x & y in datâ æquatione $a x^m \dot{x} + b y^2 \dot{x} = \dot{y}$ ſeparationem admittant, tum etiam indeterminatæ in æquatione hujuſce formulæ $\alpha z^{\frac{-m-4}{m+3}} \dot{x} + \beta v^2 \dot{z} = \dot{v}$ ſemper ſeparari poſſunt.

Cor. 1. Sit $m = 0$, & data æquatio fit $a\dot{x} + b y^2 \dot{x} = \dot{y}$, unde $\dot{x} = \frac{\dot{y}}{a + b y^2}$, in quâ ſeparantur variabiles quantitates; & conſequenter indeterminatæ in æquatione $\alpha x^{\frac{-m-4}{m+3} = \frac{-4}{3}} + \beta y^2 \dot{x} = y$ ſeparari poſſunt, reduci enim poteſt hæc æquatio in præcedentem: ſit $m = \frac{-4}{3}$ & conſequenter $\frac{-m-4}{m+3} = \frac{-8}{5}$, unde æquatio $\alpha x^{-\frac{8}{5}} + \beta y^2 \dot{x} = \dot{y}$ ſemper reduci poteſt in æquationem datam; & exinde in genere ſi $m = \frac{-4n}{2n+1}$, ubi n ſit integer numerus, ſemper reduci poteſt æquatio $\alpha x^m + \beta y^2 \dot{x} = \dot{y}$ in æquationem datæ formulæ $a\dot{x} + b y^2 \dot{x} = \dot{y}$.

Cor. 2. Fluens vero hujuſce æquationis $a\dot{x} + b y^2 \dot{x} = \dot{y}$, i. e. $\dot{x} = \frac{\dot{y}}{a + b y^2}$ generaliter inveniri poteſt circularium arcuum & logarithmorum ope, ergo fluens æquationis $\alpha x^{\frac{-4n}{2n+1}} \dot{x} + \beta y^2 \dot{x} = \dot{y}$ ſemper detegi poteſt ope finitorum terminorum, circularium arcuum & logarithmorum.

Aliter: Sit data æquatio $(P)\ \dot{y} + a y^2 \dot{x} = b x^m \dot{x}$; aſſumatur $y = c x^{n-1} + \frac{\dot{z}}{a z \dot{x}}$, ubi $\dot{x}$ ſit conſtans quantitas; tum erit $\dot{y} = (n-1) c x^{n-2} \dot{x} + \frac{\ddot{z}}{a z \dot{x}} - \frac{\dot{z}^2}{a \dot{x} z^2}$, quibus quantitatibus pro ſuis valoribus in datâ æquatione (P) ſubſtitutis, reſultat (P') $\frac{\ddot{z}}{az\dot{x}} + (n-1)cx^{n-2}\dot{x} + ac^2x^{2n-2}\dot{x} + \frac{2cx^{n-1}\dot{z}}{z} = bx^m\dot{x}$: ſcribantur $m = 2n-2$ & $b = ac^2$, & fit reſultans æquatio $(\mathcal{Q})$ $\ddot{z} + (n-1)acx^{n-2}z\dot{x}^2 + 2acx^{n-1}\dot{x}\dot{z} = 0$; fingatur $z = Ax^{\frac{-n+1}{2}} + Bx^{\frac{-3n+1}{2}} + Cx^{\frac{-5n+1}{2}}$ + &c. unde $\frac{\dot{z}}{\dot{x}} = -\left(\frac{n-1}{2}\right)Ax^{\frac{-n-1}{2}} - \left(\frac{3n-1}{2}\right)Bx^{\frac{-3n-1}{2}} - \left(\frac{5n-1}{2}\right)Cx^{\frac{-5n-1}{2}}$ − &c. & $\frac{\ddot{z}}{\dot{x}^2} = \left(\frac{n-1}{2} \times \frac{n+1}{2}\right)Ax^{\frac{-n-3}{2}} + \left(\frac{3n-1}{2} \times \frac{3n+1}{2}\right)Bx^{\frac{-3n-3}{2}} + \left(\frac{5n-1}{2} \times \frac{5n+1}{2}\right)Cx^{\frac{-5n-3}{2}}$ + &c. $= \frac{n^2-1}{4}Ax^{\frac{-n-3}{2}} + \frac{9n^2-1}{4}Bx^{\frac{-3n-3}{2}}$ + &c. quibus quantitatibus pro ſuis valoribus in æquatione $\ddot{z} + (n-1)acx^{n-2}z\dot{x}^2 + 2acx^{n-1}\dot{x}\dot{z} = 0$ ſubſtitutis, reſultat

$$\ddot{z} = +\left(\frac{n^2-1}{4}Ax^{\frac{-n-3}{2}} + \frac{9n^2-1}{4}Bx^{\frac{-3n-3}{2}} + \&c.\right)\dot{x}^2$$

$$+(n-1)acx^{n-2}z\dot{x}^2 = +\left((n-1)acAx^{\frac{n-3}{2}} + (n-1)acBx^{\frac{-n-3}{2}} + (n-1)acCx^{\frac{-3n-3}{2}} + \&c.\right)\dot{x}^2$$

$$+2acx^{n-1}\dot{x}\dot{z} = \left(-ac(n-1)Ax^{\frac{n-3}{2}} - (3n-1)acBx^{\frac{-n-3}{2}} - (5n-1)acCx^{\frac{-3n-3}{2}} - \&c.\right)\dot{x}^2$$

$$0 = 0.$$

Fiant correſpondentes termini reſultantis æquationis nihilo reſpective æquales, & reſultant $z = Ax^{\frac{-n+1}{2}} + \frac{n^2-1}{8nac}Ax^{\frac{-3n+1}{2}} + \frac{n^2-1}{8} \times \frac{9n^2-1}{16n^2a^2c^2}Ax^{\frac{-5n+1}{2}} + \frac{n^2-1}{4\times 2} \times \frac{9n^2-1}{4\times 4} \times \frac{25n^2-1}{4\times 6} \times \frac{A}{n^3a^3c^3}x^{\frac{-7n+1}{2}}$ + &c. & $\frac{\dot{z}}{\dot{x}} = \frac{1-n}{2}Ax^{\frac{-n-1}{2}} + \left(\frac{n^2-1}{8nac}\right)\left(\frac{1-3n}{2}\right)Ax^{\frac{-3n-1}{2}}$ + &c. & exinde $y = cx^{n-1} + \frac{\dot{z}}{az\dot{x}} =$

$cx^{n-1} -$

$$c x^{n-1} - \frac{1}{a}\left\{\frac{\frac{n-1}{2} A x^{\frac{-n-1}{2}} + \left(\frac{3n-1}{2}\right)\left(\frac{n^2-1}{8}\right)\frac{A}{nac} x^{\frac{-3n-1}{2}} + \&c.}{A x^{\frac{-n+1}{2}} + \frac{n^2-1}{8}\,\frac{A}{nac} x^{\frac{-3n+1}{2}} + \&c.}\right. =$$

$$c x^{n-1} - \frac{1}{ax}\left\{\frac{\frac{n-1}{2} + (3n-1)(n^2-1)\frac{x^{-n}}{16nac} + \&c.}{1 + \frac{n^2-1}{8}\,\frac{x^{-n}}{nac} + \frac{n^2-1}{8} \times \frac{9n^2-1}{16}\,\frac{x^{-2n}}{n^2a^2c^2} + \&c.}\right. .$$

Cor. . Hæc expreſſio erit finita, ſi $(2i+1)^2 n^2 - 1 = 0$, ubi i ſit integer numerus, & $m = 2n - 2 = \frac{-4i-2\mp 2}{2i+1}$: data æquatio in hoc caſu erit $\dot{y} + ay^2\dot{x} = ac^2 x^{\frac{-4i-2\pm 2}{2i+1}}\dot{x}$.

Sit $m = \frac{-4i}{2i+1}$, & conſequenter $\dot{y} + ay^2\dot{x} = ac^2 x^{\frac{-4i}{2i+1}}\dot{x}$; & erit

$$ayx = acx^n - \frac{\frac{n-1}{2} + \left(\frac{3n-1}{2}\right)\left(\frac{n^2-1}{8}\right)\frac{x^{-n}}{nac} + \&c.}{1 + \frac{n^2-1}{8}\,\frac{x^{-n}}{nac} + \frac{n^2-1}{8} \times \frac{9n^2-1}{16}\,\frac{x^{-2n}}{n^2a^2c^2} + \&c.} =$$

$$acx^n + \frac{\frac{i}{2i+1} - \frac{i(i^2-1)\,x^{-\frac{-1}{2i+1}}}{2(2i+1)^2 ac} + \&c.}{1 - \frac{i(i+1)}{2(2i+1)}\,\frac{x^{\frac{-1}{2i+1}}}{ac} + \&c.} + \&c.$$

Cor. 2. Hæc reſolutio fluxionalis æquationis $ax^m\dot{x} + by^2\dot{x} = \dot{y}$, non erit generalis, nam in eâ haud continetur invariabilis quantitas, quæ in fluxionali æquatione non invenitur.

3. Sit æquatio $\dot{y} + (ax^b + bx^{b-m} + cx^{b-2m} + \&c.)y^b\dot{x} = (Ax^i + Bx^{i'})\dot{x}$; & aſſumatur $y = \frac{ax^n + bx^{n-m} + cx^{n-2m} + \&c.}{Ax^r + Bx^{r-m} + Cx^{r-2m} + \&c.}$, ſubſtituatur

hæc quantitas pro y & ejus fluxio pro $\dot{y}$ in datâ æquatione; & ex æquatis correſpondentibus terminis reſultantis æquationis erui poſſunt caſus, in quibus terminat ſeries, i. e. fractio aſſumpta.

Eadem principia, i. e. methodus inveniendi valorem quantitatis y in fractionibus, quarum numeratorum & denominatorum termini ſecundum dimenſiones quantitatis (x) progrediuntur, ad quamplurimos caſus applicari poſſunt.

Cor. Si modo pro variabilibus quantitatibus in quâcunque datâ æquatione contentis ſcribantur quæcunque functiones novarum variabilium aſſumptarum, reſultabunt æquationes, quæ facile reduci poſſunt ad datam.

Ex. 1. Sit æquatio $ax^r\dot{x} + byx^{-1}\dot{x} + cy^2\dot{x} = \dot{y}$, ſcribantur in datâ fluxionali æquatione $z^{\frac{b}{b+1}}u$ & $z^{\frac{b}{b+1}}$ pro y & x, & nonnunquam reſultat æquatio formulæ $\alpha x^m\dot{x} + \beta y^2\dot{x} = \dot{y}$.

Ex. 2. Sit æquatio $ax^m\dot{x} + byx^p\dot{x} + cy^2\dot{x} = \dot{y}$, ſcribatur pro y ejus valor aſſumptus $\frac{a}{p}x^p + u$, ubi $p = m + 1$, & reſultat æquatio $\frac{(abp + a^2c)}{p^2}x^{2p}\dot{x} + \frac{(2ac + bp)x^p}{p}u\dot{x} + cu^2\dot{x} = \dot{u}$, quæ facile reduci poteſt in priorem.

Ex. 3. Sit $ax^m\dot{x} + byx^p\dot{x} + cy^2\dot{x} = \dot{y}$, ſcribatur in eâ pro y, u^{-1}; & pro x, $z^{\frac{1}{m+1}}$; & reſultat $\frac{-c}{m+1}z^{\frac{-m}{m+1}}\dot{z} - \frac{bu}{m+1}z^{\frac{p-m}{m+1}}\dot{z} - \frac{a}{m+1}u^2\dot{z} = \dot{u}$.

Et ſic in genere ſcribantur in datâ æquatione $ax^m\dot{x} + by^2\dot{x} = \dot{y}$ pro x & y reſpective $\alpha z^e u^b + \beta z^f u^k + \gamma z^g u^l +$ &c. & $\pi z^n u^s + \varrho x^v u^t +$ &c. & reſultant æquationes, quæ facile reduci poſſunt ad datam.

4. Si u fluat uniformiter, pro $\dot{u}$ ſcribatur in quibuſdam locis datæ æquationis data quantitas, & exinde e principiis prius traditis ſæpe deduci poteſt ejus fluens.

Ex. 1. Sit $\surd(\dot{x}^2+\dot{y}^2)=\dot{u}$ conſtans quantitas, & data æquatio $by^m=\frac{2ay\ddot{x}+a\dot{x}\dot{y}}{\dot{u}\dot{y}}$; ducatur hæc æquatio in $\frac{\dot{u}\dot{y}}{2\surd(y)}$ & reſultat $\frac{by^m\dot{y}\dot{u}}{2\surd(y)}=\frac{2ay\ddot{x}+a\dot{x}\dot{y}}{2\surd(y)}$, cujus fluens eſt $\frac{by^{m+\frac{1}{2}}}{2(m+\frac{1}{2})}=a\dot{x}\surd(y)+C\surd(\dot{x}^2+\dot{y}^2)$, ubi C eſt quantitas invariabilis.

2. Sit æquatio $f=\frac{\dot{x}^2-y\ddot{y}}{y^3\dot{x}^2}$, ubi $y\dot{x}$ eſt conſtans; ducatur hæc æquatio in $2\dot{y}$, & fiet $2f\dot{y}=\frac{2\dot{x}^2\dot{y}-2y\dot{y}\ddot{y}}{y^3\dot{x}^2}=\frac{2\dot{y}}{y^3}-\frac{2\dot{y}\ddot{y}}{\dot{x}^2y^2}$, ſed quoniam $y\dot{x}$ eſt conſtans, erit fluens $\int.2f\dot{y}=-\frac{1}{y^2}-\frac{\dot{y}^2}{y^2\dot{x}^2}+C$.

PROB. LVIII.

Invenire fluxionales æquationes, quarum dantur quædam particulares reſolutiones.

Aſſumatur quæcunque functio variabilium quantitatum x & y & earum fluxionum pro uno datæ æquationis latere: ſcribatur etiam quæcunque functio variabilis quantitatis x pro variabili quantitate y, & ejus fluxio pro $\dot{y}$, & ſic deinceps, in functione aſſumptâ; & fiat quantitas reſultans, alterum æquationis latus; & reſultat æquatio, cujus reſolutio datur.

Ex. 1. Aſſumatur functio $(y^{n+1}+ay^{n-1}+by^{n-3}\ldots Py+Q)\dot{y}$; pro quantitatibus y & $\dot{y}$ ſcribantur $(A+\surd(p^n))^{\frac{1}{n}}+(A-\surd(p^n))^{\frac{1}{n}}$ & ejus fluxio reſpective, & reſultare poteſt $(y^{n+1}+ay^{n-1}+by^{n-3}+\&c.)\dot{y}-(2Ay+(n(A^2-p^n)^{\frac{1}{n}}+a)y^{n-1}-(n.\frac{n-3}{2}(A^2-p^n)^{\frac{2}{n}}-b)y^{n-3}+(n.$

$$\frac{n-4}{2}\cdot\frac{n-5}{3}(A^2-p^n)^{\frac{3}{n}}+c)y^{n-5}+\&c.)\Bigg(\frac{\frac{1}{n}\dot{A}+\frac{1}{2}p^{\frac{1}{2}n-1}\dot{p}}{(A+\surd(A^2-p^n))^{\frac{n-1}{n}}}+$$

$$+\frac{(\frac{1}{n}\dot{A}-\frac{1}{2}p^{\frac{1}{2}n-1}\dot{p}}{(A-\surd(A^2-p^n))^{\frac{n-1}{n}}}\Bigg),$$ ubi literæ *a*, *b*, *c*, &c. *A* & *p* ſunt quæcunque functiones quantitatis *x*, tum erit particularis valor quantitatis $y=(A+\surd(p^n))^{\frac{1}{n}}+(A-\surd(p^n))^{\frac{1}{n}}$.

Aliter: Invenire fluxionales æquationes, quarum fluentes particulares dantur.

Aſſumatur fluentialis æquatio, & exinde deducatur fluxionalis; vel inveniatur fluxio aſſumptæ æquationis, & in æquatione fluxionali reſultante pro quibuſcunque quantitatibus ſubſtituantur earum valores e datâ æquatione acquiſiti, e. g. ſit $V=0$ fluentialis æquatio aſſumpta, inveniatur $\dot{V}=p\dot{x}+q\dot{y}=0$; e datâ æquatione inveniantur $y=w$ functio quantitatis *x*, & $x=u$ functio quantitatis *y*; quibus quantitatibus pro ſuis valoribus in quantitatibus *q* & *p* ſubſtitutis, reſultat æquatio $Y\dot{x}+X\dot{y}=0$, ubi literæ *Y* & *X* denotant functiones quantitatum *y* & *x* reſpective; & conſequenter relatio particularis inter *x* & *y* variabiles quantitates æquationis $\frac{\dot{x}}{X}+\frac{\dot{y}}{Y}=0$ detegi poteſt. Ex hoc principio in ſubſequentibus exemplis detegi poſſunt generales fluentes; plures enim continentur invariabiles quantitates ad libitum aſſumendæ in fluentiali quam in fluxionali æquatione.

Cor. . Sit *V* functio quantitatum *x* & *y*, in quibus ſimiliter involvuntur variabiles quantitates *x* & *y*, & erit *X* eadem functio quantitatis *x* ac *Y* quantitatis *y*.

THEOR.

THEOR. XL.

1. Sit fluxionalis æquatio $\alpha = o$, cujus fluens eſt a log. $A + b$ log. $B + c$ log. $C +$ &c. $=$ conſt.; tum erit fluens etiam $A^{a} \times B^{b} \times C^{c} \times$ &c. $=$ conſt.

Ex. Sit fluxionalis æquatio $\frac{(a'y^{r} + b'y^{r-1} + c'y^{r-2} + \&c.)\dot{y}}{ay^{n} + by^{n-1} + cy^{n-2} + \&c.} + \frac{(A'x^{s} + B'x^{s-1} + C'x^{s-2} + \&c.)\dot{x}}{Ax^{m} + Bx^{m-1} + Cx^{m-2} + \&c.} = o$, ubi r & s ſunt reſpective minores quam n & m, & ſint $\frac{(a'y^{r} + b'y^{r-1} + \&c.)}{ay^{n} + by^{n-1} + \&c.} = \frac{a'}{a}\left(\frac{\alpha'}{y - \alpha} + \frac{\beta'}{y - \beta} + \frac{\gamma'}{y - \gamma} + \&c.\right)$ & $\frac{(A'x^{s} + B'x^{s-1} + \&c.)}{Ax^{m} + Bx^{m-1} + \&c.} = \frac{A'}{A}\left(\frac{\pi'}{x - \pi} + \frac{\varrho'}{x - \varrho} + \frac{\sigma'}{x - \sigma} + \&c.\right)$; tum erit æquatio relationem inter x & y exprimens $((y - \alpha)^{\alpha'} \times (y - \beta)^{\beta'} \times (y - \gamma)^{\gamma'} \times \&c.)^{\frac{a'}{a}} \times ((x - \pi)^{\pi'} \times (x - \varrho)^{\rho'} \times (x - \sigma)^{\sigma'} \times \&c.)^{\frac{A'}{A}} = E$ invariabili quantitati.

Sit fluxionalis æquatio, in quâ ſimiliter involvuntur quantitates x, y & earum fluxiones $\dot{x}, \dot{y}$, &c. tum in ejus generali fluente & multiplicatore ſimiliter etiam involventur quantitates x, y, &c.

2. Idem etiam affirmari poteſt de pluribus fluxionalibus æquationibus plures variabiles quantitates involventibus.

Ex. 1. Sit data æquatio $a + b(x + y) + cxy = o$, cujus fluxio erit $(b + cx)\dot{y} + (b + cy)\dot{x} = o$; in hâc fluxione pro x & y ſcribantur reſpective $-\frac{a + by}{b + cy} = x$ & $-\frac{a + bx}{b + cx} = y$ earum valores ex datâ æquatione deducti; tum reſultabit fluxionalis æquatio $\frac{b^{2} - ca}{b + cy}\dot{y} +$

$b^{2} - ca$

$\frac{b^2 - ca}{b + cx}\dot{x} = o$, cujus generalis fluens erit $\frac{b^2 - ca}{c} \times (\log. (b + cy) + \log. (b + cx)) = \frac{b^2 - a}{c} \log. ((b + cy) \times (b + cx)) = A$, ubi A eſt quantitas ad libitum aſſumenda; ſed $(b + cy) \times (b + cx) = b^2 + cb(y + x) + c^2yx$, & exinde $b(y + x) + cyx$ equat invariabilem quantitatem.

Ex. 2. Sit æquatio $a + 2b(x + y) + c(x^2 + y^2) + 2dxy = o$, cujus fluxio eſt $(2b + 2cx + 2dy)\dot{x} + (2b + 2cy + 2dx)\dot{y} = o$, in hâc fluxione pro x & y ſcribantur $-\frac{b + dy \mp \surd(b^2 - ac + (2bd - 2bc)y + (d^2 - c^2)y^2)}{c}$ (P) & $-\frac{b + dx \mp \surd(b^2 - ac + (2bd - 2bc)x + (d^2 - c^2)x^2)}{c}$ $(\mathcal{Q})$; & reſultant duæ fluxionales æquationes $\frac{\dot{y}}{(b^2 - ac + (2bd - 2bc)y + (d^2 - c^2)y^2)^{\frac{1}{2}}} + \frac{\dot{x}}{(b^2 - ac + (2bd - 2bc)x + (d^2 - c^2)x^2)^{\frac{1}{2}}} = o$, & $(b + cx + + d\mathcal{Q})\dot{x} + (b + cy + dP)\dot{y} = o$.

Sit $b = o$ & $\frac{ac}{c^2 - d^2} = n$; tum reſultat æquatio $\frac{\dot{x}}{\surd(x^2 + n)} + \frac{\dot{y}}{\surd(y^2 + n)} = o$.

3. Sit æquatio $\frac{\dot{x}}{\surd(x^2 + n)} + \frac{\dot{y}}{\surd(y^2 + n)} = o$, cujus fluens eſt $\log. (x + \surd(x^2 + n)) + \log. (y + \surd(y^2 + n)) = \log. ((x + \surd(x^2 + n)) \times (y + \surd(y^2 + n))) =$ invariabili quantitati; unde æquatio $(x + \surd(x^2 + n)) \times (y + \surd(y^2 + n)) = xy + x\surd(y^2 + n) + y\surd(x^2 + n) + \surd(x^2 + n) \times \surd(y^2 + n) = E$ variabili quantitati; ſcribatur enim

ob $(x+\surd(x^2+n))\,(y+\surd(y^2+n))=E$, pro $\frac{\dot{x}}{\dot{y}}$ ejus valor $-\frac{1}{E}$ $(x+\surd(x^2+n))\times(y+\surd(y^2+n))\,\frac{\surd(x^2+n)}{\surd(y^2+n)}=-\frac{E}{E}\times\frac{\surd(x^2+n)}{\surd(y^2+n)}$, unde $\frac{\dot{x}}{\surd(x^2+n)}=-\frac{\dot{y}}{\surd(y^2+n)}$.

Reducatur æquatio $(x+\surd(x^2+n))\times(y+\surd(y^2+n))=E$, ita ut exterminentur irrationales quantitates; per noſtr. medit. algeb. erit $((x+\surd(x^2+n))\,(y+\surd(y^2+n))-E)\times((x-\surd(x^2+n))\,(y-\surd(y^2+n))-E)=n^2+E^2-2Exy-2E\surd(x^2+n)\times\surd(y^2+n)$; ducatur hæc quantitas in $n^2+E^2-2Exy+2E\surd(x^2+n)\times\surd(y^2+n)$, & reſultat æquatio quæſita $(n^2+E^2)^2-4E(n^2+E^2)xy-4E^2((x^2+y^2)+n)n=L=0$ ab irrationalibus libera.

In æquatione $\frac{\dot{y}}{\surd(y^2+n)}+\frac{\dot{x}}{\surd(x^2+n)}=0$ pro y ſcribatur $\surd(A)v+\frac{B}{2A^{\frac{1}{2}}}$, & $\surd(A)z+\frac{B}{2A^{\frac{1}{2}}}$ pro x; & ſi modo ſit $\frac{B^2}{4A}(f^2)+n=C$, tum evadet æquatio $\frac{\dot{v}}{\surd(Av^2+Bv+C)}+\frac{\dot{z}}{\surd(Az^2+Bz+C)}=0$: deinde in æquatione $L=0$, pro x, y & n ſcribantur reſpective $\surd(A)z+\frac{B}{2A^{\frac{1}{2}}}$, $\surd(A)v+\frac{B}{2A^{\frac{1}{2}}}$ & $C-f^2$; & reſultat æquatio, quæ eſt generalis fluens æquationis $\frac{\dot{v}}{\surd(Av^2+Bv+C)}+\frac{\dot{z}}{\surd(Av^2+Bv+C)}=0$: aliter generalis fluens fluxionalis æquationis $\frac{\dot{v}}{\surd(Av^2+Bv+C)}+\frac{\dot{z}}{\surd(Av^2+Bv+C)}=\frac{1}{\surd(A)}\left(\frac{\dot{v}}{\surd(v^2+\frac{B}{A}v+\frac{C}{A})}+\frac{\dot{z}}{\surd(z^2+\frac{B}{A}z+\frac{C}{A})}\right)=0$ erit $\frac{1}{\surd(A)}$(log. (v

$+\frac{1}{2}B$

$+ \frac{1}{2}\frac{B}{A} + \surd(v^2 + \frac{B}{A}v + \frac{C}{A})\big)\big) + \log.(z + \frac{1}{2}\frac{B}{A} + \surd(z^2 + \frac{B}{A}z + \frac{C}{A})\big)\big)\big)$ $= \frac{1}{\surd(A)}(\log.(v + \frac{1}{2}\frac{B}{A} + \surd(v^2 + \frac{B}{A}v + \frac{C}{A})\big)\big)(z + \frac{1}{2}\frac{B}{A} + \surd(z^2 + \frac{B}{A}z + \frac{C}{A})\big)\big) = A'$ cuicunque invariabili quantitati; conſequenter $(v + \frac{1}{2}\frac{B}{A} + \surd(v^2 + \frac{B}{A}v + \frac{C}{A})\big)\big)(z + \frac{1}{2}\frac{B}{A} + \surd(z^2 + \frac{B}{A}z + \frac{C}{A})\big)\big) = E$ invariabili quantitati erit etiam fluens æquationis prædictæ.

Per eundem modum inveniri poteſt $(v + \frac{2}{1}\frac{B}{A} + \surd(v^2 + \frac{B}{A}v + \frac{C}{A})\big)\big)^m \times (z + \frac{1}{2}\frac{B}{A} + \surd(z^2 + \frac{B}{A}z + \frac{C}{A})\big)\big)^r = E$ generalis fluens fluxionalis æquationis $\frac{m\dot{v}}{\surd(v^2 + \frac{B}{A}v + \frac{C}{A})} + \frac{r\dot{z}}{\surd(z^2 + \frac{B}{A}z + \frac{C}{A})} = 0$.

Ex. 2. Sit æquatio $a + 2b(x + y) + c(x^2 + y^2) + 2dxy + 2exy(x + y) + fx^2y^2 = A = 0$, cujus fluxio $(b + cx + dy + 2eyx + ey^2 + fy^2x)\dot{x} + (b + cy + dx + 2eyx + ex^2 + fx^2y)\dot{y} = 0$, in quantitate $b + cx + dy + 2eyx + ey^2 + fy^2x$ pro x ſcribatur $-\frac{b + dy + ey^2}{c + 2ey + fy^2} + \surd\left(\frac{(b + dy + ey^2)^2 - (c + 2ey + fy^2)(a + 2by + cy^2)}{(c + 2ey + fy^2)^2}\right)$; & in quantitate $b + cy + dx + 2eyx + ex^2 + fx^2y$ pro y ſcribatur $-\frac{b + dx + ex^2}{c + 2ex + fx^2} + \surd\left(\frac{(b + dx + ex^2)^2 - (c + 2ex + fx^2)(a + 2bx + cx^2)}{(c + 2ex + fx^2)^2}\right)$; & reſultabunt reſpective $\surd((e^2 - cf)y^4 + (2ed - 2ce - 2bf)y^3 + (2eb + d^2 - c^2 - af - 4be)y^2 + (2bd - 2ea - 2bc)y + b^2 - ca) = \surd(Ay^4 + By^3 + Cy^2 + Dy + E)$, & $\surd((e^2 - cf)x^4 + (2ed - 2ce - 2bf)x^3 + (2eb + d^2 - c^2 - af - 4be)x^2 + (2bd - 2ea - 2bc)x + b^2 - ca) = \surd(Ax^4 + Bx^3 + Cx^2 + Dx + E)$, & exinde reſultat fluxionalis æquatio $\frac{\dot{x}}{\surd(Ax^4 + Bx^3 + Cx^2 + Dx + E)}$

$+$

$+\frac{\dot{y}}{\surd(Ay^4+By^3+Cy^2+Dy+E)}=o$, cujus data æquatio $A'=o$ erit fluens.

Ex datis coefficientibus A, B, C, D & E, & ſuppoſitis quantitatibus $e^2-cf=Am$, $2ed-2ce-2bf=Bm$, $2eb+d^2-c^2-af-4be=Cm$, $2bd-2ea-2bc=Dm$ & $b^2-ca=Em$; reſultant quinque æquationes, e quibus deduci poſſunt incognitæ quantitates a, b, c, d, e, f & m; & conſequenter æquatio, quæ eſt fluens prædictæ fluxionalis æquationis, $a'+2b'(x+y)+c'(x^2+y^2)+2d'xy+2e'xy(x+y)+f'x^2y^2=o$.

2. Sint $b=o$ & $e=o$ & reſultat æquatio $a+c(x^2+y^2)+2dxy+fx^2y^2=o$, unde fluxionalis æquatio $\frac{\dot{x}}{\surd(-cfx^4+(d^2-c^2-af)x^2-ca)}+\frac{\dot{y}}{\surd(-cfy^4+(d^2-c^2-af)y^2-ca)}=\frac{\dot{x}}{\surd(Ax^4+Cx^2+E)}+\frac{\dot{y}}{\surd(Ay^4+Cy^2+E)}=o$, cujus fluens erit $-A+l(x^2+y^2)+2xy\surd(l^2+lC+AE)-Ex^2y^2=o$, ubi l eſt invariabilis quantitas ad libitum aſſumenda.

Ex. 3. Sit æquatio $\pm\frac{\ddot{p}}{\surd(a+bp^2+cp^4+dp^6)}\pm\frac{\ddot{q}}{\surd(a+bq^2+cq^4+dq^6)}$; fingatur $x=p^2$ & $y=q^2$, & exinde deduci poteſt æquatio $\pm\frac{\dot{x}}{\surd(ax+bx^2+cx^3+dx^4)}\pm\frac{\dot{y}}{\surd(ay+by^2+cy^3+dy^4)}=o$.

In fluxione $\frac{\dot{x}}{\surd(a+bx+cx^2+dx^3+ex^4)}$ pro x & $\dot{x}$ ſcribantur $\frac{\alpha+\beta z}{\gamma+\delta z}$ & ejus fluxio; & ita aſſumi poſſunt coefficientes, ut reſultet fluxio formulæ $\frac{\dot{z}}{\surd(A+Bz^2+Cz^4)}$.

THEOR. XLI.

Sit X eadem functio quantitatis x, ac Y ſit quantitatis y, & $X\dot{x} + Y\dot{y} = o$; & detur generalis fluens $\alpha = o$ hujuſce æquationis; ex æquatione $X\dot{x} + Y\dot{y} = o$ conſtat $\phi:(x) + \phi:(y) = \text{conſt.} = \phi:(c)$, unde c erit conſimilis functio quantitatum x & y; i. e. $c = \pi:(x \,\&\, y)$: in æquationibus $\pi:(x \,\&\, y) = c$ & $\phi:(x) + \phi:(y) = \phi:c$, pro x & y ſcribantur a & b & reſultabit $c = \pi:(a \,\&\, b)$ & $\phi:(a) + \phi:(b) = \phi:(c)$; deinde in duabus æquationibus $\phi:(a) + \phi:(b) = \phi:(c)$ & $c = \pi:(a \,\&\, b)$ pro b ſcribatur a, & reſultabit $\phi:(a) + \phi:(b) = 2\phi:(a) = \phi:(c)$, & $c = \pi:(a \,\&\, b) = \pi':(a)$: nunc aſſumatur æquatio $2X\dot{x} + Y\dot{y} = o$, unde $2\phi:(x) + \phi:(y) = \text{conſt.} = \phi:(d)$: in hâc æquatione $d = \pi:(x \,\&\, y)$ pro x ſcribatur $\pi':(x)$, & reſultat æquatio $d = \pi:(\pi':(x) \,\&\, y)$, quæ erit generalis fluens æquationis fluxionalis $2X\dot{x} + Y\dot{y} = o$. In æquationibus $2\phi:(x) + \phi:(y) = \phi:(d)$ & $d = \pi:(\pi'(x) \,\&\, y)$ pro x & y ſcribantur a & b & reſultant æquationes $2\phi:(a) + \phi:(b) = \phi:(d)$ & $d = \pi:(\pi':(a) \,\&\, a) = \pi'':(a)$: 3^{tia}. aſſumatur æquatio $3X\dot{x} + Y\dot{y} = o$, unde $3\phi:(x) + \phi:(y) = \phi:(e)$: in æquatione $\pi:(x \,\&\, y) = e$ pro x ſcribatur $\pi'':(x)$, & reſultat $e = \pi:(\pi'':(x) \,\&\, y)$; quæ erit generalis fluens prædictæ fluxionis $3X\dot{x} + Y\dot{y} = o$: nunc fingatur $x = a$ & $y = b$, & denique $a = b$, tum reſultabunt $2:\phi:(a) + \phi:(b) = 3\phi:(a) = \phi:(e)$ & $e = \pi:(\pi'':(a) \,\&\, a) = \pi''':(a)$, & ſic deinceps: ex hâc methodo deduci poteſt fluens generalis fluxionalis æquationis $nX\dot{x} + mY\dot{y} = o$, ubi literæ n & m integros denotant numeros.

THEOR. XLII.

Sit æquatio $A\overset{n}{\dot{y}} + B\overset{n-1}{\dot{y}}\,\dot{x} + C\overset{n-2}{\dot{y}}\,\dot{x}^2 + D\overset{n-3}{\dot{y}}\,\dot{x}^3 \ldots + L\ddot{y}\,\dot{x}^{n-2} + M\dot{y}\,\dot{x}^{n-1} + Ny\dot{x}^n = X\dot{x}^n$, ubi $A, B, C, \ldots M$ & N invariabiles denotant coefficientes, & X eſt functio quantitatis x.

Ducatur hæc æquatio in $e^{\lambda x}$, & reſultat æquatio, cujus generalis fluens eſt $e^{\lambda x}(A\overset{n-1}{\dot{y}} + (B - \lambda A)\overset{n-2}{\dot{y}}\,\dot{x} + (C - \lambda B + \lambda^2 A)\overset{n-3}{\dot{y}}\,\dot{x}^2 + (D - \lambda C + \lambda^2 B - \lambda^3 A)\overset{n-4}{\dot{y}}\,\dot{x}^3 + \ldots + (M - \lambda L + \lambda^2 K - \ldots \pm \lambda^{n-3} C \mp \lambda^{n-2} B \pm \lambda^{n-1} A)y = \dot{x}^{n-1}\int. e^{\lambda x} X\dot{x} + a\dot{x}^{n-1}$, ſi modo $N - \lambda M + \lambda^2 L - \lambda^3 K + \ldots \pm \lambda^{n-1} B \pm \lambda^n A = 0$, & conſequenter $A\lambda^n - B\lambda^{n-1} + C\lambda^{n-2} - D\lambda^{n-3} +$ &c. $= 0$; at (n) ſunt diverſi valores quantitatis λ in æquatione $A\lambda^n - B\lambda^{n-1} + C\lambda^{n-2} -$ &c. $= 0$, qui ſint reſpective λ, μ, ν, ξ, &c.; unde ex (n) valoribus quantitatis λ exoriuntur totidem (n) multiplicatores, viz. $e^{\lambda x}$, $e^{\mu x}$, $e^{\nu x}$, &c.

Ducatur prædicta generalis fluens in $e^{(\mu-\lambda)x}$, & detegi poteſt fluens reſultantis æquationis $e^{\mu x}(A\overset{n-2}{\dot{y}} + (B - (\lambda + \mu)A)\overset{n-3}{\dot{y}}\,\dot{x} + (C - (\lambda + \mu)B + (\lambda^2 + \mu\lambda + \mu^2)A)\overset{n-4}{\dot{y}}\,\dot{x}^2 + (D - (\lambda + \mu)C + (\lambda^2 + \lambda\mu + \mu^2)B - (\lambda^3 + \lambda^2\mu + \lambda\mu^2 + \lambda^3)A)\overset{n-5}{\dot{y}}\,\dot{x}^3 +$ &c. $= \dot{x}^{n-2}\int. e^{(\mu-\lambda)x}\dot{x}\int. e^{\lambda x}X\dot{x} + \frac{a}{\mu-\lambda}e^{(\mu-\lambda)x}\dot{x}^{n-2} + b\dot{x}^{n-2} = -\dot{x}^{n-2}\Big(\frac{1}{\lambda-\mu}e^{(\mu-\lambda)x}\int. e^{\lambda x}X\dot{x} + \frac{1}{\mu-\lambda}\int. e^{\mu x}X\dot{x} - \frac{a}{\mu-\lambda}e^{(\mu-\lambda)x} - b\Big) = \dot{x}^{n-2}\Big(\frac{1}{\mu-\lambda}e^{(\mu-\lambda)x}\int. e^{\lambda x}X\dot{x} + \frac{1}{\lambda-\mu}\int. e^{\mu x}X\dot{x} + a' e^{(\mu-\lambda)x} + b\Big)$.

Hìc animadvertendum eſt $M - (\lambda + \mu)L + (\lambda^2 + \lambda\mu + \mu^2)K - (\lambda^3 + \lambda^2\mu + \lambda\mu^2 + \mu^3)I - \ldots \pm (\lambda^{n-1} + \lambda^{n-2}\mu + \lambda^{n-3}\mu^2 + \lambda^{n-4}\mu^3 + \ldots \mu^{n-1})A = 0$.

Ducatur fluens reſultantis æquationis in $e^{(\nu-\mu)x}$, & fluens æquationis exinde reſultantis erit $e^{\nu x}(A\overset{n-3}{\dot{y}} + (B - (\lambda + \mu + \nu)A)\overset{n-4}{\dot{y}}\,\dot{x}^2 + (C - (\lambda + \mu + \nu)B + (\lambda^2 + \mu^2 + \nu^2 + \lambda\mu + \lambda\nu + \mu\nu)A)\overset{n-5}{\dot{y}}\,\dot{x}^2 + (D - (\lambda + \mu + \nu)C + (\lambda^2 + \mu^2 + \nu^2 + \lambda\mu + \lambda\nu + \mu\nu)B + (\lambda^3 + \mu^3 + \nu^3 + \lambda\mu^2 + \lambda\nu^2 + \mu\lambda^2 + \mu\nu^2 + \nu\lambda^2 + \nu\mu^2 + \lambda\mu\nu)A)\overset{n-6}{\dot{y}}\,\dot{x}^3 +$ &c. $= \dot{x}^{n-3}\Big(\frac{1}{\lambda-\mu} \times \frac{1}{\lambda-\nu}e^{(\nu-\lambda)x}\int. e^{\lambda x}X\dot{x} + \frac{1}{\mu-\lambda} \times \frac{1}{\mu-\nu}e^{(\nu-\mu)x}$

$\int . e^{\mu x} X\dot{x} + \frac{1}{\nu - \mu} \times \frac{1}{\nu - \lambda} \int . e^{\nu x} X\dot{x} + (d' e^{(\nu - \lambda)x} + b' e^{(\nu - \mu)x} + c) = o$, & ſic deinceps; ultimo erit $y = \pm \left(\left(\frac{1}{\lambda - \mu} \times \frac{1}{\lambda - \nu} \times \frac{1}{\lambda - \xi} \times \frac{1}{\lambda - o} \times \&c.\right)\right.$ $\times\, e^{-\lambda x} \int . e^{\lambda x} X\dot{x} + \frac{1}{\mu - \lambda} \times \frac{1}{\mu - \nu} \times \frac{1}{\mu - \xi} \times \frac{1}{\mu - o} \times \&c. \times e^{-\mu x} \int . e^{\mu x} X\dot{x}$ $+ \frac{1}{\nu - \lambda} \times \frac{1}{\nu - \mu} \times \frac{1}{\nu - \xi} \times \frac{1}{\nu - o} \times \&c. \times e^{-\nu x} \int . e^{\nu x} X\dot{x} + \&c. + A' e^{-\lambda x}$ $+ B' e^{-\mu x} + C' e^{-\nu x} + D' e^{-\xi x} + \&c.$, ubi literæ A', B', C', D', &c. reſpective denotant invariabiles coefficientes ad libitum aſſumendas; & ſic deinceps.

Erit $(\lambda - \mu) \times (\lambda - \nu) \times (\lambda - \xi) \times (\lambda - o) \times \&c. = nA\lambda^{n-1} + (n-1)B\lambda^{n-2} + (n-2)C\lambda^{n-3} + (n-3)D\lambda^{n-4} + \&c.$ & ſic de reliquis radicibus μ, ν, ξ, &c.

2. Hoc theorema ex ſubſequentibus conſtabit: ſit æquatio $N + Mx + Lx^2 + Kx^3 + Ix^4 + Hx^5 + \ldots + Dx^{n-3} + Cx^{n-2} + Bx^{n-1} + Ax^n = o$, cujus radices ſint λ, μ, ν, ξ, o', &c.; tum erit $N + M\lambda + L\lambda^2 + K\lambda^3 + I\lambda^4 + \&c. = o$, & $M + \frac{\lambda^2 - \mu^2}{\lambda - \mu} L + \frac{\lambda^3 - \mu^3}{\lambda - \mu} K + \frac{\lambda^4 - \mu^4}{\lambda - \mu} I + \&c. = M + (\lambda + \mu) L + (\lambda^2 + \mu^2 + \mu\lambda) K + (\lambda^3 + \mu^3 + \lambda^2\mu + \mu^2\lambda) I + \&c. = o$, & ſimiliter $L + (\lambda + \mu + \nu) K + (\lambda^2 + \mu^2 + \nu^2 + \lambda\mu + \lambda\nu + \mu\nu) I + (\lambda^3 + \mu^3 + \nu^3 + \lambda^2\mu + \lambda^2\nu + \mu^2\lambda + \mu^2\nu + \nu^2\lambda + \nu^2\mu + \lambda\mu\nu) H + \&c. = o$, & $K + (\lambda + \mu + \nu + \xi) + K + (\lambda^2 + \mu^2 + \nu^2 + \xi^2 + \lambda\mu + \lambda\nu + \mu\nu + \lambda\xi + \mu\xi + \nu\xi) I + (\lambda^3 + \mu^3 + \nu^3 + \xi^3 + \lambda^2\mu + \mu^2\lambda + \lambda^2\nu + \mu^2\nu + \nu^2\lambda + \&c.) H + \&c. = o$; &c.: in primâ æquatione deductâ una ſolummodo continetur radix λ vel μ vel ν, &c,; in ſecundâ duæ; in tertiâ tres; & ſic deinceps: prima prædicta æquatio incipit ab primâ datæ æquationis coefficiente, ſecunda a ſecundâ, tertia a tertiâ, & ſic deinceps.

Coefficiens primi termini prædictarum æquationum erit 1; ſecundi termini erit ſumma omnium radicum in iis reſpective contentarum;

tarum; tertii termini erit ſumma omnium rationalium & non fractionalium functionum radicum, quarum dimenſiones ſunt duæ, i. e. erit ſumma quadratorum e ſingulis radicibus & rectangulorum ſub quibuſque duabus in iis contentis; quarti termini coefficiens erit ſumma omnium conſimilium functionum radicum, quarum dimenſiones ſunt tres, i. e. erit ſumma cuborum e ſingulis radicibus, & ſingulorum valorum ex quadrato alterius radicis in alteram ducto, viz. cujus formula eadem eſt ac $\alpha^2\beta$; & contentorum ſub quibuſque tribus radicibus; & ſic deinceps: coefficientes omnium quantitatum in datis æquationibus contentæ erunt 1.

Cor. 1. Poſitis $\dot{y} = p\dot{x}$, $\dot{p} = q\dot{x}$, $\dot{q} = r\dot{x}$, &c. ſcribantur hæ quantitates pro ſuis valoribus in datâ fluxionali æquatione, & reſultat $Ny + Mp + Lq + Kr +$ &c. $= X$.

Cor. 2. In datâ prædictâ fluxionali æquatione pro y, $\dot{y}$, $\ddot{y}$, & ſcribantur $e^{\int\cdot u\dot{x}}$, $u\dot{x}e^{\int\cdot u\dot{x}}$, $(\dot{u}\dot{x} + u^2\dot{x}^2)\,e^{\int\cdot u\dot{x}}$, &c.; & reſultat fluxionalis æquatio $(n-1)$ ordinis, cujus generalis fluens facile innoteſcit; nam erit $\frac{\dot{y}}{y\dot{x}} = u$, quoniam $e^{\int\cdot u\dot{x}} = y$; & exinde e generali valore quantitatis y prius dato facile conſtat generalis valor quantitatis u.

In reſultante æquatione pro $\dot{x}$ ſcribatur $\frac{\dot{u}}{t}$, deinde pro $t + u^2$ ſcribatur z; & ſimiliter ex infinitis ſubſtitutionibus facile deduci poſſunt fluxionales æquationes, quarum fluentes innoteſcunt.

3. Sit fluxionalis æquatio $H\overset{n}{\dot{y}} + G\overset{n-2}{\dot{y}}\dot{x}^2 + F\overset{n-4}{\dot{y}}\dot{x}^4 + \ldots + C\ddot{y}\dot{x}^{n-4} + B\ddot{y}\dot{x}^{n-2} + Ay\dot{x}^n = 0$; ubi n eſt par numerus; ducatur hæc æquatio in $\dot{y}$, & reſultat æquatio, cujus fluens eſt $H(\overset{n-1}{\dot{y}}\dot{y} - \overset{n-2}{\dot{y}}\ddot{y} + \overset{n-3}{\dot{y}}\dddot{y} \ldots \frac{1}{2}\overset{\frac{1}{2}n}{\dot{y}}{}^2) + G(\overset{n-3}{\dot{y}}\dot{y} - \overset{n-4}{\dot{y}}\ddot{y} + \overset{n-5}{\dot{y}}\dddot{y} \ldots \frac{1}{2}\overset{\frac{1}{2}n-1}{\dot{y}}{}^2)\dot{x}^2 +$ &c. $+ C(\dddot{y}\dot{y} - \frac{1}{2}\ddot{y}^2)\dot{x}^{n-4} + B(\frac{1}{2}\dot{y}^2)\dot{x}^{n-2} + \frac{1}{2}Ay^2\dot{x}^n + A'\dot{x}^n = 0$, ubi A' eſt invariabilis quantitas ad libitum aſſumenda.

Si in hâc æquatione pro $\dot{y}$ ſcribatur $e^{\int\cdot v\dot{x}}$, &c., vel $\dot{y} = u\dot{x}$, &c.;

&

& exinde deduci poſſunt æquationes fluxionales, quarum fluentes innoteſcunt.

4. Sit fluxionalis æquatio $a\overset{n}{\dot y} + b\overset{n-1}{\dot y}\dot x + c\overset{n-2}{\dot y}\dot x^2 + d\overset{n-3}{\dot y}\dot x^3 + \ldots f\overset{n-m}{\dot y}\dot x^m \ldots + h\dddot y\dot x^{n-3} + i\ddot y\dot x^{n-2} + k\dot y\dot x^{n-1} + ly\dot x^n = X\dot x^n$; ducatur hæc æquatio in datam quantitatem p, & ſcribantur $ap = A$, $bp = B$, $cp = C$, $dp = D$, &c., $fp = F$, &c., $hp = H$, $ip = I$, $kp = K$ & $lp = L$; ubi literæ a, b, c, d, &c., h, i, k, l & p functiones quantitatis x denotant; tum integrari poteſt fluxionalis quantitas $pa\overset{n}{\dot y} + pb\overset{n-1}{\dot y}\dot x + pc\overset{n-2}{\dot y}\dot x^2 +$ &c., ſi modo $\overset{n}{\dot A} - \overset{n-1}{\dot B} + \overset{n-2}{\dot C} - \overset{n-3}{\dot D} + \ldots \pm \dddot H \mp \ddot I \pm \dot K \mp L = o$: ſi evaneſcant termini l, k, i, h, &c. uſque ad f; tum erit $\overset{n-m}{\dot A} - \overset{n-m-1}{\dot B} + \overset{n-m-2}{\dot C} -$ &c. $= o$, cum quantitas prædicta integrari poſſit; fluens enim erit $A\overset{n-1}{\dot y} + (B - \dot A)\overset{n-2}{\dot y}\dot x + (C - \dot B + \ddot A)\overset{n-3}{\dot y}\dot x^2 +$ &c. $= \dot x^{n-1}\int.pX\dot x =$ conſt.

Cor. Si dentur omnes præter unam vel l vel k vel i vel h, &c. quantitates prædictæ l, k, i, h, &c. . . c, b, a & p; tum facile ex æquatione fluxionali $\overset{n}{\dot A} - \overset{n-1}{\dot B} + \overset{n-2}{\dot C} -$ &c. $= o$ acquiri poteſt quantitas vel L vel K vel I vel H, &c. ei correſpondens, & exinde vel $l = \frac{L}{p}$, vel $h = \frac{H}{p}$, vel $i = \frac{I}{p}$, &c.

Cor. 2. Sint $a = a'x^n$, $b = b'x^{n-1}$, $c = c'x^{n-2}$, &c., $h = h'x^3$, $i = i'x^2$, $k = k'x$; ubi literæ a', b', c', d', &c., h', i', k' & l' invariabiles denotant quantitates; ducatur data æquatio $a'x^n\overset{n}{\dot y} + b'x^{n-1}\overset{n-1}{\dot y}\dot x + c'x^{n-2}\overset{n-2}{\dot y}\dot x^2 +$ &c. $+ h'x^3\dddot y\dot x^{n-3} + i'x^2\ddot y\dot x^{n-2} + k'x\dot y\dot x^{n-1} + l'y\dot x^n = X\dot x^n$ in x^λ; & reſultat æquatio, cujus fluens eſt $a'x^{n+\lambda}\overset{n-1}{\dot y} + (b' - (n+\lambda)a')x^{n+\lambda-1}\overset{n-2}{\dot y}\dot x + (c' - (n+\lambda-1)b' + (n+\lambda) \times (n+\lambda-1)a')x^{n+\lambda-2}\dot y\dot x^2 +$

$\overset{n-3}{\dot{y}}\,\dot{x}^2 + (d' - (n+\lambda-2)\,c' + (n+\lambda-1)\times(n+\lambda-2)\,b' - (n+\lambda)\times(n+\lambda-1)\times(n+\lambda-2)\,a')\,x^{n+\lambda-3}\,\overset{n-4}{\dot{y}}\,\dot{x}^3 +$ &c. (cujus ultimi termini coefficiens $l' - (\lambda+1)\,k' + (\lambda+1)\times(\lambda+2)\,i' - (\lambda+1)(\lambda+2)(\lambda+3)\,b' + \ldots\ldots \pm (\lambda+1)\times(\lambda+2)\times(\lambda+3)\times\ldots\ldots\times(\lambda+n)\,a' = o) = \dot{x}^{n-1}\int.\,x^{\lambda}X\dot{x} +$ conſt. $\dot{x}^{n-1}$.

Eadem fluens ſic exprimi poteſt, viz. $\dot{x}^{n-1}\int.\,x^{\lambda}X\dot{x} +$ conſt. $\dot{x}^{n-1} = \frac{l'}{\lambda+1}\,x^{\lambda+1}\,y\,\dot{x}^{n-1} + \left(\frac{k'-\pi}{\lambda+2}\right)x^{\lambda+2}\,\dot{y}\,\dot{x}^{n-2} + \left(\frac{i'-\varrho}{\lambda+3}\right)x^{\lambda+3}\,\ddot{y}\,\dot{x}^{n-3} + \left(\frac{b'-\sigma}{\lambda+4}\right)x^{\lambda+4}\,\dddot{y}\,\dot{x}^{n-4} +$ &c., ubi literæ π, ϱ, σ, &c. præcedentes coefficientes reſpective denotant.

Cor. 3. Sit $l' - (\lambda+1)\,k' + (\lambda+1)(\lambda+2)\,i' - (\lambda+1)(\lambda+2)(\lambda+3)\,b' + \ldots\ldots \pm (\lambda+1)(\lambda+2)(\lambda+3)\ldots\ldots(\lambda+n)\,a' = a'(\lambda-\alpha)(\lambda-\beta)(\lambda-\gamma)(\lambda-\delta)\times$ &c.; unde λ vel $=\alpha$, vel $=\beta$, vel $=\gamma$, &c.: & exinde, ſi modo ducatur data æquatio in x^{α}, &c. reſultabit æquatio, cujus fluens eſt datæ ſimilis, ordinis vero inferioris $(n-1)$; & ſic deprimi poteſt reſultans æquatio in fluxionalem æquationem $(n-2)$ ordinis, ſi modo ea ducatur in x^{β}; & ſic eâdem operatione (n) vicibus repetitâ, i. e. ductis reſultantibus æquationibus in x^{γ}, x^{δ}, &c. ſucceſſive, & inventis fluxionum reſultantium fluentibus, tandem invenietur fluens quæſita.

5. Sit $y = a^n \times e^{-ax}\int.\,\dot{x}\int.\,\dot{x}\int.\,\dot{x}$ ad $n-1$ numerum $\times\int.\,e^{ax}X\dot{x}$; reducatur hæc æquatio, ita ut exterminentur fluentiales quantitates, & reſultat $y + \frac{n\dot{y}}{a\dot{x}} + \frac{n(n-1)\ddot{y}}{2a^2\dot{x}^2} + \frac{n(n-1)(n-2)\dddot{y}}{2\cdot3a^3\dot{x}^3} + \frac{n(n-1)}{2\cdot3}$
$\frac{(n-2)(n-3)\ddddot{y}}{\cdot4a^4\dot{x}^4} +$ &c. $= X$.

Hoc ſequitur etiam ex problemate; nulli in hoc caſu requiruntur multiplicatores, ni $\dot{x}$.

6. Sit æquatio $X = a\overset{n}{\dot{y}} + b\overset{n-1}{\dot{y}}\,\dot{x} + c\overset{n-2}{\dot{y}}\,\dot{x}^2 +$ &c., ſubſtituatur $y = v + Z$, ubi Z eſt functio quantitatis x, quæ eſt particularis valor

quantitatis y, & reſultabit fluxionalis æquatio $A v \dot{x}^n + B \dot{v} \dot{x}^{n-1} + C \ddot{v} \dot{x}^{n-2} + \&c. = o$, ubi a, b, c, &c., A, B, C, &c. ſunt functiones quantitatis x.

7. Sit æquatio $X = ay + \frac{b\dot{y}}{\dot{x}} + \frac{c\ddot{y}}{\dot{x}^2} + \frac{d\dddot{y}}{\dot{x}^3} + \&c.$; ſtatuatur $x = \log. v$, unde $\dot{x} = \frac{\dot{v}}{v}$, & erit $X = \phi : (v)$; deinde inveniatur æquatio relationem inter v & y & earum fluxiones exprimens, quæ habet formulam $V = ay + \frac{bv\dot{y}}{\dot{v}} + \frac{cv^2\ddot{y}}{\dot{v}^2} + \frac{dv^3\dddot{y}}{\dot{v}^3} + \&c.$: & ſimiliter ſit æquatio $V = ay + \frac{bv\dot{y}}{\dot{v}} + \frac{cv^2\ddot{y}}{\dot{v}^2} + \&c.$; ſtatuatur $v = e^x$, & reducitur data æquatio ad æquationem formulæ $X = ay + \frac{b\dot{y}}{\dot{x}} + \frac{c\ddot{y}}{\dot{x}^2} + \&c.$

8. Sint $X = Ay + B\frac{\dot{y}}{\dot{x}} + C\frac{\ddot{y}}{\dot{x}^2} \ldots \frac{N\overset{m+n}{\dot{y}}}{\dot{x}^{m+n}}$ & $P = A + Bz + Cz^2 \ldots Nz^{m+n} = \mathcal{Q}R$, ubi $\mathcal{Q} = a + bz + cz^2 \ldots lz^m$ & $R = \alpha + \beta z + \gamma z^2 \ldots \nu z^n$: fingatur $X = av + b\frac{\dot{v}}{\dot{x}} + c\frac{\ddot{v}}{\dot{x}^2} \ldots l\frac{\overset{m}{\dot{v}}}{\dot{x}^m}$, & erit $v = \alpha y + \beta\frac{\dot{y}}{\dot{x}} + \gamma\frac{\ddot{y}}{\dot{x}^2} \ldots + \nu\frac{\overset{n}{\dot{y}}}{\dot{x}^n}$.

9. Sint n diverſi valores variabilis y in fluxionale æquatione $\overset{n}{\dot{y}} + p\overset{n-1}{\dot{y}}\dot{x} + q\overset{n-2}{\dot{y}}\dot{x}^2 \ldots + Sy\dot{x}^n = o$ (ubi p, q, &c. S datas functiones quantitatis x denotant) reſpective $\pi, \varrho, \sigma, \tau$, &c. & erit generalis valor ipſius y in prædicta æquatione $= a\pi + b\varrho + c\sigma + d\tau + \&c.$ ubi a, b, c, &c. invariabiles quantitates ad libitum aſſumendas reſpective deſignant.

Cor. 4. Sit ϑ valor variabilis y in fluxionale æquatione $(A)\, \overset{n}{\dot{y}} + p\overset{n-1}{\dot{y}}\dot{x} + q\overset{n-2}{\dot{y}}\dot{x}^2 \ldots Sy\dot{x}^n + X\dot{x}^n = o$, (n) vero diverſi valores quantitatis y in æquatione $\overset{n}{\dot{y}} + p\overset{n-1}{\dot{y}}\dot{x} + q\overset{n-2}{\dot{y}}\dot{x}^2 \ldots Sy\dot{x}^n = o$ reſpective $\pi, \varrho, \sigma, \tau$, &c.

σ, τ, &c. tum erit generalis valor ipſius y in fluxionali æquatione (A) $= a\pi + b\rho + c\sigma + d\tau +$ &c. $+ \vartheta_{,}$, ubi a, b, c, d, &c. ſunt quæcunque invariabiles quantitates ad libitum aſſumendæ.

Cor. 2. Sint b valores variabilis y fluxionalis æquationis $(B)\, \overset{n}{\dot{y}} + p\overset{n-1}{\dot{y}}\dot{x} + q\overset{n-2}{\dot{y}}\dot{x}^2 \ldots Sy\dot{x}^n = X\dot{x}^n$ (ubi $p, q \ldots S$ & X ſunt functiones ipſius x) $\alpha, \beta, \gamma, \delta$, &c. tum $b - 1$ diverſi valores variabilis y in æquatione (A), cum $X = o$; erunt reſpective $\alpha - \beta$, $\alpha - \gamma$, $\alpha - \delta$, &c. ſint l valores variabilis y in fluxionali æquatione $\overset{n}{\dot{y}} + p\overset{n-1}{\dot{y}}\dot{x} + q\overset{n-2}{\dot{y}}\dot{x}^2 \ldots Sy\dot{x}^n = \daleth\dot{x}^n$, ubi $\daleth$ etiam eſt functio ipſius x, haud vero eadem ac X, reſpective π, ρ, σ, τ, &c. & erunt $\pi - \rho$, $\pi - \sigma$, $\pi - \tau$, &c. reſpective $l - 1$ diverſi valores ipſius y in æquatione (A), ubi $\daleth = o$: & ex n diverſis valoribus ipſius y in æquatione (A), & uno valore variabilis y in æquatione (B); facile erui poteſt generalis valor ipſius y in fluxionali æquatione B: &c. Omnia hæc conſtant e ſcribendo pro y & ejus fluxionibus in datâ fluxionali æquatione earum valores prius aſſignatos.

10. Sit fluxionalis æquatio n ordinis $X\dot{x}^n = Py\dot{x}^n + Q\dot{y}\dot{x}^{n-1} + R\ddot{y}\dot{x}^{n-2} + S\dddot{y}\dot{x}^{n-3} + T\ddddot{y}\dot{x}^{n-4} + V\overset{5}{\dot{y}}\dot{x}^{n-5} + \ldots A\overset{n}{\dot{y}}$, & ſi $P\dot{x}^m - m\dot{Q}\dot{x}^{m-1} + m \cdot \frac{m+1}{2}\ddot{R}\dot{x}^{m-2} - m \cdot \frac{m+1}{2} \cdot \frac{m+2}{3}\dddot{S}\dot{x}^{m-3} + m \cdot \frac{m+1}{2} \cdot \frac{m+2}{3} \cdot \frac{m+3}{4}\ddddot{T}\dot{x}^{m-4} -$ &c. $= o$, cum m vel 1 vel 2 vel 3, &c. ad numerum m; tum reduci poteſt data æquatio ad fluxionalem æquationem $n - m$ ordinis ex m repetitis iiſdem multiplicatoribus $(\dot{x})$.

11. Sit fluxionalis æquatio $p\overset{n}{\dot{y}} + q\overset{n-1}{\dot{y}}\dot{x} + r\overset{n-2}{\dot{y}}\dot{x}^2 + s\overset{n-3}{\dot{y}}\dot{x}^3 + t\overset{n-4}{\dot{y}}\dot{x}^4 \ldots + P\overset{n-m}{\dot{y}}\dot{x}^m = X\dot{x}^n$, ubi omnes termini poſt terminum $P\overset{n-m}{\dot{y}}$ ad terminum $X\dot{x}^n$ deſunt; ejus fluens deduci poteſt ex m repetitis multiplicatoribus, quorum primus inveniri poteſt e fluxionali æquatione

$p\overset{m}{\dot{M}} +$

$$\begin{array}{lllll}
p\overset{m}{M} & +m\dot{p}\overset{m-1}{M} & +m.\frac{m-1}{2}\ddot{p}\overset{m-2}{M} & +m.\frac{m-1}{2}\times\frac{m-2}{3}\dddot{p}\overset{m-3}{M} & +m.\frac{m-1}{2}.\frac{m-2}{3}.\frac{m-3}{4}\ddddot{p}\overset{m-4}{M}+\&c. \\
-q & -(m-1)\dot{q} & -(m-1)\times\frac{m-2}{2}\ddot{q} & -(m-1)\times\frac{m-2}{2}\times\frac{m-3}{3}\dddot{q} & -\&c. \\
 & +\quad r & +(m-2)\dot{r} & +(m-2)\times\frac{m-3}{2}\ddot{r} & \\
 & & -\quad s & -(m-3)\quad\dot{s} & \\
 & & & + &
\end{array}$$

(m) ordinis; ſecundus, tertius, &c. e fluxionali æquatione ($m-1$, $m-2$, &c.) ordinis deduci poteſt.

THEOR. XLIII.

Sit $\pi = o$ fluxionalis æquatio, in quâ continentur variabiles x & y & earum fluxiones $\dot{x}, \dot{y}, \ddot{y}, \dddot{y}, \ldots \overset{n}{\dot{y}}$; reducatur data æquatio, ita ut in eâ contineatur tantummodo una dimenſio fluxionis maximi ordinis $\overset{n}{\dot{y}}$ & reſultet $\alpha = o$, cujus multiplicator ſit M; i. e. ſit $M\alpha = \rho = o$; deinde ita reducatur $\rho = o$, ut ſolummodo ſimplex dimenſio fluxionis $\overset{n-1}{\dot{y}}$ in eâ contineatur; & ſic deinceps redintegratâ operatione, uſque donec inveniatur fluens quæſita. Hìc animadvertendum eſt, quod ſingula extractio radicum tot diverſas fluentes præbet, quot diverſæ radices exinde reſultant.

Ex. 1. Sit fluxionalis æquatio $\ddot{y} + \left(\frac{\alpha}{x+a} + \frac{\beta}{x+b}\right)\dot{y}\dot{x} + \left(\frac{\alpha}{(x+a)}\frac{\beta}{(x+b)} - \frac{\alpha}{(x+a)^2}\right)y\dot{x}^2 = X\dot{x}^2$; tum erit $y = \frac{1}{(x+a)^{\alpha}}\int.\frac{(x+a)^{\alpha}}{(x+b)^{\beta}}\dot{x}\int.(x+b)^{\beta}X\dot{x}$.

2. Sit fluxionalis æquatio $\dddot{y} + \left(\frac{\alpha}{x+a} + \frac{\beta}{x+b} + \frac{\gamma}{x+c}\right)\ddot{y}\dot{x} +$

$\alpha\beta +$

$\left(\frac{\alpha\beta}{(x+\alpha)(x+\beta)} + \frac{\alpha\gamma}{(x+a)(x+c)} + \frac{\beta\gamma}{(x+b)(x+c)} - \frac{2\alpha}{(x+a)^2} - \frac{\beta}{(x+b)^2}\right)\dot{y}\dot{x} + \left(\frac{\alpha\beta\gamma}{(x+a)(x+b)(x+c)} - \frac{\alpha\beta}{(x+a)^2(x+b)} - \frac{\alpha\beta}{(x+a)(x+b)^2} - \frac{\alpha\gamma}{(x+a)^2\times(x+c)} + \frac{1.2\alpha}{(x+a)^3}\right)y\dot{x}^3 = X\dot{x}^3$; tum erit $y = \frac{1}{(x+a)^\alpha}\times\int.\frac{(x+a)^\alpha}{(x+b)^\beta}\dot{x}\int.\frac{(x+b)^\beta}{(x+c)^\gamma}\dot{x}\int.(x+c)^\gamma X\dot{x}$.

3. Sit fluxionalis æquatio $\overset{n}{\dot{y}} + \left(\frac{\alpha}{x+a} + \frac{\beta}{x+b} + \frac{\gamma}{x+\gamma} + \frac{\delta}{x+\delta} + \&c.\right)\overset{n-1}{\dot{y}}\dot{x} + \left(\frac{\alpha\beta}{(x+a)(x+b)} + \frac{\alpha\gamma}{(x+a)(x+c)} + \frac{\alpha\delta}{(x+a)(x+d)} + \frac{\beta\gamma}{(x+b)(x+c)} + \frac{\beta\delta}{(x+b)(x+d)} + \frac{\gamma\delta}{(x+c)(x+d)} + \&c. - \frac{(n-1)\alpha}{(x+a)^2} - \frac{(n-2)\beta}{(x+b)^2} - \frac{(n-3)\gamma}{(x+c)^2} - \&c.\right)\overset{n-2}{\dot{y}}\dot{x}^2 + \left(\frac{\alpha\beta\gamma}{(x+a)(x+b)(x+c)} + \frac{\alpha\beta\delta}{(x+a)(x+b)(x+d)} + \frac{\alpha\gamma\delta}{(x+a)(x+c)(x+d)} + \frac{\beta\gamma\delta}{(x+b)(x+c)(x+d)} + \&c. - \frac{(n-2)\alpha\beta}{(x+a)^2\times(x+b)} - \frac{(n-2)\alpha\beta}{(x+a)(x+b)^2} - \frac{(n-2)\alpha\gamma}{(x+a)^2(x+c)} - \frac{(n-3)\alpha\gamma}{(x+a)(x+c)^2} - \frac{(n-3)\beta\gamma}{(x+b)^2(x+c)} - \frac{(n-3)\beta\gamma}{(x+b)(x+c)^2} - \frac{(n-3)\alpha\delta}{(x+a)^2(x+d)} - \frac{(n-3)\alpha\delta}{(x+a)(x+d)^2} - \&c. + \frac{(n-1).(n-2)\alpha}{(x+a)^3} + \frac{(n-2).(n-3)\beta}{(x+b)^3} + \frac{(n-3).(n-4)\gamma}{(x+c)^3} + \&c.\right)\overset{n-3}{\dot{y}}\dot{x}^3\ \&c. = X\dot{x}^n$; tum erit $y = \frac{1}{(x+a)^\alpha}\times\int.\frac{(x+a)^\alpha}{(x+b)^\beta}\dot{x}\int.\frac{(x+b)^\beta}{(x+c)^\gamma}\dot{x}\int.\frac{(x+c)^\gamma}{(x+d)^\delta}\dot{x}\times\int.\&c.\times X\dot{x}$.

THEOR. XLIV.

Sit quæcunque fluxionalis æquatio $a\overset{n}{\dot{y}} + b = o$, ubi a & b ſunt functiones quantitatis x, ejus incrementi $\dot{x}$; $y, \dot{y}, \ddot{y}$, &c. ad $\overset{n-1}{\dot{y}}$, in quâ nullæ continentur fluentes; ducatur hæc æquatio in functionem p quantitatum x & $\dot{x}$, & ſi reſultans æquatio $pa\overset{n}{\dot{y}} + pb = o$ ſit integrabilis, tum reduci poteſt data fluxionalis æquatio ad fluxionalem æquationem $n - 1$ ordinis ope multiplicatoris p, qui etiam deduci poteſt ope fluxionalis æquationis $n - 1$ ordinis.

Inveniatur enim fluens fluxionis $pa\overset{n}{\dot{y}}$ ex hypotheſi quod omnes quantitates $x, \dot{x}, \dot{y}, \ldots \overset{n-2}{\dot{y}}$ præter $\overset{n-1}{\dot{y}}$ ſint conſtantes, quæ ſit A; deinde ſupponantur omnes quantitates $x, y, \dot{y}, \ldots \overset{n-2}{\dot{y}}$ præter $\overset{n-1}{\dot{y}}$ & $\dot{x}$ in quantitate A variabiles, & ex hâc hypotheſi inveniatur fluxio B quantitatis A, quæ auferatur de pb; & deinde ex eodem proceſſu repetito, i. e. ex ſupponendo omnes quantitates $x, \dot{x}, y, \dot{y}, \ldots \overset{n-3}{\dot{y}}$ præter $\overset{n-2}{\dot{y}}$ conſtantes, inveniatur fluens quantitatis $pb - B$, & ſic deinceps uſque donec ſolummodo contineantur in inventâ fluente A' variabiles y, x & $\dot{x}$: inveniatur ejus fluxio H ex hypotheſi quod x ſolummodo ſit variabilis; fiant correſpondentes termini, i. e. termini, in quibus eædem inveniuntur dimenſiones quantitatis y, &c. inter ſe æquales, &c. & reſultant fluxionales æquationes haud majoris quam n ordinis, quarum variabiles quantitates ſunt p & x; pro p in his æquationibus ſcribatur $e^{\int \cdot v\dot{x}}$, &c. & æquatio reſultans relationem inter $x, \dot{x}, v$ & ejus fluxiones deſignans haud majoris erit quam $n - 1$ ordinis.

Ex. . Sit fluxionalis æquatio $\alpha\dot{y} + \beta\dot{x} = o$, ducatur hæc æquatio in p, quæ ſit functio quantitatis x, & reſultat $p\alpha\dot{y} + p\beta\dot{y} = o$; inveniatur fluens fluxionis $\alpha\dot{y}$ ex hypotheſi quod y ſolummodo ſit variabilis, quæ ſit π; deinde inveniatur fluxio rectanguli $p \times \pi$ ex hypotheſi,

pothesi, quod x ſolummodo ſit variabilis, quæ erit $p\dot{x}\left(\frac{\dot{\pi}}{\dot{x}}\right)+\pi\dot{p}=p\beta\dot{x}$; in hâc æquatione pro p ſcribatur $e^{\int v\dot{x}}$ & reſultat $\left(\frac{\dot{\pi}}{\dot{x}}\right)+\pi v=\beta$, in quâ nullæ continentur fluxiones, unde conſequitur valor quantitatis v.

Cor. . Si duæ vel plures reſultent æquationes ex correſpondentibus terminis inter ſe æqualibus eſſe ſuppoſitis, tum plerumque e methodo prius traditâ reducendi plures æquationes in unam, ita ut variabiles & earum fluxiones exterminentur, vel erui poteſt quantitas p, vel ſaltem erui poteſt fluxionalis æquatio inferioris ordinis.

THEOR. XLV.

1. Si in datâ æquatione algebraicâ vel fluxionali contineatur generalis functio quantitatis (π), quæ ſit functio quantitatum x & y, & earum fluxionum; tum in multis caſibus, ſi modo ſcribatur $v=\pi$, & ita reducantur duæ æquationes, ut exterminentur altera variabilis x vel y & ejus fluxiones; reſultabit æquatio, cujus fluens innoteſcit.

2. Si vero fluens æquationis reſultantis vel ex hâc ſubſtitutione vel ex quâcunque aliâ haud generaliter inveniri poſſit; in multis caſibus ita aſſumi poſſunt coefficientes, exponentes, &c. ut conſtabunt caſus particulares, in quibus ſit reſultans æquatio formulæ, cujus fluens innoteſcit.

3. 1. Dividatur data fluxionalis æquatio in duas partes, & ſi fluens (P) ex unâ parte exprimi poſſit, ducatur data æquatio in functionem fluentis P, & inveniantur caſus, in quibus fluens ex alterâ parte etiam exprimi poteſt; tum conſequuntur æquationes, quarum fluentes generales deduci poſſunt.

2do. Ducatur prior pars ($\dot{P}$) datæ æquationis in functionem quantitatis P; ſecunda vero in quamcunque quantitatem quantitati P æqualem ſed haud eandem, & inveniantur caſus in quibus fluens producti inveniri poteſt; & conſequuntur æquationes, quarum particulares fluentes deduci poſſunt.

PROB. LIX.

Datâ æquatione fluxionali; invenire alias, quarum fluentes e datâ fluxionali æquatione deduci poſſunt.

Aſſumantur æquationes relationem inter variabiles datæ & quæſitæ æquationis exprimentes, ita quidem ut relatio inter variabiles quæſitæ e relatione inter variabiles datæ æquationis aſſignari poteſt; & exinde deducatur æquatio relationem inter variabiles quæſitæ æquationis quantitates deſignans, reſultabit æquatio quæſita.

Ex. Sit data æquatio $\dot{y} + ay^2\dot{x} = bx^m\dot{x}$, in hâc æquatione ſcribatur $x^n z = y$; tum exorietur æquatio $\dot{z} + \frac{nz\dot{x}}{x} + ax^n z^2\dot{x} = bx^m\dot{x}$: & facile patebit, ſi modo detur fluens æquationis $\dot{y} + ay^2\dot{x} = bx^m\dot{x}$, exinde deduci poſſe fluentem æquationis $\dot{z} + \frac{nz\dot{x}}{x} + ax^n z^2\dot{x} = bx^m\dot{x}$; etiamque ſi modo cognoſcantur caſus in quibus deduci poteſt fluens prioris, facile etiam conſtari caſus, in quibus deduci poteſt fluens poſterioris æquationis.

PROB. LX.

Dato valore M *variabilis quantitatis* y *in fluxionali æquatione contentæ, i. e. ſit* M *data functio alterius variabilis quantitatis* x; *in nonnullis caſibus per ſubſequentes methodos deducere generalem datæ æquationis fluentis valorem, i. e. in genere corrigere datam æquationem.*

Scribatur pro y ejus valor aſſumptus $M + z$ vel Mz, &c. in datâ fluxionali æquatione; & reſultantis æquationis nonnunquam detegi poteſt generalis fluens.

Ex. 1. Sit $\dot{y} + Py\dot{x} + Qy^2\dot{x} + R\dot{x} = 0$, ubi literæ P, Q & R ſint functiones quantitatis x; ſed per hypotheſin M eſt valor quantitatis y, & conſequenter $\dot{M} + PM\dot{x} + QM^2\dot{x} + R\dot{x} = 0$; ſcribatur $M + z$ pro y in datâ æquatione, & reſultat $\dot{z} + Pz\dot{x} + 2QMz\dot{x} + Qz^2\dot{x} = 0$; deinde ſcribatur $\frac{1}{v}$ pro z, & proveniet $-\frac{\dot{v}}{v^2} + \frac{P\dot{x}}{v} + \frac{2QM\dot{x}}{v} + \frac{Q\dot{x}}{v^2} = 0$, quâ ductâ in v^2, reſultat $-\dot{v} + (P + 2QM)v\dot{x} + Q\dot{x} = 0$, quæ eſt æquatio formulæ prius traditæ; ducatur hæc æquatio in $S = e^{-\int.(P+2QM)\dot{x}}$, & exorietur æquatio, cujus fluens erit $ve^{-\int.(P+2QM)\dot{x}} - \int.QS\dot{x} =$ conſt. ſed $v = \frac{1}{y-M}$; & conſequenter ducatur data æquatio in $v^2 e^{-\int.(P+2QM)\dot{x}} = \frac{1}{(y-M)^2} e^{-\int.(P+2QM)\dot{x}} = \frac{S}{(y-M)^2}$; & reſultat æquatio integralis $Sv - \int.QS\dot{x} =$ conſt.

Cor. 1. Omnes quantitates quæ in datam æquationem ductæ, eam reddunt integrabilem, in hâc formulâ $\frac{S}{(y-M)^2} \times$ in functionem quamcunque quantitatis $\left(\frac{S}{y-M} - \int.QS\dot{x}\right)$, ubi M eſt cognita functio quantitatis x, & $S = e^{-\int.(P+2QM)\dot{x}}$, continentur.

Cor. 2. Exhinc deduci poteſt generalis fluens æquationis $\ddot{z} + S\dot{z}\dot{x} + Tz\dot{x}^2 = \frac{E\dot{x}^2}{z^3} e^{-2\int.S\dot{x}}$; inveniatur fluens v æquationis $\dot{v} + v^2\dot{x} + Sv\dot{x} + T\dot{x} = 0$, fingatur $V = \int.e^{-2\int.v\dot{x} - \int.S\dot{x}}\dot{x}$, eritque $z = e^{\int.v\dot{x}} \sqrt{(A + BV + CV^2)}$, ubi literæ A, B, C, &c. ſunt invariabiles quantitates, & $AC - \frac{1}{4}B^2 = E$.

Cor. 3. Sit æquatio fluxionalis $P = q$, ubi P ſit functio variabilium quantitatum y & x & earum fluxionum, q vero functio quantitatum x & $\dot{x}$; ſcribantur in datâ æquatione $P = q$ pro y, $\dot{y}$, $\ddot{y}$, &c. reſpective $z + E$, $\dot{z} + \dot{E}$, $\ddot{z} + \ddot{E}$, &c. ubi E ſit particularis valor quantitatis y, tum reſultabit æquatio, in quâ deficit terminus, qui fuit functio ſolummodo quantitatum x & $\dot{x}$.

Ex. g. Sit $P\ddot{y} + Q\dot{x}\dot{y} + Ry\dot{x}^2 = o$, ubi literæ P, Q & R reſpective denotant functiones quantitatis x: ſit X functio quantitatis x, quæ ſit particularis valor quantitatis y in datâ æquatione integrali; pro $\dot{y}, \dot{y}$ & $\ddot{y}$ in datâ æquatione ſcribantur reſpective Xz, $X\dot{z} + z\dot{X}$, $z\ddot{X} + 2\dot{z}\dot{X} + X\ddot{z}$, & reſultat $(Pz\ddot{X} + Qz\dot{X}\dot{x} + RzX\dot{x}^2) + 2P\dot{X}\dot{z} + QX\dot{x}\dot{z} + PX\ddot{z} = o$, ſed quoniam X eſt valor quantitatis y in datâ æquatione, erit $Pz\ddot{X} + Qz\dot{X}\dot{x} + RzX\dot{x}^2 = o$, unde $2P\dot{X}\dot{z} + QX\dot{x}\dot{z} + PX\ddot{z} = o$.

PROB. LXI.

Sint duæ vel plures n *æquationes tres vel plures* (n + 1) *variabiles quantitates & earum fluxiones habentes; ita reducere has æquationes in unam, ut exterminentur omnes præter duas variabiles quantitates & earum fluxiones, & haud nunquam vel reſultent æquationes, quarum fluentes innoteſcunt; vel ita aſſumi poſſint coefficientes, ut evadant æquationes, quarum fluentes dantur: exemplis docebitur prob. reſolutio.*

1. Sint $\dot{x} + (Cx + Dy)\dot{t} = o$ & $\dot{y} + (Kx + Ly)\dot{t} = o$, unde $-\dot{t} = \frac{\dot{x}}{Cx + Dy} = \frac{\dot{y}}{Kx + Ly}$ homogenea æquatio, cujus fluens innoteſcit.

2. Subſtituantur pro quibuſdam functionibus quantitatum in datis æquationibus contentarum quædam functiones novarum variabilium aſſumptarum; & exinde nonnunquam deduci poſſunt æquationes, quarum fluentes dantur.

Ex. 1. Sint æquationes $T'\dot{x} + T''\dot{t}\,(Cx + Dy) + \theta\dot{t} = o$ & $T'\dot{y} + T''\dot{t}\,(Kx + Ly) + \theta'\dot{t} = o$; ubi T', T'', θ, θ' ſunt functiones quantitatis t; ducatur poſterior æquatio in v, & ad productum reſultans addatur prior, & exorietur $T'\dot{x} + vT'\dot{y} + \dot{t}\,(T''\,(Cx + Dy) + vT''\,(Kx + Ly) + \theta + v\theta') = o$: ſit $(C + Kv)x + (D + Lv)y = n(x + vy)$,

vy), unde $C+Kv : D+Lv :: 1 : v$, & exinde acquiri poteſt quantitas v; pro $x+vy$ in æquatione exortâ ſcribatur u, & reſultat $T'\dot{u} + T''nu\dot{t} + (\theta + v\theta')\dot{t} = o$, cujus fluens innoteſcit.

Ex. 2. Sint $\dot{x} + (ax+by+cz)\dot{t} = o$, $\dot{y} + (ex+fy+gz)\dot{t} = o$ & $\dot{z} + (hx+my+nz)\dot{t} = o$; ducantur ſecunda & tertia æquationes in coefficientes invariabiles deducendas ν & μ, & ad ſummam productorum reſultantium addatur prima æquatio; deinde ſupponatur $a + e\nu + h\mu = \frac{b + f\nu + m\mu}{\nu} = \frac{c + g\nu + n\mu}{\mu}$, & fingatur $x + \nu y + \mu z = u$; & reſultabit æquatio $\dot{u} + (a + e\nu + h\mu)u\dot{t} = o$ cujus fluens inveniri poteſt. Hic ν & μ tres habent valores, in præcedente exemplo v duos habet valores.

Ex. 3. Sint n fluxionales æquationes $T(a\dot{x} + b\dot{y} + c\dot{z} + d\dot{v} + \&c.) + T'(ex + fy + gz + hv + \&c.)\dot{t} + \theta\dot{t} = o$, $T(a'\dot{x} + b'\dot{y} + c'\dot{z} + d'\dot{v} + \&c.) + T'(e'x + f'y + g'z + h'v + \&c.)\dot{t} + \theta'\dot{t} = o$, $T(a''\dot{x} + b''\dot{y} + c''\dot{z} + d''\dot{v} + \&c.) + T'(e''x + f''y + g''z + h''v + \&c.)\dot{t} + \theta''\dot{t} = o$, &c.; ducantur hæ æquationes reſpective in $1, \lambda, \mu, \nu$, &c.; & pro $a + \lambda a' + \mu a'' + \nu a''' + \&c.$, $b + \lambda b' + \mu b'' + \nu b''' + \&c.$, $c + \lambda c' + \mu c'' + \nu c''' + \&c.$, &c. ſcribantur reſpective $\alpha, \beta, \gamma, \delta$, &c.; deinde aſſumatur $\frac{e + \lambda e' + \mu e'' + \nu e''' + \&c.}{\alpha} = \frac{f + \lambda f' + \mu f'' + \nu f''' + \&c.}{\beta} = \frac{g + \lambda g' + \mu g'' + \nu g''' + \&c.}{\gamma} = \&c.$: ex hiſce æquationibus inveniantur valores quantitatum λ, μ, ν, &c.; & exinde quantitates α, β, γ, &c.; quibus inventis, reſultat æquatio $T\dot{u} + \frac{1}{\alpha}(e + \lambda e' + \mu e'' + \nu e''' + \&c.)T'u\dot{t} + (\theta + \lambda\theta' + \mu\theta'' + \&c.)\dot{t} = o$, cujus fluens innoteſcit, ubi $u = (a + \lambda a' + \mu a'' + \&c.)x + (b + \lambda b' + \mu b'' + \&c.)y + (c + \lambda c' + \mu c'' + \&c.)z + \&c.$

Per quantitates $T, T', \theta, \theta', \theta''$, &c. deſignantur datæ functiones quantitatis t.

Ex. 4. Sint n fluxionales æquationes ſuperiorum (m) ordinum, viz. $T k \overset{m}{\dot{v}} + T' l \overset{m-1}{\dot{v}} + T'' n \overset{m-2}{\dot{v}} + \ldots + \overset{m}{T}'(ex + fy + gz + \&c.)\dot{t} +$

$\theta\dot t = o$, $T k'\overset{m}{\dot v} + T' l'\overset{m-1}{\dot v} + T'' n'\overset{m-2}{\dot v} + \&c. + \ldots + \overset{m}{T'}(e'x + f'y + g'z + \&c.)\dot t + \theta'\dot t = o$, $T k''\overset{m}{\dot v} + T' l''\overset{m-1}{\dot v} + T'' n''\overset{m-2}{\dot v} + \ldots + \overset{m}{T'}(e''x + f''y + g''z + \&c.)\dot t + \theta''\dot t = o$, ubi $v = ax + by + cz + dv + \&c.$; ducantur hæ æquationes reſpective in 1, λ, μ, ν, &c.; & aſſumatur $\frac{e + \lambda e' + \mu e'' + \&c.}{a} = \frac{f + \lambda f' + \mu f'' + \&c.}{b} = \frac{g + \lambda g' + \mu g'' + \&c.}{c}$ = &c. & exinde per ſimplices æquationes inveniantur quantitates λ, μ, ν, &c.; quibus inventis reſultat fluxionalis æquatio $T(k + \lambda k' + \mu k'' + \&c.)\overset{m}{\dot v} + T'(l + \lambda l' + \mu l'' + \&c.)\overset{m-1}{\dot v} + T''(n + \lambda n' + \mu n'' + \&c.)\overset{m-2}{\dot v} + \&c. + \overset{m}{T'} \times \left(\frac{e + \lambda e' + \mu e'' + \&c.}{a}\right) v\dot t + (\theta + \lambda\theta' + \mu\theta'' + \&c.)\dot t = o$.

Ex. 5. Sint m æquationes $(m+1)$ variabiles quantitates (x, y, z, &c. & t) involventes, & ſi in ſingulis terminis datæ æquationis ſolummodo contineatur vel una dimenſio quantitatum (x, y, z, v, &c.), vel earum fluxionum quorumcunque ordinum; vel nulla; & contineatur unus terminus in ſingulis æquationibus, qui eſt functio quantitatis t in $\dot t^n$, ubi t eſt quantitas, quæ fluit uniformiter; i. e. ſint æquationes hujuſce formulæ $T\overset{n}{\dot x} + t\overset{n}{\dot y} + \tau\overset{n}{\dot z} + \&c. + T'\overset{n-1}{\dot x}\dot t + t'\overset{n-1}{\dot y}\dot t + \tau'\overset{n-1}{\dot z}\dot t + \&c. + \&c. + \ldots + \overset{n-1}{T'}\dot x\dot t^{n-1} + \overset{n-1}{t'}\dot y\dot t^{n-1} + \overset{n-1}{\tau'}\dot z\dot t^{n-1} + \&c. + \overset{n}{T'}x\dot t^n + \overset{n}{t'}y\dot t^n + \overset{n}{\tau'}z\dot t^n + \&c. + \theta\dot t^n = o$, ubi T, T', T'', &c., t, t', t'', &c., τ, τ', τ'', &c. θ ſunt functiones quantitatis t; reducantur hæ æquationes per prob. 26. in unam, ita ut exterminentur omnes quantitates y, z, v, &c., & earum fluxiones præter unam x & ejus fluxiones & t & $\dot t$, tum reſultabit æquatio hujuſce formulæ $P\overset{a}{\dot x} + Q\overset{a-1}{\dot x}\dot t + R\overset{a-2}{\dot x}\dot t^2 + S\overset{a-3}{\dot x}\dot t^3 + \&c. = L\dot t^a$, ubi P, Q, R, S, &c. & L denotant functiones quantitatis t, quæ facile deduci poſſunt.

Et ſimiliter erui poſſunt formulæ æquationum reſultantium ex datis æquationibus, quarum formulæ haud ſunt multo magis compoſitæ quam præcedentes.

Cor. Sint m æquationes $(m+1)$ variabiles quantitates $(x, y, z, v,$ &c., & $t)$ involventes, quarum formulæ ſint $a\overset{n}{\dot{x}} + b\overset{n}{\dot{y}} + c\overset{n}{\dot{z}} +$ &c. $+ a'\overset{n-1}{\dot{x}}\dot{t} + b'\overset{n-1}{\dot{y}}\dot{t} + c'\overset{n-1}{\dot{y}}\dot{t} +$ &c. $+ a''\overset{n-2}{\dot{x}}\dot{t}^2 + b''\overset{n-2}{\dot{y}}\dot{t}^2 + c''\overset{n-2}{\dot{y}}\dot{t}^2 +$ &c. $+ \dots + \overset{n-1}{a'}\dot{x}\dot{t}^{n-1} + \overset{n-1}{b'}\dot{y}\dot{t}^{n-1} + \overset{n-1}{c'}\dot{z}\dot{t}^{n-1} +$ &c. $+ \overset{n}{a'}x\dot{t}^n + \overset{n}{b'}y\dot{t}^n + \overset{n}{c'}z\dot{t}^n +$ &c. $+ T\dot{t}^n = 0$, ubi literæ $a, a', a'', \dots \overset{n}{a'}$; b, b', b'', &c., $\dots \overset{n}{b'}$; c, c', c'', &c., $\dots \overset{n}{c'}$; quaſcunque invariabiles quantitates reſpective denotant, & T eſt quæcunque functio quantitatis t; reducantur hæ m æquationes per prob. 26. in unam, ita ut exterminentur omnes variabiles & earum fluxiones præter duas, viz. x & ejus fluxiones, & t & $\dot{t}$; tum reſultabit æquatio formulæ $A\overset{\alpha}{\dot{x}} + B\overset{\alpha-1}{\dot{x}}\dot{t} + C\overset{\alpha-2}{\dot{x}}\dot{t}^2 + D\overset{\alpha-3}{\dot{x}}\dot{t}^3 + \dots + H\dot{x}\dot{t}^{\alpha-1} + Ix\dot{t}^{\alpha} = T'\dot{t}^{\alpha}$; literis A, B, C, D, &c., H & I invariabiles quantitates denotantibus, & T denotante functionem quantitatis t: hujuſce æquationis fluens in theor. 42. tradita fuit.

3. Reſolvi etiam poſſunt duæ vel plures n æquationes tres vel plures $(n+1)$ variabiles quantitates habentes, aſſumendo generales quantitates, quæ in ſe continent reſolutiones quantitatum quæſitas; & eas pro valoribus e ſingulis variabilibus quantitatibus in datis æquationibus contentis ſcribendo, & correſpondentes terminos reſultantes æquando.

Ex. 1. Sint $\dot{x} + (ax + by + cz)\dot{t} = 0$, $\dot{y} + (e'x + fy + gz)\dot{t} = 0$, $\dot{z} + (hx + my + nz)\dot{t} = 0$; aſſumantur $x = Ae^{ft} + Be^{gt} + He^{bt}$; $y = A'e^{ft} + B'e^{gt} + C'e^{bt}$; & $z = A''e^{ft} + B''e^{gt} + C''e^{bt}$, ubi A, B, C, &c.; A', B', C', &c.; A'', B'', &c. ſunt invariabiles quantitates; ſcribantur hæ quantitates pro ſuis valoribus in datis æquationibus, & e reſultantibus æquationibus inveſtigari poteſt fluens quæſita.

Ex. 2. Sint $\dot{x} + a\dot{y} + T(cx + ay)\dot{t} + \theta\dot{t} = 0$, & $\dot{y} + b\dot{x} + T(fx + gy)\dot{t} + \varepsilon\dot{t} = 0$; ſupponatur $x = Ae^{f\int T\dot{t}} + Be^{g\int T\dot{t}} + e^{f\int T\dot{t}}\int e^{-f\int T\dot{t}}(E\theta + H\varepsilon)\dot{t} + e^{g\int T\dot{t}}\int e^{-g\int T\dot{t}}(L\theta + M\varepsilon)\dot{t}$; $y = A'e^{f\int T\dot{t}} + B'e^{g\int T\dot{t}} + e^{f\int T\dot{t}}\int e^{-f\int T\dot{t}}(E'\theta + H'\varepsilon)\dot{t} + e^{g\int T\dot{t}}\int e^{-g\int T\dot{t}}(L'\theta + M'\varepsilon)\dot{t}$, ubi $A, A', B, B', E, H, E', H', L, M, L', M'$, ſunt invariabiles; & T, θ & ε ſunt functiones quantitatis t; & deinde ſcribantur hæ quantitates, &c. pro ſuis valoribus $x, y, \dot{x}, \dot{y}$, in datis æquationibus, & exinde inveniri poteſt fluens quæſita.

Facile deduci poſſunt infinitæ æquationes hujuſmodi, quarum fluentes innoteſcunt; aſſumantur enim æquationes pro fluentibus quæſitis, quarum inveniantur fluxiones, & methodis prius traditis conſequuntur infinitæ æquationes fluxionales, quarum fluentes acquiri poſſunt. Aliter: aſſumantur quæcunque æquationes relationem inter variabiles quantitates, &c. exprimentes pro fluentibus ipſis; deinde aſſumantur quæcunque fluxionales quantitates l, m, n, &c. & ex aſſumptis æquationibus & earum fluxionibus, &c. inveniantur quantitates (P, Q, R, &c.) quæ æquant aſſumptas quantitates l, m, n, &c. reſpective, tum $l = P$, $m = Q$, $n = R$, &c. erunt æquationes, quarum fluens particularis erit aſſumptæ æquationes prædictæ.

THEOR. XLVI.

Sit æquatio fluxionalis $A = o$ ordinis (n), in quâ fluit uniformiter x, cujus fluens ſit $B = o$; deinde transformentur utræque æquationes $A = o$ & $B = o$ reſpective in duas alias $C = o$ & $D = o$, ſcribendo pro $\ddot{y}$, $-\frac{\ddot{y}\ddot{x}}{\dot{x}}$; & ſic deinceps, ut docetur in prob. 34; tum $D = o$ erit fluens fluxionalis æquationis $C = o$.

Ex. Sit æquatio $\dot{y}\dot{x} - x\ddot{y} - a\ddot{y} - \frac{x\dot{y}^2}{b} = o$, in quâ fluit uniformiter x; ſcribatur pro $\ddot{y}$ ejus valor e prob. 34. deductus $-\frac{\dot{y}\ddot{x}}{\dot{x}}$, & reſultat $\dot{x}\dot{y} + \frac{x\dot{y}\ddot{x}}{\dot{x}} + \frac{a\dot{y}\ddot{x}}{\dot{x}} - \frac{x\dot{y}^2}{b} = o$; ducatur hæc æquatio in $\dot{x}$ & reſultat $\dot{x}^2\dot{y} + x\dot{y}\ddot{x} + a\dot{y}\ddot{x} - \frac{x\dot{x}\dot{y}^2}{b} = o$, cujus fluens eſt $x\dot{x}\dot{y} + a\dot{y}\dot{x} - \frac{x^2\dot{y}^2}{2b} = o$; quæ ſi modo generaliter corrigatur, evadet $x\dot{x}\dot{y} + a\dot{y}\dot{x} - \frac{x^2\dot{y}^2}{2b} + A\dot{y}^2 = o$, ubi A denotat quantitatem invariabilem ad libitum aſſumendam: aliter ducatur data æquatio in $\frac{\dot{x}}{\dot{y}^2}$, & reſultat $\frac{\dot{x}^2\dot{y} - x\dot{x}\ddot{y}}{\dot{y}^2} - \frac{a\dot{x}\ddot{y}}{\dot{y}^2} - \frac{x\dot{x}}{b} = o$, cujus fluens generalis erit $\frac{x\dot{x}}{\dot{y}} + \frac{a\dot{x}}{\dot{y}} - \frac{x^2}{2b} = A$ conſtant. quant. quæ æquatio eadem eſt ac præcedens.

Cor. . Sit æquatio $p\dot{x} + q\dot{y} = o$, inveniatur ejus fluxio ex hypotheſi quod $\dot{x}$ ſit conſtans, & reſultat $\dot{p}\dot{x} + \dot{q}\dot{y} + q\ddot{y} = o$, cujus generalis fluens erit $p\dot{x} + q\dot{y} = C\dot{x}$; deinde inveniatur ejus fluxio ex hypotheſi quod $\dot{y}$ ſit conſtans & reſultat $\dot{p}\dot{x} + \dot{q}\dot{y} + p\ddot{x} = o$, cujus fluens erit $p\dot{x} + q\dot{y} = C\dot{y}$; quæ fluentes minime eædem ſunt: in æquatione $\dot{p}\dot{x} + \dot{q}\dot{y} + q\ddot{y} = o$ pro $\ddot{y}$ ſcribatur ejus valor prædictus $-\frac{\ddot{x}\dot{y}}{\dot{x}}$, & reſultat $\dot{p}\dot{x} + \dot{q}\dot{y} - \frac{q\ddot{x}\dot{y}}{\dot{x}} = o$, ducatur hæc æquatio in $\frac{1}{\dot{x}}$, & reſultat fluxio, cujus fluens eſt $p + \frac{q\dot{y}}{\dot{x}} = C$; vel quod idem eſt $p\dot{x} + q\dot{y} = C\dot{x}$; quæ eadem eſt ac fluens æquationis $p\dot{x} + q\dot{y} + q\ddot{y} = o$.

Idem etiam affirmari poteſt de fluentibus quarumcunque fluxionum hoc modo transformatarum: & e principiis prius traditis & datis multiplicatoribus datarum fluxionalium æquationum facile erui poſſunt multiplicatores ex conſimili ſubſtitutione fluxionalium æquationum hoc modo transformatarum.

PROB. LXII.

Datâ fluxionali æquatione duas variabiles quantitates x *&* y *& earum fluxiones habente, invenire utrum altera* (y) *exprimi poteſt in algebraicis & haud exponentialibus terminis alterius* (x), *necne.*

Inveniantur ſinguli valores (A, B, C, &c.) quantitatis y (eâdem correctione adhibitâ) in terminis progredientibus ſecundum dimenſiones alterius x; deinde per Med. Algeb. inveniatur annon ſumma e ſingulis hiſce valoribus ſit algebraica & rationalis functio literæ x.

Ducantur quique duo e ſingulis prædictis valoribus in ſeſe, deinde inveniatur utrum aggregatum e ſingulis prædictis rectangulis ſit algebraica & rationalis functio prædictæ literæ x, necne.

Et ſic ducantur quique tres, quatuor, &c. prædicti valores in ſeſe, & inveniatur, utrum aggregata e ſingulis hiſce contentis ſint rationales & algebraicæ functiones literæ x, necne; ſi vero detegantur prædictæ rationales functiones, tum datur æquatio algebraica, quæ exprimit relationem inter x & y; ſin haud exprimi poſſint prædicta aggregata per illas rationales functiones, tum nulla aſſignari poteſt æquatio algebraica, quæ exprimit relationem inter x & y.

Et ſic de inveniendis algebraicis æquationibus exprimentibus relationes inter quaſcunque functiones literarum x & y.

Omnia hæc etiam ad plures fluxionales æquationes applicari poſſunt.

PROB. LXIII.

Datâ fluxionali æquatione exponentiales quantitates involvente; invenire fluentem datæ æquationis, ſi modo finitis terminis exprimi poſſit.

Per prob. 3. inveniri poteſt quantitas, quæ in datam exponentialem datæ æquationis quantitatem ducta, exponentialem fluxionem dat, cujus fluens inveniri poteſt; ducatur data æquatio in quantitatem inventam, & reſultantis æquationis inveniatur fluens; & conſit problema.

Sæpe vero exigit problema, ut prius transferantur exponentiales quantitates e terminis datæ æquationis, in quibus continentur, in reliquos; deinde per præcedentem methodum progrediendum eſt.

Ex. 1. Sit exponentialis æquatio fluxionalis $y^2\dot{x} + y x \log. x \times \dot{y} + x\dot{y} = \frac{p\dot{x} + q\dot{y}}{x^{y-1}}$, ubi p & q ſunt functiones literarum x & y, & ſit fluxio $p\dot{x} + q\dot{y}$ integrabilis; ducatur data æquatio in x^{y-1}, & reſultat æquatio, cujus fluens invenietur $x^v \times y = \int.(p\dot{x} + q\dot{y}) + a$, ubi a denotat invariabilem quantitatem ad libitum aſſumendam.

PROB. LXIV.

Datâ fluxionali æquatione, in quâ fluentes continentur; invenire ejus fluentialem æquationem.

Hoc ſæpe perfici poteſt ex iiſdem principiis, ac ea; quæ in præcedente problemate de exponentialibus & fluxionalibus æquationibus tradita fuere.

Ex. . Sit æquatio fluxionalis $r\dot{x}\int. x^m\dot{y} + x^{m+1}\dot{y} = p\dot{x} \times x^{-r+1}$; ducatur hæc æquatio in x^{r-1} & reſultat $rx^{r-1}\dot{x}\int. x^m\dot{y} + x^{m+r}\dot{y} = p\dot{x}$, cujus fluens eſt $x^r\int. x^m\dot{y} = \int. p\dot{x} + a$.

Æquationes in quibus fluentiales quantitates involvuntur, facile reduci poſſunt, ita ut fluentiales quantitates exterminentur.

Eadem etiam applicari poſſunt ad plures fluxionales æquationes fluentiales & exponentiales quantitates involventes.

PROB. LXV.

Datâ fluxionali æquatione duas variabiles quantitates x & y *& earum fluxiones involvente; invenire utrum* x & y *ſint algebraicæ, exponentiales vel fluentiales functiones novæ aſſumptæ quantitatis* z.

Aſſumantur pro variabilibus x & y functiones quantitatis z generalibus terminis expreſſæ, quæ neceſſario exprimunt valores quantitatum x & y; ſcribantur hæ functiones pro ſuis valoribus x & y in datâ æquatione, & ex æquatis terminis reſultantis æquationis correſpondentibus inveniantur functiones quantitatis z quæſitæ, ſi modo tales recipiat data æquatio; & perficitur problema.

Cor. . Hinc inveniri poſſunt infinitæ æquationes fluxionales relationem inter duas variabiles quantitates x, y & earum fluxiones exprimentes, quarum variabiles (x & y) majori facilitate exprimi poſſunt in terminis aſſumptæ variabilis z; quam variabilis x exprimi poteſt in terminis quantitatis y, vel y in terminis quantitatis x.

Aſſumantur valores variabilium x & y in terminis novæ aſſumptæ quantitatis z inter ſe reſpective reſpondentes; reducantur hæ duæ aſſumptæ æquationes in unam, ita ut exterminetur variabilis quantitas z, & reſultat quæſita æquatio relationem inter x & y & earum fluxiones exprimens, cujus fluens datur in terminis aſſumptæ quantitatis z.

Ex. 1. Aſſumantur $\log. z + az = y$, & $\log. z + fz = x$, unde $(a-f)z = y - x$, & $z = \frac{y-x}{a-f}$; ſed e primâ æquatione ſequitur $\frac{\dot{z}}{z} + a\dot{z} = \dot{y}$; ſcribantur igitur $\frac{y-x}{a-f}$ & $\frac{\dot{y}-\dot{x}}{a-f}$ pro z & $\dot{z}$ in hâc æquatione & reſultat æquatio quæſita $\frac{\dot{y}-\dot{x}}{y-x} + \frac{a\dot{y}-a\dot{x}}{a-f} = \dot{y}$.

Ex. 2. Aſſumantur $(\log. z)^2 + az = y$, & $\log. z + fz = x$; unde $2fz \log. z + f^2z^2 - az = x^2 - y$, & exinde $f^2z^2 + az = -x^2 + 2fxz + y$, & conſequenter $z = \frac{2fx-a}{2f^2} \mp \sqrt{\left(\frac{(2fx-a)^2}{4f^4} - x^2 + y\right)}$; ſcribatur hæc quantitas pro z, & ejus fluxio pro $\dot{z}$ in æquatione $\frac{\dot{z}}{z} + f\dot{z} = \dot{x}$; & reſultat æquatio quæſita.

Ex. 3. Aſſumantur æquationes ſ. $(e+fz^n)^m\dot{z} + ax^n = y$ & ſ. $\frac{\dot{z}}{z} + fz = x$, unde $\frac{\dot{z}}{z} + f\dot{z} = \dot{x}$ & $\dot{z} = \frac{z\dot{x}}{1+fz}$; ſcribatur valor $\frac{(1+fz)\dot{z}}{z}$, &c. pro $\dot{x}$, &c. in æquatione $(e+fz^n)^m\dot{z} + nax^{n-1}\dot{x} = \dot{y}$; deinde per prob. 26. ita reducantur æquationes reſultantes, ut exterminetur z; & reſultat æquatio quæſita.

Et ſic inveniri poſſunt plures (n) æquationes $(n+1)$ variabiles quantitates & earum fluxiones involventes, quarum variabiles quantitates

titates exprimuntur per datas functiones aſſumptarum variabilium quantitatum.

Hic animadvertendum eſt has reſolutiones non eſſe generales reſolutiones æquationis reſultantis: ſi vero in aſſumptis reſolutionibus, i. e. duabus æquationibus involventibus quantitates x, y, & z contineatur quantitas ad libitum aſſumenda; & ea quantitas ex reſultante æquatione evaneſcat, cum evaneſcat quantitas z, tum aſſumptæ erunt generales reſolutiones reſultantis æquationis relationem inter x & y exprimentis; ſi modo ſit fluxionalis æquatio primi ordinis: ſi autem duæ vel tres, &c. independentes & invariabiles quantitates in aſſumptis reſolutionibus contineantur, tum ſi modo reducantur aſſumptæ æquationes relationes inter x, y, z, &c. exprimentes in unam, ita ut exterminentur omnes præter x & y, &c.; & ſi modo evaneſcant (m) invariabiles quantitates ex fluxionali æquatione (m) ordinis, tum erunt aſſumptæ æquationes generales reſolutiones æquationis reſultantis (m) ordinis.

Sint duæ datæ æquationes ($A=o$ & $B=o$) exprimentes relationes inter variabiles x, y & z; in quibus continetur invariabilis quantitas ad libitum aſſumenda (a); tum ita reducantur tres æquationes $A=o$, $B=o$ & $\dot{A}=o$, ut exterminentur duæ quantitates (z & a), & reſultat æquatio relationem inter x & y exprimens, cujus duæ datæ æquationes erunt generalis reſolutio.

Sint tres vel quatuor vel plures (n) æquationes ($A=o$, $B=o$, $C=o$, &c.) exprimentes relationes inter $(h+1)$ variabiles (x, y, u, v, &c.), & $(n-h)$ variabiles (z, w, Z, &c.), & duas vel tres vel plures $(n-h)$ invariabiles quantitates (a, b, c, &c.) ad libitum aſſumendas: ita reducantur æquationes $A=o$, $\dot{A}=o$, $B=o$, $\dot{B}=o$, $C=o$, $\dot{C}=o$, &c. in (h) æquationes, ut exterminentur quantitates variabiles (z, w, Z, &c.) & invariabiles a, b, c, &c.; tum reſultant (h) æquationes, quarum datæ erunt reſolutio.

Et ſic de æquationibus aſſumptis, in quibus variabiles habent fluxiones ſuperiorum ordinum.

THEOR. XLVII.

In detegendâ fluxione n ordinis contenti $a \times b \times c \times d \times e \times$ &c. $f \times g \times h \times$ &c. coefficientes cujuſcunque contenti $\overset{m}{\dot{a}}\ \overset{r}{\dot{b}}\ \overset{s}{\dot{c}}\ \overset{t}{\dot{d}}$, &c. $f g h$, &c. erit æqualis fractioni, cujus numerator eſt $n\,(n-1)\,(n-2)\,(n-3)\ .\ .\ .\ .\ .\ (n-m-r-s-t-\&c.+1)$, denominator vero $1\,.\,2\,.\,3\,.\,.\,m \times 1\,.\,2\,.\,3\ .\,.\,.\ r \times 1\,.\,2\,.\,3\ .\,.\,.\ s \times$ &c. ubi $n = m + r + s + t +$ &c.

Cor. . Hinc ſi fluxio reduci poſſit in prædictam formulam $= o$, ejus fluens erit $a \times b \times c \times d \times \&c. = A x^{n+1} + B x^{n} + C x^{n-1} +$ &c. ubi A, B, C, &c. ſunt quæcunque invariabiles quantitates, & x quantitas quæ fluit uniformiter.

THEOR. XLVIII.

Sit fluxionalis æquatio, &c. $(\dot{d} \times (\dot{c} \times (\dot{b}\, \dot{a}))) = \overset{n}{\dot{a}}\, b\, c\, d$ &c. $+ ((n-1)\,\dot{b} + (n-2)\,\dot{c} + (n-3)\,\dot{d} + \&c.)\,\overset{n-1}{\dot{a}} + \&c. = o$, & erit ejus generalis fluens $A + \int .\, \frac{B}{b} + \int .\, \frac{\int .\, \frac{C}{c}}{b} + \int .\, \frac{\int .\, \frac{\int .\, \frac{D}{d}}{c}}{b} + \&c. = o$, ubi A, B, C, D, &c. ſunt conſtantes quantitates ad libitum aſſumendæ nullius, primi, ſecundi, tertii, &c. fluxionalis ordinis.

Cor. . Si vero b, c, d, e, &c. ſint functiones quantitatis, quæ fluit uniformiter, tum ejus fluens facile inveniri poteſt.

THEOR. XLIX.

1. Sint x & y reſpective quæcunque algebraicæ functiones quantitatum t & u & earum fluxionum, & ſi modo detur relatio algebraica inter quantitates t & u; tum dabitur algebraica relatio inter quantitates x & y.

Facile conſtat.

Cor. . Hinc facile deduci poſſunt quantitates x & y, quæ erunt algebraicæ functiones ex ſe ipſis, ſi modo duæ aliæ (t & u) etiam ſint algebraicæ functiones ex ſe ipſis: aſſumantur enim functiones quantitatum t, u, & earum fluxionum, quæ ſint x, & y; deinde inveniantur t & u in terminis quantitatum x & y, & reſolvi poteſt problema.

Ex. . Sint $y=\frac{u\sqrt{(\dot{t}^2-\dot{u}^2)}}{\dot{t}}$, & $x=t-\frac{u\dot{u}}{\dot{t}}$; ita reducantur hæ æquationes ut inveniantur (u & t) termini quantitatum x & y, & reſultant $u=\frac{y\sqrt{(\dot{y}^2+\dot{x}^2)}}{\dot{x}}$ & $t=x+\frac{y\dot{y}}{\dot{x}}$; & conſtat ſi t ſit algebraica functio quantitatis u, tum erit x algebraica functio quantitatis y.

2. Sit æquatio $\frac{\ddot{y}}{\phi:(\dot{x}\,\&\,\dot{y})}=\frac{1}{ax+by}$; ubi per $\phi:(\dot{x}\,\&\,\dot{y})$ denotatur functio fluxionum $\dot{x}$ & $\dot{y}$ duarum dimenſionum, & x fluit uniformiter; in eâ pro $\dot{y}$ ſcribatur $z\dot{x}$, & reſultat $(ax+b\textstyle\int.(z\dot{x}))\times\dot{z}=\phi':(z)\times\dot{x}$, & conſequenter $b\textstyle\int.z\dot{x}=\frac{\phi':(z)\,\dot{x}}{\dot{z}}-ax$, & exinde $bz=\phi'':(z)-\frac{\phi':(z)\,\ddot{z}}{\dot{z}^2}-a$, & $\frac{\ddot{z}}{\dot{z}}=(\phi'':(z)-bz-a)\,\dot{z}$, unde log. $\frac{\dot{z}}{\dot{x}}=\textstyle\int.(\phi'':(z)-bz-a)\,\dot{z}+A=V+A$, & conſequenter $\dot{z}=e^{V+A}\dot{x}$, unde $\dot{z}e^{-V-A}=\dot{x}$, cujus fluens $x=\textstyle\int.\dot{z}e^{-V-A}$ & $y=\textstyle\int.z\dot{x}=\textstyle\int.z\dot{z}e^{-V-A}$.

Ex. 1. Sit æquatio $\frac{-\ddot{y}}{\dot{x}^2+\dot{y}^2}=\frac{1}{y}$, pro $\dot{y}$ ſcribatur $z\dot{x}$, & reſultat æquatio $-y\ddot{y}=\dot{z}\dot{x}\times-\textstyle\int.\dot{z}\dot{x}=(1+z^2)\dot{x}^2$, unde $-\textstyle\int.z\dot{x}=\frac{(1+z^2)\dot{x}}{\dot{z}}$, & exinde $-z\dot{x}=2z\dot{x}-\frac{(1+z^2)\ddot{z}}{\dot{z}^2}\dot{x}$, & $\frac{\ddot{z}}{\dot{z}}=\frac{3z\dot{z}}{1+z^2}$, & log. $\frac{\dot{z}}{\dot{x}}=\frac{3}{2}$ log. $(1+z^2)$, unde $\dot{z}=a(1+z^2)^{\frac{3}{2}}\dot{x}$; & exinde $a\dot{x}$

$= \frac{\dot{z}}{(1+z^2)^{\frac{3}{2}}}$, & $\dot{y} = z\dot{x} = \frac{z\dot{z}}{a(1+z^2)^{\frac{3}{2}}}$, & conſequenter $x = \frac{az}{\sqrt{(1+z^2)}} + c$ & $y = -\frac{1}{a}(1+z^2)^{-\frac{1}{2}} + b$. Aliter: ob $y\ddot{y} + \dot{y}^2 = -\dot{x}^2$ erit $y\dot{y} = -x\dot{x} + a\dot{x}$, & conſequenter $y^2 = -x^2 + 2ax + b$.

Ex. 2. Sit $\frac{\ddot{y}}{\dot{x}^2 + \dot{y}^2} = \frac{1}{y}$, pro $\dot{y}$ ſcribatur $z\dot{x}$, & reſultat æquatio $y\ddot{y} = \dot{z}\dot{x}\int . z\dot{x} = (1+z^2)\dot{x}^2$, unde $\int . z\dot{x} = \frac{(1+z^2)\dot{x}}{\dot{z}}$, & exinde $\frac{\ddot{z}}{\dot{z}} = \frac{z\dot{z}}{1+z^2}$, & log. $\frac{\dot{z}}{a\dot{x}} = \frac{1}{2}$ log. $(1+z^2)$, unde $\frac{\dot{z}}{a(1+z^2)^{\frac{1}{2}}} = \dot{x}$, & $ax = \frac{1}{2} \times$ log. $b \times (z + \sqrt{(1+z^2)})$, & $\dot{y} = z\dot{x} = \frac{1}{a}\frac{z\dot{z}}{(1+z^2)^{\frac{1}{2}}}$, cujus fluens eſt $y = \frac{1}{a} \times (1+z^2)^{\frac{1}{2}} + c$. Eadem principia ulterius promovere liceat.

3. Sit fluxionalis æquatio $A = o$ ordinis (m), in quâ continentur (r) diverſæ invariabiles quantitates ad libitum aſſumendæ, quæ in datâ $B = o$ æquatione $(m+r)$ ordinis eaſdem variabiles quantitates & earum fluxiones involvente non inveniuntur: ſcribantur quantitates & earum fluxiones ex æquatione $A = o$ acquiſitæ pro ſuis valoribus in æquatione $B = o$, & ſi ex nonnullis radicibus æquationis $A = o$ pro ſuis valoribus in æquatione $B = o$ ſubſtitutis evaneſcant prædictæ (r) invariabiles quantitates, ex nonnullis vero non; tum æquatio $A = o$ reſolvi poteſt in plures, quarum nonnullæ ſunt generales fluentes datæ fluxionalis æquationis $B = o$, nonnullæ vero non.

4. Sit $V = o$ fluens fluxionalis æquationis $W = o$; tum, ut prius probatur, omnis functio quantitatis V erit etiam fluens æquationis $B = o$; unde fluens cujuſcunque fluxionalis æquationis contineat in ſe quemlibet numerum invariabilium & quodammodo variabilium quantitatum ad libitum aſſumendarum. Si æquationis $V = o$ diviſor $(P = o)$ maxime ſimplex ſit fluens (m) ordinis fluxionalis æquationis $W = o$,

$W=o$, & in ſe contineat (m) invariabiles quantitates ad libitum aſſumendas, quæ in æquatione $W=o$ haud inveniuntur, tum $P=o$ erit generalis fluens prædictæ æquationis $W=o$, ſin aliter particularis.

THEOR. L.

1. Sit $\dot{V}=p\dot{x}+q\dot{y}=\left(\frac{\dot{V}}{\dot{x}}\right)\dot{x}+\left(\frac{\dot{V}}{\dot{y}}\right)\dot{y}=D$; & detur æquatio relationem inter p, q, x & y exprimens; tum ex eâ non datur valor quantitatis (V).

2. Sint duæ æquationes relationes inter prædictas quantitates exprimentes; tum ex iis deduci poſſunt valores quantitatum p & q reſpective in terminis quantitatum x & y; & conſequenter facile conſtabit, annon fluxio $p\dot{x}+q\dot{y}$ ſit integrabilis, necne.

Sint p & q datæ functiones quantitatum x & y; ita reducantur hæ functiones, ut exterminetur una incognita quantitas vel p vel q vel x vel y; & reſultabit æquatio relationem inter tres reliquas exprimens, unde inveniri poteſt quantitas p in terminis quantitatum x & y; & exinde inveniri poteſt quantitas q in terminis quantitatum x & y, & tum ex reſultante æquatione aſſignari poteſt valor quantitatis (V).

Si detur relatio inter p & x vel y vel q, tum ex eâ non deduci poteſt valor quantitatis V.

3. Et ſimiliter cum dentur relationes inter has quantitates $\left(\frac{\dot{V}}{\dot{x}}\right)=p$, $\left(\frac{\dot{V}}{\dot{y}}\right)=q$, $\left(\frac{\dot{V}}{\dot{z}}\right)=r$, $\left(\frac{\ddot{V}}{\dot{x}^2}\right)=\alpha$, $\left(\frac{\ddot{V}}{\dot{x}\dot{y}}\right)=\beta$, &c.; tum ſi duæ ſolummodo ſint variabiles quantitates x & y & deduci poſſint duæ ex his quantitatibus in terminis quantitatum x & y & earum fluxionum, exprimi poteſt V in terminis quantitatum x & y & earum fluxionum: ſi vero tres invariabiles quantitates in datis æquationibus contineantur; tum, ſi tres prædictæ quantitates exprimi poſſint in terminis quantitatum x, y & z; V exprimi poteſt in terminis quan-

titatum x, y & z & earum fluxionum, ſin aliter vero non: & ſic deinceps.

Omnes æquationes hujuſce generis reduci poſſunt ad fluxionales æquationes; ſcribantur enim in iis earum valores e fluxionalibus æquationibus deducti, & reſultant fluxionales æquationes quæſitæ.

PROB. LXVI.

Datis quibuſdam fluxionibus $(\dot{V})$, *in quibus continentur duæ vel plures variabiles quantitates; invenire caſus, in quibus earum fluentes* (V) *exprimi poſſunt: ſi data fluxio dividi poſſit in fluxiones integrabiles hujuſce generis* $P\dot{x}$, *tum erit* P *functio quantitatis* $\dot{x}$; *& ſi vero reduci poſſit in formulam* $p\dot{x} + q\dot{y} + r\dot{z} + s\dot{v} +$ *&c. tum erunt* p, q, r, s, *&c. functiones quantitatum* x, y, z, v, *&c. Conſimiles etiam propoſitiones etiam ad fluxionales æquationes ſuperiorum ordinum applicari poſſunt.*

1. Sit $\dot{z} = P\dot{x}$, & erit P functio quantitatis x; & ſic ſit $\dot{z} = p(\dot{x} + a\dot{y})$, & erit p functio quantitatis $x + ay$; vel ſimiliter ſit $\dot{z} = px\left(\frac{\dot{x}}{x} + \frac{\dot{y}}{a}\right)$ & erit $px = \varphi : \left(\text{log.}\, x + \frac{y}{a}\right)$.

2. Sit $\dot{z} = \dot{w} + P\dot{x}$, & erit P functio quantitatis x; unde ſit $\dot{z} = p\dot{x} + \frac{\gamma + \alpha p}{\beta}\dot{y} = \frac{\gamma}{\beta}\dot{y} + p(\dot{x} + \frac{\alpha}{\beta}\dot{y})$, & erit $p = \varphi : (x + \frac{\alpha}{\beta}y)$.

3. Sit $\dot{z} = p\dot{x} - p\dot{y} + \frac{z\dot{y}}{a}$, & erit $\frac{p}{z}(\dot{x} - \dot{y}) = \frac{\dot{z}}{z} - \frac{\dot{y}}{a}$ & conſequenter $\frac{p}{z} = \varphi : (x - y)$.

4. Sit $\dot{z} = p\dot{x} + q\dot{y}$, tum fingatur y conſtans, & inveniatur $\int . p\dot{x} = \alpha$; & erit $z = \alpha + \varphi : (y)$. Et ſic de pluribus variabilibus quantitatibus in data æquatione contentis.

5. Sit $\dot{z} = p\dot{x} + q\dot{y}$; & erit $z = px + qy - \int . (x\dot{p} + y\dot{q})$.

6. Sit $p\dot{x} + q\dot{y} = r\dot{v} + s\dot{z} + t\dot{w}$, ubi p & q ſunt functiones quantitatum x & y; & r, s & t ſunt functiones quantitatum v, z & w:

ſint $\dot{S} = \frac{M}{b} \times (p\dot{x} + q\dot{y})$ & $\dot{R} = \frac{L}{k}(r\dot{v} + s\dot{z} + t\dot{w})$; tum erit $\dot{R} = \frac{bL\dot{S}}{kM}$, & conſequenter R & $\frac{bL}{kM}$ erunt functiones quantitatis S: & exinde S & R erunt functiones quantitatis $\frac{kL}{bM}$.

7. Sit $\dot{z} = p\dot{x} + q\dot{y}$, ubi p eſt functio quantitatis q; tum erit $\dot{p} = \mathcal{Q}\dot{q}$, ubi $\mathcal{Q}$ eſt functio quantitatis q; unde $z = px + qy - \int.(x\dot{p} + y\dot{q} = x\mathcal{Q}\dot{q} + y\dot{q} = (x\mathcal{Q} + y)\dot{q})$; & conſequenter $x\mathcal{Q} + y$ & x erunt functiones quantitatis q.

Sit $\dot{z} = p\dot{x} + \frac{px}{a}\dot{y} = px\left(\frac{\dot{x}}{x} + \frac{\dot{y}}{a}\right) = p\dot{x} + x\dot{p} - xp\left(\frac{\dot{p}}{p} - \frac{\dot{y}}{a}\right)$; unde xp & z erunt functiones quantitatum $\frac{y}{a} + \log. x$ & $\log. p - \frac{y}{a}$, cum $q = \frac{px}{a}$.

8. Sæpe e ſubſtitutione deduci poſſunt hujuſce generis problematum reſolutiones. E. g. ſit $\dot{z} = p\dot{x} + (pX + T)\dot{y}$, ubi X & T ſunt functiones quantitatis x; ſtatuatur $p = r - \frac{T}{X}$, & prodit $\dot{z} = r\dot{x} - \frac{T}{X}\dot{x} + rX\dot{y} = \frac{-T\dot{x}}{X} + rX\left(\frac{\dot{x}}{X} + \dot{y}\right)$, unde rX erit functio quantitatis $\int.\frac{\dot{x}}{X} + y$: E. 2. Sit $\dot{V} = p\dot{x} + Px\dot{y} + \Pi\dot{y}$, ubi P & Π ſunt functiones ipſius p, unde $V = px + \int.(Px\dot{y} + \Pi\dot{y} - x\dot{p})$; ſtatuatur $Px + \Pi = v$, & reſultat $V = px + \int.\left(v\dot{y} - \frac{v\dot{p}}{P} + \frac{\Pi\dot{p}}{P}\right) = px + \int.\frac{\Pi\dot{p}}{P} + \int.v\left(\dot{y} - \frac{\dot{p}}{P}\right)$, & conſequenter $v = \phi:\left(y - \int.\frac{\dot{p}}{P}\right)$.

9. Nonnunquam in his reſolutionibus e dato particulari valore erui poteſt ejus generalis valor. E. g. Sit $\dot{z} = p\dot{x} + q\dot{y}$, ubi $z = Mp + Nq$ (exiſtentibus M & N functionibus quibuſve variabilium

x & y),

x & y), & ſit V particularis valor quantitatis (z); ſtatuantur $z = Vv$, & $\dot{v} = r\dot{x} + s\dot{y}$, & erunt $p = Pv + Vr$ & $q = Qv + Vs$, ubi $\dot{V} = p\dot{x} + q\dot{y} =$ in hoc caſu $P\dot{x} + Q\dot{y}$, ideoque $z = Mp + Nq = (MP + NQ)v + V(Mr + Ns) = Vv$; at $V = MP + NQ$, ergo $Mr + Ns = 0$, unde $\dot{v} = r\left(\dot{x} - \frac{M\dot{y}}{N}\right) = \frac{r}{N}(N\dot{x} - M\dot{y})$; inveniatur multiplicator R, ita ut $R(N\dot{x} - M\dot{y}) = \dot{T}$, & erit $\dot{v} = \frac{r\dot{T}}{NR}$, & exinde $\frac{r}{NR} = \phi : (T)$.

10. Nonnunquam e notis methodis deduci poteſt formula particularis valoris. E. g. Sit $\dot{z} = p\dot{x} + q\dot{y}$ & $z = p(\alpha x + \beta y) + q(\gamma x + \delta y)$, ſtatuatur $u = \frac{y}{x}$ functio nullius dimenſionis, & $z = \phi : (u)$, unde $\dot{z} = \dot{u}\phi' : u = \left(\frac{\dot{y}}{x} - \frac{y\dot{x}}{x^2} = \frac{\dot{y}}{x} - \frac{u\dot{x}}{x}\right)\phi' : (u)$; & exinde $p = -\frac{u}{x}\phi' : (u) = -\frac{u\dot{z}}{x\dot{u}}$ & $q = \frac{1}{x}\phi' : (u) = \frac{\dot{z}}{x\dot{u}}$; ſcribantur hæ quantitates pro ſuis valoribus in æquatione $z = p(\alpha x + \beta y) + q(\gamma x + \delta y)$, & reſultabit $\frac{\dot{z}}{z} = \frac{\dot{u}}{\gamma + (\delta - \alpha)u - \beta u^2} =$ flux. log. V; eritque V particularis valor quantitatis z.

Hoc particulare valore dato, generalis facile per præcedentem methodum inveniri poteſt.

11. Sit $\dot{z} = p\dot{x} + q\dot{y}$, ubi p & q denotant functiones quantitatum x, y & z; deinde ex æquatione $\left(\frac{\dot{p}}{\dot{y}}\right) = \left(\frac{\dot{q}}{\dot{x}}\right)$ inveniatur z in terminis quantitatum x & y; & exinde p & q in terminis earundem quantitatum x & y, quæ ſint P & Q: tum detegatur multiplicator M, qui in $P\dot{x} + Q\dot{x}$ ductus, eam reddit integrabilem; unde $MP\dot{x} + MQ\dot{y} = M\dot{z} = \dot{S}$; & conſequenter M erit functio quantitatis z.

Infinitæ conſimiles deduci poſſunt propoſitiones: primo enim aſſumantur æquationes, quarum reſolutiones hujuſce formulæ dantur; deinde ex ſubſtitutione, &c. reducantur hæ æquationes ad alias, quarum reſolutiones non apparent; tum facile reduci poſſunt poſteriores æquationes ad priores.

Eadem principia etiam ad fluxionales æquationes ſuperiorum ordinum extendi poſſunt.

THEOR. LI.

Si S ſit functio quantitatum x, y, z, v, w, &c.; quæ evaneſcat; cum $z = a$, $v = b$, $w = c$, &c.; tum inveniatur fluxio $\dot{S}$ ex hypotheſi quod z, v, w, &c. ſint invariabiles & $z = a$, $v = b$, $w = c$, &c. & reliquæ x, y, &c. variabiles; & erit $\dot{S} = o$; ſi enim fluens $S = o$, tum ejus fluxio a fortiori $\dot{S}$ nihilo erit æqualis.

PROB. LXVII.

Quaſcunque fluxionales quantitates per quantitates diverſæ formulæ denotare.

Sit V functio quantitatum x, y, v, &c. tum erit $\dot{V} = p\dot{x} + q\dot{y} + r\dot{z} +$ &c. $= \dot{x}\left(\frac{\dot{V}}{\dot{x}}\right) + \dot{y}\left(\frac{\dot{V}}{\dot{y}}\right) + \dot{z}\left(\frac{\dot{V}}{\dot{z}}\right) +$ &c. & $\ddot{V} = p\ddot{x} + q\ddot{y} + r\ddot{z} + \dot{p}\dot{x} + \dot{q}\dot{y} + \dot{r}\dot{z} +$ &c. Sint vero $\dot{p} = \alpha\dot{x} + \beta\dot{y} + \gamma\dot{z} +$ &c. $\dot{q} = \delta\dot{x} + \varepsilon\dot{y} + \zeta\dot{z} +$ &c. $\dot{r} = \lambda\dot{x} + \mu\dot{y} + \nu\dot{z} +$ &c. unde per theor. 2. erunt $\beta = \delta$, $\gamma = \lambda$, $\zeta = \mu$, &c. & conſequenter $\ddot{V} = p\ddot{x} + q\ddot{y} + r\ddot{z} + \dot{p}\dot{x} + \dot{q}\dot{y} + \dot{r}\dot{z} +$ &c. $= p\ddot{x} + q\ddot{y} + r\ddot{z} + \alpha\dot{x}^2 + \varepsilon\dot{y}^2 + \nu\dot{z}^2 + 2\beta\dot{y}\dot{x} + 2\gamma\dot{z}\dot{x} + 2\mu\dot{z}\dot{y} +$ &c. $=$ (per præcedent. notat.) $\ddot{x}\left(\frac{\dot{V}}{\dot{x}}\right) + \ddot{y}\left(\frac{\dot{V}}{\dot{y}}\right) + \ddot{z}\left(\frac{\dot{V}}{\dot{z}}\right) + \dot{x}^2\left(\frac{\ddot{V}}{\dot{x}^2}\right) + \dot{y}^2\left(\frac{\ddot{V}}{\dot{y}^2}\right) + \dot{z}^2\left(\frac{\ddot{V}}{\dot{z}^2}\right) + 2\dot{y}\dot{x}\left(\frac{\ddot{V}}{\dot{y}\dot{x}}\right) + 2\dot{z}\dot{x}\left(\frac{\ddot{V}}{\dot{x}\dot{z}}\right) + 2\dot{y}\dot{z}\left(\frac{\ddot{V}}{\dot{y}\dot{z}}\right) +$ &c. ubi per $\left(\frac{\ddot{V}}{\dot{x}\dot{y}}\right)$ denotatur ſecunda fluxio quantitatis V per $\dot{x}\dot{y}$ diviſa, in quâ inveniendâ primum ſolummodo habetur x variabilis, deinde ſolummodo habetur y variabilis: & ſic de omnibus quantitatibus hoc modo denotatis: eâdem vero methodo inveniri poſſunt valores fluxionum $\dddot{V}$, $\ddddot{V}$, &c. ſed hic animadvertendum eſt, ſi modo ſit quantitas $\pi \dot{x}^n \ddot{x}^m \dddot{x}^r \times \&c. \times \dot{y}^s \ddot{y}^t \times \&c. \times \dot{z}^k \ddot{z}^l \times \&c.$ tum per præcedentem notationem ſic denotari debet, viz. $\dot{x}^n \ddot{x}^m \dddot{x}^r \times \&c. \times \dot{y}^s \times \ddot{y}^t \times \&c. \times \dot{z}^k \times \ddot{z}^l \times \&c. \times$

$$\left(\frac{\overset{n+m+r+\&c.+s+t+\&c.+k+l+\&c.}{V}}{\dot{x}^{n+m+r+\&c.}\,\dot{y}^{s+t+\&c.}\,\dot{z}^{k+l+\&c.}\ \&c.}\right).$$

Hinc transformari poteſt quæcunque quantitas V & ejus fluxionum functio in functiones ejuſdem quantitatis V & ejus fluxionum ex hypotheſi quod, 1mo. una quantitas fluat uniformiter, deinde altera; & ſic deinceps.

PROB. LXVIII.

Si vero detur quantitas, quæ ſit fluens $\int. P\dot{x}$ *fluxionis* $P\dot{x}$, *duas vel plures quantitates* (x, y, z, *&c.*) *involvens, ex hypotheſi quod una* x *ſolummodo ſit variabilis; invenire fluxionem fluentis* $\int. P\dot{x}$ *ex hypotheſi, quod* x, y, z, *&c. ſint variabiles.*

Inveniatur $\dot{P} = Q\dot{x} + R\dot{y} + S\dot{z} + \&c.$ tum erit fluxio quæſita $= P\dot{x} + \dot{y}\int. R\dot{x} + \dot{z}\int. S\dot{x} + \&c.$ ubi $\int. R\dot{x}$ & $\int. S\dot{x}$, &c. inveniri debent ex hypotheſi, ut x ſolummodo ſit variabilis, quod facile conſtat e theor. 2. ſit enim $V = \int. P\dot{x}$, & exinde $\dot{V} = P\dot{x} + q\dot{y} +$ &c. unde $\dot{P} = \alpha\dot{x} + \beta\dot{y} + \&c.$ & $\dot{q} = \beta\dot{x} + \gamma\dot{y} + \&c.$ & conſequenter fluxio quantitatis P ex hypotheſi, quod y ſolummodo ſit variabilis, erit $\beta\dot{y}$; ergo $R\dot{y} = \beta\dot{y}$; ſed $\dot{q} = \beta\dot{x} = R\dot{x}$, ſi modo x tantum ſit variabilis; & conſequenter $q = \int. R\dot{x}$, & $\dot{V} = P\dot{x} + \dot{y}\int. R\dot{x} + \&c.$ $= \left(\frac{\dot{V}}{\dot{x}}\right)\dot{x} + \left(\frac{\dot{V}}{\dot{y}}\right)\dot{y} + \&c.$

Cor. 1. Sit quantitas $V = H\int. P\dot{x}$, & erit $\dot{V} = \dot{H}\int. P\dot{x} + HP\dot{x} + H\dot{y}\int. R\dot{x} + H\dot{z}\int. S\dot{x} + \&c.$ ubi $\dot{P} = Q\dot{x} + R\dot{y} + S\dot{z} + \&c.$

Cor. 2. Sit V functio quantitatum x & u, & $y = \int. V\dot{x}$; tum erit $\dot{y} = \dot{u}\int.\dot{x}\left(\frac{\dot{V}}{\dot{u}}\right)$, ſi modo x habeatur invariabilis: ſupponatur enim

$\dot y = p\dot x + q\dot u$, tum $\dot p = \alpha\dot x + \beta\dot u = \dot V$, unde $\left(\frac{\dot V}{\dot u}\right) = \beta$, ſed $\dot q = \beta\dot x + \gamma\dot u$; ſi vero x ſolummodo habeatur variabilis, erit $\dot q = \beta\dot x = \left(\frac{\dot V}{\dot u}\right)\dot x$, & exinde $q = \int.\dot x\left(\frac{\dot V}{\dot u}\right)$; unde $q\dot u = \dot u\int.\dot x\left(\frac{\dot V}{\dot u}\right) = \dot y$ ſi modo x ſit invariabilis; & ſic inveniri poteſt $\ddot y = \dot u^2\int.\dot x\left(\frac{\ddot V}{\dot u^2}\right)$.

Cor. 3. Sit quantitas $V = \int.\pi\dot x\int.P\dot x$, ubi x ſolummodo habetur variabilis; aſſumantur $\dot\pi = \alpha\dot x + \beta\dot y + \gamma\dot z + \&c.$ & $\dot P = \lambda\dot x + \mu\dot y + \nu\dot z + \&c.$ tum erit fluxio quantitatis $(\pi\int.P\dot x) = (\alpha\dot x + \beta\dot y + \gamma\dot z + \&c.)\int.P\dot x + \pi P\dot x + \pi\dot y\int.\mu\dot x + \pi\dot z\int.\nu\dot x + \&c.$ exinde $\dot V = \pi\dot x\int.P\dot x + \dot y\int.\dot x(\beta\int.P\dot x + \pi\int.\mu\dot x) + \dot z\int.\dot x(\gamma\int.P\dot x + \pi\int.\nu\dot x) + \&c.$ ubi in fluxionibus $\dot x(\beta\int.P\dot x + \pi\int.\mu\dot x)$ & $\dot x(\gamma\int.P\dot x + \pi\int.\nu\dot x)$, &c. ſolummodo habetur x variabilis.

Et ſic inveniri poſſunt fluxiones cujuſcunque ordinis quantitatis V, quæ continet in ſe fluentem cujuſcunque ordinis.

THEOR. LII.

Sit V functio quantitatum x, y, &c.; tum erit $\dot V = p\dot x + q\dot y + \&c.$; ſint etiam y, x, &c. functiones quantitatum t, u, &c. unde $\dot x = a\dot t + b\dot u + \&c.$, & $\dot y = a'\dot t + b'\dot u + \&c.$; pro $\dot x$ & $\dot y$ in prædictâ æquatione $\dot V = p\dot x + q\dot y + \&c.$ ſcribantur earum valores, & reſultat $\dot V = (pa + qa' + \&c.)\dot t + (pb + qb' + \&c.)\dot u + \&c.$: in hâc æquatione pro p, q, &c., a', b', &c. ſcribantur earum valores $\left(\frac{\dot V}{\dot x}\right)$, $\left(\frac{\dot V}{\dot y}\right)$, &c.; $\left(\frac{\dot y}{\dot t}\right)$, $\left(\frac{\dot y}{\dot u}\right)$, &c.; & transformabitur data æquatio in aliam $\dot V = \dot t\left(\frac{\dot V}{\dot x}\right)\left(\frac{\dot x}{\dot t}\right) + \dot t\left(\frac{\dot V}{\dot y}\right)\left(\frac{\dot y}{\dot t}\right) + \dot u\left(\frac{\dot V}{\dot x}\right)\left(\frac{\dot x}{\dot u}\right) + \dot u\left(\frac{\dot V}{\dot y}\right)\left(\frac{\dot y}{\dot u}\right) + \&c.$

Et ſic facile transformari poteſt hæc æquatio in aliam, in quâ involvuntur quantitates t', u', &c. ubi t, u, &c. ſunt functiones quantitatum t', u', &c.; & ſic deinceps: vel transformari poteſt data æquatio $\ddot V = p\ddot x + q\ddot y + r\dot x\dot y +$ &c. in aliam præcedentis formulæ, in quâ etiam involvuntur t, u, &c.; vel etiam t', u', &c.; & earum fluxiones: ſit enim $\dot x = a\dot t + b\dot u +$ &c., & $\dot y = a'\dot t + b'\dot u +$ &c., unde $\ddot x = a\ddot t + \dot t\dot a + b\ddot u + \dot b\dot u +$ &c., &c. ſint $\dot a = c\dot t + d\dot u +$ &c., & $\dot b = c'\dot t + d'\dot u +$ &c.: ſcribantur hi valores pro $\dot a$ & $\dot b$ in præcedente æquatione & reſultat $\ddot x = a\ddot t + b\ddot u + c\dot t^2 + (d + c')\dot t\dot u + d'\dot u^2$, &c.; & ſimiliter detegi poteſt $\ddot y = a'\ddot t + b'\ddot u + e\dot t^2 + (f + e')\dot t\dot u + f'\dot u^2$, ubi $\ddot y = a'\ddot t + \dot t\dot a' + b'\ddot u + \dot b'\dot u +$ &c., & $\dot a' = e\dot t + f\dot u +$ &c., & $\dot b' = e'\dot t + f'\dot u +$ &c.: ſubſtituantur hi valores, & $a\dot t + b\dot u +$ &c., & $a'\dot t + b'\dot u +$ &c. pro ſuis valoribus $\ddot x$, $\ddot y$, $\dot x$ & $\dot y$, in datâ æquatione; & reſultat æquatio in quâ ſolummodo continentur fluxiones quantitatum t & u.

Hæc fluxio facile transformari poteſt in terminos præcedentis formulæ ſcribendo pro p, q, r, &c. reſpective $\left(\frac{\ddot V}{\ddot x}\right)$, $\left(\frac{\ddot V}{\ddot y}\right)$, $\left(\frac{\ddot V}{\dot x\dot y}\right)$, &c.; & pro a, b, &c.; a', b', &c.; ſubſtituendo $\left(\frac{\dot x}{\dot t}\right)$, $\left(\frac{\dot x}{\dot u}\right)$, &c., $\left(\frac{\dot y}{\dot t}\right)$, $\left(\frac{\dot y}{\dot u}\right)$, &c.; deinde pro c, d, &c.; c', d', &c.; e, f, &c.; e', f', &c.; ſcribendo $\left(\frac{\ddot x}{\dot t^2}\right) = \left(\frac{\dot a}{\dot t}\right)$ ob $a = \left(\frac{\dot x}{\dot t}\right)$, $\left(\frac{\ddot x}{\dot t\dot u}\right) = \left(\frac{\dot a}{\dot u}\right)$, &c.; $\left(\frac{\ddot x}{\dot u\dot t}\right) = \left(\frac{\dot b}{\dot t}\right)$ ob $b = \left(\frac{\dot x}{\dot u}\right)$, $\left(\frac{\ddot x}{\dot u^2}\right) = \left(\frac{\dot b}{\dot u}\right)$, &c.; $\left(\frac{\ddot y}{\dot t^2}\right) = \left(\frac{\dot a'}{\dot t}\right)$ ob $a' = \left(\frac{\dot y}{\dot t}\right)$, $\left(\frac{\ddot y}{\dot t\dot u}\right) = \left(\frac{\dot a'}{\dot u}\right)$, &c.; $\left(\frac{\ddot y}{\dot u\dot t}\right) = \left(\frac{\dot b'}{\dot t}\right)$ ob $b = \left(\frac{\dot y}{\dot u}\right)$, $\left(\frac{\ddot y}{\dot u^2}\right) = \left(\frac{\dot b'}{\dot u}\right)$, &c.; &c.; in reſultante æquatione.

Conſimiles etiam æquationes ex iiſdem principiis deduci poſſunt, cum t, u, &c. ſint functiones quantitatum x, y, &c. i. e. $\dot t = b\dot x + k\dot y$, & $\dot u = b'\dot x + k'\dot y$, &c.; facile enim deduci poſſunt fluxiones $\dot x$, $\dot y$, &c. ex fluxionibus $\dot t$, $\dot u$, &c.; quibus pro ſuis valoribus in datâ æquatione ſubſtitutis, reſultat æquatio, quæ per methodum prius traditam reduci poteſt ad alteram præcedentis formulæ.

PROB.

PROB. LXIX.

1. Sit æquatio relationem inter quantitates $\left(\frac{\dot{V}}{\dot{x}}\right)$, $\left(\frac{\dot{V}}{\dot{y}}\right)$, $\left(\frac{\dot{V}}{\dot{z}}\right)$, &c. $\left(\frac{\ddot{V}}{\dot{y}^2}\right)$, $\left(\frac{\ddot{V}}{\dot{x}\dot{z}}\right)$, &c. $\left(\frac{\dddot{V}}{\dot{y}^3}\right)$, &c. deſignans, i. e. ſint $\dot{V} = p\dot{x} + q\dot{y} + r\dot{z} +$ &c. & $\dot{p} = \alpha\dot{x} + \beta\dot{y} + \gamma\dot{z} +$ &c. $\dot{q} = \delta\dot{x} + \varepsilon\dot{y} + \zeta\dot{z} +$ &c. $\dot{r} = \lambda\dot{x} + \mu\dot{y} + \sigma\dot{z} +$ &c. & etiamque $\dot{\alpha} = a\dot{x} + b\dot{y} +$ &c. $\dot{\beta} = d\dot{x} + e\dot{y} +$ &c. &c. $\dot{\delta} = f\dot{x} + g\dot{y} +$ &c. &c. &c. & ſit data æquatio relationem inter quaſcunque quantitates p, q, r, &c. α, β, γ; δ, ε, ζ; λ, μ, ν; a, b; d, e; f, g, &c. deſignans; invenire utrum ea transformari poteſt in æquationem relationem inter V, x, y, z, &c. & earum fluxiones exprimentem: aſſumantur omnes quantitates præter V & alteram x tanquam invariabiles, & æquationis reſultantis abjiciantur parentheſes, & fluxionalis æquationis exinde reſultantis inveniatur fluens; & ſic de ſingulis reliquis (y, z, &c.) variabilibus quantitatibus; & ex fluxionibus fluentium ſic inventarum ad prædictas formulas reductarum conſtabit, utrum data æquatio reduci poteſt in fluxionalem, necne. E. g.

Si data æquatio relationem inter quantitates hujuſce generis $\left(\frac{\dot{V}}{\dot{x}}\right)$, $\left(\frac{\dot{V}}{\dot{y}}\right)$, $\left(\frac{\ddot{V}}{\dot{x}^2}\right)$, $\left(\frac{\ddot{V}}{\dot{x}\dot{y}}\right)$, $\left(\frac{\ddot{V}}{\dot{y}^2}\right)$, $\left(\frac{\dddot{V}}{\dot{x}^3}\right)$, &c. V, x, y, &c. & earum fluxiones deſignet: aſſumantur omnes quantitates præter duas (V & x vel y, &c.) tanquam conſtantes, & in æquatione reſultante pro $\left(\frac{\dot{V}}{\dot{x}}\right)$ ſcribatur $\frac{\dot{V}}{\dot{x}}$, & pro $\left(\frac{\ddot{V}}{\dot{x}^2}\right)$ ſcribatur $\frac{\ddot{V}}{\dot{x}^2}$, & ſic deinceps; fluxionalis æquationis reſultantis inveniatur fluens; vel ducatur ea in multiplicatorem M, ita ut ejus fluens inveniri poſſit, quæ dicatur V; nunc ſupponantur omnes prædictæ quantitates iterum variabiles, & inveniatur $\dot{V}$, subducatur hæc fluxio de datâ æquatione in fluxionales terminos mutatâ & in multiplicatorem M ductâ; & ſi modo reſultet fluxio, cujus fluens inveniri poteſt, quæ ſit W; tum $V \pm W = 0$ exprimet relationem quæſitam, ſi modo per fluxionales æquationes exprimi poſſit, quam etiam deſignat data æquatio.

2. Eadem principia, quæ prius edita fuerunt de fluxionalibus æquationibus tres vel plures variabiles quantitates & earum fluxiones habentibus, etiam applicari poſſunt ad inveniendas conſimiles propoſitiones de æquationibus hujuſce generis. E. g. Omnis æquatio, quæ eſt generalis reſolutio datæ æquationis prædictæ formulæ habet tot vel plures invariabiles quantitates ad libitum aſſumendas, quot ſit maximus ordo fluxionis in datâ æquatione contentus.

3. Duæ æquationes exprimentes relationes inter quantitates hujuſce generis $\left(\frac{\dot{z}}{\dot{x}}\right)$, $\left(\frac{\dot{z}}{\dot{y}}\right)$, &c. reduci poſſunt ad unam, ita ut una variabilis quantitas & ejus fluxiones exterminentur ex eâdem methodo, &c. quæ prius tradita fuit de duabus æquationibus fluxionalibus in unam reducendis, ita ut exterminentur variabilis quantitas & ejus fluxiones, &c.

4. Fere omnia etiam proferre liceat de his, quæ in priori hujus capitis parte de fluxionalibus æquationibus conſcripta fuere.

E. g. Datâ generali reſolutione detegi poteſt æquatio, cujus fluens datur per methodum in fluxionalibus æquationibus traditam; & per conſimiles methodos deduci poſſunt quotcunque æquationes hujuſmodi, quarum particulares valores fluentium dantur: e. 2. aſſumatur quæcunque functio (π) quantitatum x, y, z, &c. pro V; etiamque quantitas W, quæ eſt data functio quantitatum y, x, z, & V, & earum fluxionum; vel quantitatum formularum prædictarum $\left(\frac{\dot{V}}{\dot{x}}\right)$, $\left(\frac{\dot{V}}{\dot{y}}\right)$, $\left(\frac{\ddot{V}}{\dot{x}^2}\right)$, &c.: in hâc quantitate W pro $V, \dot{V}, \ddot{V}$, &c.; $\left(\frac{\dot{V}}{\dot{x}}\right)$, $\left(\frac{\dot{V}}{\dot{y}}\right)$, &c. ſcribantur earum valores ex dato valore π quantitatis V

deducti; & reſultet quantitas H, quæ eſt functio quantitatum x & y, & earum fluxionum; tum plerumque erit $V = \pi$ particularis valor æquationis $W = H$.

5. Si hæc vel quæcunque alia æquatio, cujus fluens innoteſcit, transformetur in aliam per datas ſubſtitutiones novarum pro variabilibus in datâ æquatione contentis; ex reſolutione datæ æquationis facile erui poteſt reſolutio reſultantis æquationis.

6. Ex aſſumptis multiplicatoribus vel ſubſtitutionibus deduci poſſunt æquationes datæ formulæ, quarum fluentes exprimi poſſunt; & generaliter omnia quæ tradita fuere de fluxionalibus æquationibus mutatis mutandis ad æquationes hujuſce formulæ applicari poſſunt.

7. Principia in fluxionalibus æquationibus reſolvendis uſitata, ut prius aſſeritur, etiam applicari poſſunt ad fluentes quantitatum hujuſce generis inveniendas.

Ex. Sit $p\left(\frac{\dot{V}}{\dot{x}}\right) + q\left(\frac{\dot{V}}{\dot{y}}\right) = o$, tum erit fluens hujuſce æquationis eadem ac fluens æquationis $\frac{1}{p}\dot{x} - \frac{1}{q}\dot{y} = o$.

2. Nonnunquam ſubſtituendo $\dot{V}$ pro $\dot{x}\left(\frac{\dot{V}}{\dot{x}}\right) + \dot{y}\left(\frac{\dot{V}}{\dot{y}}\right) + \dot{z}\left(\frac{\dot{V}}{\dot{z}}\right) +$ &c. in data æquatione; ſi modo x, y, z, &c. ſint variabiles; vel $\ddot{V}$ pro $\ddot{x}\left(\frac{\dot{V}}{\dot{x}}\right) + \ddot{y}\left(\frac{\dot{V}}{\dot{y}}\right) + a\dot{x}\dot{y}\left(\frac{\ddot{V}}{\dot{x}\dot{y}}\right) +$ &c.; transformari poteſt data æquatio in fluxionalem, vel in aliam ſimplicioris formulæ; e. g. ſit data æquatio $M\dot{x}\left(\frac{\dot{V}}{\dot{x}}\right) + M\dot{y}\left(\frac{\dot{V}}{\dot{y}}\right) = W\dot{x} + Z\dot{y}$, ſcribatur $\dot{V}$ pro $\dot{x}\left(\frac{\dot{V}}{\dot{x}}\right) + \dot{y}\left(\frac{\dot{V}}{\dot{y}}\right)$, & reſultat fluxionalis æquatio $M\dot{V} = W\dot{x} + Z\dot{y}$.

3. Si V ſit functio unius ſolummodo quantitatis x, & in datâ æquatione tantummodo fluxiones quantitatum x & V contineantur;

tum rejiciendo parentheſes reduci poteſt data ad fluxionalem æquationem.

4. Si vero fluxiones plurium quam duarum variabilium (V, x, y, z, &c.) in datâ æquatione contineantur, & conſequenter plures quam duæ in datâ æquatione habeantur variabiles; tum, ſi ſemper occurrant quæcunque quantitates hujuſce formulæ $M\left(\frac{\overset{n+m+r+\&c.}{\dot{V}}}{\dot{x}^{n}\dot{y}^{m}\dot{z}^{r}\,\&c.}\right)$; & nulla alia; in quâ continentur V, vel ejus fluxiones; ubi M eſt functio quantitatum (x, y, z, &c.) & earum fluxionum; & n', m', r', &c. ſunt minimi indices fluxionum ($\dot{x}$, $\dot{y}$, $\dot{z}$, &c.) in denominatoribus fractionum $\left(\frac{\overset{n+m+r+\&c.}{\dot{V}}}{\dot{x}^{n}\dot{y}^{m}\dot{z}^{r}\,\&c.}\right)$; reduci poteſt data æquatio ad alteram, in quâ ordo fluxionis quantitatis V minor eſt per $n' + m' + r' +$ &c. quam ordo fluxionis ejuſdem quantitatis V in æquatione; ſubſtituantur enim $v = \left(\frac{\dot{V}}{\dot{x}}\right)$, $v' = \left(\frac{\dot{v}}{\dot{x}}\right)$, $v'' = \left(\frac{\dot{v}'}{\dot{x}}\right)$, . . ., $\overset{n-1}{v'} = \left(\frac{\overset{n-2}{\dot{v}'}}{\dot{x}}\right)$; deinde $w = \left(\frac{\overset{n-1}{\dot{v}'}}{\dot{y}}\right)$, $w' = \left(\frac{\dot{w}}{\dot{y}}\right)$, $w'' = \left(\frac{\dot{w}'}{\dot{y}}\right)$, . . ., $\overset{m-1}{w'} = \left(\frac{\overset{m-2}{\dot{w}'}}{\dot{y}}\right)$; tum $u = \left(\frac{\overset{m-1}{\dot{w}'}}{\dot{z}}\right)$, $u' = \left(\frac{\dot{u}}{\dot{z}}\right)$, $u'' = \left(\frac{\dot{u}'}{\dot{z}}\right)$, . . ., $\overset{r-1}{u'} = \left(\frac{\overset{r-2}{\dot{u}'}}{\dot{z}}\right)$, &c.: æquatio in quâ introducuntur variabilis $\overset{r-1}{u'}$ & ejus fluxiones loco variabilis V & ejus fluxionum non involvit fluxionem quantitatis $\overset{r-1}{u'}$ majoris ordinis quam maximus ordo fluxionis quantitatis V in datâ æquatione per $n' + m' + r'$ diminutus.

Hoc facilius perfici poteſt, ſi modo pro $\left(\frac{\overset{n'}{\dot{V}}}{\dot{x}^{n'}}\right)$ ſcribatur U; deinde in æquatione reſultante pro $\left(\frac{\overset{m'}{\dot{U}}}{\dot{y}^{m'}}\right)$ ſcribatur W; tum pro $\left(\frac{\overset{r'}{\dot{W}}}{\dot{z}^{r'}}\right)$ ſcribatur T; & ſic deinceps.

E. g. Sit $\left(\frac{\dddot{V}}{\dot{x}^2\dot{y}}\right) = P$, & erit $\left(\frac{\ddot{V}}{\dot{x}\dot{y}}\right) = \int . P\dot{x} + \phi : y$, ubi y aſſumitur conſtans; deinde $\left(\frac{\dot{V}}{\dot{y}}\right) = \int . \int . P\dot{x} + x\phi : y + \phi' : y$, ubi y iterum aſſumitur conſtans; ultimo $z = \int .^3 (P\dot{x}) + x\int . \dot{y}\phi : y + \int . \dot{y}\phi' : y + \phi'' : (x)$ ubi x aſſumitur conſtans; & ſic de fluxionalibus æquationibus ſuperiorum ordinum.

Ex. 2. Sit $\left(\frac{\ddot{z}}{\dot{x}^2}\right) = P\left(\frac{\dot{z}}{\dot{x}}\right) + Q$; ponatur $\left(\frac{\dot{z}}{\dot{x}}\right) = v$, unde $\left(\frac{\ddot{z}}{\dot{x}^2}\right) = \left(\frac{\dot{v}}{\dot{x}}\right)$ & data æquatio fit $\dot{v} = Pv\dot{x} + Q\dot{x}$, ubi x ſumitur conſtans; eodem modo ſit $\left(\frac{\ddot{z}}{\dot{x}\dot{y}}\right) = P\left(\frac{\dot{z}}{\dot{x}}\right) + Q$, ſcribatur $\left(\frac{\dot{z}}{\dot{x}}\right) = v$, & data æquatio fit $\left(\frac{\dot{v}}{\dot{y}}\right) = Pv + Q$, in quo caſu habetur x tanquam invariabilis, & reſultat æquatio $\dot{v} + Pv\dot{y} + Q\dot{y} = o$, cujus e methodo prius traditâ inveniri poteſt fluens.

5. Nonnunquam facile conſtat formula quantitatis, in quâ continetur fluens vel reſolutio datæ æquationis.

Ex. 1. Sit fluxionalis homogenea æquatio prædictæ formulæ $A\left(\frac{\overset{n}{\dot{V}}}{\dot{x}^n}\right) + B\left(\frac{\overset{n}{\dot{V}}}{\dot{x}^{n-1}\dot{y}}\right) + C\left(\frac{\overset{n}{\dot{V}}}{\dot{x}^{n-2}\dot{y}^2}\right) + D\left(\frac{\overset{n}{\dot{V}}}{\dot{x}^{n-3}\dot{y}^3}\right) + \&c. = o$: ubi A, B, C, D, &c. ſunt invariabiles quantitates; haud difficilis eſt obſervatio ut particularis fluens hujuſce æquationis contineatur in formulâ $\phi : (ax + by) = V$; ſubſtituatur hæc functio pro V, & valores quantatitum $\left(\frac{\dot{V}}{\dot{x}^r\dot{y}^s}\right)$, &c. exinde deducti pro ſuis valoribus in datâ æquatione; & reſultat æquatio $(Aa^n + Ba^{n-1}b + Ca^{n-2}b^2 + Da^{n-3}b^3 + \&c.) \times \phi' : (ax + by) = o$, unde $a^nA + a^{n-1}bB + a^{n-2}b^2C + a^{n-3}b^3D + \&c. = o$; ſint $\alpha, \beta, \gamma, \delta$, &c. radices æquationis $Ax^n + Bx^{n-1} + Cx^{n-2} +$

$Dx^{n-3} +$ &c. $= o$, tum erunt $\phi : (\alpha x - y)$, $\phi' : (\beta x - y)$, $\phi'' : (\gamma x - y)$, &c. reſolutiones datæ æquationis; & magis generaliter $V = \phi : (\alpha x - y) + \phi' : (\beta x - y) + \phi'' : (\gamma x - y) +$ &c. erit reſolutio datæ æquationis.

Subſequens eſt elegans reſolutio particularis caſus hujuſce exempli, quem primus reſolvere docuit Clar. D'Alembert: ſit æquatio $\left(\frac{\ddot{V}}{\dot{y}^2}\right) = a^2 \left(\frac{\ddot{V}}{\dot{x}^2}\right)$, tum $\dot{V} = p\dot{x} + q\dot{y}$, $\dot{p} = r\dot{x} + s\dot{y}$, $\dot{q} = s\dot{x} + t\dot{y}$; & per æquationem erit $t = a^2 r$; unde ex his æquationibus deduci poteſt $a\dot{p} + \dot{q} = (ar + s)(\dot{x} + a\dot{y})$, & exinde $ar + s = \phi : (x + ay)$; & quoniam a vel affirmative vel negative accipi poteſt, erit $q + ap = 2a\phi' : (x + ay)$, & $q - ap = 2a\phi' : (x - ay)$; unde $q = a\phi : (x + ay) + a\phi' : (x - ay)$, & $p = \phi : (x + ay) - \phi' : (x - ay)$, & exinde $V = \pi : (x + ay) + \pi' : (x - ay)$.

Ex. 2. Sit homogenea æquatio $A\left(\frac{\overset{n}{\dot{V}}}{\dot{x}^n}\right) + B\left(\frac{\overset{n}{\dot{V}}}{\dot{x}^{n-1}\dot{y}}\right) + C\left(\frac{\overset{n}{\dot{V}}}{\dot{x}^{n-1}\dot{z}}\right) + D\left(\frac{\overset{n}{\dot{V}}}{\dot{x}^{n-1}\dot{v}}\right) + E\left(\frac{\overset{n}{\dot{V}}}{\dot{x}^{n-2}\dot{y}^2}\right) + F\left(\frac{\overset{n}{\dot{V}}}{\dot{x}^{n-2}\dot{y}\dot{z}}\right) +$ &c. $= o$, in quâ plures variabiles x, y, z, v, &c. continentur, tum aſſumatur $V = \phi : (ax + by + cz + dv +$ &c.$)$, ſcribatur hæc functio pro V, & pro terminis $\left(\frac{\overset{n}{\dot{V}}}{\dot{x}^r \dot{y}^s \dot{z}^t \text{ \&c.}}\right)$, &c. eorum valores exinde deducti; reſultans æquatio $Aa^n + Ba^{n-1}b + Ca^{n-1}c + Da^{n-1}d + Ea^{n-2}b^2 + Fa^{n-2}bc +$ &c. $= H = o$; ſi $H = (fa + gb + bc +$ &c.$) \times (f'a + g'b + b'c +$ &c.$) \times (f''a + g''b + b''c +$ &c.$) \times$ &c.; tum ex hâc methodo conſtat datæ æquationis reſolutio; vel ſi $H = (fa + gb + hc +$ &c.$)(f'a^2 + g'b^2$ $h'c^2 + l'ab + m'ac + n'bc +$ &c.$)$ (&c.); & ſic deinceps; i. e. ſi modo algebraica expreſſio H haud in ſimplices diviſores reſolvi poſſit, ſed in ſimplices, quadraticos, cubicos, &c. etiam conſtat reſolutio.

Si hæ algebraicæ expreſſiones e fluxionalibus æquationibus ſuperiorum ordinum deductæ haud recipiant ſimplices diviſores; ſed quadraticos, cubicos, &c.; tum ſemper reduci poteſt data æquatio ad fluxionales ſecundi, tertii, &c. ordinum æquationes.

Ex. 3. Sit $\left(\frac{\ddot{z}}{\dot{x}\dot{y}}\right) = az$, & ſtatuatur $z = e^{p}q$, ut quantitas exponentialis e^{p} e calculo evaneſcat, quoniam quantitas z unam ubique tenet dimenſionem; ponatur $z = e^{\alpha x}Y$, unde $\left(\frac{\ddot{z}}{\dot{x}\dot{y}}\right) = \alpha e^{\alpha x}\frac{\dot{Y}}{\dot{y}} = ae^{\alpha x}Y$, & ſi $Y' = Y$, exinde $\frac{\alpha\dot{Y}}{Y} = a\dot{y}$ & $Y = e^{\frac{ay}{\alpha}}$, unde particularis valor ipſius $z = Ae^{\alpha x + \frac{ay}{\alpha}}$; & ſic generaliter $z = Ae^{\alpha x + \frac{a}{\alpha}y} + Be^{\beta x + \frac{b}{\beta}y} + Ce^{\gamma x + \frac{c}{\gamma}y} +$ &c. ubi literæ A, B, C, &c. α, β, γ, &c. &c. omnes recipere poſſunt poſſibiles valores.

Ex. 4. Sit æquatio $P\left(\frac{\overset{n}{\dot{V}}}{\dot{x}^{n}}\right) + Q\left(\frac{\overset{n}{\dot{V}}}{\dot{x}^{n-1}\dot{y}}\right) + R\left(\frac{\overset{n}{\dot{V}}}{\dot{x}^{n-2}\dot{y}^{2}}\right) +$ &c. $+$ $P'\left(\frac{\overset{n-1}{\dot{V}}}{\dot{x}^{n-1}}\right) + Q'\left(\frac{\overset{n-1}{\dot{V}}}{\dot{x}^{n-2}\dot{y}}\right) + R\left(\frac{\overset{n-1}{\dot{V}}}{\dot{x}\dot{y}^{n-2}}\right) +$ &c. $+ P''\left(\frac{\overset{n-2}{\dot{V}}}{\dot{x}^{n-2}}\right) +$ &c. $+ LV = 0$; ubi in ſingulis terminis V vel ejus fluxio unam ſolummodo habet dimenſionem; in hâc æquatione pro V & ejus fluxionibus ſcribantur $e^{v} \times u$ & ejus correſpondentes fluxiones, ubi v & u ſunt functiones quantitatum x & y: functiones v & u pendent e functionibus P, Q, R, &c., P', Q', &c., P'', &c. quo magis ſimplices ſunt priores functiones, eo magis plerumque ſimplices erunt poſteriores: ſi P, Q, R, &c., P', &c. ſint invariabiles; tum aſſumatur $V = e^{x}(\varphi : (ax + by) + x\varphi' : (y) + \varphi'' : y) = e^{x}H$; vel $e^{a'x + b'y}H$; vel prior vel poſterior evadat quantitas magis compoſita; ſubſtituantur hæ quantitates pro ſuis valoribus in datâ æquatione, & ex terminis inter ſe & nihilo reſpective æquatis forſan deduci poteſt æquatio, cujus particularis reſolutio innoteſcit.

6. In pleriſque caſibus haud neceſſe eſt, ut introducatur exponentialis quantitas: in quibuſdam caſibus ſimplex algebraica expreſſio particularem datæ æquationis relationem præbebit. E. g. Sit $(x + y)^{2}$

$\left(\frac{\ddot{V}}{\dot{x}\dot{y}}\right) + m(x+y)\left(\frac{\dot{V}}{\dot{x}}\right) + m(x+y)\left(\frac{\dot{V}}{\dot{y}}\right) + nz = 0$: hujuſce æquationis reſolutio erit $V = A(x+y)^{\lambda}\phi:(x) + B(x+y)^{\lambda+1}\phi':(x) + C(x+y)^{\lambda+2}\phi'':(x) +$ &c.; ſubſtituatur enim hæc quantitas pro ejus valore in datâ æquatione, & ex æquatis correſpondentibus terminis reſultant $n + 2m\lambda + \lambda^2 - \lambda = 0$, $(n + 2m\lambda + 2m + \lambda^2 + \lambda)B + (m+\lambda)A = 0$, &c., unde $B = -\frac{(m+\lambda)}{2(m+\lambda)}A$, $C = -\frac{(m+\lambda+1)}{2(2m+2\lambda+1)}B$, $D = -\frac{(m+\lambda+2)}{3(2m+2\lambda+2)}C$, &c.; hæc ſeries terminat, quoties $m + \lambda + i = 0$, ubi i ſit integer poſitivus numerus.

Cor. E datis valoribus quantitatum n & m duo exoriuntur valores quantitatis λ, & exinde duæ ſeries, quæ ad eandem reduci poſſunt.

Conſimilis æquationis (n) ordinis reſolutio erit conſimilis.

7. Si in datâ æquatione fluxionali hujuſce formulæ quantitates x & y ſimiliter involvantur; tum in ejus fluente vel reſolutione ſimiliter etiam involventur.

8. Nonnunquam aſſumatur quantitas compoſita ex exponentialibus, algebraicis, &c. E. g. Sit æquatio $\left(\frac{\ddot{V}}{\dot{y}^2}\right) = x^2\left(\frac{\ddot{V}}{\dot{x}^2}\right)$, fingatur $V = x^{\lambda}e^{\mu y}(m \times \log. x + ny)$ & ſubſtituantur pro $\left(\frac{\ddot{V}}{\dot{y}^2}\right)$ & $\left(\frac{\ddot{V}}{\dot{x}^2}\right)$ earum valores in datâ æquatione; & ex æquatis correſpondentibus terminis reſultantis æquationis deduci poteſt fluens vel reſolutio quæſita.

9. Aliquando ex aſſumptâ relatione inter variabiles datæ & quæſitæ æquationum reduci poteſt data æquatio ad alteram, cujus reſolutio innoteſcit. E. g. Sit æquatio $\left(\frac{\ddot{V}}{\dot{y}\dot{x}}\right) + P\left(\frac{\dot{V}}{\dot{x}}\right) + Q\left(\frac{\dot{V}}{\dot{y}}\right) + Rz + S = 0$; ponatur $V = e^{u}v$, ubi $u = \phi:(x \;\&\; y)$; ſubſtituatur hæc quantitas pro ejus valore V in datâ æquatione, &c.; & ita aſſumantur termini reſultantis æquationis, ut evadat $\left(\frac{\ddot{v}}{\dot{x}\dot{y}}\right) + T\left(\frac{\dot{v}}{\dot{x}}\right) + e^{-u}S = 0$, cujus fluens innoteſcit.

Ex. 2. Sit $\left(\frac{\ddot{V}}{\dot{y}^2}\right) = P^2\left(\frac{\ddot{V}}{\dot{x}^2}\right)$: introducantur binæ variabiles t & u, quæ ſunt functiones quantitatum x & u; & æquationis reſultantis termini inter ſe ita æquentur, ut evadat æquatio, cujus fluens innoteſcit.

Per eundem modum introducendo variabiles t & u & æquando terminos reſultantis æquationis inveniri poſſunt caſus quantitatum P, Q, R, &c. in æquatione $\left(\frac{\ddot{V}}{\dot{y}^2}\right) + P\left(\frac{\ddot{V}}{\dot{x}^2}\right) + Q\left(\frac{\dot{V}}{\dot{y}}\right) + R\left(\frac{\dot{V}}{\dot{x}}\right) + SV + T = o$ contentarum, in quibus fluens ope reductionis hìc dictæ deduci poteſt.

10. Methodus hìc tradita in hoc conſiſtit, ut hujuſmodi æquationes ope introductionis binarum novarum variabilium t & u ad hanc formulam $\left(\frac{\ddot{V}}{\dot{t}\dot{u}}\right) + P\left(\frac{\dot{V}}{\dot{t}}\right) + Q\left(\frac{\dot{V}}{\dot{u}}\right) + RV + S = o$, vel ad quamcunque aliam formulam, cujus reſolutio innoteſcit, reducantur.

Ex conſimili methodo reduci poteſt data fluxionalis æquatio hujuſce formulæ ad alteram; æquationis, cujus ordo minor eſt quam ordo datæ fluxionalis æquationis, ope. E. g. Data æquatio $\left(\frac{\ddot{V}}{\dot{y}^2}\right) + P\left(\frac{\ddot{V}}{\dot{x}^2}\right) + Q\left(\frac{\dot{V}}{\dot{x}}\right) + Rv = o$ in æquationem $\left(\frac{\ddot{V}}{\dot{y}^2}\right) + F\left(\frac{\ddot{v}}{\dot{x}^2}\right) + G\left(\frac{\dot{v}}{\dot{x}}\right) + Hv = o$ transformari poteſt, æquationis $V = M\left(\frac{\dot{v}}{\dot{x}}\right) + Nv$ ope, ubi literæ P, Q, R, F, G, H, M & N ſunt functiones quantitatis x.

Hoc conficitur reducendo duas vel plures æquationes in unam, ita ut quædam variabiles & earum fluxiones exterminentur; & æquando terminos reſultantis æquationis, ita ut evadat æquatio, cujus reſolutio innoteſcit.

In æquationibus hujuſce generis minime adhuc datur methodus inveniendi; annon data æquatio ſit generalis reſolutio datæ fluxionalis æquationis.

Æquatio fluxionalis $a \times b \times c \times \&c. = o$ hujuſce formulæ dividi poteſt in æquationes $a = o$, $b = o$, $c = o$, &c.

THEOR. LIII.

Sit π particularis fluens fluxionis $p\dot{x}$, tum erit ejus generalis fluens $= \pi + a$.

Sit ϱ particularis fluens fluxionis $p\dot{x}\int.q\dot{x}$, tum erit ejus generalis fluens $= \varrho + a'\pi + b$.

Sit σ particularis fluens fluxionis $p\dot{x}\int.q\dot{x}\int.r\dot{x}$, & erit ejus generalis fluens $\sigma + a''\varrho + b'\pi + c$.

Sit τ particularis fluens fluxionis $p\dot{x}\int.q\dot{x}\int.r\dot{x}\int.s\dot{x}$, & erit ejus generalis fluens $\tau + a'''\sigma + b''\varrho + c'\pi + d$, & ſic deinceps; ubi literæ a, b, c, d, &c; a', b', c', &c.; a'', b'', &c.; a''', &c. invariabiles quantitates ad libitum aſſumendas reſpective denotant.

Et ſic de generalibus fluentibus ſuperiorum ordinum deducendis.

PROB. LXX.

Sit fluxionalis æquatio $\frac{\dot{\alpha}}{1} = 0$, *cujus fluens ſit* $\alpha = C$ (*conſt.*), *deducere quamcunque æquationem, in quâ variabiles ſeparantur: aſſumatur* $z + v = \alpha$, *& fingantur* z *&* v *quæcunque algebraicæ functiones quantitatum* π *&* ϱ *reſpective; e datâ æquatione* $\frac{\dot{\alpha}}{1} = 0$ *inveniatur æquatio relationem inter duas variabiles quantitates* π *&* ϱ *exprimens, & reſultat æquatio, in quâ ſeparantur variabiles* π *&* ϱ *& earum fluxiones.*

Et ex eâdem methodo progredi liceat ad ſubſtitutiones pro reducendis fluxionalibus æquationibus ſuperiorum (n) ordinum ad fluxionales æquationes inferiorum m ordinum, in quibus ſeparantur variabiles: ſed plerumque reductio fluxionalis æquationis n ordinis ad ordinem inferiorem $n - 1$, in quâ ſeparantur variabiles, &c. exigit unam ſubſtitutionem, ad ordinem inferiorem $n - 2$ exigit duplicem ſubſtitutionem, & ſic deinceps.

THEOR.

THEOR. LIV.

Sit algebraica æquatio $A = o$, in quâ x & z ſimiliter involvuntur; & fluxionalis æquatio $p\dot{x} + q\dot{y} = o$, ubi p & q ſunt functiones quantitatum x & y; reducantur hæ duæ æquationes $A = o$ & $p\dot{x} + q\dot{y} = o$ in unam, ita ut exterminentur x & $\dot{x}$, & reſultet æquatio $P\dot{z} + Q\dot{y} = o$; in æquatione $A = o$ pro x ſcribantur α & β, quarum correſpondentes valores quantitatis (z) ſint π & ϱ: ſit $y = \phi : (x)$, etiamque $y = \phi' : (z)$: tum, ſi modo ſcribantur α, β, π & ϱ pro x in quantitate $\phi : (x) + \phi' : (x)$, & ſint quantitates reſultantes A, B, C & D, erit $A - B = C - D$; modo correſpondentes fluentes fluxionalium æquationum $p\dot{x} + q\dot{y} = o$ & $P\dot{x} + Q\dot{y} = o$ aſſumantur.

Aſſignari poſſunt fluentiales æquationes, quæ ſunt fluentes datarum fluxionalium; at revera nulli uſui inſerviunt: e. g. ſi in datâ fluentiali æquatione contineantur fluentiales quantitates, quarum fluxiones ſunt functiones quantitatum x, y & earum fluxionum non integrabiles, tum vix ulli uſui inſervit integratio hujuſmodi; e. g. ſit fluentialis æquatio $y = ſ.\frac{\dot{x}}{y} ſ.\frac{\dot{x}}{y}$, cujus fluxionalis erit $y^2\ddot{y} + y\dot{y}^2 = \dot{x}^2$; 2. ſit $y = ſ.\frac{\dot{x}}{y} ſ.\frac{\dot{x}}{y} ſ.\frac{\dot{x}}{y}$, cujus fluxionalis erit $y^3\dddot{y} + 4y^2\dot{y}\ddot{y} + y\dot{y}^3 = \dot{x}^3$, &c. hæ fluentiales æquationes non pro fluentibus datarum fluxionalium æquationum deſiderandis habendæ ſunt.

SCHOLIUM.

1. In detegendis fluentibus fluxionalium æquationum plerumque generales functiones variabilium tanquam independentes variabiles tractandæ ſunt; & exinde per regulas prius traditas de iis ratiocinari liceat.

2. Prius docetur methodus inveniendi fluentes omnium fluxionalium æquationum per aſſumptiones rationalium functionum, &

rationalium & irrationalium quantitatum, vel earum radicum in datis æquationibus contentarum, ni faſtigium calculi prohibeat.

Conſimilia principia etiam applicari poſſunt ad fluentes fluxionalium æquationum detegendas algebraicarum quantitatum, circularium arcuum & logarithmorum ope.

3. Facile demonſtrari poteſt, quod fluentes fere omnium fluxionum (comparative loquendo) nec exprimi poſſint per finitos terminos, logarithmos, circulares, nec per ellipticos nec hyperbolicos arcus; &c.: hinc, ſi ſeparari poſſint variabiles quantitates in datâ fluxionali æquatione, plerumque non inveniri poſſunt per prædictas methodos fluentes: ſed per ſubſtitutiones algebraicarum functionum, &c. novarum variabilium pro variabilibus in datâ æquatione contentis perraro ſeparari poſſunt variabiles quantitates; ergo in pleriſque caſibus ad infinitas ſeries confugiendum eſt.

4. In æquationibus quantitates formularum $\left(\frac{\overset{n}{\dot{V}}}{\dot{x}^{m}\dot{y}^{r}\ \&c.}\right)$ involventibus non adhuc datur generalis nota; ex quâ dici poteſt, annon data ſit generalis datarum æquationum reſolutio.

5. In detegendis fluentibus fluxionum, cum variabilis (x) evadat infinita: ſi fluens exprimatur per algebraicam functionem variabilis (x); tum ex ſubſtitutione infinitæ & finitæ (a) quantitatis pro x facile conſtat fluens inter prædictos valores (a) & infinitum poſita; ſi ſolummodo in ſingulis quantitatibus addendis vel ſubtrahendis rejiciantur omnes quantitates, quæ ſunt infinite minores quam quæcunque aliæ in eâdem ſummâ vel differentiâ contentæ; & ſi modo numeratores & denominatores nonnullarum fractionum nihilo evadant æquales, tum earum valores per methodum prius traditam inveſtigandi ſunt.

6. Si fluens exprimi poſſit per circulares arcus, &c. variabilis, vel ſit tangens vel ſecans, &c.; tum facile per circularem arcum acquiri poteſt fluens, cum x evadat infinita.

7. Si vero fluens exprimi poſſit per plures logarithmos; viz. $a \times$ log.

a +

$\alpha + b \times \log. \beta + c \times \log. \gamma +$ &c.; cujus nonnulli; e. g. log. α, log. β, log. γ, &c. evadunt infiniti; reducantur omnes hi logarithmi in logarithmum unius ſummæ, viz. $a \times \log. \alpha + b \times \log. \beta + c \times \log. \gamma +$ &c. $= \log. \alpha^a \times \beta^b \times \gamma^c$ &c. $= z$; & ſi z ſit finita quantitas, tum log. prædictæ ſummæ erit finita; ſin aliter non: inveniatur valor algebraicæ quantitatis z, & exinde acquiri poteſt ejus log.: ſi fluens ſit quæcunque algebraica functio quantitatis x, & circularium arcuum & logarithmorum ejus functionum; tum per principia prius tradita erui poteſt ejus valor, cum x evadat infinita.

Si autem fluens exprimatur per datas alias fluentes; tum ex datis curvis, ſolidis, &c., per quæ innoteſcunt datæ fluentes, erui poteſt ejus valor inter quoſcunque valores variabilis in eâ contentæ.

Et ſic rationari liceat de fluxionalibus æquationibus, &c.

8. Sint n fluxionales æquationes $a = o$, $a' = o$, $a'' = o$, &c. quarum ordo ſit l, $(n + 1)$ variabiles quantitates $(x, y, z,$ &c.$)$ involventes; tum ſint m, m', m'', &c. functiones quantitatum $(x, y, z,$ &c.$)$ & earum fluxionum, quarum ordines minores ſunt quam l; & ita aſſumi poſſunt multiplicatores m, m', m'', &c., ut exoriantur (n) diverſæ & independentes fluxionales æquationes $ma + m'a' + m''a'' +$ &c. $= o$, quæ integrari poſſunt.

9. Generalis fluens fluxionalis æquationis (n) ordinis partiales fluxiones involvens (n) arbitrarias functiones detegendas continet.

DE
METHODO INCREMENTORUM.

LIBER. I.

Def. 1. DENOTENT quantitates $\underset{.}{x}, \underset{.}{y}, \underset{.}{z}$, &c. prima incrementa quantitatum x, y, &c. reſpective; eodem modo deſignent quantitates ita ſcriptæ $\underset{..}{x}, \underset{..}{y}, \underset{..}{z}$, &c. ſecunda incrementa quantitatum x, y, z, &c. vel prima incrementa quantitatum $\underset{.}{x}, \underset{.}{y}, \underset{.}{z}$, &c. & ſic deinceps.

Def. 2. Denotent quantitates $\underset{'}{x}, \underset{'}{y}$, &c. integrales quantitatum x, y, &c. & ſic $\underset{''}{x}, \underset{''}{y}$, &c. ſecundas integrales quantitatum x, y, &c. & primas integrales quantitatum $\underset{'}{x}, \underset{'}{y}$, &c.

PROB. I.

Datâ quantitate P *vel algebraicâ vel fluxionali, quæ ſit functio quantitatum* x, y, z, *&c. quarum incrementa ſint* $\underset{.}{x}, \underset{.}{y}, \underset{.}{z}$, *&c. ejus incrementum inveſtigare.* 1. *Scribantur* $x + \underset{.}{x}$, $y + \underset{.}{y}$, $z + \underset{.}{z}$, *&c. pro literis* x, y, z, *&c. reſpective, &* $\underset{.}{x} + \underset{..}{x}$, $\underset{.}{y} + \underset{..}{y}$, *&c. pro* $\underset{.}{x}$ *&* $\underset{.}{y}$, *&c. in datâ æquatione, & ſit quantitas reſultans* Q; *tum erit* Q — P *incrementum quæſitum, i. e. differentia inter duos proxime ſucceſſivos valores integralis dicitur incrementum.*

Secunda incrementa e primis eodem modo ac prima ex integralibus deduci poſſunt; & ſic deinceps.

Ex. 1. Incrementum rectanguli xy erit $(x + \underset{.}{x})(y + \underset{.}{y}) - xy = x\underset{.}{y} + y\underset{.}{x} + \underset{.}{x}\underset{.}{y}$.

Ex. 2. Incrementum logarithmi quantitatis (x), cujus fluxio eſt $\frac{\dot{x}}{x}$, erit $\log.(x+\underset{\cdot}{x}) - \log.(x) = \log.\frac{x+\underset{\cdot}{x}}{x}$, cujus fluxio erit $-\frac{\underset{\cdot}{x}\dot{x}}{x(x+\underset{\cdot}{x})} = \frac{\dot{x}}{x+\underset{\cdot}{x}} - \frac{\dot{x}}{x}$, ſi modo $\underset{\cdot}{x}$ ſit conſtans.

Ex. 3. Incrementum circularis arcus $\int.\frac{\dot{x}}{1+x^2}$, cujus radius eſt 1, & tangens x, erit differentia inter duos circulares arcus, quorum radius eſt 1, & tangentes reſpective x & $x+\underset{\cdot}{x}$, i. e. $\int.\left(\frac{\dot{x}}{1+(x+\dot{x})^2} - \frac{\dot{x}}{1+x^2}\right) = \int.-\frac{2x\underset{\cdot}{x}+\underset{\cdot}{x}^2}{(1+x^2)(1+(x+\underset{\cdot}{x})^2)}\times\dot{x}$.

Ex. 4. Sit integralis $a\,x_{\underset{,}{n}}\times x_{\underset{,}{n+1}}\times x_{\underset{,}{n+2}}\,\&c.\times x_{\underset{,}{n+m}}$, ejus vero incrementum erit, ſi modo x creſcat uniformiter, $a\times\left(x_{\underset{,}{n+1}}\times x_{\underset{,}{n+2}}\,\&\,..\,x_{\underset{,}{n+m+1}} - x_{\underset{,}{n}}\times x_{\underset{,}{n+1}}\,\&c.\times x_{\underset{,}{n+m}}\right) = (m+1)\,a\underset{\cdot}{x}\left(x_{\underset{,}{n+1}}\times x_{\underset{,}{n+2}}\,...\,x_{\underset{,}{n+m}}\right)$; & ex eâdem methodo conſtat incrementum integralis $\frac{a}{x_{\underset{,}{n}}\times x_{\underset{,}{n+1}}\times\&c.\,..\,x_{\underset{,}{n+m}}}$ eſſe $\frac{a}{x_{n+1}\times x_{\underset{,}{n+2}}\times\&c.\;x_{\underset{,}{n+m+1}}} - \frac{a}{x_{\underset{,}{n}}\times x_{\underset{,}{n+1}}\,..\,x_{\underset{,}{n+m}}} = \frac{-(m+1)\,a\underset{\cdot}{x}}{x_{\underset{,}{n}}\times x_{\underset{,}{n+1}}\,...\,x_{\underset{,}{n+m+1}}}$, ſi modo x fluat uniformiter.

Cor. 1. Sint S, S', S'', S''', &c. (n) ſucceſſivi valores integralis, tum erit incrementum n ordinis $= S - nS' + n.\frac{n-1}{2}S'' - n.\frac{n-1}{2}.\frac{n-2}{3}S''' +$ &c. incrementa enim primi ordinis ſunt reſpective $S - S$, $S' - S''$, $S'' - S'''$, &c. ſecundi vero $S - S' - (S' - S'') = S - 2S' + S''$, &c.

Cor. 2. Hinc deduci poteſt integralis primi ſubſequentis ordinis quantitatis $(z) = z + \underset{\cdot}{z}$, ſecundi vero ſubſequentis ordinis $= z + \underset{\cdot}{z} + (\underset{\cdot}{z} + \underset{\cdot\cdot}{z}) = z + 2\underset{\cdot}{z} + \underset{\cdot\cdot}{z}$, & exinde integralis ſubſequentis n ordinis erit $z + n\underset{\cdot}{z} + n \cdot \frac{n-1}{2}\underset{\cdot\cdot}{z} + n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3}\underset{\cdot\cdot\cdot}{z} + \&c.$

Ex hoc corollario ſequitur incrementum contenti $(z \times \underset{'}{z} \times \underset{''}{z} \ldots \underset{n}{z}) = \underset{'}{z} \times \underset{''}{z} \ldots \underset{n+1}{z} - z \times \underset{'}{z} \times \underset{''}{z} \ldots \underset{n}{z} = \underset{'}{z} \times \underset{''}{z} \ldots \underset{n}{z} \cdot (z + (n+1)\underset{\cdot}{z} + (n+1) \cdot \frac{n}{2}\underset{\cdot\cdot}{z} + \&c. - z)$; etiamque incrementum fractionis $\left(\frac{1}{z \cdot \underset{'}{z} \cdot \underset{''}{z} \ldots \underset{n}{z}}\right)$ eſſe $\frac{1}{\underset{'}{z} \times \underset{''}{z} \ldots \underset{n+1}{z}} - \frac{1}{z \times \underset{'}{z} \ldots \underset{n}{z}} = \frac{z - \underset{n+1}{z}}{z \times \underset{'}{z} \times \underset{''}{z} \ldots \underset{n+1}{z}} = -\frac{z + (n+1) \times}{z \times}$ $\frac{\underset{\cdot}{z} + (n+1) \cdot \frac{n}{2}\underset{\cdot\cdot}{z} + \&c. - z}{\underset{'}{z} \ldots \underset{n+1}{z}}$.

Cor. 2. Et ſic inveniri poteſt $\frac{1}{\underset{m}{\underset{'}{z}}} = \frac{1}{z} - \frac{m\underset{\cdot}{z}}{z\underset{'}{z}} + \frac{m \times (m-1)\underset{\cdot}{z}^2}{z\underset{'}{z}\underset{''}{z}} - \frac{m \times (m-1) \times (m-2)\underset{\cdot}{z}^3}{z\underset{'}{z}\underset{''}{z}z} + \&c.$ ſi modo $\bar{z}$ fluat uniformiter; ſupponatur enim $\frac{1}{\underset{m-1}{z}} = \frac{1}{z} - \frac{(m-1)\underset{\cdot}{z}}{z\underset{'}{z}} + \frac{(m-1) \cdot (m-2)\underset{\cdot}{z}^2}{z\underset{'}{z}\underset{''}{z}} - \&c.$ & proximum incrementum ſuperſcriptarum quantitatum erit $-\frac{\underset{\cdot}{z}}{z\underset{'}{z}} + \frac{2(m-1)\underset{\cdot}{z}^2}{z\underset{'}{z}\underset{''}{z}} - \&c.$ harum ſumma erit $\frac{1}{\underset{m}{z}} = \frac{1}{z} - \frac{m\underset{\cdot}{z}}{z\underset{'}{z}} + \frac{m(m-1)\underset{\cdot}{z}^2}{z\underset{'}{z}\underset{''}{z}} - \&c.$ ſi igitur vere aſſignetur inferior fluentialis quantitas $\underset{m-1}{x}$, vere etiam aſſignatur ſuperior $\underset{m}{x}$.

Et generaliter erit $\frac{1}{\underset{m}{z}} = \frac{1}{z} - \frac{m\underset{.}{z}}{z\underset{,}{z}} + m \cdot \frac{m-1}{2} \frac{2\underset{.}{z}^2 - z\underset{..}{z} + \underset{.}{z}\underset{..}{z}}{z \cdot \underset{,}{z} \cdot \underset{,,}{z}} -$ &c.

Ex. 3. Sit integralis $1 \cdot 2 \cdot 3 \cdot 4 \ldots z$, & ejus incrementum erit $1 \cdot 2 \cdot 3 \cdot 4 \ldots z \cdot (1 - z)^2 = 1 \cdot 2 \cdot 3 \cdot 4 \ldots z \cdot (z + 1) - 1 \cdot 2 \cdot 3 \cdot 4 \ldots z$, ſi modo incrementum quantitatis z ſit 1.

Ex. 4. Sit integralis $\frac{1}{1 \cdot 2 \cdot 3 \cdot 4 \ldots z}$, & ejus incrementum erit $- \frac{1}{1 \cdot 2 \cdot 3 \ldots z - 1 \cdot z + 1} = \frac{1}{1 \cdot 2 \cdot 3 \ldots z + 1} - \frac{1}{1 \cdot 2 \cdot 3 \ldots z}$.

Et ſic ex incrementis inveniri poſſunt integrales.

2. Sit α functio quantitatum x, y, z, &c.; inveniantur $\dot{\alpha}, \ddot{\alpha}, \dddot{\alpha}, \ddddot{\alpha}$, &c.; ex hypotheſi; quod x, y, z, &c. fluunt uniformiter; & conſequenter earum ſecundæ, tertiæ, &c. fluxiones nihilo ſunt æquales; in his quantitatibus ($\dot{\alpha}, \ddot{\alpha}, \dddot{\alpha}$, &c.) pro $\dot{x}, \dot{y}, \dot{z}$, &c. ſcribantur reſpective $\underset{.}{x}, \underset{.}{y}, \underset{.}{z}$, &c.; & reſultent quantitates $\underset{.}{\alpha}, \underset{..}{\alpha}, \underset{...}{\alpha}$, &c.; tum erit primum incrementum quantitatis $(\alpha) = \underset{.}{\alpha} + \frac{1}{1 \cdot 2}\underset{..}{\alpha} + \frac{1}{1 \cdot 2 \cdot 3}\underset{...}{\alpha} + \frac{1}{1 \cdot 2 \cdot 3 \cdot 4}\underset{::}{\alpha} +$ &c. Ex hoc primo incremento per eandem methodum detegi poteſt ſecundum, ſi modo ſupponantur $\ddot{x}, \ddot{y}, \ddot{z}$, &c. invariabiles, & pro iis ſcribantur reſpective $\underset{..}{x}, \underset{..}{y}, \underset{..}{z}$, &c. &c.

PROB. II.

Invenire incrementa fluentium, quarum fluxiones ſunt algebraicæ functiones quantitatis x *in* ẋ *ductæ: inveniri poſſunt e reductione fluxionum in ſeriem, cujus termini ſecundum dimenſiones quantitatis* x *progrediuntur: hujuſce ſeriei inveniatur fluens, & deinde hujuſce fluentis detegatur incrementum quæſitum.*

Ex. . Invenire incrementum fluentis $\int . \frac{\dot{x}}{1 - x}$: erit $\int . \frac{\dot{x}}{1 - x} =$

$x +$

$x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 +$ &c. cujus incrementum quæſitum erit $(1 + x + x^2 + \&c.)\,\underset{\cdot}{x} + \frac{1}{2}(1 + 2x + 3x^2 + \&c.)\,\underset{\cdot}{x^2} + \frac{1}{3}(1 + 3x + 6x^2 + \&c.)\,\underset{\cdot}{x^3} + \frac{1}{4}(1 + 4x + \&c.)\,\underset{\cdot}{x^4} + \&c. = \frac{\underset{\cdot}{x}}{1 - x} + \frac{\frac{1}{2}\underset{\cdot}{x^2}}{(1 - x)^2} + \frac{\frac{1}{3}\underset{\cdot}{x^3}}{(1 - x)^3} + \frac{\frac{1}{4}\underset{\cdot}{x^4}}{(1 - x)^4} + \&c.$ ſi modo incrementum $\underset{\cdot}{x}$ ſit conſtans, fingatur $z = 1 - x$ & reſultat incrementum fluentis $\int.\frac{\dot{z}}{z} = \frac{\underset{\cdot}{z}}{z} - \frac{\underset{\cdot}{z^2}}{2z^2} + \frac{\underset{\cdot}{z^3}}{3z^3} + \&c.$

Et ſic de incrementis deducendis duplicatarum, &c. fluentium, vel quarumcunque aliarum quantitatum, quæ ſint functio finita vel infinita algebraicarum vel exponentialium, &c. quantitatum. Aliter: Earum incrementa detegi poſſunt e methodo in caſu ſecundo præcedentis problematis traditâ.

2. Si vero e dato incremento fluentis (v) requiratur incrementum quantitatis x; inveniatur per infinitas ſeries quantitas x in terminis ſecundum dimenſiones quantitatis v progredientibus, hujuſce quantitatis incrementum erit quæſitum.

E. g. Sit $v = \int.\frac{\dot{x}}{1 + x}$, & erit $x = v + \frac{v^2}{1.2} + \frac{v^3}{1.2.3} + \&c.$ cujus incrementum $\underset{\cdot}{x} = \underset{\cdot}{v}(1 + v + \frac{1}{2}v^2 + \frac{1}{1.2.3}v^3 + \&c.) + \frac{1}{2}\underset{\cdot}{v^2}(1 + v + \frac{1}{1.2}v^2 + \frac{1}{1.2.3}v^3 + \&c.) + \frac{1}{1.2.3}\underset{\cdot}{v^3}(1 + v + \frac{1}{2}v^2 + \frac{1}{2.3}v^3 + \&c.) + \&c. = (1 + x)(\underset{\cdot}{v} + \frac{1}{1.2}\underset{\cdot}{v^2} + \frac{1}{1.2.3}\underset{\cdot}{v^3} + \frac{1}{1.2.3.4}\underset{\cdot}{v^4} + \&c.)$. ſi modo $\underset{\cdot}{v}$ ſit conſtans.

THEOR. I.

Sit quantitas P functio quantitatis x, cujus incrementum ſit Q; tum erit fluxio $\dot{Q}$ quantitatis (Q) æqualis incremento fluxionis $\dot{P}$, ſi modo fluxio $(\dot{o})$ incrementi (o) quantitatis x æqualis ſit incremento fluxionis $\dot{x}$.

Scribatur in quantitate P pro x valor aſſumptus $x + \dot{x} + o + \dot{o}$, & collocentur termini quantitatis reſultantis ſecundum dimenſiones fluxionum $\dot{x}$ & $\dot{o}$, quæ reſpective denotant fluxiones quantitatum x & o, i. e. quantitas reſultans ſit hujuſce formulæ $l + m(\dot{x} + \dot{o}) + (p\dot{x}^2 + q\dot{x}\dot{o} + p\dot{o}^2) +$ &c.; deinde in quantitate P pro x ſcribatur $x + o$ & conſequitur ejus incrementum (α) ipsâ quantitate P auctum, hujus incrementi α inveniatur fluxio quæ erit $m(\dot{x} + \dot{o}) - \dot{P}$, ſicut e ſubſtitutione prædictâ facile conſtat; tertio inveniatur fluxio quantitatis P, vel quod ad idem redit in datâ quantitate P pro x ſcribatur $x + \dot{x}$, & collocentur termimi quantitatis reſultantis ſecundum dimenſiones fluxionis $\dot{x}$, i. e. ſint $a + b\dot{x} + c\dot{x}^2 +$ &c. tum erit $b\dot{x}$ fluxio quantitatis P; in hâc fluxione pro x ſcribatur $x + o$ & pro $\dot{x}$ ejus valor $\dot{x} + \dot{o}$, & e ſubſtitutione facile conſtat eſſe $m(\dot{x} + \dot{o})$, & exinde poteſt theor.

Idem vero mutatis mutandis affirmari poteſt, ſi P ſit functio quarumcunque quantitatum x, y, z, &c. mutatis etiam mutandis erit incrementum n ordinis fluxionis m ordinis quantitatis P æquale fluxioni m ordinis incrementi n ordinis ejuſdem quantitatis P.

PROB. III.

1. *Invenire incrementum cujuſcunque fluentialis quantitatis* $\int . W$.

Sit $\int . W = u$, & incrementum quantitatis $\int . W = \int . (W + \underset{.}{W}) - \int . W = \int . \underset{.}{W} = u$.

Sit W functio quantitatis x in $\dot{x}$, & in eâ pro x ſcribatur $x + \underset{.}{x}$ & reſultet quantitas W', tum erit incrementum quæſitum $\int . W' - \int . W$.

Ex. Sit fluentialis quantitas $\int . V\dot{x}$, cujus incrementum requiritur; inveniatur incrementum quantitatis $V\dot{x}$, quæ erit $\dot{x}\underset{.}{V} + V(\underset{.}{\dot{x}}) + \underset{.}{V}(\underset{.}{\dot{x}})$, & exinde incrementum fluentis $\int . V\dot{x} = \int . \dot{x}\underset{.}{V} + \int . V(\underset{.}{\dot{x}}) + \int . \underset{.}{V}(\underset{.}{\dot{x}})$; at $\int . V\underset{.}{\dot{x}} = V\underset{.}{x} - \int . \underset{.}{x}\dot{V}$, unde $\underset{.}{\underline{\int . V\dot{x}}} = \int . \dot{x}\underset{.}{V} + V\underset{.}{x} - \int . \underset{.}{x}\dot{V} + \int . \underset{.}{V}(\underset{.}{\dot{x}})$.

2. Si requiratur incrementum duplicis fluentialis formulæ $\int . \int . \underline{W}$, tum erit $(\int . \int . W) = \int . (\int . W) = \int . \int . W$, & ſic deinceps.

3. Sint incrementa evaneſcentia, & ponantur $\dot{y} = p\dot{x}$, $\dot{p} = q\dot{x}$, $\dot{q} = r\dot{x}$, &c.; cum fuerit V functio quæcunque quantitatum $x, y, p, q, r. s$, &c.; tum erit $\dot{V} = M\dot{x} + N\dot{y} + P\dot{p} + Q\dot{q} +$ &c., & ſimili modo erit $\underset{\cdot}{V} = M\underset{\cdot}{x} + N\underset{\cdot}{y} + P\underset{\cdot}{p} + Q\underset{\cdot}{q} +$ &c.

Ex inveniendis incrementis quantitatum $\int. V\dot{x}$, &c. detegi poſſunt maxima & minima, ſi modo in incrementis reſultantibus nihilo fiant æqualia incrementa, & exinde fluentiales quantitates etiam nihilo evadant æquales; ſed hic non de maximis & minimis, nec de evaneſcentibus incrementis agitur.

PROB. IV.

Transformare datum incrementum in alterum, cujus variabiles quantitates (z, v, *&c.*) *datam habeant relationem ad variabiles* (x, y, *&c.*) *in dato incremento contentas. Subſtituantur valores quantitatum* (x, y, *&c.*) *in terminis quantitatum* (z *&* v, *&c.*) *expreſſi pro quantitatibus ipſis* (x, y, *&c.*) *in dato incremento, & eorum incrementa pro reſpectivis incrementis quantitatum* (x, y. *&c.*) *& reſultat incrementum quæſitum.*

PROB. V.

Datum incrementum in alterum datæ formulæ, ſi modo fieri poſſit, reducere.

Aſſumantur generaliter incrementa datæ formulæ, ſupponatur eorum ſumma dato incremento æqualis, & exinde erui poſſunt coefficientes, &c. incrementorum aſſumptorum.

Ex. 1. Sit $\dot{x}$ conſtans; reducere quantitates x^n ad incrementa formulæ $x \times (x - \underset{\cdot}{x}) \times (x - 2\dot{x}) \,..\, (x - m\underset{\cdot}{x})$.

Aſſumantur $x^n = x \times (x - \underset{\cdot}{x}) \times (x - 2\underset{\cdot}{x}) \,..\, (x - (n-1)\underset{\cdot}{x}) + b\underset{\cdot}{x}x \times (x - \underset{\cdot}{x}) . (x - 2\underset{\cdot}{x}) \,..\, (x - (n-2)\underset{\cdot}{x}) + c\underset{\cdot}{x}^2 x \times (x - \underset{\cdot}{x}) \,...\, (x - (n-3)\underset{\cdot}{x}) +$ &c. e reductione facile conſtat $b = n . \frac{n-1}{2}$, $c = \frac{5n^3 - 42n^2 + 127n - 135}{3}$, &c.

Ex. 2. Sit $\underset{.}{x}$ conſtans, & inveniri poteſt $\frac{1}{x^2} = \frac{1}{x\times(x+\underset{.}{x})} + \frac{\underset{.}{x}}{x\times(x+\underset{.}{x})\times(x+2\underset{.}{x})} + \frac{2\underset{.}{x}^2}{x(x+\underset{.}{x})(x+2\underset{.}{x})(x+3\underset{.}{x})} + \frac{6\underset{.}{x}^3}{x(x+\underset{.}{x})\dots(x+4\underset{.}{x})} + \frac{24\ \&c.}{\&c.}$. & $\frac{1}{x^n} = \frac{1}{x(x+\underset{.}{x})\dots(x+(n-1)\underset{.}{x})} + \frac{n\,.\,\frac{n-1}{2}\underset{.}{x}}{x(x+\underset{.}{x})\dots(x+n\underset{.}{x})}$ + &c.

PROB. VI.

1. *Sit* $\underset{.}{\mathrm{x}}$ *incrementum quantitatis* x *conſtans, invenire integralem quantitatis* $\mathrm{A\,x}\times(\mathrm{x}-\underset{.}{\mathrm{x}})\,.\,(\mathrm{x}-2\underset{.}{\mathrm{x}})\dots(\mathrm{x}-\mathrm{n}\underset{.}{\mathrm{x}})$.

Integralis erit $\frac{A}{(n+2)\underset{.}{x}}(x+\underset{.}{x})\times x\times(x-\underset{.}{x})\,..\,(x-n\underset{.}{x})$; inveniantur enim duæ ſucceſſivæ integrales $\frac{A}{(n+2)\underset{.}{x}}\times(x+\underset{.}{x})\times x\,.\,(x-\underset{.}{x})\,..\,(x-n\underset{.}{x})$ & $\frac{A}{(n+2)\underset{.}{x}}x\,.\,(x-\underset{.}{x})\,.\,(x-2\underset{.}{x})\,..\,(x-n\underset{.}{x})\times(n-n\underset{.}{x}-\underset{.}{x})$, & earum differentia erit quantitas data.

Co. . Dato incremento $\underset{.}{x}$; omnis quantitas hujuſce formulæ $Ax^n + Bx^{n-1} + Cx^{n-2} + Dx^{n-3}$ + &c. ubi n eſt integer affirmativus numerus, & A, B, C, D, &c. invariabiles denotant quantitates, tranſformari poteſt in quantitates prædictæ formulæ $ax\times(x-\underset{.}{x})\times(x-2\underset{.}{x})\dots(x-(n-1)\underset{.}{x}) + bx\times(x-\underset{.}{x})\,..\,(x-(n-2)\underset{.}{x}) + cx\times(x-\underset{.}{x})\,..\,(x-(n-3)\underset{.}{x}) + d\times$ &c. æquatis autem terminis correſpondentibus hujuſce & datæ æquationis, i. e. ſint $a=A$, $(n-1)\times\frac{n}{2}a\underset{.}{x}+b=B$, &c. inveniri poſſunt coefficientes a, b, &c. & conſequenter reduci poteſt data quantitas ad incrementa formularum, quarum innoteſcunt integrales.

Aliter: Invenire integralem prædictæ incrementialis quantitatis $Ax^n + Bx^{n-1} + Cx^{n-2} + \&c.$ aſſumatur quantitas $ax^{n+1} + bx^n + cx^{n-1} + dx^{n-2} + \&c.$ pro integrali deducendâ, cujus inveniatur proxime ſucceſſivus valor, qui erit $a(x+\underset{\cdot}{x})^{n+1} + b(x+\underset{\cdot}{x})^n + c(x+\underset{\cdot}{x})^{n-1} + \&c.$ unde differentia inter duos ſucceſſivos valores erit $a((x+\underset{\cdot}{x})^{n+1} - x^{n+1}) + b((x+\underset{\cdot}{x})^n - x^n) + c((x+\underset{\cdot}{x})^{n-1} - x^{n-1}) + \&c. =$

$$(n+1)ax^n\underset{\cdot}{x} + \left.\begin{matrix}(n+1)\cdot\frac{n}{2}a\underset{\cdot}{x}^2\\ +nb\underset{\cdot}{x}\end{matrix}\right\} x^{n-1} + \left.\begin{matrix}(n+1)\cdot\frac{n}{2}\cdot\frac{n-1}{3}a\underset{\cdot}{x}^3\\ +n\cdot\frac{n-1}{2}b\underset{\cdot}{x}^2\\ +(n-1)c\underset{\cdot}{x}\end{matrix}\right\} x^{n-2} + \&c.$$

æquatis autem terminis hujus & datæ æquationis, reſultant $(n+1)$ æquationes $a\underset{\cdot}{x} = A$, $(n+1)\cdot\frac{n}{2}a\underset{\cdot}{x}^2 + nb\underset{\cdot}{x} = B$, &c. unde facile conſtant coefficientes a, b, c, &c. & exinde integralis datæ quantitatis.

Aliter: Aſſumatur quantitas $ax^{n+1} + p$ pro integrali quæſitâ, cujus incrementum erit $(n+1)a\underset{\cdot}{x}x^n + (n+1)\times\frac{n}{2}a\underset{\cdot}{x}^2x^{n-1} + \&c. + \underset{\cdot}{p} = Ax^n + Bx^{n-1} + Cx^{n-2} + \&c.$ & exinde $(n+1)a\underset{\cdot}{x} = A$, & $\underset{\cdot}{p} = (B - (n+1)\cdot\frac{n}{2}a\underset{\cdot}{x}^2)x^{n-1} + \&c.$ & conſequenter $n\underset{\cdot}{x}p = (B - (n+1)\times\frac{n}{2}a\underset{\cdot}{x}^2)x^n$ prope; ſcribatur igitur in præcedente æquatione $(B - (n+1)\times\frac{n}{2}a\underset{\cdot}{x}^2)x^n + q$ pro p, & ejus incrementum pro $\underset{\cdot}{p}$, & exinde deduci poteſt propinquus valor quantitatis q; & ſic deinceps uſque donec inveniatur integralis quæſita.

2. Sit $\underset{\cdot}{x}$ incrementum quantitatis x conſtans, & incrementialis quantitatis $\frac{\underset{\cdot}{x}}{x\times(x-\underset{\cdot}{x})\times(x-2\underset{\cdot}{x})\ldots(x-n\underset{\cdot}{x})}$ integralis invenietur

$$\frac{\underset{\cdot}{x}}{n\underset{\cdot}{x}\times(x-\underset{\cdot}{x})\times(x-2\underset{\cdot}{x})\ldots(x-n\underset{\cdot}{x})} = \frac{1}{n\times(x-\underset{\cdot}{x})\ldots(x-n\underset{\cdot}{x})}.$$

Facile conſtat hic caſus ex differentiâ inter duos ſucceſſivos valores integralis prædictæ.

3. Sit incrementum $\frac{ax^{m}+bx^{m-1}+cx^{m-2}+dx^{m-3}+ex^{m-4}+\&c.}{x\times(x+\underset{\cdot}{x})\times(x+2\underset{\cdot}{x})\times(x+3\underset{\cdot}{x})\ldots.(x+(n-1)\underset{\cdot}{x})}$ rationalis functio quantitatum x & $\underset{\cdot}{x}$ ad minimos terminos reducta; ubi m eſt integer numerus, & æqualis vel minor quam n, & $\underset{\cdot}{x}$ eſt conſtans; invenire annon integralis incrementi prædicti exprimi poteſt in finitis terminis.

Reducatur datum incrementum ad incrementa hujuſce generis $\alpha+\left(\frac{\beta}{x}\right)+\frac{\gamma}{x.(x+\underset{\cdot}{x})}+\frac{\delta}{x.(x+\underset{\cdot}{x})(x+2\underset{\cdot}{x})}+\frac{\epsilon}{x(x+\underset{\cdot}{x})(x+2\underset{\cdot}{x})(x+3\underset{\cdot}{x})}+\ldots.\frac{\theta}{x(x+\underset{\cdot}{x})(x+2\underset{\cdot}{x})\ldots(x+(n-1)\underset{\cdot}{x})}$; ſi $m=n$, tum $\alpha=a$, $\beta=b-\alpha A$, $\gamma=c-\beta A'-\alpha B$, $\delta=d-\gamma A''-\beta B'-\alpha C$, $\epsilon=e-\delta A'''-\gamma B''-\beta C'-\alpha D$, &c.; ubi $x(x+\underset{\cdot}{x})(x+2\underset{\cdot}{x})\ldots(x+(n-1)\underset{\cdot}{x})=x^{n}+Ax^{n-1}+Bx^{n-2}+Cx^{n-3}+Dx^{n-4}+\&c.$; $(x+\underset{\cdot}{x})(x+2\underset{\cdot}{x})\,.\,.\,(x+(n-1)\underset{\cdot}{x})=x^{n-1}+A'x^{n-2}+B'x^{n-3}+C'x^{n-4}+\&c.$, $(x+2\underset{\cdot}{x})(x+3\underset{\cdot}{x})\ldots(x+(n-1)\underset{\cdot}{x})=x^{n-2}+A''x^{n-3}+B''x^{n-4}+D''x^{n-5}+\&c.$, $(x+3\underset{\cdot}{x})(x+4\underset{\cdot}{x})\ldots(x+(n-1)\underset{\cdot}{x})=x^{n-3}+A'''x^{n-4}+B'''x^{n-5}+C'''x^{n-6}+\&c.$, &c.

Si $\beta=b-n.\frac{n-1}{2}a\underset{\cdot}{x}$ haud ſit nihilo æqualis, tum integralis non exprimi poteſt, ni detur integralis incrementi $\frac{\beta}{x}$; in reliquis caſibus per hanc methodum ſemper inveniri poteſt integralis; erit enim $\alpha\frac{x}{\underset{\cdot}{x}}+\frac{\gamma}{\underset{\cdot}{x}x}+\frac{\delta}{2\underset{\cdot}{x}x(x+\underset{\cdot}{x})}+\frac{\epsilon}{3\underset{\cdot}{x}x(x+\underset{\cdot}{x})(x+2\underset{\cdot}{x})}+\ldots+\frac{\theta}{(n-1)\underset{\cdot}{x}x(x+\underset{\cdot}{x})(x+2\underset{\cdot}{x})\,.\,.\,(x+(n-2)\underset{\cdot}{x})}$, ubi α, β, γ, δ, &c. ſunt invariabiles quantitates.

Si $m = n - 1$; tum integralis non exprimi poteſt, ni detur integralis incrementi $\left(\frac{\beta}{x}\right)$: ſi autem m minor ſit quam $n - 1$, tum ſemper per hanc regulam inveniri poteſt integralis.

Si vero m major ſit quam n, tum reducenda eſt data fractio ad propriam, ita ut maxima poteſtas quantitatis x in denominatore major ſit quam maxima poteſtas in numeratore.

4. Sit incrementum fractio

$$\frac{a x^{m'} + b x^{m'+1} + c x^{m'+2} + \&c.}{x.(x+\underset{\cdot}{x})(x+2\underset{\cdot}{x})\dots(x+(n-1)\underset{\cdot}{x}) \times (x+p)(x+p+\underset{\cdot}{x})(x+p+2\underset{\cdot}{x})\dots(x+p+(m-1)\underset{\cdot}{x}) \times (x+q) \times (x+q+\underset{\cdot}{x})(x+q+2\underset{\cdot}{x})\dots(x+q+(r-1)\underset{\cdot}{x}) \times \&c.}$$

Semper reduci poteſt hæc fractio ad terminos progredientes ſecundum formulas ſubſequentes

$$\alpha + \left(\frac{\beta}{x}\right) + \frac{\gamma}{x.(x+\underset{\cdot}{x})} + \frac{\delta}{x(x+\underset{\cdot}{x})(x+2\underset{\cdot}{x})} + \frac{\varepsilon}{x(x+\underset{\cdot}{x})(x+2\underset{\cdot}{x})(x+3\underset{\cdot}{x})} + \dots + \frac{\theta}{x(x+\underset{\cdot}{x})(x+2\underset{\cdot}{x})\dots(x+(n-1)\underset{\cdot}{x})}$$

$$+ \left(\frac{\beta'}{x+p}\right) + \frac{\gamma'}{(x+p)(x+p+\underset{\cdot}{x})} + \frac{\delta'}{(x+p)(x+p+\underset{\cdot}{x})(x+p+2\underset{\cdot}{x})} + \dots + \frac{\theta'}{(x+p)(x+p+\underset{\cdot}{x})(x+p+2\underset{\cdot}{x})\dots(x+p+(m-1)\underset{\cdot}{x})}$$

$$+ \left(\frac{\beta''}{x+q}\right) + \frac{\gamma''}{(x+q)(x+q+\underset{\cdot}{x})} + \frac{\delta''}{(x+q)(x+q+\underset{\cdot}{x})(x+q+2\underset{\cdot}{x})} + \dots + \frac{\theta''}{(x+q)(x+q+\underset{\cdot}{x})(x+q+2\underset{\cdot}{x})\dots(x+q+(r-1)\underset{\cdot}{x})}$$

+ &c., ubi $\alpha, \beta, \gamma, \dots \theta$; β', γ', &c.; β'', γ'', &c.; p, q, &c; ſunt invariabiles quantitates.

Conſtat ex eo quod continentur $n + m + r +$ &c. $+ 1$ invariabiles, independentes & incognitæ coefficientes $\alpha, \beta, \gamma, \delta$, &c.; β', γ', &c.; β'', γ'', &c.; in quantitate aſſumptâ.

Si β, β', β'', &c. nihilo ſint reſpective æquales, vel integralis incrementi $\frac{\beta}{x} + \frac{\beta'}{x+p} + \frac{\beta''}{x+q} + \&c.$ finitis terminis exprimi poſſit, tum integralis dati incrementi ſemper iiſdem exprimi poteſt; ſin aliter non.

Hinc, ſi modo numerus literarum o, p, q, &c. ſit λ; ex λ independentibus integralibus hujuſce formulæ datis deduci poſſunt omnes ejuſdem formulæ.

Si $m' = n + m + r + \&c. - 1$, tum nunquam exprimi poteſt integralis; ſi m' major ſit quam $n + m + r + \&c.$; tum reducenda eſt data ad propriam fractionem: ſi $m' = n + m + r + \&c. - 2$ vel minor; tum ſemper $\beta + \beta' + \beta'' + \&c. = 0$.

5. Sit incrementum $\frac{a x^m + b x^{m-1} + c x^{m-2} + \&c.}{x^\pi (x+\underset{.}{x})^\pi (x+2\underset{.}{x})^\pi \ldots (x+(n-1)\underset{.}{x})^\pi \times (x^{\pi'} (x+p+\underset{.}{x})^{\pi'} \times (x+p+2\underset{.}{x})^{\pi'} \ldots (x+p+(r-1)\underset{.}{x})^{\pi'} \times \&c.}$; hoc incrementum reduci poteſt ad terminos progredientes ſecundum ſubſequentes formulas

$$\alpha + \left(\frac{\beta}{x} + \frac{\gamma}{x^2} + \frac{\delta}{x^3} + \ldots + \frac{\lambda}{x^\pi}\right) + \left(\frac{\beta'}{x+p} + \frac{\gamma'}{(x+p)^2} + \frac{\delta'}{(x+p)^3} \ldots + \frac{\lambda'}{(x+p)^\pi}\right) + \left(\frac{\varepsilon}{x(x+\underset{.}{x})} + \frac{\varepsilon'}{x(x+\underset{.}{x})(x+2\underset{.}{x})}\right.$$

$$\left. + \ldots + \frac{\varepsilon^{\overset{n-2}{\prime}}}{x(x+\underset{.}{x}) \ldots (x+(n-1)\underset{.}{x})}\right) + \left(\frac{\zeta}{(x+p)(x+p+\underset{.}{x})} + \right.$$

$$\left.\frac{\zeta''}{(x+p)(x+p+\underset{.}{x})(x+p+2\underset{.}{x})} + \ldots + \frac{\zeta^{\overset{r-2}{\prime}}}{x(x+p+\underset{.}{x}) \ldots (x+p+(r-1)\underset{.}{x})}\right)$$

$$+ \frac{a'((x+\underset{.}{x})^2 - x^2)}{x^2 \times (x+\underset{.}{x})^2} + \frac{a''((x+\underset{.}{x})^3 - x^3)}{x^3 (x+\underset{.}{x})^3} + \ldots + \frac{a^{\overset{\pi-2}{\prime}}((x+(n-1)\underset{.}{x})^\pi - x^\pi)}{x^\pi \times (x+\underset{.}{x})^\pi}$$

$$+ \frac{b'(x+p+\underset{.}{x})^2 - (x+p)^2}{(x+p)^2 (x+p+\underset{.}{x})^2} + \frac{b''(x+p+\underset{.}{x})^3 - (x+p)^3}{(x+p)^3 (x+p+\underset{.}{x})^3} + \ldots +$$

$b'(x+$

$$\frac{\overset{\pi'-3}{b}(x+p+\underset{\cdot}{x})^{\pi'}-(x+p)^{\pi'}}{(x+p)^{\pi'}(x+p+\underset{\cdot}{x})^{\pi'}}+\&c.\ldots;\frac{d'(x+2\underset{\cdot}{x})^2-x^2}{x^2\times(x+\underset{\cdot}{x})^2\times(x+2\underset{\cdot}{x})^2}+$$

$$\frac{d''(x+2\underset{\cdot}{x})^3-x^3}{x^3(x+\underset{\cdot}{x})^3(x+2\underset{\cdot}{x})^3}+\ldots\ldots+\frac{\overset{\pi-3}{d'}(x+2\underset{\cdot}{x})^{\pi}-x^{\pi}}{x^{\pi}\times(x+\underset{\cdot}{x})^{\pi}\times(x+2\underset{\cdot}{x})^{\pi}}+$$

$$\frac{e\times(x+p+2\underset{\cdot}{x})^2-(x+p)^2}{(x+p)^2\times(x+p+\underset{\cdot}{x})^2\times(x+p+2\underset{\cdot}{x})^2}+\frac{e'(x+p+2\underset{\cdot}{x})^3-(x+p)^3}{(x+p)^3\times(x+p+\underset{\cdot}{x})^3\times(x+p+2\underset{\cdot}{x})^3}$$

$$+\ldots\ldots+\frac{\overset{\pi'-3}{e'}(x+p+2\underset{\cdot}{x})^{\pi'}-(x+p)^{\pi'}}{(x+p)^{\pi'}\times(x+p+\underset{\cdot}{x})^{\pi'}\times(x+p+2\underset{\cdot}{x})^{\pi'}}+\frac{f(x+3\underset{\cdot}{x})^2-x^2}{x^2(x+\underset{\cdot}{x})^2\times(x+2\underset{\cdot}{x})^2\times(x+3\underset{\cdot}{x})^2}+\&c.+$$

$$\frac{g(x+p+3\underset{\cdot}{x})^2-(x+p)^2}{(x+p)^2\times(x+p+\underset{\cdot}{x})^2\times(x+p+2\underset{\cdot}{x})^2\times(x+p+3\underset{\cdot}{x})^2}+\&c.+$$

$$\frac{l(x+(n-1)\underset{\cdot}{x})^2-x^2}{(x^2)(x+\underset{\cdot}{x})^2(x+2\underset{\cdot}{x})^2(x+3\underset{\cdot}{x})^2\ldots(x+(n-1)\underset{\cdot}{x})^2}+\ldots\ldots+\frac{\overset{\pi}{l'}(x+(n-1)\underset{\cdot}{x})^{\pi}-x^{\pi}}{x^{\pi}(x+\underset{\cdot}{x})^{\pi}(x+2\underset{\cdot}{x})^{\pi}\ldots(x+(n-1)\underset{\cdot}{x})^{\pi}}$$

$$+\frac{\overset{\pi}{l''}(x+p+(r-1)\underset{\cdot}{x})^2-x^2}{(x+p)^2(x+p+\underset{\cdot}{x})^2(x+p+2\underset{\cdot}{x})^2(x+p+3\underset{\cdot}{x})^2\ldots(x+p+(r-1)\underset{\cdot}{x})^2}$$

$$+\ldots\ldots+\frac{\overset{\pi'}{l'}(x+p+(r-1)\underset{\cdot}{x})^{\pi'}-x^{\pi'}}{(x+p)^{\pi'}(x+p+\underset{\cdot}{x})^{\pi'}\ldots(x+p+(r-1)\underset{\cdot}{x})^{\pi'}},\ \&c.;$$

quorum ſingulorum integrales detegi poſſunt, ſi integralis quantitatis $\frac{\beta}{x}+\frac{\gamma}{x^2}+\frac{\delta}{x^3}+\&c.+\frac{\beta'}{x+p}+\frac{\gamma'}{(x+p)^2}+\frac{\delta'}{(x+p)^3}+\&c.$ deduci poſſit.

Si data fractio ſit impropria, tum reducatur ea ad propriam fractionem.

Hæc principia etiam ad inveniendas integrales omnium rationalium functionum variabilis (x) applicari poſſunt.

Si vero in denominatore contineantur duo factores $x+l\underset{\cdot}{x}$ & $x+(l+r)\underset{\cdot}{x}$,

$(l+r)\underset{.}{x}$, vel quicunque factores inter hos poſiti, ubi r ſit integer numerus: tum reducenda eſt data fractio ad conſimiles terminos, i. e. eoſdem denominatores habentes ac ii, in quibus habetur contentum $(x+l\underset{.}{x})\,(x+(l+1)\underset{.}{x})\,(x+(l+2)\underset{.}{x})\,(x+(l+3)\underset{.}{x})\ldots(x+(l+r)\underset{.}{x})$. Et ſic de pluribus contentis hujuſce generis & eorum poteſtatibus.

7. Dimenſiones quantitatis x in incremento ſemper erunt minores quam dimenſiones ejuſdem quantitatis in integrali per unitatem, ni dimenſiones quantitatis x in numeratore integralis æquales ſint ejus dimenſionibus in ejus denominatore, in quo caſu dimenſiones quantitatis x in numeratore incrementi minores erunt per quantitatem majorem quam unitatem quam dimenſiones quantitatis x in denominatore.

Cor. Si dimenſiones quantitatis x in numeratore incrementi ſint minores quam dimenſiones quantitatis x in denominatore per unitatem, tum ejus integralis non deduci poteſt.

Et ſic de fractionibus reducendis, &c. in quaſcunque formulas, quarum integrales innoteſcunt.

8. Inveniantur ſinguli diviſores denominatoris fractionis $Ax^n+Bx^{n-1}+$ &c. qui ſint α, β, γ, δ, &c. & ſi quicunque diviſor inveniatur in denominatore, qui non habet alterum a ſe diſtantem per integrum numerum r in $\underset{.}{x}$ ductum, tum integralis dati incrementi finitis terminis haud exprimi poteſt. E. g. Sit denominator $(x+\frac{1}{2}\underset{.}{x})\times(x-4\underset{.}{x})\times(x-5\underset{.}{x})$, nunc diviſor $x+\frac{1}{2}\underset{.}{x}$ non habet alterum in denominatore a ſe diſtantem per integrum numerum in $\underset{.}{x}$, ergo integralis dati incrementi finitis terminis haud exprimi poteſt.

Eadem principia etiam ad irrationales quantitates applicari poſſunt.

1. Contineantur ſolummodo in dato incrementi $\dfrac{a'x^m+b'x^{m-1}+\text{\&c.}}{Ax^n+Bx^{n-1}+\text{\&c.}}$ denominatore duo ſimplices diviſores $x+l\underset{.}{x}$ & $x+(l+r)\underset{.}{x}$; vel hi duo & quicunque alii quorum numerus ſit r inter eos poſiti, ubi r

ſit integer numerus; tum aſſumenda eſt quantitas $((x + l\underset{.}{x}) \times (x + (l + 1)\underset{.}{x}) \times (x + (l + 2)\underset{.}{x}) \ldots (x + (l + r - 1)\underset{.}{x}))^{-1} \times (\alpha x^{m+r+1-n} + \beta x^{m+r-n} + \&c.)$ ubi α, β, &c. ſunt invariabiles coefficientes inveſtigandæ, pro integrali quæſitâ; inveniatur ejus incrementum, quod dato incremento fiat æquale, & ex æquatis eorum correſpondentibus terminis deduci poſſunt invariables coefficientes α, β, γ, &c.

2. Sit denominator $P = (x + a\underset{.}{x})(x + (a + b)\underset{.}{x})(x + (a + b + c)\underset{.}{x})$ $(x + e\underset{.}{x})(x + (e + f)\underset{.}{x})(x + (e + f + g)\underset{.}{x})(x + k\underset{.}{x})(x + (k + l)\underset{.}{x})$ &c.) ubi b, c; f, g; l, &c. reſpective denotant integros numeros; tum aſſumenda eſt quantitas $((x + a\underset{.}{x})(x + (a + 1)\underset{.}{x})(x + (a + 2)\underset{.}{x}) \ldots (x + (a + b + c - 1)\underset{.}{x})(x + e\underset{.}{x})(x + (e + 1)\underset{.}{x})(x + (e + 2)\underset{.}{x}) \ldots (x + e + f + g - 1)\underset{.}{x})(x + k\underset{.}{x})(x + (k + 1)\underset{.}{x}) \ldots (x + (k + l - 1)\underset{.}{x}) \times \&c.)^{-1} \times (\alpha x^{\pi} + \beta x^{\pi - 1} + \&c.)$ ubi $\pi = m - n + 1 + b + c + f + g + l + \&c.$ pro integrali quæſita, cujus inveniatur incrementum, & ex æquatis inter ſe incrementi dati & inventi correſpondentibus terminis, inveſtigari poſſunt coefficientes α, β, γ, &c.

Cor. 2. Si modo numerus quantitatum a, e, l, &c. ſit π, & dentur $\pi - 1$ diverſæ independentes integrales incrementorum; tum ex iis deduci poſſunt integrales e quibuſcunque incrementis prædicti generis.

3. Sit quantitas $\dfrac{a'x^{m} + b'x^{m-1} + \&c.}{(x + \alpha\underset{.}{x})^{n} \times (x + (\alpha + r)\underset{.}{x})^{n}}$, ubi m, n & r ſunt integri numeri: aſſumatur $((x + \alpha\underset{.}{x})(x + (\alpha + 1)\underset{.}{x})(x + (\alpha + 2)\underset{.}{x}) \ldots (x + (\alpha + r - 1)\underset{.}{x}))^{-n} \times (\alpha x^{m+(r-2)n+1} + \beta x^{m+(r-2)n} + \&c.)$ pro integrali quæſitâ, cujus inveniatur incrementum; & ex æquatis dati & inventi incrementi correſpondentibus terminis deduci poſſunt coefficientes α, β, &c.

Conſimilis erit ſubſtitutio, ſi modo inter $(x + \alpha\underset{.}{x})^{n}$ & $(x + (\alpha + r)\underset{.}{x})^{n}$ interponantur quidam termini formulæ $(x + (\alpha + s)\underset{.}{x})^{n}$, ubi s eſt integer numerus minor vero quam r.

Si vero plures horum generum, &c. quantitates in denominatore contineantur. E. g. Sint $(x + \alpha\underset{.}{x})^{n}$ & $(x + (\alpha + r)\underset{.}{x})^{n}$, $(x + \beta\underset{.}{x})^{p}$ &

& $(x + (\beta + s)\underset{\cdot}{x})^p$, &c. in denominatore contentæ, tum aſſumatur pro denominatore integralis $((x + \alpha\underset{\cdot}{x})(x + (\alpha + 1)\underset{\cdot}{x}) \,.\,.\, (x + (\alpha + r - 1)\underset{\cdot}{x}))^{-n} \times ((x + \beta\underset{\cdot}{x})(x + (\beta + 1)\underset{\cdot}{x}) \ldots (x + (\beta + s - 1)\underset{\cdot}{x}))^{-p}$ $\times$ &c. & per principia prius tradita deduci poteſt integralis incrementi dati, ſi modo in finitis terminis exprimi poſſit.

Cor. 3. Hinc facile conſtat numerus diverſarum independentium incrementorum prædicti generis integralium, e quibus deduci poſſunt integrales ſingulorum ejuſdem generis incrementorum.

4. Dato irrationali incremento; primo inveniendum eſt, annon quædam irrationales quantitates ſint ſucceſſivi valores aliarum, & exinde conſtabit ſubſtitutio, quam integralis dati incrementi exigit; i. e. abjiciantur ultimi ſucceſſivi valores ſingulorum factorum in denominatore, etiamque ſingulorum irrationalium in numeratore contentorum: horum factorum deducatur primus; tum aſſumantur pro integrali prædicti factores (ultimis abjectis) in ſeſe & in factores deducendos ducti; & exinde erui poteſt integralis quæſita.

Ex. Sit incrementum, cujus numerator ſit $(e + 2f + 4g)^l (e + 3f + 9g)^l (e + 4f + 16g)^l \ldots (e + (z + 1)f + (z + 1)^2 g)^l \times (\alpha \times (z + 1) + \beta) - ((a + (z^2 + 1)b)^m (a + (z^2 + 2)b)^m (a + (z^2 + 3)b)^m \ldots (a + (z + 1)^2 b)^m \times (e + f + g)^l \ldots (e + zf + z^2 g)^l \times (\alpha z + \beta)$ & denominator $a^m (a + b)^m (a + 2b)^m (a + 3b)^m \,.\,.\, (a + (z + 1)^2 b)^m$; pro denominatore integralis aſſumenda eſt quantitas $a^m (a + b)^m (a + 2b)^m \ldots (a + z^2 b)^m$; ubi ultimi factores, qui ex denominatore evaneſcunt, cum z evadat minor per ejus incrementum; viz. $(a + (z^2 + 1)b)^m (a + (z^2 + 2)b)^m \,.\,.\, (a + (z + 1)^2 b)^m$ rejiciuntur; & pro numeratore $= (e + 2f + 4g)^l (e + 3f + 9g)^l \,.\,.\, (e + zf + z^2 g)^l \times$ in terminum præcedentem hujuſce ſeriei, qui facile cernitur, viz. $(e + f + g) \times (k + hz)$, ubi k & b ſunt invariabiles coefficientes inveſtigandæ; hujuſce integralis inveniatur incrementum, & fiat dato incremento æquale, & invenientur $h = \alpha$ & $k = \beta$: obſervandum eſt, ſi ſint quædam irrationales quantitates in denominatore dati incrementi contentæ, quæ non habent etiam in eo earum ſucceſſivos valores, tum minime exprimi poteſt in finitis terminis integralis dati incrementi.

Per ſucceſſivos valores quantitatis A, intelligo valores reſultantes e ſubſtitutione quantitatum $x + \dot{x}$, $x + 2\dot{x}$, &c. pro x in quantitate A. Sit incrementum $B - A$, & ſi B ſit ſucceſſivus valor quantitatis A; tum erit A integralis quæſita.

Hæc methodus, cum ſeries non terminet, ſemper dabit ſeriem terminorum ſecundum reciprocas dimenſiones quantitatis x progredientium.

5. Hæ vero irrationales quantitates reduci poſſunt ad plures infinitas ſeries ſecundum dimenſiones quantitatis x progredientes vel in ſeries formulæ $a + bx + c \times x \times (x - \dot{x}) + dx \times (x - \dot{x}) \times (x - 2\dot{x}) +$ &c. vel formulæ $a + \frac{b}{x} + \frac{c}{x \times (x + \dot{x})} + \frac{d}{x \times (x + \dot{x}) \times (x + 2\dot{x})} +$ &c. &c. vel $a\dot{x} + 2bx\dot{x} + (4c + 2d\dot{x}^2)x^3\dot{x} +$ &c. vel formulæ $a + bx^r + cx^r \times (x - \dot{x})^r + dx^r \times (x - \dot{x})^r \times (x - 2\dot{x})^r +$ &c. vel formulæ $a + \frac{b}{x^r} + \frac{c}{x^r \times (x - \dot{x})^r} +$ &c. in quibus ſingulis eædem adhibentur radices; tum inveniatur aggregatum (P) ex integralibus ſingularum ſerierum; ducantur ſingulæ duæ prædictæ infinitæ integrales in ſeſe, & inveniatur aggregatum e ſingulis productis; ducantur quæque tres, quatuor, &c. prædictæ integrales in ſeſe, & inveniantur aggregata ſingularum quantitatum reſultantium, quæ dicantur reſpective Q, R, S, T, &c.; tum erit $1 - \frac{P}{v} + \frac{Q}{v^2} - \frac{R}{v^3} +$ &c. $= 0$ æquatio, cujus radix v eſt integralis quæſita.

6. Si vero in incremento dato contineantur incrementa duorum vel plurium ordinum quantitatis x; tum abjiciantur omnes termini, in quibus inveniuntur incrementa ſuperiorum ordinum; & e terminis, in quibus ſolummodo contineatur incrementum primi ordinis, inveniatur integralis; inter incrementum inventæ integralis & datum ſumatur differentia, cujus integralis e præcedente methodo detegatur, & ſic deinceps; & tandem detegetur integralis dati incrementi.

THEOR. II.

Sit incrementum $\left(\underset{\overset{\cdot}{n}}{V}\right)$ n ordinis & $\underset{\cdot}{x}$ conſtans; tum erit ejus integralis generaliter correcta $V + ax^{n-1} + bx^{n-2} + cx^{n-3} \dots kx + l$, ubi $a, b, c; k, l$ denotant quaſcunque invariabiles coefficientes ad libitum aſſumendas. Hinc ad integrales quantitates applicari poſſunt ea, quæ edita fuere de fluentibus fluxionum in prob. 8, & 9. libri præcedentis.

THEOR. III.

1. Sint duæ vel plures variabiles quantitates (x, y, z, &c.) in data incrementiali quantitate contentæ, aſſumantur omnes variabiles quantitates præter quamcunque unam tanquam invariables & inveniatur integralis quantitatis reſultantis; & ſic de ſingulis reliquis variabilibus quantitatibus; & ſi omnes termini integralium inventarum ex aſſumendo omnes variabiles præter unam tanquam invariabiles, in quibus inveniuntur plures quam una variabiles quantitates (x, y, z, &c.) haud ſint iidem, tum haud admittet integralem datum incrementum: ſi ſint iidem, tum inveniantur eorum incrementa, & invenietur, utrum integralis datæ quantitatis inveniri poteſt, necne.

2. Si incrementum unius variabilis quantitatis ſit invariabilis, tum incrementa cæterarum haud poſſunt eſſe dati numeri, nec exprimi poſſunt, niſi per literas deſignantes earum incrementa; ſi enim dentur relationes inter incrementa unius variabilis quantitatis & alterius, tum etiam deduci poteſt relatio inter earum integrales.

Ex. 1. Sit incrementum $xy\underset{\cdot}{z} + xz\underset{\cdot}{y} + yz\underset{\cdot}{x} + y\underset{\cdot}{x}\underset{\cdot}{z} + x\underset{\cdot}{y}\underset{\cdot}{z} + z\underset{\cdot}{x}\underset{\cdot}{y} + \underset{\cdot}{x}\underset{\cdot}{y}\underset{\cdot}{z} + a\underset{\cdot}{x} + b\underset{\cdot}{y} + c\underset{\cdot}{z}$; ſupponantur omnes termini præter eos, in quibus inveniuntur minimæ dimenſiones incrementorum quantitatum x, y, z, &c. nihilo æquales, & reſultat quantitas incrementialis $xy\underset{\cdot}{z} + xz\underset{\cdot}{y} + zy\underset{\cdot}{x} + a\underset{\cdot}{x} + b\underset{\cdot}{y} + c\underset{\cdot}{z}$: nunc ſupponantur omnes præter x invariabiles, & reſultat quantitas $yz\underset{\cdot}{x} + a\underset{\cdot}{x}$, cujus integralis eſt $yzx + ax$; & ſic ſupponantur omnes quantitates præter

ter y invariabiles, & reſultat incrementum $zx\underset{\cdot}{y} + b\underset{\cdot}{y}$, cujus integralis inveniatur $xzy + by$, & ſic ſupponantur omnes præter tertiam z invariabiles, & reſultat incrementum $xy\underset{\cdot}{z} + c\underset{\cdot}{z}$, cujus integralis erit $xyz + cz$: hinc aſſumenda eſt quantitas $xyz + ax + by + cz$ pro integrali; cujus incrementum erit datum.

Ex. 2. Sit incrementum datum $x\underset{\cdot}{y} + y\underset{\cdot}{x} + 2\underset{\cdot}{y}\underset{\cdot}{x}$; aſſumantur x & y invariabiles alternatim, & reſultabit integralis xy, cujus incrementum erit $x\underset{\cdot}{y} + y\underset{\cdot}{x} + \underset{\cdot}{x}\underset{\cdot}{y}$; ſubtrahatur hoc incrementum de dato, & reſultat differentia $\underset{\cdot}{x}\underset{\cdot}{y}$, cujus integralis haud deduci poteſt, ni vel $\underset{\cdot}{x}$ vel $\underset{\cdot}{y}$ ſit conſtans.

3. Si vero incrementum quantitatis variabilis x ſit datus numerus, & conſequenter haud exprimatur in dato incremento; tum ſupponantur omnes præter unam e reliquis variabilibus invariabiles, & incrementi reſultantis inveniatur integralis, cujus deinde inveniatur incrementum ex hypotheſi quod omnes mox aſſumptæ invariabiles ſint variabiles: ſubducatur hoc incrementum de dato, & conſimili methodo inueniatur integralis differentiæ reſultantis; & tandem inveniri poteſt, utrum integralis dati incrementi exprimi poteſt, necne.

4. Et ſic inveniri poteſt, utrum integralis cujuſcunque quantitatis duas vel plures variabiles quantitates, & earum incrementa quorumcunque ordinum involventis exprimi poteſt; necne.

5. Si una ſolummodo contineantur variabilis quantitas x & ejus incrementa in dato incremento, tum e principiis prius traditis detegi poteſt, utrum ejus integralis exprimi poteſt, necne; e. g. ſit datum incrementum $P\underset{\cdot}{x} + Q\underset{\cdot}{x}^2 + R\underset{\cdot}{x}^3 +$ &c. ubi P, Q, R, &c. ſunt functiones quantitatis x; nunc ſupponantur omnes termini præter $P\underset{\cdot}{x}$ nihilo æquales, & inveniatur fluens π fluxionis $P\dot{x}$, quæ erit integralis incrementi dati, ſi in eo haud contineantur incrementa quantitatis $\underset{\cdot}{x}$; inveniatur incrementum quantitatis π, quæ erit $P\underset{\cdot}{x} + Q\underset{\cdot}{x}^2 + R\underset{\cdot}{x}^3 +$ &c. $= \frac{\dot{\pi}\underset{\cdot}{x}}{\dot{x}} + \frac{1}{2}\frac{\ddot{\pi}\underset{\cdot}{x}^2}{\dot{x}^2} + \frac{1}{6}\frac{\dddot{\pi}\underset{\cdot}{x}^3}{\dot{x}^3} +$ &c. ſi modo $\dot{x}$ & $\underset{\cdot}{x}$ ſint invariabiles.

Si vero contineantur incrementa ſuperiorum ordinum quantitatis x in dato incremento, ex præcedentibus principiis etiam deduci poteſt ejus integralis, ſi modo integrari poſſit. E principiis in prob. 4. l. 1. mutatis mutandis, & iis quæ ſupra tradita fuere, inveſtigari poteſt, annon integralis cujuſcunque dati incrementi exprimi poteſt.

Ea, quæ de fluxionibus in theor. 2. tradita fuere mutatis mutandis ad incrementa applicari poſſunt, e. g. inveniatur incrementum quantitatis *V* ordinis *m* ex hypotheſi, quod *x* ſolummodo ſit variabilis, & reſultet quantitas *W*; deinde inveniatur incrementum quantitatis *W* ordinis *n* ex hypotheſi quod *y* ſolummodo ſit variabilis; & eadem reſultabit quantitas, ac ſi modo inveniatur 1^{mo}. incrementum quantitatis *V* ordinis *n* ex hypotheſi quod *y* ſolummodo ſit variabilis, & reſultet quantitas *u*, & deinde inveniatur incrementum *m* ordinis quantitatis *u* ex hypotheſi quod *x* ſolummodo ſit variabilis.

Hinc e principiis in prædicto theor. 2. deduci poteſt, annon datum incrementum duas vel plures variabiles quantitates & earum incrementa involvens, integrari poteſt.

In inveſtigandis integralibus incrementorum eædem radices in integrali & incremento ſemper adhibendæ ſunt, ut prius docetur de fluentibus.

Si variabilis quantitas in denominatore aſcendat ad dimenſiones majores per quantitatem majorem quam unitatem quam ejus dimenſiones in numeratore; tum integralis inter finitam & infinitam diſtantiam erit finita; ſin aliter non; &c. ut de fluentibus prius docetur.

PROB. VII.

Transformare data incrementa in alia, quorum variabiles datas habent relationes ad variabiles datarum quantitatum:

Ex datis relationibus inter variabiles datarum & quæſitarum quantitatum ſubſtituantur illarum valores in terminis variabilium quæſitarum quantitatum, & exinde illarum incrementa pro incrementis variabilium datarum quantitatum, & reſultant quæſita incrementa.

Ex pleriſque ſubſtitutionibus in prob. 14. l. 1. datis transformari poſſunt irrationalia incrementa in rationalia.

Ex. Sit relatio inter variabiles z & x datæ & quæſitæ quantitatis $x = \surd(z^2 + a^2)$, unde $\underset{.}{x} = \surd((z + \underset{.}{z})^2 + a^2) - \surd(z^2 + a^2)$; & $z = \surd(x^2 - a^2)$ & $\underset{.}{z} = \surd((\underset{.}{x} + x)^2 - a^2) - \surd(x^2 - a^2)$; & ſic inveniri poſſunt incrementa $\underset{..}{z}$, $\underset{...}{z}$, &c.: in quantitate A, quæ eſt functio quantitatis z & ejus incrementorum, pro z & ejus incrementis ſcribantur eorum prædicti valores, & transformatur data quantitas in alteram B, quæ eſt functio quantitatis x & ejus incrementorum.

Cor. 1. Sint α & π, etiamque β & ϱ correſpondentes valores prædictarum variabilium; tum integralis quantitatis A inter α & β contenta eadem erit ac correſpondens integralis quantitatis B inter π & ϱ.

Cor. 2. Sit A data integralis, quæ eſt functio quantitatis z & ejus incrementorum, & aſſumatur æquatio in quâ x & z ſimiliter involvuntur; deinde inveniantur quantitas z & ejus incrementa in terminis quantitatis x & ejus incrementorum, quæ ſit B; in quantitate B pro x ſcribatur z & reſultet C; ſint π & ϱ valores quantitatis x, qui correſpondent valoribus α & β quantitatis z; tum integralis inter valores α & β variabilis x quantitatis $A + C$ contenta eadem erit ac integralis ejuſdem quantitatis $A + C$ inter valores π & ϱ ejuſdem quantitatis (x) contenta.

Principia e quibus detegi poteſt, utrum fluens fluxionalis æquationis tres vel plures variabiles quantitates involventis, exprimi poteſt, necne; etiam ad incrementiales æquationes applicari poſſunt.

THEOR. IV.

1. Sit incrementum $\dfrac{Px^m + Qx^{m-1} + Rx^{m-2} + \&c.}{A \times (x + \alpha\underset{.}{x}) \times (x + (\alpha + 1)\underset{.}{x}) \,..\, (x + (\alpha + a)\underset{.}{x}}$ ubi a & m ſint integri numeri, & ejus integralis inveniri poteſt vel e finitis terminis, vel e finitis terminis & integrali formulæ $\dfrac{1}{x + a\underset{.}{x}}$, vel formulæ $\dfrac{1}{x}$, quæ revera eadem haberi poteſt.

2. Et ſic ſit denominator $(x + \alpha\underset{.}{x}) \times (x + (\alpha + 1)\underset{.}{x}) \,..\, (x + (\alpha + a)\underset{.}{x})$ $\times (x +$

$\times (x + \beta \underset{\cdot}{x}) \times (x + (\beta + 1)\underset{\cdot}{x}) \,..\, (x + (\beta + b)\underset{\cdot}{x}) \times (x + \gamma) \times (x + (\gamma + 1)\underset{\cdot}{x}) \times (x + (\gamma + 2)\underset{\cdot}{x}) \ldots (x + (\gamma + c)\underset{\cdot}{x}) \times$ &c. ubi a, b, c, &c. ſunt integri numeri; tum integralis cujuſcunque rationalis incrementi denominatorem prædictum habentis ſemper detegi poteſt vel e finitis terminis, vel e finitis terminis & integralibus incrementorum hujuſce formulæ $\frac{1}{x}$.

3. Sit denominator $(Ax^n + Bx^{n-1} + \&c.) \times (x + \gamma)^m \times \&c.\, (x + \delta)^r \times$ &c. ubi r major eſt quam m, &c. & quantitates n, m & r, &c. integri ſunt numeri, tum integralis cujuſcunque rationalis incrementi denominatorem prædictum habentis ſemper detegi poteſt vel e finitis terminis, vel e finitis terminis & integralibus incrementorum harum formularum $\frac{1}{x}$, $\frac{1}{x^2}$, $\frac{1}{x^3}$, $\frac{1}{x^4} \ldots \frac{1}{x^r}$.

Et in genere integralis cujuſcunque rationalis incrementi prædicti generis ſemper detegi poteſt vel e finitis terminis, vel e finitis terminis & integralibus incrementorum formularum $\frac{1}{x^2 + ax + b}$, $\frac{1}{(x^2 + ax + b)^2}$, $\frac{1}{(x^2 + ax + b)^3} \ldots \frac{1}{(x^2 + ax + b)^t}$, & $\frac{1}{x}$, $\frac{1}{x^2} \ldots \frac{1}{x^s}$, ubi s ſit maximus numerus æqualium diviſorum in denominatore contentorum, & t vel $= \frac{s}{2}$ vel $\frac{s - 1}{2}$.

Omnia hæc facile e prob. 6. deduci poſſunt; facile enim e prædicto problemate reduci poteſt quodcunque rationale incrementum ad ejus magis ſimplices diviſores.

Idem etiam vere affirmari poteſt de quocunque incremento, quod ad prædictum rationale reduci poteſt.

PROB. VIII.

Invenire, annon datæ incrementialis quantitatis integralis ope aliarum (L, M, N, *&c.*) *detegi poteſt: ducantur incrementiales* (L, M, N, *&c.*)

quantitates in invariabiles (α, β, γ, &c.) coefficientes generaliter ſumptas, & quantitatum reſultantium ſumma e datâ incrementiali ſubtrahatur; & in quantitate reſultante ita aſſumantur coefficientes α, β, γ, &c. ut inveniatur ejus integralis, ſi modo fieri poſſit; & id quod requiritur, conſit.

Eadem principia, quæ generaliter inveniunt, annon fluens datæ fluxionis inveniri poteſt ope datarum fluxionum fluentium; etiam detegent utrum integralis cujuſcunque incrementi detegi poteſt ope integralium datorum incrementorum, necne.

PROB. IX.

Dato incremento, quæ in ſe continet integralem v, *quæ haud exprimi poteſt in finitis algebraicis terminis variabilis* x, *&c. invenire utrum integralis dati incrementi ſit finita algebraica functio quantitatis* x *& prædictæ integralis & earum incrementorum, necne.*

Sit incrementum $v \times \underset{.}{\pi}$ cujus integralis requiritur; aſſumatur pro integrali $v\pi$, cujus incrementum erit $v\underset{.}{\pi}$ (datum) $+ \pi\underset{.}{v} + \underset{.}{\pi}\underset{.}{v}$; ergo $v\pi$ — integ. increm. $(\pi\underset{.}{v} + \underset{.}{\pi}\underset{.}{v})$ erit integralis quæſita.

Si vero in dato incremento vel contineantur plures dimenſiones integralis v, vel quæcunque aliæ integrales quantitates; tum per methodum (mutatis mutandis) in prob. 5. libri primi traditam pro fluentibus conſimilium fluxionum progrediendum eſt.

Et fere eadem erit methodus detegendi integrales exponentialium incrementorum, ac ea quæ prius tradita fuit ad inveſtigandas integrales incrementorum præcedentium. Ex. g. Sit incrementum e^{z}, & ejus integralis erit $\frac{e^{z}}{e^{\underset{.}{z}} - 1}$, ſi modo $\underset{.}{z}$ ſit conſtans.

Ex. Sit $z = -x$, & erit incrementi e^{-x} integralis $\frac{e^{-x}}{e^{-\underset{.}{x}} - 1}$.

Facile acquiri poſſunt infinita hujuſmodi incrementa, quorum integrales innoteſcunt; aſſumantur enim integrales, & deducantur earum incrementa, quorum igitur integrales dantur.

Datis

2. Datis incrementis, quæ in ſe involvunt irrationales quantitates in prob. 15, &c. libri præcedentis traditas, facile e ſubſtitutionibus in problemate prædicto contentis reduci poſſunt ad incrementa, quæ nullas irrationales quantitates continent.

E ſubſtitutione duorum valorum (α & β) variabilis quantitatis x pro ipſâ x in datâ integrali reſultabit valor integralis inter duos prædictos valores variabilis x contentæ.

In his integralibus æque ac in fluentibus deducendis eædem radices ſemper uſurpandæ ſunt.

PROB. X.

Invenire integralem incrementi $v r^z \underset{\cdot}{z}$, *ubi* $\underset{\cdot}{z}$ *ſit invariabilis quantitas.*

Aſſumatur pro integrali $\frac{v r^z \underset{\cdot}{z}}{r^{\underset{\cdot}{z}} - 1\,(R)} - P$, cujus incrementum eſt $v r^z \underset{\cdot}{z} + \underset{\cdot}{v} r^z \underset{\cdot}{z} + \underset{\cdot}{v} \frac{r^z \underset{\cdot}{z}}{R} - \underset{\cdot}{P}$; unde $\underset{\cdot}{P} = \underset{\cdot}{v} \frac{r^z \underset{\cdot}{z}}{R} + \underset{\cdot}{v} r^z \underset{\cdot}{z} = \underset{\cdot}{v} \frac{r^z r^{\underset{\cdot}{z}} \underset{\cdot}{z}}{R}$, cujus integralis ſit $\underset{\cdot}{v} \frac{r^z \underset{\cdot}{z}}{R^2} - Q$, & ex iiſdem principiis invenietur $Q = \frac{r^{2\underset{\cdot}{z}} \times r^z}{R^3} - S$, & ſic deinceps; unde integralis quæſita eſt $\frac{v r^z \underset{\cdot}{z}}{R} + \frac{r^{\underset{\cdot}{z}} \times r^z \underset{\cdot}{z}}{R^2} \underset{\cdot}{v} + \frac{r^{2\underset{\cdot}{z}} \times r^z \underset{\cdot}{z}}{R^3} \underset{\cdot\cdot}{v} +$ &c.

Eadem methodus, quæ generaliter detegit vel fluentem fluxionis vel integralem incrementi in finitis terminis, cum ita exprimi poſſit, ſemper eam exprimet per infinitam ſeriem, ſi finitis terminis exprimi nequeat.

2. Data incrementiali quantitate, cujus integralis per ſeriem $A + Bx^m + Cx^{2m} +$ &c. exprimatur, & per methodum in Medit. Algebr. traditam ſcribantur pro x & ejus incrementis quantitates αx, βx, γx, &c. & earum incrementa; ubi α, β, γ, &c. ſint reſpective radices æquationis $x^n - 1 = 0$, & e quantitatibus reſultantibus per Medit.

Algeb. facile deduci poteſt quantitas æqualis ſummæ e ſingulis terminis prædictæ ſeriei, quorum diſtantiæ a ſe invicem ſint n, $2n$, $3n$, &c.

Eadem etiam applicari poſſunt ad fluentiales & integrales æquationes.

Per methodum in fluxionibus vel algebraicis æquationibus prius traditam irrationales ex numeratore in denominatorem & vice versâ ex denominatore in numeratorem transformari poſſunt: & exinde nonnunquam facilius deduci poſſunt integrales.

PROB. XI.

Invenire, utrum integralis logarith. $\frac{A}{B}$ *detegi poteſt ope logarithmorum, necne; ubi* A & B *ſunt functiones quantitatis* x.

Inveniantur ſimplices diviſores numeratoris A & denominatoris B; qui ſint reſpective α, β, γ, δ, &c.; π, ϱ, σ, τ, &c.; & ſi in numeratore vel denominatore inveniatur diviſor, qui non habet alterum ei correſpondentem in denominatore vel numeratore, tum non inveniri poteſt integralis per logarithmos; ſin aliter ſemper exprimi poteſt.

Sint a & b correſpondentes diviſores; tum, ſi in a pro x ſcribatur $x + r\underset{.}{x}$, reſultabit b; ubi r eſt integer numerus.

Si in diviſoribus α, β, &c. numeratoris pro x ſcribantur $x + r\underset{.}{x}$, $x + r'\underset{.}{x}$, &c., reſultant π, ϱ, &c. diviſores denominatoris; & vice versâ, ſi in reliquis σ, τ, &c. diviſoribus denominatoris ſcribantur $x + s\underset{.}{x}$, $x + s'\underset{.}{x}$, &c. pro x, reſultant reliqui diviſores numeratoris; tum integralis dati incrementi log. $\frac{A}{B}$ erit log.

$$\frac{\sigma \times \sigma' \times \sigma'' \times \sigma''' \ldots \sigma^{\overset{s-1}{\prime}} \times \tau \times \tau' \times \tau'' \times \ldots \tau^{\overset{s'-1}{\prime}} \times \&c.}{\alpha \times \alpha' \times \alpha'' \ldots \alpha^{\overset{r-1}{\prime}} \times \beta \times \beta' \times \beta'' \times \ldots \beta^{\overset{r'-1}{\prime}} \times \&c.}$$

: ubi, ſi modo pro x in terminis σ, σ', σ'', σ''', &c. ſcribatur $x + \underset{.}{x}$, reſultabunt quantitates σ', σ'', σ''', σ'''', &c.; & ſic de reliquis τ, τ', τ'', &c.; α, α', α'', &c.; β, β', β'', &c., &c.

Ex. Sit incrementum log. $\frac{x \times (x+p+3\underset{\cdot}{x})}{(x+4\underset{\cdot}{x})(x+p)}$; tum per hoc prob. erit integralis log. $\frac{(x+p)(x+p+\underset{\cdot}{x})(x+p+2\underset{\cdot}{x})}{x(x+\underset{\cdot}{x})(x+2\underset{\cdot}{x})(x+3\underset{\cdot}{x})}$: erit enim ejus incrementum log. $\frac{(x+p+\underset{\cdot}{x})(x+p+2\underset{\cdot}{x})(x+p+3\underset{\cdot}{x})}{(x+\underset{\cdot}{x})(x+2\underset{\cdot}{x})(x+3\underset{\cdot}{x})(x+4\underset{\cdot}{x})}$ — log. $\frac{(x+p)(x+p+\underset{\cdot}{x})(x+p+2\underset{\cdot}{x})}{x(x+\underset{\cdot}{x})(x+2\underset{\cdot}{x})(x+3\underset{\cdot}{x})} =$ log. $\frac{x \times (x+p+3\underset{\cdot}{x})}{(x+4\underset{\cdot}{x})(x+p)}$.

Eadem principia etiam ad circulares arcus, qui ex impoſſibilibus logarithmis deduci poſſunt; ad ellipticos, hyperbolicos, &c. arcus applicari poſſunt.

PROB. XII.

Datâ æquatione relationem inter x *&* y *exprimente, & conceſsâ radicum extractione; invenire aggregatum e ſingulis valoribus cujuſcunque algebraicæ functionis quantitatum* x *&* y, *earum incrementorum & fluxionum.*

Inveniatur e prædictis conceſſis primum incrementum quantitatis y, & ſic ſecundum, tertium, &c. etiamque ejus fluxiones; quibus valoribus in datâ incrementiali quantitate ſubſtitutis, reſultat algebraica quantitas, cujus ſumma e ſingulis valoribus erui poteſt per Medit. Algebr. Et ſic aggregatum rectangulorum e quibuſque duobus valoribus datæ functionis; contenta e quibuſque tribus, quatuor, &c. valoribus datæ functionis deduci poſſunt; unde detegetur æquatio ipſa, cujus radix eſt prædicta functio.

2. Sit æquatio $Ay^n + (a+bx)y^{n-1} + (c+dx+ex^2)y^{n-2} +$ &c. $= o$, & fluat x uniformiter, cujus incrementum ſit $\underset{\cdot}{x}$; tum pro ſingulis datis correſpondentibus valoribus quantitatum x, y & $\underset{\cdot}{x}$, reſultare poſſunt n diverſi correſpondentes valores incrementi $\underset{\cdot}{y}$, quorum ſumma erit $-\frac{a+bx+b\underset{\cdot}{x}}{A} - ny$.

3. Datâ quantitate x & ejus incremento $\underset{\cdot}{x}$, & numerus valorum incrementorum y poteſt eſſe $n \times n$, ut facile conſtat e Medit. Algeb.

4. Datâ quantitate x & ejus incremento $\underset{\cdot}{x}$, & ſumma n valorum incrementi $y\underset{\cdot}{x} = -\frac{a+bx}{A}\underset{\cdot}{x}$, cujus integralis erit $-\left(\frac{a}{A}x + \frac{b}{A} \times \text{int.}\, x\underset{\cdot}{x}\right)$ $= \left(\frac{a}{A}x + \frac{b}{A}x \times \frac{x - \underset{\cdot}{x}}{2} + \frac{b}{A}x + c\right)$.

Et ſic de ſummis integralium incrementialium $y^2\underset{\cdot}{x}$, $y^3\underset{\cdot}{x}$, &c.

5. Sit æquatio $(A + Bx)y^{n-1} + (C + Dx + Ex^2)y^{n-2} + \&c. = 0$, & ſi $A + Bx$ haud ſit diviſor quantitatis $C + Dx + Ex^2$, tum ſumma e ſingulis integralibus quantitatis incrementialis $y\underset{\cdot}{x}$ haud exprimi poteſt per finitos terminos qauntitatum x & y.

PROB. XIII.

Datâ æquatione algebraicâ (n) *dimenſionum* $y^n + (a + bx)y^{n-1} + \&c. = 0$, *invenire æquationem cujus radix* (z) *ſit integralis quantitatis* $y\underset{\cdot}{x}$, *ſi modo ea inveniri poſſit.*

Aſſumatur æquatio, in quâ dimenſiones quantitatis z inveniuntur n, quoniam omni valori quantitatis x correſpondent n diverſi valores quantitatis z; viz. $z^n + \left(ax + \frac{b}{2}x^2 - \frac{b}{2}x\underset{\cdot}{x} + c\right)z^{n-1} + \&c. = 0$, ubi c eſt invariabilis quantitas ad libitum aſſumenda, & $\underset{\cdot}{x}$ conſtans; exinde inveniatur æquatio cujus radix erit $\frac{z}{\underset{\cdot}{x}}$, & fiant coefficientes reſultantis & datæ æquationis inter ſe æquales, & exinde deduci poſſunt coefficientes, ſi modo integralis exprimi poſſit in finitis terminis.

Aliter: reduci poteſt data æquatio, ita ut inveniantur n valores quantitatis y progredientes ſecundum dimenſiones qnantitatis x; deinde inveniatur per prædictas methodos integrales e ſingulis hiſce ſeriebus, quarum ſumma erit ſumma radicum quæſitæ æquationis; ducantur quæque duæ, tres, &c. ex his integralibus in ſeſe & inveniantur aggregata e ſingulis hiſce reſultantibus ſeriebus, & erunt reſpective coefficientes tertii, quarti, &c. terminorum quæſitæ æquationis, & ſic deinceps; & exinde conſtabit æquatio quæſita.

Cor. In omnibus hiſce reſolutionibus irrepit in terminos æquationum quantitas ad libitum aſſumenda, quæ quantitas denotat correctionem quantitatis vel æquationis quæſitæ.

2. Eædem etiam methodi applicari poſſunt ad inveniendas æquationes, quarum radices ſunt quæcunque algebraicæ functiones quantitatum x & y incognitarum in datâ æquatione contentarum, & earum incrementorum.

3. Et ſic de pluribus m æquationibus plures $(m+1)$ incognitas quantitates habentibus, ſed obſervandum eſt in omnibus hiſce æquationibus ac in æquationibus exprimentibus fluentes fluxionum, ſi modo ſolummodo ſit fluxio vel incrementum primi ordinis in quantitate, cujus æquatio exprimens ejus fluentem vel integralem requiritur; tum una ſolummodo quantitas in æquatione reſultanti aſſumi poteſt ad libitum; ſi vero fluxiones vel incrementa ſecundi, tertii, &c. ordinis in prædictâ quantitate contineantur, tum duæ, tres, &c. quantitates ad libitum aſſumi poſſunt.

THEOR. V.

1. Sit æquatio integralis $ax^r + bx^s \times y^t = cy^v$, e prob. 32. l. 1. conſtant quantitates x & y in terminis tertiæ z, & exinde earum incrementa in terminis tertiæ z & ejus incrementorum; unde deduci poteſt quæcunque functio quantitatum x & y, & earum incrementorum in terminis quantitatis z & ejus incrementorum.

2. Sit algebraica æquatio homogenea, i. e. ejus ſinguli termini eaſdem conficiant dimenſiones, & inveniri poteſt quæcunque functio ejus variabilium x & y, & earum incrementorum in terminis novæ variabilis z, & ejus incrementorum.

Omnia, quæ de algebraicis æquationibus & fluxionalibus quantitatibus in prob. 32, & 33. l. 1. tradita fuere, ad algebraicas æquationes

&

& incrementiales quantitates (mutatis mutandis) applicari poſſunt; & eadem principia quæ tradita fuere in fluxionalibus æquationibus de detegendis rationalibus functionibus tertiæ z, quæ reſpective æquant variabiles x & y, ad incrementiales æque applicari poſſunt.

PROB. XIV.

Datam æquationem, in quâ integralis continetur quantitas, in incrementialem reducere, ita ut exterminetur integralis quantitas.

Si integralis ducatur in variabilem quantitatem; dividatur æquatio per eam quantitatem, & inveniatur incrementum quantitatis reſultantis, & exterminabitur integralis.

Si autem majores ſint dimenſiones (r) integralis quantitatis exterminandæ, vel altior ſit ordo (n) ejus integralis; in poſteriori caſu neceſſe eſt invenire incrementum (n) ordinis datæ æquationis, in priori vero caſu conſtabit reductio e prob. 28. l. 1. i. e. integrales ex æquationibus exterminabuntur omnino per eandem methodum ac fluentes e datis æquationibus.

Ex. . Sit data æquatio $(x+y) \times$ integ. $\frac{x\underset{\cdot}{x} + yx^2\underset{\cdot}{y}}{\surd(x^2+y^2)} = \surd(b^2+y^2) - cx^2$; dividatur æquatio per $x+y$, & reſultat integ. $\frac{x\underset{\cdot}{x} + yx^2\underset{\cdot}{y}}{\surd(x^2+y^2)} = \frac{\surd(b^2+y^2) - cx^2}{x+y}$; inveniatur incrementum reſultantis æquationis, & reſultat $\frac{x\underset{\cdot}{x} + yx^2\underset{\cdot}{y}}{\surd(x^2+y^2)} =$ increm. $\frac{\surd(b^2+y^2) - cx^2}{x+y}$.

PROB. XV.

Reducere duas incrementiales æquationes A = 0 & B = 0 *in unam, ita ut exterminentur variabilis quantitas* z & *ejus incrementa.*

Sit infima incrementialis quantitas literæ z in unâ æquatione $\underset{n}{\underset{\cdot}{z}}$, in alterâ vero $\underset{n+p}{\underset{\cdot}{z}}$; inveniatur p incrementum e ſingulis terminis prioris æquationis, & reſultat infima incrementialis quantitas literæ z in æquatione reſultante $\underset{n+p}{z}$; deinde e $(p-1)$ incremento e ſingulis terminis prioris æquationis exterminetur incrementialis quantitas $\underset{n+p-1}{x}$; & ſic reduci poteſt poſterior æquatio ad incrementialem æquationem, in quâ infimum incrementum quantitatis z ſit $\underset{n}{z}$; unde duæ dantur æquationes, in quibus infima incrementialis quantitas literæ z erit $\underset{n}{z}$; ex his duabus æquationibus deduci poteſt æquatio, cujus infima incrementialis quantitas literæ z erit $\underset{n-1}{z}$, & ex hâc & priori datâ æquatione deduci poteſt altera æquatio, cujus infima incrementialis quantitas literæ z eſt $\underset{n-1}{z}$, & ex his duabus æquationibus conſequitur incrementialis æquatio, in quâ infimum incrementum quantitatis z eſt $\underset{n-2}{z}$; & ſic deinceps; & tandem ita reducetur æquatio, ut exterminentur quantitas z, & ejus incrementa.

Cor. . In reducenda infimâ incrementiali quantitate $\underset{r}{z}$ in utriſque æquationibus ad unam ſuperiorem $\underset{r-1}{z}$ ſemper inveniendum eſt primum incrementum quantitatum in utriſque æquationibus contentarum; unde ſi in duabus prædictis æquationibus $A=0$ & $B=0$, in quibus $\underset{n}{z}$ & $\underset{n+p}{z}$ reſpective ſint infima incrementa quantitatis z, etiamque ſint $\underset{\alpha}{y}$ & $\underset{\beta}{y}$ infima incrementa quantitatis y; & infimum incrementum quantitatis y in æquatione reſultante, in quâ haud inveniuntur quantitas z vel ejus incrementa, haud poteſt eſſe majus quam $\alpha+n+p$ vel $\beta+n$.

Sed e prob. 36. l. 1. conſtat, ſi æquatio reſultans ſit incrementialis cujuſcunque ordinis $\alpha+n+p$ vel $\beta+n$, &c. tot aſſumi poſſe correſpondentes valores variabilium x & y in eâ contentarum, & incrementorum variabilis y diverſorum, quot ſit ordo datæ æquationis, ſi $\underset{\cdot}{x}$ ſit conſtans.

2. Et ſic de pluribus (m) incrementialibus vel fluxionalibus æquationibus

tionibus in unam reducendis; hinc enim fluxionalis vel incrementialis æquationis reſultantis deduci poteſt ordo.

3. Æquationes incrementiales vel fluxionales poſſunt eſſe inter ſe contradictoriæ; hoc autem ſemper conſtabit ex earum reductione in unam, ita ut variabiles, &c. quantitates exterminentur, & ſi hæc una reſultans æquatio aliquid abſurdi in ſe contineat, vel forſan ejus diviſores; tum aliquid contradictorium continent in ſe æquationes incrementiales.

4. Ex hâc methodo transformandi æquationes, ita ut variabiles quantitates & earum incrementa exterminentur, irrepunt in æquationes reſultantes radices, quæ in datis haud inveniuntur; quomodo hæ radices irrepant in reſultantes æquationes, & radices ipſæ, e principiis in medit. algebr. traditis, erui poſſunt.

THEOR. VI.

1. Sit incrementialis æquatio $A \times B \times C \times$ &c. $= 0$, quæ admittat diviſores A, B, C, &c. hæc incrementialis æquatio dividi poteſt in plures incrementiales æquationes $A = 0$, $B = 0$, $C = 0$, &c.

2. Sint duæ incrementiales æquationes $A \times B \times C \times$ &c. $= 0$, & $a \times b \times c \times$ &c. $= 0$; tum hæ duæ incrementiales æquationes reduci poſſunt ad alias, viz. $A = 0$ & $a = 0$; $A = 0$ & $b = 0$; $A = 0$ & $c = 0$, &c. $B = 0$ & $a = 0$; $B = 0$ & $b = 0$, &c. &c.

THEOR. VII.

E duabus datis incrementialibus æquationibus facile formari poſſunt infinitæ aliæ, quarum variabiles quantitates eandem habent inter ſe rationem. Ducantur duæ æquationes in invariabiles quantitates, & addantur vel ſubducantur æquationes reſultantes de ſe ipſis; & reſultant novæ æquationes quæſitæ.

THEOR. VIII.

Datâ incrementiali æquatione relationem inter x & y & earum incrementa n ordinum exprimente, ſi modo ex eâ inveniatur æquatio, quæ haud continet incrementa majorum quam $n-1$ ordinum, tum aſſumi poteſt una quantitas invariabilis ad libitum aſſumenda, & ſic deinceps; unde in æquatione ex incrementis liberâ erunt n invariabiles quantitates ad libitum aſſumendæ.

Sit quantitas ipſa, vel ea ducta in quamcunque incrementialem quantitatem inferioris ordinis, perfectum incrementum, cujus integralis dicatur V; & erit $V+C=o$ æquatio vere correcta, ubi C denotat invariabilem quantitatem: ſi modo ſit incrementialis æquatio ordinis n, & $\underset{m}{\dot{x}}$ ſit incrementum ordinis m, qui minor ſit quam n; etiamque ſit $\underset{m}{\dot{x}}$ conſtans; tum erit æquatio vere correcta $V+A(\underset{m}{\dot{x}})^{\frac{n-1}{m}}$ ubi A eſt invariabilis quantitas.

Et ſic progredi liceat ad æquationes integrales ſuperiorum ordinum.

Quot invariabiles quantitates ad libitum aſſumi poſſunt, tot correſpondentes valores e ſingulis variabilibus quantitatibus, & e diverſis incrementis variabilis quantitatis, quæ haud creſcit uniformiter, aſſumi poſſunt.

PROB. XVI.

Datâ æquatione relationem inter x *&* y *& earum incrementa exprimente; invenire æquationem, cujus radix eſt correctio ipſa: ſcribatur* v + z *pro* y *in datâ æquatione, ubi* v *denotat particularem valorem quantitatis* y; *ex æquationibus reſultantibus conſtat æquatio, cujus radix eſt* z *correctio quæſita: vel ſi modo per infinitas ſeries convergentes inveniatur generalis valor quantitatis* y *in terminis quantitatis* x, *conſequetur correctio quæſita.*

Omnis mothodus, quæ detegit generalem correctionem datæ æquationis, etiam deducet generalem ejus integralem.

Et ſic de pluribus (m) æquationibus plures ($m+1$) variabiles quantitates & earum incrementa habentibus.

PROB. XVII.

Invenire quamplurimas incrementiales æquationes, quarum integrales cognoſcuntur.

1. Aſſumantur integrales æquationes, & ex iis deducantur incrementiales.

2. Deducantur æquationes, quarum radices quamcunque habeant relationem ad radices aſſumptarum vel inventarum æquationum, & e radicibus aſſumptarum deduci poſſunt radices deductarum æquationum.

3. Aſſumatur α pro y, quæ ſit quæcunque functio quantitatis x, deinde ſcribantur hæc quantitas & ejus incrementa pro ſuis valoribus in quantitate π, quæ ſit functio variabilium x, y & earum incrementorum, & ſit quantitas exinde reſultans ϱ; tum erit α particularis valor incrementialis æquationis $\pi = \varrho$.

4. Datâ incrementiali æquatione, e quâ deduci poteſt F integralis incrementi (P); pro variabilibus & earum incrementis in datâ æquatione & datâ quantitate P ſcribantur quæcunque functiones novarum variabilium, & earum incrementa; & reſultant æquatio inter has novas variabiles & earum incrementa relationem exprimens, & nova quantitas π, cujus integralis deduci poteſt ex integrali quantitatis P.

5. Sit quæcunque incrementialis æquatio, dividatur ea in duas æquales partes, ita ut integralis alterius partis inveniri poteſt, ea erit etiam integralis alterius.

PROB. XVIII.

1. *Datâ integrali* $(P = 0)$ *generali primi ordinis incrementialis æquationis, invenire æquationem ipſam.*

Sit a quantitas invariabilis in datâ integrali $P = o$ ad libitum aſſumenda, & ex eâ deducatur $a = \pi$, & erit $\pi = o$ incrementialis æquatio quæſita.

Ex. Sit integralis $y = \frac{z - z}{\underset{\cdot}{z} + n} y + a$, ubi z eſt conſtans & a invariabilis quantitas ad libitum aſſumenda; hujus æquationis inveniatur incrementum, & evadet $\underset{\cdot}{y} = \frac{z - z}{\underset{\cdot}{z} + n} \underset{\cdot}{y} + \frac{\underset{\cdot}{z}}{\underset{\cdot}{z} + n} (y + \underset{\cdot}{y})$, quod reductum fit $\frac{\underset{\cdot}{z} + n - z}{\underset{\cdot}{z} + n} \underset{\cdot}{y} = \frac{\underset{\cdot}{z}}{\underset{\cdot}{z} + n} y$, unde $(\underset{\cdot}{z} + n - z) \underset{\cdot}{y} = z \underset{\cdot}{y}$ æquatio, cujus integralis generalis eſt data.

2. Datâ integrali ($P = 0$) generali n ordinis incrementialis æquationis ($\alpha = 0$); ex æquatione $P = 0$, quæ continet n invariabiles quantitates (a, b, c, &c.) ad libitum aſſumendas, inveniantur n æquationes $a = \pi$, $b = \varrho$, $c = \sigma$, &c. tum erunt $\underset{\cdot}{\pi} = 0$, $\underset{\cdot}{\varrho} = 0$, $\underset{\cdot}{\sigma} = 0$, &c. ($n$) diverſæ æquationes, quæ ſunt integrales $n - 1$ ordinis incrementialis æquationis $\alpha = 0$; ex his æquationibus inveniri poſſunt $n \cdot \frac{n-1}{2}$ aliæ, quæ ſunt integrales $n - 2$ ordinis æquationis $\alpha = 0$; & ſic deinceps uſque ad n æquationes detegendas, quæ ſunt integrales primi ordinis æquationis $\alpha = 0$.

Si non inveniri poſſint $a = \pi$, tum reducantur duæ æquationes $P = 0$ & $\dot{P} = 0$ in unam, ita ut exterminetur invariabilis quantitas; & ſic deinceps.

Cor. . Sunt n multiplicatores, qui ducti in datam æquationem $\alpha = 0$ prebent incrementa, quorum integrales cognoſcuntur; & ſic deinceps, ut conſtat e theor. 29. libri præcedentis.

Sint m, m', m'', &c. reſpectivi multiplicatores, qui ducti in æquationem incrementialem ($\alpha = 0$) prebent (n) æquationes, quarum integrales reſpective ſunt λ, λ', λ'', λ''', &c.; tum erit generalis integralis quæcunque functio (n) quantitatum λ, λ', λ'', λ''', &c. Et ſic de particularibus integralibus.

Facile etiam deduci poſſunt conſimiles propoſitiones de hâc re, quæ prius de fluentialibus æquationibus tradita fuere.

Hæc etiam applicari poſſunt ad fluentes vel integrales æquationum fluxiones, & incrementa ſimul involventium.

Sint duæ incrementiales æquationes $\pi = 0$ & $\varrho = 0$, quarum communis ſit multiplicator; tum ex integrali alterius æquationis datâ facile acquiri poteſt integralis alterius.

THEOR. IX.

1. Sit incrementialis æquatio $P\underset{.}{z} = 0$; quæ haud continet z vel $\underset{.}{z}$, &c. tum erit ejus integralis $z =$ conſt. haud vero $P = 0$; ſi vero ſit $\underset{.}{y} = P\underset{.}{x}$ & ſit $P = 0$ functio quantitatis $y =$ conſt. tum erit $y =$ conſt. particularis valor incrementialis æquationis $\underset{.}{y} = P\underset{.}{x}$, & ſic de pluribus hujuſcemodi æquationibus.

2. Sit incrementialis æquatio $\pi = 0$; & M quantitas, cujus integralis deduci poteſt; tum ſemper detegi poteſt integralis quantitatis $\phi : (\pi) + M$ ubi $\phi : (\pi)$ ſit functio quantitatis π, in quâ haud invenitur terminus, cujus dimenſio quantitatis π vel nulla eſt, vel negativa.

3. Eadem etiam applicari poſſunt ad duas vel plures incrementiales æquationes $\pi = 0$, $\varrho = 0$, &c. ſemper enim detegi poteſt integralis quantitatis $\phi : (\pi$ & ϱ, &c.$) + M$, ubi $\phi : (\pi, \varrho,$ &c.$)$ deſignat functionem quantitatum π, ϱ, &c. in quâ haud invenitur terminus, in quo nulla continetur dimenſio e ſingulis quantitatibus π, ϱ, &c. vel negativa.

4. Cujuſcunque incrementialis æquationis $(\pi = 0)$ n ordinis, in quâ continentur duæ variabiles x & y & earum incrementa, ubi alterius (x) primum incrementum eſt conſtans, ſemper datur (L) functio variabilium x, y, & earum incrementorum primi, ſecundi, &c. $(n - 1)$ ordinis, quæ ducta in datam æquationem præbet quantitatem, cujus integralis (α) datur.

5. Sit α generalis integralis æquationis $\pi = 0$, tum quæcunque functio quantitatis α etiam erit integralis datæ æquationis $\pi = 0$; & idem etiam valet, cum α ſit particularis integralis.

6. Sit quæcunque integrabilis æquatio $\pi = \rho$, cujus incrementum ſit $\underset{.}{\pi} = \underset{.}{\rho}$, hujuſce æquationis ducatur alterum latus in π, alterum vero in $\underset{.}{\rho}$, & evadit $\pi\underset{.}{\pi} = \rho\underset{.}{\rho}$: generalis integralis hujuſce æquationis haud erit $\pi = \rho$; hoc & conſimilia conſequuntur e libro præced.

7. Sint π, ρ, σ, &c. incrementa, quæ ducta in eundem multiplicatorem L, præbent incrementa, quorum integrales innoteſcunt; tum erit $L(a\pi + b\rho + c\sigma +$ &c.) incrementum, cujus integralis etiam inveniri poteſt, ſi modo a, b, c, &c. ſint quæcunque invariabiles quantitates.

8. Datis formulis æquationis fluxionalis & ejus multiplicatoris, inveſtigari poſſunt caſus, in quibus ejus integralis deduci poteſt.

Ducatur æquatio datæ formulæ in multiplicatorem prædictum, & quantitatis reſultantis per methodos prius traditas inveniatur integralis, ſi modo integralem recipiat quantitas reſultans; vel e notis integrabilitatis conſtare poteſt, annon quantitas reſultans integrationem admittit; vel aliter ex datâ incrementiali æquatione & formulâ ejus multiplicatoris datâ facile conſtat formula ejus integralis, cujus inveniatur incrementum, dividatur hoc incrementum per datum; & exinde deduci poſſunt incognitæ quantitates in aſſumptâ integrali, ſi modo ea recipere poſſit formulam prædictam.

9. E ſubſtitutionibus ſæpe reduci poſſunt datæ incrementiales æquationes ad alias, quarum integrales innoteſcunt; unde ex aſſumptis æquationibus incrementialibus, quarum innoteſcunt integrales, facile e ſubſtitutionibus deduci poſſunt aliæ, quarum integrales e prædictis deduci poſſunt.

10. Sit æquatio incrementialis $P + Q = 0$; e ſubſtitutione reducatur hæc æquatio ad ſubſequentem $R + S = 0$, unde $P + Q = R + S$: hinc ſi vero $\frac{R+S}{V}$ ſit incrementum, cujus integralis innoteſcit, tum etiam $\frac{P+Q}{V}$ erit incrementum cujus integralis etiam innoteſcit;

teſcit; & $\frac{1}{V}$ erit multiplicator, qui in datam æquationem ductus, præbet quantitatem, quæ integrari poteſt.

11. Ex irrationalibus quantitatibus in datâ incrementiali æquatione contentis conſtabunt irrationales quantitates in multiplicatore M, qui ductus in datam æquationem, eam reddit integrabilem; hæ irrationales quantitates enim deduci poſſunt ex iiſdem principiis, quæ prius tradita fuere de inveniendis irrationalibus quantitatibus in multiplicatore datæ fluxionalis æquationis contentis.

12. Ex formulis etiam fluxionalium æquationum, quarum fluentes innoteſcunt, conſtabunt formulæ incrementialium æquationum, quarum integrales correſpondent fluentibus prædictis.

13. Et ſic in quibuſdam caſibus deduci poſſunt integrales generales incrementialium æquationum ex earum particularibus valoribus.

14. Si modo datæ quantitatis incrementum ſit conſtans; ſcribantur in datâ æquatione quæcunque quantitates pro ſuis valoribus ex conſtante incremento, & earum incrementis diverſorum ordinum deductæ; & haud nunquam exinde conſequitur incrementialis æquatio, cujus integralis innoteſcit.

15. Aſſumatur integralis æquatio duas variabiles x & y involvens; quarum incrementa reſpective ſint x & y; datæ æquationis inveniatur incrementum, in hoc vero incremento ſcribantur quæcunque quantitates pro ſuis valoribus ex aſſumptâ integrali æquatione deductæ, & reſultat æquatio, cujus particularis integralis erit aſſumpta æquatio.

16. Si vero detur incrementialis æquatio, in quâ ſimiliter involvuntur variabiles x & y, & earum incrementa; tum in generali integrale ſimiliter etiam involventur variabiles quantitates x, y, &c. & ſi α ſit valor quantitatis y, cum $x = \beta$; tum erit β valor quantitatis y, cum $x = \alpha$.

17. In quibuſdam caſibus deduci poſſunt integrales duarum vel plurium (n) æquationum incrementialium tres vel plures variabiles quantitates,

titates, & earum incrementa involventium: vel e reductione harum (n) æquationum in unam, &c. ita ut exterminentur variabiles quantitates & earum fluxiones: vel ex aſſumptis generalibus quantitatibus pro correſpondentibus valoribus e ſingulis variabilibus quantitatibus, quibus pro ſuis valoribus in datis æquationibus incrementialibus ſubſtitutis; e reſultantibus æquationibus deduci poſſunt valores generalium quantitatum aſſumptarum.

2. Facile deduci poſſunt (n) incrementiales æquationes $n+1$ variabiles quantitates & earum incrementa involventes; quarum particulares valores ſingularum variabilium in prædictis æquationibus contentarum, innoteſcunt: aſſumantur enim quæcunque (n) functiones unius variabilis quantitatis & ejus incrementorum pro n reliquis, ſcribantur hæ quantitates & earum incrementa pro ſuis valoribus in quibuſcunque n diverſis quantitatibus a, b, c, d, &c. quæ ſint functiones ($n+1$) variabilium prædictarum & earum incrementorum, & ſint quantitates reſultantes P, Q, R, S, &c. reſpective; tum erunt $a=P$, $b=Q$, $c=R$, $d=S$, &c. n diverſæ æquationes, quarum particulares valores erunt quantitates aſſumptæ.

18. Datâ incrementiali æquatione duas variabiles x & y & earum incrementa involvente, ex principiis prius traditis, i. e. ex infinitis ſeriebus deduci poteſt, annon una x exprimi poteſt in algebraicis terminis alterius y.

19. Per infinitas ſeries detegi poteſt ex datâ incrementiali æquatione ($\alpha=0$) duas variabiles quantitates x & y & earum incrementa involvente generalis valor quantitatis y in terminis quantitatis x, qui continet invariabilem quantitatem a ad libitum aſſumendam; etiamque ſi modo data incrementialis æquatio ſit majoris ordinis (n), tum in prædicto valore continebuntur (n) invariabiles quantitates a, b, c, &c. ad libitum aſſumendæ; deinde ex infinitâ ſerie, quæ exprimit valorem quantitatis y in terminis quantitatis x, inveniantur $a=\pi$, $b=\varrho$, $c=\gamma$, &c.; & exinde facile deduci poſſunt n multiplicatores, qui reddunt datam æquationem $\alpha=0$ integrabilem.

20. Sit $\underset{.}{V}=P\underset{.}{x}$, tum P erit functio quantitatum x & $\underset{.}{x}$.

Et ſic, mutatis mutandis, applicari licet reliqua principia de fluxionalibus æquationibus tradita ad æquationes incrementiales, vel conjunctim fluxionales & incrementiales.

THEOR. X.

Sit æquatio $\underset{\underset{n}{1}}{y} + a\underset{\underset{n-1}{1}}{y} + b\underset{\underset{n-2}{1}}{y} \ldots, d\underset{\prime}{y} + fy = 0$, ubi $\underset{\prime}{y}, \underset{\prime\prime}{y}$, &c. ſunt præcedentes integrales quantitatis y; fingatur $y = Ae^{\alpha x}$, ubi $\underset{\cdot}{x}$ ſit conſtans; tum erit $Ae^{\alpha x + n\alpha\underset{\cdot}{x}} = \underset{\underset{n}{1}}{y}$, & $Ae^{\alpha x + n\alpha\underset{\cdot}{x}} + aAe^{\alpha x + (n-1)\alpha\underset{\cdot}{x}} \ldots dAe^{\alpha x + \alpha\underset{\cdot}{x}} + fAe^{\alpha x} = 0$, dividatur hæc æquatio per $Ae^{\alpha x}$, & in æquatione reſultante pro $e^{\alpha\underset{\cdot}{x}}$ ſcribatur v & reſultat æquatio $v^n + av^{n-1} + bv^{n-2} +$ &c. $= 0$, cujus radices ſint $\pi, \varrho, \sigma, \tau$, &c. inveniantur $\alpha, \beta, \gamma, \delta$, &c., ita ut $e^{\alpha\underset{\cdot}{x}} = \pi$, $e^{\beta\underset{\cdot}{x}} = \varrho$, $e^{\gamma\underset{\cdot}{x}} = \sigma$, $e^{\delta\underset{\cdot}{x}} = \tau$, &c. tum erit $Ae^{\alpha x} + Be^{\beta x} + Ce^{\gamma x} + De^{\delta x} +$ &c. $= y$, (ubi A, B, C, &c. ſunt quæcunque invariabiles quantitates ad libitum aſſumendæ) integralis datæ æquationis.

Et ſic de infinitis conſimilibus æquationibus.

Sit æquatio $a\underset{\underset{n}{1}}{y} + b\underset{\underset{n-1}{1}}{y} + \ldots + d\underset{1}{y} + fy = X$, vel $a\underset{n}{y} + b\underset{n-1}{y} + \ldots + d\underset{\cdot}{y} + f = X$; ubi a, b, &c., d, f & X ſunt functiones quantitatum x & $\underset{\cdot}{x}$; & $\underset{\cdot}{x}$ eſt conſtans; tum multiplicatores harum æquationum ſunt functiones quantitatum x & $\underset{\cdot}{x}$.

SCHOLIUM.

Facilies eſt obſervatu, ut haud poſſibile ſit ulterius promovere generale problema, quam ejus particulare; methodum generalium incrementorum, quam methodum fluxionum: haud difficile etiam erit pleraque in methodo fluxionum contenta reddere magis generalia & ad methodum incrementorum applicare; quæ præcipue in hoc libro perficiuntur: ſæpe etiam in primis incrementi terminis ſi mutetur incrementum in fluxionem, & inveniatur fluens fluxionis reſultantis, exinde deduci poteſt integralis dati incrementi.

Omnia in hoc libro (mutatis mutandis) etiam applicari poſſunt ad æquationes, in quibus incrementa & fluxiones ſimul continentur.

DE
INFINITIS SERIEBUS.

LIBER III.

Def. SIT data infinita ſeries $a+b+c+d+e+f+g+h+$ &c. & ſi ſucceſſivæ ſummæ a, $a+b$, $a+b+c$, $a+b+c+d$, &c. continuo ad ſummam ſeriei vergant, & ultimo propius accedant, quam data quævis differentia; tum hæc ſeries dici poteſt convergens.

Cor. Invenire, utrum data ſeries ſit convergens, necne: continuo inveniantur limites, inter quos conſiſtit quæſitæ ſeriei ſumma; & ſi hi limites continuo ad ſe vergant, & ultimo propius ad ſe invicem accedant, quam quævis data differentia; tum hæc ſeries dici poteſt convergens.

2. Convergentia termini (d) revera erit in eâdem ratione, quam habet terminus ipſe d ad ſummam termini ipſius & reliquorum ſeriei terminorum $d+e+f+g+h+$ &c.

3. Sint A & P quantitates inventæ, inter quas conſiſtit ſumma terminorum $c+d+e+f+g+$ &c. i. e. ſit A major quam prædicta ſumma & P minor, & B & Q quantitates inventæ inter quas conſiſtit ſumma $d+e+f+g+$ &c. apparens convergentia termini (d) ſemper erit in eâdem ratione, quam habet $(A-P)-(B-Q) : A-P$, i. e. $\frac{(A-P)-(B-Q)}{A-P} : 1$.

4. Convergentia totius ſeriei proportionalis erit ultimæ terminorum convergentiæ, i. e. terminorum ad infinitam diſtantiam; ea enim ſeries magis celeriter convergere dici poteſt, cujus termini ultimo celerius convergunt ad ſummam ſeriei quæſitam.

5. In ſerierum convergentiâ dijudicandâ, neceſſe eſt ut continuo inveniantur quantitates, inter quas ponitur ſeriei ſumma: in ſeriebus convergentibus, quarum termini alternatim ſunt negativi & affirmativi, plerumque continuo conſequuntur quantitates, quæ alternatim ſunt majores & minores quam ſumma quæſita; hæ vero ſeries facile mutari poſſunt in ſeries affirmativas, ſi modo continuo ſumantur differentiæ inter affirmativos terminos & negativos proxime ſubſequentes pro terminis novæ ſeriei quæſitis. E. g. Sit ſeries $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} +$ &c. mutari poteſt hæc ſeries in ſubſequentem affirmativam (ex ſumendo differentiam continuo inter duos ſucceſſivos terminos) viz. $\frac{1}{1 \cdot 2} + \frac{1}{3 \cdot 4} + \frac{1}{5 \cdot 6} +$ &c. Series vero affirmativæ haud eâdem facilitate in ſeries regulariter alternatim affirmativas & negativas mutari poſſunt, i. e. ita quidem ut omnes ſucceſſivi termini ultimo continuo fiant minores, & denique minores evadant quam quævis datæ quantitates; hoc vero in multis caſibus facile perfici poteſt: E. g. ſit ſeries $S = \frac{1}{m \times (m+n)} + \frac{1}{(m+1) \times (m+n+1)} + \frac{1}{(m+2) \cdot (m+n+2)} +$ &c. facile ſeſe dividet in ſeries $\frac{1}{n} \times \left(\frac{1}{m} - \frac{1}{m+n} + \frac{1}{m+1} - \frac{1}{m+n+1} + \frac{1}{m+2} - \frac{1}{m+n+2} + \text{\&c.} \right)$; unde terminis ſeriei recte diſpoſitis reſultat $S = \frac{1}{n} \left(\frac{1}{m} + \frac{1}{m+1} + \frac{1}{m+2} \ldots + \frac{1}{m+n} - \frac{1}{m+n} + \frac{1}{m+n+1} - \frac{1}{m+n+1} + \frac{1}{m+n+2} - \frac{1}{m+n+2} + \text{\&c.} \right) = \frac{1}{n} \left(\frac{1}{m} + \frac{1}{m+1} + \frac{1}{m+2} \ldots + \frac{1}{m+n-1} \right)$, ſi n ſit integer affirmativus numerus.

6. Seriem affirmativam $t + t' + t'' +$ &c. cujus terminus generalis ſit T quæcunque data functio quantitatis z diſtantiæ a primo ſeriei termino, in regularem alteram alternatim affirmativam & negativam transformare.

Fingatur T incrementum functionis quantitatis z, cujus incrementum ſupponitur 1; aſſumatur $\phi : (z) - \phi : (z+1) = T$, ubi $\phi : (z)$ denotat eandem functionem quantitatis z, ac $\phi : (z+1)$ eſt functio quantitatis $z+1$; ex hâc æquatione inveniatur $\phi : (z)$, & perficitur problema.

Et ſic interponere liceat n terminos inter duos ſucceſſivos datæ ſeriei terminos.

7. Sæpe vero ex aliis methodis inveſtigari poſſunt quantitates, quæ exſuperant datæ ſeriei ſummam; ſi enim ſummam ſeriei vel ſerierum novimus, quarum termini exſuperant terminos datæ ſeriei, tum ſumma data exſuperat ſummam ſeriei datæ: & ſic ſæpe inveniri poſſunt limites inter quos conſiſtit ſumma ſeriei quæſita.

Ex. 1. Sit ſeries $\frac{1}{1.2.3} + \frac{1}{4.5.6} + \frac{1}{7.8.9} + \frac{1}{10.11.12} +$ &c. & ſit $\frac{1}{1.2} + \frac{1}{4.5} + \frac{1}{7.8} +$ &c. $= S$, tum erit $\frac{1}{3}S$ major quam datæ ſeriei ſumma, quod facile conſtat ex hoc, nempe ſinguli ejus $(\frac{1}{3}S)$ termini majores ſunt quam datæ ſeriei correſpondentes termini: & ſic ſumma $\frac{1}{4.5} + \frac{1}{7.8} +$ &c. ducta in $\frac{1}{6}$ major erit quam ſumma $\frac{1}{4.5.6} + \frac{1}{7.8.9} +$ &c. & ſic deinceps.

Ex. 2. Summa ſeriei $1 + \frac{1}{2} + \frac{1}{1.2.3} + \frac{1}{1.2.3.4} +$ &c. minor erit quam $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} +$ &c. & ſumma $\frac{1}{1.2} + \frac{1}{1.2.3} + \frac{1}{1.2.3.4} +$ &c. minor erit quam $\frac{1}{2}(1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} +$ &c.); &c.

PROB. I.

FIG. 4. *Conceſſis curvilinearum quadraturis; invenire limites, inter quos conſiſtit ſeriei ſumma.*

E terminis generaliter expreſſis inveniatur curva, cujus inſcripta & circumſcripta pologona ſint reſpectivi datæ ſeriei termini, & area curvæ minor erit quam ſumma totius ſeriei, major vero quam ſumma totius ſeriei a primo termino diminutæ: ſi modo curva inter initium & finem abſciſſæ non habeat punctum contrariæ flexuræ vel ordinatam maximam vel minimam.

Ex. . Sit ſeries data $\frac{1}{2^{\frac{3}{2}}} + \frac{1}{3^{\frac{3}{2}}} + \frac{1}{4^{\frac{3}{2}}} \ldots \frac{1}{z^{\frac{3}{2}}}$, ubi $z - 2$ ſit diſtantia termini $\frac{1}{z^{\frac{3}{2}}}$ a primo: aſſumatur curva $FAPQRS$, &c. ſit $AL = AB = BC = CD = DE =$ &c. in infinitum $= 1$, & $AP = \frac{1}{2^{\frac{3}{2}}} = \frac{1}{OA^{\frac{3}{2}}}$, $BQ = \frac{1}{3^{\frac{3}{2}}} = \frac{1}{OB^{\frac{3}{2}}}$, $CR = \frac{1}{4^{\frac{3}{2}}} = \frac{1}{OC^{\frac{3}{2}}}$, &c. ergo ſumma datæ ſeriei $\frac{1}{2^{\frac{3}{2}}} + \frac{1}{3^{\frac{3}{2}}} + \frac{1}{4^{\frac{3}{2}}} +$ &c. $=$ ſummæ arearum circumſcriptorum parallelogrammorum $PB + CQ + DR + ES +$ &c. $=$ ſummæ arearum inſcriptorum parallelogrammorum $LP + AQ + BR + CS +$ &c. ſed curvilinea area $LNPQRS$, &c. major eſt quam prædicta ſumma circumſcriptorum vel inſcriptorum parallelogrammorum, & curvilinea area $APQRS$, &c. minor: hæ vero curvilineæ areæ erunt reſpective $\frac{2}{OL^{\frac{1}{2}}}$ & $\frac{2}{OA^{\frac{1}{2}}}$ $\left(2 \text{ \& } \frac{2}{2^{\frac{1}{2}}}\right)$; ergo ſumma datæ ſeriei $\frac{1}{2^{1\frac{1}{2}}} + \frac{1}{3^{1\frac{1}{2}}} +$ &c. in infinitum inter duas quantitates 2 & $\frac{1}{2^{\frac{1}{2}}}$ ponitur.

Et ſic ſumma ſeriei $\frac{1}{3^{1\frac{1}{2}}} + \frac{1}{4^{1\frac{1}{2}}} +$ &c. quæ eſt data ſeries ab primo termino diminuta inter quantitates $\frac{2}{2^{\frac{1}{2}}}$ & $\frac{2}{3^{\frac{1}{2}}}$ continetur; & in genere ſumma ſeriei $\frac{1}{z^{1\frac{1}{2}}} + \frac{1}{(z+1)^{1\frac{1}{2}}} + \frac{1}{(z+2)^{1\frac{1}{2}}} +$ &c. in infinitum inter quantitates $\frac{2}{(z-1)^{\frac{1}{2}}}$ & $\frac{2}{z^{\frac{1}{2}}}$ continetur; & ſic ſumma $(z-2)$ terminorum $\frac{1}{2^{1\frac{1}{2}}} + \frac{1}{3^{1\frac{1}{2}}} + \frac{1}{4^{1\frac{1}{2}}} \ldots \frac{1}{(z-1)^{1\frac{1}{2}}}$ inter quantitates $2 - \frac{2}{(z-1)^{\frac{1}{2}}}$ & $\frac{2}{2^{\frac{1}{2}}} - \frac{2}{z^{\frac{1}{2}}}$ ponitur.

Et ſic inveniri poſſunt limites, inter quos cujuſcunque conſimilis ſeriei conſiſtit ſumma; &c.

Cor. 1. Sit ſeries $\frac{1}{2^{n}} + \frac{1}{3^{n}} + \frac{1}{4^{n}} +$ &c. in infinitum; & curvilineæ areæ, inter quas conſiſtit ſumma hujus ſeriei, erunt infinitæ magnæ, necne; prout n vel ſit unitas vel minor ſit quam unitas, necne.

Cor. 2. Si hæ curvæ habeant ordinatas maximas vel minimas, vel puncta contrariæ flexuræ; tum ad ſingulam maximam vel minimam, &c. corrigenda eſt area inventa.

PROB. II.

Datâ lege, quam obſervant termini ſeriei in infinitum progredientis; invenire utrum ea ſit finita vel infinite magna.

E datâ lege conſtant termini ad infinitam diſtantiam; ſi vero terminus (A) ad infinitam diſtantiam conſtitutus ſit minor quam quæcunque quantitas hujuſce formulæ $\frac{1}{nz}$, ubi n ſit finita quantitas, & z diſtantia termini a primo ſeriei termino; tum ſumma ſeriei erit finita quantitas, ſin aliter vero non.

Conſtat e cor. 1. prob. præced.

2. Sit z diſtantia cujuſcunque termini a primo ſeriei termino, etiamque

que T & T' duo ſucceſſivi termini datæ ſeriei; & ſit data æquatio relationem inter T & T' exprimens, viz. $T' = T \times (az^r + \frac{b}{z^s} + \frac{c}{z^t} + \&c.)$ ubi $az^r + \frac{b}{z^s} + \frac{c}{z^t} + \&c.$ ſit ſeries ſecundum dimenſiones quantitatis z deſcendens; tum, ſi $1 : -z + az^{r+1} + \frac{b}{z^{s-1}} + \&c.$ ubi z ſupponitur infinita quantitas, habet affirmativam & minorem quam rationem æqualitatis per finitam rationem, tum ſumma ſeriei prædictæ erit finita quantitas; ſin aliter non.

Nam per coroll. prædict: ſeries erit finita, ſi modo duo ſucceſſivi termini (T & T') ſint reſpective $\frac{1}{z^n}$ & $\frac{1}{(z+1)^n}$; unde $\frac{T}{z^n} = \frac{T'}{(z+1)^n}$; & exinde $T' = (1 + \frac{n}{z} + n \cdot \frac{n-1}{2z^2} + \&c.)\, T$; cum n major ſit quam 1, & affirmativa; ſin aliter infinita: ducatur quantitas $1 + \frac{n}{z} + n \cdot \frac{n-1}{2z^2} + \&c.$ in z & reſultat $z + n + n \cdot \frac{n-1}{2z} + \&c.$; ad hanc quantitatem addatur quantitas $-z$; & ſumma erit $n + n \cdot \frac{n-1}{2z} + \&c.$; at ſeries erit finita, cum n major ſit quam unitas, &c. ergo conſtat propoſitio.

3. Sint T & T^m ſucceſſivi termini, quorum diſtantia a ſe invicem ſit m, & $T^m = T(az^r + \frac{b}{z^s} + \frac{c}{z^t} + \&c.)$ quæ ſit ſeries ſecundum dimenſiones quantitatis z deſcendens; tum, ſi $1 : -z + az^{r+1} + \frac{b}{z^{s-1}} + \&c.$ ubi z ſupponitur infinita quantitas, habet affirmativam & minorem rationem quam $1 : m$ per finitam rationem, ſumma ſeriei prædictæ erit finita quantitas; ſin aliter non.

4. Datâ æquatione inter ſucceſſivos datæ ſeriei terminos relationem

exprimente, facile e terminis datæ æquationis qui maximi ſunt ad infinitam diſtantiam & principiis in hoc problemate traditis erui poteſt, annon ſeriei ſumma ſit finita quantitas.

Inveniatur enim ratio, quam habent termini ſucceſſivi ad infinitam diſtantiam poſiti; i. e. cum z evadat infinita quantitas, & per methodos prius traditas detegi poteſt, annon ſeries ſit finita.

5. Si ad infinitam diſtantiam $T : T'$ habeat rationem majorem per finitam quam æqualitatis, tum ſeries erit finita: ſi termini ſeriei ſint alternatim affirmativi & negativi, reducendi ſunt ad ſeriem affirmativorum terminorum ex inveniendo differentias inter affirmativos & negativos terminos ſucceſſive.

Ex. 1. Sit $t' = at^n$, ubi t^n eſt (n) poteſtas quantitatis t & a invariabilis quantitas; tum, ſi at^{n-1} minor ſit quam 1, ſeries converget; ſin major non.

Ex. 2. Sit terminus ad infinitam diſtantiam (i. e. cum z evadat infinitus numerus) $= \frac{z^n}{e^{mz}}$; ubi literæ e, n & m denotant invariabiles quantitates, quarum una m ſaltem eſt affirmativa; tum ſeries ſemper erit finita, ſi e major ſit quam 1; ſin aliter non; nam ad infinitam diſtantiam erit $\frac{T'}{T} = \overline{\frac{z^n}{e^{mz}}}\Big|^{-1} \times \frac{(z+1)^n}{e^{mz+m}} = \frac{1}{e^m}\left(1 + \frac{n}{z} + n . \frac{n-1}{2z^2} + n . \frac{n-1}{2z} . \frac{n-2}{3z^2} + \&c.\right) =$ (ob z infinitum numerum) $\frac{1}{e^m}$, unde conſtat exemp.

Hinc, ſi z ſit infinitus numerus, conſtat e^{mz} majorem eſſe quam z^n, cum e major ſit quam 1.

Ex. 3. Sit terminus ad infinitam diſtantiam $\frac{z^{\frac{1}{z}}}{z^z}$, tum, ob $z^{\frac{1}{z}}$ (cum z evadat infinitus numerus) $= 1$; duo ſucceſſivi termini T & T' erunt $\frac{1}{z^z}$ & $\frac{1}{(z+1)^{z+1}}$ prope, unde $T' : T :: 1 : (1 + 1 + \frac{1}{1.2} + \frac{1}{1.2.3} +$

1 . 2

$\frac{1}{1.2.3.4} + \&c.)\, z = (1 + n)\, z$, ubi $1 + n$ eſt numerus, cujus hyper. log. eſt 1; & conſtat ſeriem eſſe finitam.

In omnibus his excipiendus eſt caſus, cum terminus ad finitam diſtantiam evadat infinitus.

Idem etiam deduci poteſt ex æquatione datâ inter ſucceſſivas ſummas relationem deſignante; facile enim reduci poteſt hæc æquatio ad æquationem relationem inter ſucceſſivos terminos exprimentem: & de æquatione relationem inter ſucceſſivas ſummas & terminos exprimente.

THEOR. I.

1. Sit ſeries $A + \frac{b x^p}{a^p} + \frac{c x^{2p}}{a^{2p}} + \frac{d x^{3p}}{a^{3p}} + \&c. = y$, & 1^{mo} in terminis (A, b, c, d, &c.) continuo augeantur differentiæ inter dimenſiones quantitatis z diſtantiæ a primo ſeriei termino per quantitatem, quæ in terminis ad infinitam diſtantiam evadit infinite magna; tum ſi dimenſiones numeratoris ſuperent dimenſiones denominatoris, ſeries erit divergens, ſin aliter erit convergens, e. g. ſeries $1 + \alpha x^p + \alpha . (\alpha + 1) x^{2p} + \alpha . (\alpha + 1) . (\alpha + 2) x^{3p} + \&c.$ ſemper diverget; ſeries vero $1 + \frac{1}{\alpha} \frac{x^p}{a^p} + \frac{1}{\alpha . (\alpha + 1)} \frac{x^{2p}}{a^{2p}} + \frac{1}{\alpha . (\alpha + 1) . (\alpha + 2)} \frac{x^{3p}}{a^{3p}} + \&c.$ ſemper converget, quicunque ſit valor quantitatis $\frac{x^p}{a^p}$.

2. Sit p affirmativa quantitas, & differentiæ inter dimenſiones quantitatis z in denominatore quantitatum (A, b, c, d, &c.) & earum numeratore contentas ad infinitam diſtantiam continuo eædem maneant, i. e. ſit finita, tum ſeries ſemper converget, ſi x inter $+ a$ & $- a$ ponatur; ſin aliter diverget; ni $x = \pm a$, in quo caſu ſeries converget vel diverget prout dimenſiones quantitatis z ad infinitam diſtantiam in denominatore ſuperant ejus dimenſiones in numeratore

per quantitatem majorem quam unitatem, necne. Sit p negativa & ſeries ſemper converget, cum x major ſit quam a vel $-a$; &c.

3. In fluentibus fluxionalium æquationum detegendis, neceſſe eſt ut convergens ſit ſeries inter duos valores (α & β) quantitatis x contenta, i. e. $A + b(\alpha \pm \beta) + c(\alpha^2 \pm \beta^2) + d(\alpha^3 \pm \beta^3) +$ &c. ſit convergens ſeries; quod in ſeriebus præcedentibus haud fieri poteſt, ni utræque ſeries $A + b\alpha + c\alpha^2 + d\alpha^3 +$ &c. & $A + b\beta + c\beta^2 + d\beta^3 +$ &c. ſint convergentes.

Facile ex principiis hic traditis, ſi modo ſcribatur $v + \alpha$ pro x in datâ ſerie $A + bx + cx^2 + dx^3 +$ &c. erui poteſt, annon ſeries exinde reſultans $(A + b\alpha + c\alpha^2 +$ &c.$) + (b + 2c\alpha + 3d\alpha^2 +$ &c.$)v + (c + 3d\alpha +$ &c.$)v^2 +$ &c. ſit convergens.

Et ex iiſdem principiis erui poteſt; annon ſeries, quæ ſit quæcunque functio datæ ſeriei, ſit convergens.

Cor. 1. Series, cujus termini generaliter exprimuntur per algebraicam & determinatam functionem haud vero exponentialem literæ z diſtantiæ a primo ſeriei termino, nunquam tam celeriter convergit; quam ſeries, cujus termini exprimuntur etiam per algebraicam & determinatam functionem quantitatis z in x^{nz}; ubi x ſit data quantitas, minor vero quam unitas, & n quæcunque affirmativa quantitas; vel x major quam unitas & n negativa quantitas.

In priori enim ſerie termini T & T' ſucceſſivi ad infinitam diſtantiam haud habent inter ſe rationem majorem vel minorem quam æqualitatis per finitam rationem; in poſteriori vero erit ultimo $T : T' :: 1 : x^n$.

Cor. 2. Poſterior vero ſeries nunquam converget tam celeriter, quam ſeries formulæ hujuſce generis $x + \frac{x^2}{1.2} + \frac{x^3}{1.2.3} + \frac{x^4}{1.2.3.4} + \ldots \frac{x^z}{1.2.3.4\ldots z}$. Erit enim in hâc ſerie ad infinitam diſtantiam $T : T' :: z : x$; i. e. in eadem ratione quam habet infinita quantitas ad finitam.

Hæc ſeries vel ſeries $x \pm \frac{x^2}{2^2} + \frac{x^3}{3^3} + \frac{x^4}{4^4} +$ &c. ſemper convergit, quicunque ſit valor quantitatis x; & ſeries $x + 1.2x^2 + 1.2.3x^3 +$ &c. vel $x + 4x^2 + 27x^3 + 256x^4 +$ &c. ſemper diverget, quicunque ſit valor quantitatis x.

Et ſic e convergentiâ ſeriei terminorum ad infinitam diſtantiam dijudicari poteſt convergentia ipſius ſeriei.

THEOR. II.

Datâ quantitate A, quæ eſt functio quantitatis x, cujus termini expanſi per ſeriem $ax^r + bx^{r+s} + cx^{r+2s} +$ &c. progrediuntur, tum erit $\frac{\dot{A}}{\dot{x}} = rax^{r-1} + (r+s)bx^{r+s-1} +$ &c. & ſic erit ſ. $A\dot{x} = \frac{1}{r+1}ax^{r+1} + \frac{1}{r+s+1}bx^{r+s+1} +$ &c. ſeries vero $\frac{1}{r+1}ax^{r+1} + \frac{1}{r+s+1}bx^{r+s+1} +$ &c. converget, ſi ſeries $ax^r + bx^{r+s} +$ &c. convergat; plerumque etiam ſeries $rax^{r-1} + (r+s)bx^{r+s-1} +$ &c. converget, ſi convergens ſit prædicta ſeries, ſed non ſemper; quæ facile conſtabunt e principiis prius traditis; & ſic deinceps.

THEOR. III.

1. Quamvis infinita ſeries data (A) haud convergat, tamen ſi detur finita quantitas p, quæ in ſeriem convergentem B ducta præbet datam quantitatem (A), i. e. $pB = A$; tum ſumma datæ ſeriei A quodammodo dici poteſt æqualis producto $p \times B$: vel magis generaliter, ſi data ſeries (A) ſit functio quantitatis x ad ſeriem ſecundum ejus dimenſiones progredientem reducta, tum ſeries A quodammodo dici poteſt æqualis prædictæ functioni quantitatis x.

2. In caſibus, cum ſeries A nec convergat nec divergat, & α ſit valor quantitatis x; tum plerumque inveniri poteſt ſumma ſeriei (A), cum valor ejus incognitæ quantitatis x quam minime differt a dato valore (α), i. e. ſeries erunt convergentes; & exinde deduci poteſt valor ſummæ quæſitæ, cum $x = \alpha$.

E. g. Series $1 - x + x^2 - x^3 + x^4 - x^5 + \&c.$ ſi $x = 1$, haud convergit; facile etiamque conſtat, ut $\frac{1}{1+x} \times 1$ erit $1 - x + x^2 - \&c.$ ergo quodammodo erit $1 - 1 + 1 - 1 + \&c.$ in infinitum $= \frac{1}{1+1} = \frac{1}{1+1+1} = \frac{1}{1+1+1+1} = \&c. =$ vel $\frac{1}{2}$ vel $\frac{1}{3}$ vel $\frac{1}{4}$, &c.

Erit $\frac{1}{2}$, ſi e ſerie $\frac{1}{1+x} = 1 - x + x^2 - \&c.$ exoriatur; erit $\frac{1}{3}$, ſi e ſerie $\frac{1}{1+x+x^2} = 1 - x + x^3 - x^4 + x^6 - x^7 + x^9 - x^{10} + x^{12} - \&c.$; & generaliter erit $\frac{1}{n+1}$, ſi e ſerie $1 - x + x^{n+1} - x^{n+2} + x^{2n+2} - x^{2n+3} + x^{3n+3} - x^{3n+4} + x^{4n+4} - \&c.$ exoriatur.

Cor. . Hinc ſummæ ſerierum, quæ tanquamd ivergentes apparent, ſæpe deduci poſſunt: revera forſan hiſce ſeriebus haud aſſignari poſſunt ſummæ.

3. Ex diviſione alterius quantitatis per alteram deduci poteſt quæcunque ſeries; at animadvertendum eſt, ut hæc ſeries non reverâ dici poſſit quotiens, ni reſiduum nihilo ſit æquale; ſi reſiduum augeatur, tum ſeries divergit; quamvis quotientes ſunt eædem, attamen fractiones non ſint æquales, nam reſidua poſſunt eſſe diverſa: e. g. ſint

$$1+1)\,1\,(1-1+1-1+1-1+\&c. = 1+1+1)\,1\,(1-1+1-1+1-\&c.$$

$$\begin{array}{l} 1+1 \\ \hline -1 \\ -1-1 \\ \hline \quad +1 \\ \quad \&c. \end{array} \qquad\qquad \begin{array}{l} 1+1+1 \\ \hline -1-1 \\ -1-1-1 \\ \hline \quad +1 \\ \quad \&c. \end{array}$$

$= 1+1+1$ (n ſit numerus) $(1)\ 1 - 1 + 1 - 1 + 1 - \&c.$ in primo caſu reſidua ſunt $+1$ & -1, & conſequenter quotientes inventæ diſtant a verâ per quantitates $\frac{1}{2}$ & $-\frac{1}{2}$: in ultimo caſu reſidua ſunt $-n+1$ & 1, & conſequenter quotientes inventæ diſtant a verâ per quantitates $-\frac{n-1}{n}$ & $\frac{1}{n}$. Si ſeries ſint reciprocæ, viz. $a + b + c + \&c. -$

$a - b - c - \&c. + a + b + c + \&c. - a - b - c - \&c. + \&c.$, tum ſeries non convergit; & quamvis quotientes ſint eædem, attamen, ſi reſidua per diviſores diviſa non ſint eadem, valores non iidem erunt.

THEOR. IV.

1. Convergentes ſeries, cujus termini ſunt in geometricâ progreſſione, uniformiter convergunt; omnis enim terminus eandem habet rationem ad reliquam ſeriei ſummam.

2. Sit ſeries $a - b + c - d + \&c.$; ſi a minor ſit quam b, b quam c, c quam d, &c,; tum ſeries quodammodo dici poteſt eſſe divergens; uſque donec ſubſequens terminus minor ſit quam præcedens; etiamque quodammodo at plerumque non revera convergentia termini ſeriei dici poteſt eſſe in ratione quam habet datus terminus ad ejus ſubſequentem.

3. Sint R, S, T tres ſucceſſivi termini datæ ſeriei generaliter expreſſi; & mutetur ſeries de prædictâ divergentiâ in convergentiam cum $R = S$, &c. & in ſtatum prædictæ celerrimæ convergentiæ cum $\frac{S}{R} = \frac{T}{S}$, vel cum fluxio fractionis $\frac{S}{R}$ ſit nihilo æqualis, &c. Plures vero poſſunt eſſe ſtatus celerioris convergentiæ prædicti generis, &c. facile conſtabit numerus punctorum celerioris convergentiæ, &c. e prædictis æquationibus, &c.

PROB. III.

Invenire convergentes ſeries, quæ dividi poſſunt in alias convergentes.

1. Aſſumantur duæ ſeries convergentes, ducantur hæ ſeries in ſeſe; & inveniatur lex, quam obſervant termini ſeriei reſultantis; & facile reduci poteſt hæc ſeries ad duas alias.

2. Et ſic inveniri poſſunt e datis ſeriebus convergentibus quam plurimæ æquationes convergentes; inveniantur enim functiones datarum ſerierum, quæ ſint convergentes: e principiis prius traditis facile conſequuntur infinitæ functiones convergentium ſerierum, quæ convergentes ſeries præbent: exhinc deduci poſſunt quædam ſeries, quæ methodis prius traditis haud erui poſſunt.

REGULA.

REGULA.

1. Sit x incognita quantitas, & detur algebraica functio quantitatis x, reducenda eſt data functio ad terminos ſecundum dimenſiones quantitatis x progredientes. Hoc fit operando in literis ad eundem modum, quo arithmetici in numeris decimalibus dividunt, radices extrahunt vel affectas æquationes ſolvunt.

Ex. 1. Sit functio data $(a + bx^r + cx^{2r} + \&c.)^m$, & per multinomiale theorema evadet

$$a^m + m a^{m-1} b x^r + \left.\begin{array}{l} m a^{m-1} c \\ + m \cdot \frac{m-1}{2} a^{m-2} b^2 \end{array}\right\} x^{2r} + \&c.$$

quantitas quæſita.

Ex. 2. Sit quantitas $(a + bx^r + cx^{2r} + \&c.)^n (h + kx^r + lx^{2r} + \&c.)^m$, per prædictum theorema inveniantur $(a + bx^r + cx^{2r} + \&c.)^n = a^n + n a^{n-1} b x^r + \&c.$ $(h + kx^r + lx^{2r} + \&c.)^m = h^m + m h^{m-1} k x^r + \&c.$ ducantur hæ ſeries in ſeſe & reſultat $h^m a^n + (nbh + mka) a^{n-1} h^{m-1} x^r + \&c.$ quantitas quæſita.

Ex. 3. Sit quantitas $(a + bx^r + cx^{2r} + \&c. + ((A + Bx^r + Cx^{2r} + \&c.)(\alpha + \beta x^r + \gamma x^{2r} + \&c.)^n)^m + (\pi + \varrho x^r + \sigma x^{2r} + \&c.)^h)^k$; reducatur ea ad ſimplices terminos ſecundum dimenſiones quantitatis x progredientes, tum evadet

$$((a + (A\alpha^n)^m + \pi^h)^k \, (= P^k) + kP^{k-1}(b + m(A\alpha^n)^{m-1}(\alpha^n B + nA\alpha^{n-1}\beta) + h\pi^{h-1}\varrho)x^r \, (kP^{k-1}Q) + \begin{array}{l} k \times \frac{k-1}{2} P^{k-2} Q^2 + \\ + kP^{k-1} R \end{array}$$

$+ \&c.$ Series facile conſtat ex reducendo quantitatem ad poteſtatem k elevandam in infinitonomialem quantitatem ſecundum dimenſiones quantitatis x^r progredientem, & deinde infinitonomialem quantitatem ad k poteſtatem elevando; & ſic de omnibus algebraicis quantitatibus ad ſeries ſecundum dimenſiones quantitatis x progredientes reducendis.

2. Facile etiam transformari poſſunt hæ quantitates, ut progrediantur ſecundum dimenſiones reciprocas quantitatis x. E. g. Sit

quantitas $(bx^r + a)^m$, & erit $(bx^r + a)^m = b^m x^m + m b^{m-1} a x^{(r-1)m} + m \cdot \frac{m-1}{2} b^{m-2} a^2 x^{(r-2)m} +$ &c.; & ſic deinceps.

Sæpe ſeries, quæ progrediuntur ſecundum dimenſiones directas quantitatis x convergunt; ſæpe vero ſeries, quæ progrediuntur ſecundum ejus reciprocas convergunt; ſæpe autem nec hæ, nec illæ convergunt, quæ ſatis manifeſto conſtabunt e propoſ. poſtea traditis.

THEOR. V.

Sit data ſeries $a + bx + cx^2 + dx^3 +$ &c. quæ infinita evadit, cum $x = p$, vel $x = q$, &c. tum aſſumatur $\frac{1}{(p-x)^m \times (q-x)^r} \times$ &c. $\times (A + Bx + Cx^2 +$ &c.$) = a + bx + cx^2 +$ &c. & e reductione quantitatum $(p-x)^{-m}$ & $(q-x)^{-r}$, &c. in ſimplices terminos ſecundum dimenſiones quantitatis x progredientes, & ex æquatis correſpondentibus terminis utriuſque æquationis partis conſequentur coefficientes A, B, C, &c. & in quibuſdam caſibus ita aſſumi poſſunt indices (m, r, &c.) ut æquatio deducta magis celeriter convergat; vel aliæ functiones quantitatis x, quæ evadunt infinitæ, cum $x = p$ vel $x = q$, &c.

Eadem principia ad plurimas conſimiles ſeries applicari poſſunt.

THEOR. VI.

1. Sit æquatio $0 = p - qx + rx^2 - sx^3 +$ &c. in infinitum, cujus infinitæ radices ſunt α, β, γ, δ, &c. tum erunt $\frac{q}{p} = \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta} +$ &c. ſumma e ſingulis reciprocis radicibus; $\frac{r}{p} = \frac{1}{\alpha\beta} + \frac{1}{\alpha\gamma} + \frac{1}{\beta\gamma} + \frac{1}{\alpha\delta} +$ &c. ſumma e quibuſque duabus reciprocis radicibus in ſeſe ductis; & ſic deinceps.

Cor. Hinc e prob. 1. & 3. medit. algebraic. deduci poteſt aggregatum e ſingulis valoribus quantitatum reciprocarum $\frac{1}{\alpha^m} + \frac{1}{\beta^m} + \frac{1}{\gamma^m} + \frac{1}{\delta^m} +$ &c. vel e ſingulis valoribus reciprocarum quantitatum hujuſce generis $\frac{1}{\alpha^m \beta^n \gamma^r \text{ \&c.}} + \frac{1}{\alpha^n \beta^m \gamma^r \text{ \&c.}} + \frac{1}{\alpha^r \beta^m \gamma^n \text{ \&c.}} +$ &c. & ſic deinceps; ubi m, n, r, &c. ſunt affirmativæ quantitates.

Et ſic transformari poteſt data æquatio in alteram, cujus radices ſunt $\frac{1}{\alpha^m \beta^n \gamma^r \text{ \&c.}} +$ &c.

2. Sit æquatio $0 = p - \frac{q}{x} + \frac{r}{x^2} - \frac{s}{x^3} +$ &c. in infinitum, cujus infinitæ radices ſint α, β, γ, δ, &c. tum erunt $\frac{q}{p} = \alpha + \beta + \gamma + \delta +$ &c. $\frac{r}{p} = \alpha\beta + \alpha\gamma + \beta\gamma + \alpha\delta + \beta\delta + \gamma\delta +$ &c. $\frac{s}{p} = \alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta +$ &c. &c.

Cor. . Hinc e prædict. prob. medit. algebr. deduci poteſt aggregatum e ſingulis valoribus contentorum $\alpha^m \beta^n \gamma^r$ &c. $+ \alpha^n \beta^m \gamma^r$ &c. $+ \alpha^r \beta^m \gamma^n$ &c. ubi m, n, r, &c. ſunt affirmativæ quantitates.

Et ſic transformari poteſt data æquatio in alteram, cujus radices ſunt $\alpha^m \beta^n \gamma^r$ &c. + &c.

3. Sint duæ infinitæ æquationes, unam ſolummodo incognitam quantitatem x & y reſpective habentes; & quarum radices ex unâ ſunt α, β, γ, δ, &c. ex alterâ vero π, ϱ, σ, τ, &c. & ſi modo ex unâ æquatione deduci poſſit aggregatum (P) e ſingulis quantitatibus hujuſce formulæ $\alpha^m \beta^n \gamma^r$ &c. ex alterâ vero æquatione aggregatum (Q) e ſingulis quantitatibus hujuſce formulæ $\pi^a \varrho^b \sigma^c$ &c. ubi omnes literæ m, n, r, &c. vel ſimul denotant affirmativas quantitates vel negativas, & ſic omnes literæ a, b, c, &c. ſimul ſunt affirmativæ vel ſimul negativæ;

tum erit aggregatum e ſingulis quantitatibus hujuſce formulæ ($\alpha^m \beta^n \gamma^r$ &c. $\pi^a \varrho^b \sigma^c$ &c.) $= P \times Q$.

4. Ducatur vel æquatio $o = p - qx + rx^2 - sx^3 + tx^4 -$ &c. vel æquatio $o = p - \frac{q}{x} + \frac{r}{x^2} - \frac{s}{x^3} +$ &c. in arithmeticam ſeriem 0, 1, 2, 3, 4, &c. & reſultant æquationes, quarum radices ſunt limites inter datarum æquationum radices, &c.

5. E principiis in medit. algebr. traditis ſæpe conſequuntur notæ radicum impoſſibilium, affirmativarum & negativarum in datis æquationibus contentarum. E. g. Sub primo termino ſcribatur +; ſub ſecundo ſcribatur + vel — prout $\frac{1}{2}q^2$ major ſit quam pr vel non; ſub tertio ſcribatur + vel — prout $\frac{1}{3}r^2$ major ſit quam qs, necne; & ſub quarto ſcribatur + vel — prout $\frac{1}{4}s^2$ major ſit quam rt; & ſic deinceps; & tot ſaltem erunt impoſſibiles radices, quot ſunt mutationes ſignorum de + in — & — in +.

6. E medit. algebr. conſtat, quod ſi modo quædam n radices æquationis $p - qx + rx^2 -$ &c. $= o$ vel $p - \frac{q}{x} + \frac{r}{x^2} -$ &c. $= o$ ſint poſſibiles, tum etiam reperientur $n - 1$ radices poſſibiles æquationum reſultantium e multiplicatione ejus terminorum ſucceſſivorum in prædictæ arithmeticæ ſeriei terminos.

7. Hinc facile formari poteſt infinita æquatio, cujus radices haud dantur; & tamen affirmari poteſt, ut nullas habeat poſſibiles radices.

Aſſumantur enim coefficientes, ita ut per prædictas regulas omnes radices notas habeant impoſſibilitatis; & id, quod requiritur, fit.

8. Si modo in datâ infinitâ æquatione pro x ſcribantur quæcunque duæ quantitates α & β, & quantitates reſultantes mutentur de + in — vel — in +, tum impar numerus radicum datæ æquationis inter quantitates α & β interponitur; ſin aliter nulla radix vel par numerus radicum inter α & β invenietur; ni e ſcribendo γ valorem inter α & β contentum pro x ſeries evadat divergens.

THEOR.

THEOR. VII.

1. Sit infinita æquatio $A = ax^{r} + bx^{r+s} + cx^{r+2s} + \&c. \ldots + Px^{l} + Qx^{l+s} + \&c.$; & ſi coefficientes haud creſcant in majori quam quâcunque geometricâ ratione aſſignabili, & ſint omnes affirmativæ; tum ſemper datur una affirmativa radix & non plures, quæ quidem erit minor quam quæcunque negativa in datâ æquatione contenta.

2. In affirmativâ ſerie $ax^{r} + bx^{r+s} + cx^{r+2s} + \&c.$ fingantur duo ſucceſſivi termini ad infinitam diſtantiam (l) poſiti inter ſe æquales, i. e. $Px^{l} = Qx^{l+s}$, & conſequenter $x = \left.\overline{\frac{P}{Q}}\right|^{\frac{1}{s}} = \alpha$; & exinde omnis quantitas minor quam α vel $-\alpha$ pro x in prædictâ ſerie conſtituta ſemper præbebit finitam ſummam; ſin major ſit quam α, tum ſemper præbebit infinitam.

2. 2. Si ſeries ſit negativa, tum ex pluribus terminis ad infinitam diſtantiam poſitis inter ſe æqualibus eſſe ſuppoſitis acquiri poſſunt limites quantitatum, quæ in datâ ſerie pro x ſubſtitutæ ſemper præbent finitas vel infinitas quantitates.

PROB. IV.

Invenire æquationem, quæ infinitas habet cognitas radices.

Sint cognitæ radices $\alpha, \beta, \gamma, \delta, \epsilon$, &c. & ducantur factores $(x - \alpha) \times (x - \beta) \times (x - \gamma) \times (x - \delta) \times \&c.$ in ſeſe, & contentum nihilo fiat æquale; erit æquatio deſiderata.

Ex. Sint radices $\alpha, \alpha^{2}, \alpha^{3}, \alpha^{4}$, &c. in infinitum; æquatio quæſita erit

$$1 - \frac{\alpha}{1-\alpha} \times \frac{1}{x} + \frac{\alpha^{3}}{(1-\alpha).(1-\alpha^{2})} \times \frac{1}{x^{2}} - \frac{\alpha^{6}}{(1-\alpha).(1-\alpha^{2}).(1-\alpha^{3})} \times \frac{1}{x^{3}} + \frac{\alpha^{10}}{(1-\alpha).(1-\alpha^{2}).(1-\alpha^{3}).(1-\alpha^{4})} \times \frac{1}{x^{4}} - \frac{\alpha^{15}}{(1-\alpha).(1-\alpha^{2}).(1-\alpha^{3}).(1-\alpha^{4}).(1-\alpha^{5})} \times \frac{1}{x^{5}} + \&c. = 0.$$

Summa radicum eſt

$\frac{\alpha}{1-\alpha}$, ſumma rectangulorum ſub ſingulis binis $\frac{\alpha^3}{(1-\alpha)\times(1-\alpha^2)}$, & ſic deinceps.

Cor. Inveniatur æquatio, cujus radices ſint quæcunque algebraica functio hujuſce æquationis radicum; reſultat æquatio, quæ etiam habet infinitas radices.

Ex. 1. Ducatur hæc æquatio $1-\frac{\alpha}{1-\alpha}\times\frac{1}{x}+\frac{\alpha^3}{(1-\alpha).(1-\alpha^2)}\times\frac{1}{x^2}-$ &c. $=0$ in alteram æquationem $1-\frac{\beta}{1-\beta}\times\frac{1}{x}+\frac{\beta^3}{(1-\beta)\times(1-\beta^2)}\times\frac{1}{x^2}-$ &c. $=0$, & reſultat æquatio $1-\frac{\alpha+\beta-2\alpha\beta}{(1-\alpha)\times(1-\beta)}\times\frac{1}{x}+$ &c. $=0$, cujus radices erunt α, α^2, α^3, &c. in infinitum & β, β^2, β^3, &c. in infinitum; & ſic deinceps.

PROB. V.

Datis duabus infinitis æquationibus duas vel plures incognitas quantitates (x & y) *habentibus; eas in unam reducere, ita ut incognita quantitas* (y) *exterminetur.*

1mo. Collocentur termini utriuſque æquationis, ita ut illi termini primum locum occupent, qui maximi ſunt; ii vero proximum locum, qui proxime majores ſunt, & ſic deinceps. i. e. Sint termini in unâ æquatione reſultantes reſpective $a+b+c+d+e+$ &c. $=0$, in alterâ vero $p+q+r+s+$ &c. $=0$. 1mo. Supponantur $a=0$ & $p=0$, & ita reducantur hæ duæ æquationes, ut exterminetur incognita quantitas (y); deinde ſupponantur pro propriori valore æquationis quæſito $a+b=0$ & $p+q=0$, & reducantur hæ duæ æquationes, ita ut exterminetur incognita quantitas y; & ſic deinceps; & continuo ad æquationem quæſitam propius accedamus.

2. Datam æquationem unam (x) ſolummodo incognitam quanti-

tatem habentem in duas vel plures alias habentes duas vel plures incognitas quantitates dividere.

Aſſumatur æquatio duas vel plures incognitas quantitates (x, y, &c.) habens; deinde addatur, ſubtrahatur, &c. aſſumpta æquatio e datâ; & exoriuntur duæ æquationes duas vel plures incognitas quantitates habentes, quæ facile reduci poſſunt ad datam. Et ſic de pluribus æquationibus.

THEOR. VIII.

Sint duæ vel plures infinitæ æquationes duas vel plures incognitas quantitates habentes, quarum ſit una integer numerus vel fractio; tum altera etiam erit integer numerus vel fractio, ni duo vel tres vel plures valores reliquarum incognitarum quantitatum ſint inter ſe æquales.

Multarum infinitarum æquationum inveniri poſſunt radices integri numeri ex principiis, quæ tradita fuere pro inveniendis radicibus, qui ſunt integri numeri, in finitis algebraicis æquationibus.

PROB. VII.

Sint duæ infinitæ ſeries, quarum termini ſecundum dimenſiones quantitatis x *collocantur; invenire nonnunquam infinitas ſeries, quæ ſint earum communes diviſores.*

Dividantur hæ æquationes juxta methodum, per quam inveniuntur communes diviſores in algebraicis finitis æquationibus, & nonnunquam reſultant communes diviſores. E. g. Sint duæ infinitæ ſeries $1 - \frac{5}{3\cdot 2}x + \frac{5}{3\cdot 4}x^2 - \frac{5}{3\cdot 8}x^3 + \text{&c.}$ & $1 - \frac{3}{4}x + \frac{3}{8}x^2 - \frac{3}{16}x^3 + \text{&c.}$ dividantur hæ quantitates ſecundum methodum inveniendi communes diviſores, & operatio erit hujuſmodi

$$
\begin{array}{l}
1 - \frac{5}{6}x + \frac{5}{12}x^2 - \frac{5}{24}x^3 + \text{&c.}\,)\ 1 - \frac{3}{4}x + \frac{3}{8}x^2 - \frac{3}{16}x^3 + \text{&c.}\,)\ 1 \\
\phantom{1 - \frac{5}{6}x + \frac{5}{12}x^2 - \frac{5}{24}x^3 + \text{&c.}\,)\ } \underline{1 - \frac{5}{6}x + \frac{5}{12}x^2 - \frac{5}{24}x^3 + \text{&c.}} \\
\phantom{1 - \frac{5}{6}x + \frac{5}{12}x^2 - \frac{5}{24}x^3 + \text{&c.}\,)\ 1 - } \frac{1}{12}x - \frac{1}{24}x^2 + \frac{1}{48}x^3 - \text{&c.}
\end{array}
$$

ducatur

hoc reſiduum, fractionem evitandi gratiâ, in $\frac{12}{x}$, & reſultat ſeries $1 - \frac{1}{2}x + \frac{1}{4}x^2 - \frac{1}{8}x^3 +$ &c. dividatur prior diviſor per reſiduum, & operatio erit

$$\begin{array}{l} 1 - \frac{1}{2}x + \frac{1}{4}x^2 - \&c.)\ 1 - \frac{5}{6}x + \frac{5}{12}x^2 - \frac{5}{24}x^3 + \&c.)\ 1 - \frac{1}{3}x \\ \qquad\qquad\qquad\qquad\quad 1 - \frac{1}{2}x + \frac{1}{4}x^2 - \frac{1}{8}x^3 + \&c. \\ \hline \qquad\qquad\qquad\qquad\quad - \frac{1}{3}x + \frac{1}{6}x^2 - \frac{1}{12}x^3 + \frac{1}{24}x^4 - \&c. \\ \qquad\qquad\qquad\qquad\quad - \frac{1}{3}x + \frac{1}{6}x^2 - \frac{1}{12}x^3 + \frac{1}{24}x^4 - \&c. \\ \hline \qquad\qquad\qquad\qquad\quad \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \end{array}$$

unde conſtat has duas infinitas ſeries eandem præbere ſeriem $1 - \frac{1}{2}x + \frac{1}{4}x^2 -$ &c. ductam reſpective in $1 - \frac{1}{3}x$ & $1 - \frac{1}{4}x$.

THEOR. IX.

1. Sint α & β duæ radices æquationis infinitæ $o = a + bx + cx^2 + dx^3 +$ &c. $= A$, ſcribatur in hâc æquatione pro x ejus valor aſſumptus π, qui inter duas proximas radices α & β ponitur; tum plerumque erit quantitas reſultans finita quantitas; ſi A evadat infinita quantitas, cum $x = \lambda$ inter valores α & β poſita; tum evadet iterum finita quantitas, cum $x = \mu$ quantitas etiam inter α & β poſita: & plures poſſunt eſſe progreſſus hujuſce generis de finito ad infinitum, &c. inter valores α & β quantitatis x.

2. Si modo ſint α, β, γ, &c. ſucceſſivæ radices datæ infinitæ æquationis; & π quantitas inter α & β, & ϱ inter β & γ: ſcribantur π & ϱ pro x in datâ infinitâ æquatione; & quantitatum exinde reſultantium mutabuntur ſigna de + in − & − in +, ni detur quantitas inter π & ϱ contenta, quæ ſubſtituta pro x præbet divergentem ſeriem.

3. Si duæ æquationes $A + bx + cx^2 + dx^3 +$ &c. $= o$ & $A' + bx + cx^2 + dx^3 +$ &c. $= o$ habeant poſſibilem radicem (α & β) reſpective; & A'' ſit quantitas inter A & A' contenta, tum æquatio $A'' + bx + cx^2 + dx^3 +$ &c. $= o$ habet poſſibilem radicem; ni inter valores α & β quantitatis x ſeries evadat divergens.

4. Divergens ſeries in quantitatem nihilo æqualem ducta aliquando

fiet convergens; in finitam quantitatem ducta nunquam producet ſeriem, quæ proprie dici poteſt convergens.

5. Inveniantur termini ſeriei ad infinitam diſtantiam poſiti, quæ eſt productum divergentis ſeriei in quantitatem nihilo æqualem ductæ; & ex iis per principia prius tradita ſemper detegi poteſt, annon ſeries reſultans ſit convergens.

6. In ſeriebus cujuſcunque generis fluens convergit magis celeriter quam ejus fluxio; & ſic de conſimilibus.

PROB. VII.

Datâ reſolutione æquationis, quæ ſit infinita ſeries irrationalium quantitatum; invenire æquationem ipſam.

Ita reducatur per prob. 26. medit. algebr. data reſolutio, ut exterminentur irrationales quantitates, & reſultat æquatio quæſita.

PROB. VIII.

Invenire, utrum data quantitas ſit radix datæ infinitæ æquationis, necne: ſcribatur data radix pro ejus valore in datâ æquatione, & ſi e quibuſcunque principiis conſtet quantitatem reſultantem ultimo ad nihil vergere, tum conſtat quantitatem datam eſſe radicem datæ infinitæ æquationis.

E. g. Sit æquatio $o = 1 - x - x^2 - x^3 -$ &c. invenire, annon $\frac{1}{2}$ ſit radix prædictæ æquationis: ſcribantur $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, &c. pro x, x^2, x^3; &c. in datâ æquatione, & reſultant ſucceſſiva reſidua $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, &c. in infinitum, unde ultimo vergit ad nihil, & conſequenter $x = \frac{1}{2}$ erit radix datæ æquationis. Dividatur data æquatio per $\frac{1}{2} - x$, & quotiens invenietur $o = 2 + 2x + 2x^2 + 2x^3 +$ &c., cujus omnes radices ſunt impoſſibiles.

Cor. Series $1 - x - x^2 - x^3 -$ &c. $= \frac{1 - 2x}{1 - x}$; hujuſce ſeriei numerator $1 - 2x = o$, cum $x = \frac{1}{2}$: etiamque ſeries $1 + x + x^2 + x^3$ + &

$+$ &c. $= \frac{1}{1-x}$ erit convergens, cum $x = \frac{1}{2}$ ergo; $x = \frac{1}{2}$ erit radix datæ æquationis $\frac{1-2x}{1-x} = 1 - x - x^2 -$ &c. $= 0$. Et ſic generaliter ratiocinari liceat; i.e. ſi numerator $= 0$, & denominator ad ſeriem ſecundum dimenſiones quantitatis x progredientem reductus præbeat convergentem ſeriem, cum $x = \alpha$; tum α erit radix ſeriei, quæ oritur ab expanſione datæ fractionis in terminos ſecundum dimenſiones quantitatis x progredientes.

2. Æquatio $0 = 1 + x + x^2 + x^3 +$ &c. $= \frac{1}{1-x}$ habet unam radicem $x = -1$; & æquatio $0 = 1 - x + x^2 - x^3 +$ &c. $= \frac{1}{1+x}$ habet radicem $x = 1$; nam ſi pro x in utriſque æquationibus ſcribatur quantitas inter 1 & -1 poſita, tum ſemper evadent finitæ quantitates ad 0 magis appropinquantes, quo minus diſtet (x) ab -1 in priori & ab $+1$ in poſteriori æquatione.

THEOR. X.

Sit ſeries $A = 0$, quæ ſit finita functio (P) algebraicæ quantitatis x; tum radices æquationis $A = p$ erunt etiam radices æquationis $P = p$; & vice verſâ radices æquationis $P = p$ erunt radices æquationis $A = p$.

Ex. . Sit ſeries $1 + x + x^2 + x^3 +$ &c. $= a + \sqrt{(-b)}$, ſed eſt $\frac{1}{1-x} = 1 + x + x^2 +$ &c. $= a + \sqrt{(-b)}$, ergo nullæ radices inveniuntur in æquatione $1 + x + x^2 +$ &c. $= a + \sqrt{(-b)}$, quæ in æquatione $\frac{1}{1-x} = a + \sqrt{(-b)}$ haud inveniuntur.

Scribatur enim in ſerie $1 + x + x^2 + x^3 +$ &c. pro (x) valor aſſumptus $c + \sqrt{(-d)}$, & reſultat $1 + c + c^2 + c^3 +$ &c. $- d(1 +$

$3c + 6c^2 + \&c.) + d^2(1 + 5c + 15c^2 + \&c.) - d^3(1 + 7c + 28c^2 + \&c.) + \&c. \left(= \frac{1}{1-c} - \frac{d}{(1-c)^3} + \frac{d^2}{(1-c)^5} - \&c.\right) + \sqrt{(-d)}$ $(1 + 2c + 3c^2 + \&c. - d(1 + 4c + 10c^2 + \&c.) + d^2(1 + 6c + 21c^2 + \&c.) - \&c.) \left(= \sqrt{(-d)}\left(\frac{1}{(1-c)^2} - \frac{d}{(1-c)^4} + \frac{d^2}{(1-c)^6} - \&c.\right) + \&c.\right)$; unde $\frac{1}{1-c} - \frac{d}{(1-c)^3} + \frac{d^2}{(1-c)^5} - \&c. = a$ & $\sqrt{(-d)}\left(\frac{1}{(1-c)^2} - \frac{d}{(1-c)^4} + \frac{d^2}{(1-c)^6} - \&c.\right) = \sqrt{(-b)}$, & exinde $\frac{1-c}{(1-c)^2+d} = a$ & $\frac{1}{(1-c)^2+d} = \sqrt{\left(\frac{b}{d}\right)}$; & ex his duabus æquationibus deduci poſſunt iidem & nulli alii valores quantitatum c & d, præter eos, qui ex æquatione $\frac{1}{1-x} = a + \sqrt{(-b)}$, i. e. $x = \frac{a - 1 + \sqrt{(-b)}}{a + \sqrt{(-b)}}$, reſultant.

THEOR. XI.

Sit infinita æquatio $0 = a - bx + cx^2 - dx^3 + ex^4 - \&c.$; ubi $\frac{b}{a}$ multo major ſit quam $\frac{c}{b}$; & $\frac{c}{b}$ multo major quam $\frac{d}{c}$; & $\frac{d}{c}$ multo major quam $\frac{e}{d}$; & $\frac{e}{d}$ multo major quam $\frac{f}{e}$, & ſic deinceps in infinitum: tum erunt omnes radices datæ æquationis poſſibiles; & approximatio (α) ad minimam radicem erit $\frac{a}{b}$; & approximatio ad radicem (β), quæ eſt minor quam omnes præter α erit $\frac{b}{c}$; vel radices æquationis quadraticæ $a - bx + cx^2 = 0$ erunt propriores approximationes ad α & β; & ſimiliter $\frac{c}{d}$ erit approximatio ad tertiam radicem (γ) datæ æquationis; quæ eſt minor quam omnes radices datæ æquationis præter α & β; vel radices cubicæ æquationis $a - bx + cx^2 - dx^3 = o$ erunt propriores approximationes ad radices α, β & γ datæ æquationis; & ſimiliter approximatio ad quartam minimam radicem erit $\frac{d}{e}$, & quatuor radices datæ æquationis $a - bx + cx^2 - dx^3 + ex^4 = o$ erunt propriores approximationes ad quatuor minimas radices (α, β, γ & δ) datæ æquationis; & ſic deinceps.

THEOR. XII.

Sit æquatio $y^n + ay^{n-1} + by^{n-2} + cy^{n-3} \ldots fy^{n-m} + gy^{n-m-1} + hy^{n-m-2} \ldots + ky^{n-m-r} + ly^{n-m-r-1} \ldots + py^{s+1} + qy^s + \&c. = o$, cujus m radices ſint multo majores quam reliquæ, tum ex ſummâ $m+1$ primorum terminorum nihilo æquali eſſe ſuppoſitâ, viz. $y^m + ay^{m-1} + by^{m-2} + cy^{m-3} \ldots + f = o$ reſultat æquatio, cujus radices ſunt prædictæ m radices prope: ſi vero r radices datæ æquationis ſint multo minores quam prædictæ m radices, multo vero majores quam reliquæ; tum ex ſummâ $r+1$ terminorum, quorum primus ſit ultimus æquationis prius traditæ terminus fy^{n-m}, cæteri vero r ſubſequentes, viz. $fy^r + gy^{r-1} + hy^{r-2} \ldots + k = o$, nihilo æquali eſſe ſuppoſitâ reſultat æquatio, cujus radices ſunt r prædictæ radices prope; & ſic deinceps; & denique ſi s radices datæ æquationis ſint multo minores quam reliquæ, tum ex ſummâ $s+1$ ultimorum terminorum nihilo æquali eſſe ſuppoſitâ reſultat æquatio, cujus radices ſunt s radices prædictæ.

Cor. Sint α, β, γ, δ, &c. radices datæ æquationis $y^n - py^{n-1} + qy^{n-2} - ry^{n-3} + \&c. = o$, quarum α multo major ſit quam β, β quam γ, γ quam δ, & ſic deinceps; tum erit $\alpha = p$ prope, $\beta = \frac{q}{p}$ prope, $\gamma = \frac{r}{q}$ prope, & ſic deinceps.

THEOR.

THEOR. XIII.

1. Sit æquatio $y^n + ay^{n-1} + by^{n-2} + cy^{n-3} + \&c. = o$, ubi a, b, c, &c. ſunt functiones ipſius x, nunc ex hypotheſi quod x ſit perparva vel permagna quantitas vel ex quâcunque aliâ hypotheſi ſint α, β, γ, &c. proximi valores quantitatum a, b, c, d, &c. tum aſſumatur æquatio $v^n + \alpha v^{n-1} + \beta v^{n-2} + \gamma v^{n-3} + \delta v^{n-4} + \&c. = o$, & n radices hujuſce æquationis erunt primæ approximationes ad n diverſas datæ æquationis radices.

2. Si vero quæcunque una radix multo minor ſit quam quædam m radices in datâ æquatione $y^n + ay^{n-1} + by^{n-2} + \ldots fy^{n-m} + gy^{n-m-1} + hy^{n-m-2} + \&c. = o$ contentæ, multo vero major quam ſingulæ reliquæ $(n - m - 1)$; tum erit $-\frac{g}{f}$ prima approximatio ad prædictam radicem; ſi vero duæ radices ε & ζ ſint multo minores quam m radices prædictæ, multo vero majores quam $n - m - 2$ reliquæ radices, erunt radices æquationis $fy^{n-m} + gy^{n-m-1} + hy^{n-m-2} = o$ primæ approximationes ad duas radices prædictas; & ſic de pluribus.

3. Datis primis approximationibus π, ϱ, σ, &c. ad quaſcunque (m) radices datæ æquationis; alias π', ϱ', σ', &c. adhuc magis appropinquantes detegere: ſcribatur $(y - \pi - \pi') \cdot (y - \varrho - \varrho') \cdot (y - \sigma - \sigma') \cdot \&c. \times (y^{n-m} + py^{n-m-1} + qy^{n-m-2} + \&c.) = y^n + ay^{n-1} + by^{n-2} + \&c. = o$, & ex æquatis correſpondentibus terminis reſultantis æquationis exorientur n æquationes totidem n incognitas quantitates π', ϱ', σ', &c. p, q, r, &c. habentes; nunc abjectis omnibus terminis tanquam prope nihilo æqualibus, in quibus plures quam una dimenſio quantitatum π', ϱ', σ', &c. inveniuntur, & reſultantes n æquationes in m alias reducantur, ita ut exterminentur incognitæ quantitates p, q, r, s, &c. & exorientur m æquationes totidem incognitas quantitates π', ϱ', σ', &c. habentes; quibus in unam reductis, ita ut exterminentur omnes præter unam incognitæ quantitates, reſultant valores approximationum quæſitarum π', ϱ', σ', &c. quæ invenientur reſpective $\pi' = -$

$$\frac{\pi^n + a\pi^{n-1} + b\pi^{n-2} + c\pi^{n-3} + \&c.}{n\pi^{n-1} + (n-1)a\pi^{n-2} + (n-2)b\pi^{n-3} + \&c.} \text{ prope, } \varrho' = -$$
$$\frac{\varrho^n + a\varrho^{n-1} + b\varrho^{n-2} + c\varrho^{n-3} + \&c.}{n\varrho^{n-1} + (n-1)a\varrho^{n-2} + (n-2)b\varrho^{n-3} + \&c.} \text{ prope, } \sigma' = -$$
$$\frac{\sigma^n + a\sigma^{n-1} + b\sigma^{n-2} + \&c.}{n\sigma^{n-1} + (n-1)a\sigma^{n-2} + (n-2)b\sigma^{n-3} + \&c.} \text{ prope, \&c.}$$

In denominatoribus & numeratoribus harum fractionum rejiciantur omnes termini tanquam nihilo æquales, qui ex hypotheſi aſſumptâ multo minores evadunt quam reliqui; & e comparandis reſultantis æquationis $v^r + \alpha v^{r-1} + \beta v^{r-2} + \&c. = 0$ terminis, ſæpe inveniri poſſunt ejus radices. E. g. E comparandis quibuſque duobus proxime ſucceſſivis erui poteſt radix prope, ſi modo longe diſtet a reliquis; & ſic detegi poteſt quadratica æquatio, quæ continet duas radices prope a reliquis longe diſtantes; & ſic deinceps.

Eadem fere principia ad fluxionales & incrementiales æquationes applicari poſſunt.

Si plures dentur æquationes plures habentes incognitas quantitates; tum ex iiſdem principiis erui poſſunt plures æquationes, quarum radices ſunt primæ approximationes ad radices datarum æquationum.

4. Eadem principia etiam applicari poſſunt ad æquationes, in quibus quæcunque irrationales continentur quantitates; rejectis enim omnibus quantitatibus in irrationalibus terminis contentis, qui haud maximi evadant ex aſſumptâ hypotheſi, e reliquis erui poſſunt approximationes ad ſingulas datæ æquationis radices, & ex iis primæ approximationes ad quantitatem vel quantitates quæſitas; & ex primis approximationibus deduci poſſunt ſecundæ approximationes e principiis prius traditis; vel rejectis omnibus, quæ haud maximæ vel proximæ evadant ex aſſumptâ hypotheſi, e terminis reſultantibus inveniri poſſunt approximationes ad radices quæſitas vergentes; & ſic deinceps. Facile etiam conſtat numerus terminorum, qui vere obtineri poſſunt ex rejectis quibuſdam datæ æquationis terminis.

Si vero duo vel plures (n) valores quantitatis quæſitæ ſint prope $= e$ & inter ſe æquales; tum inveniendi ſunt termini datarum æquationum, qui nullam, unam, duas, vel denique (n) habent dimenſiones; qui ſint reſpective A, Be, Ce^2, De^3, $\ldots He^n$; tum radices æquationum $A + Be + Ce^2 = 0$ vel $A + Be + Ce^2 + De^3 + \ldots He^n = 0$ erunt approximationes ad duas vel denique n radices æquationum, quæ ſunt prope inter ſe æquales.

THEOR. XIV.

1. Datâ æquatione algebraicâ $x^n - px^{n-1} + qx^{n-2} - rx^{n-3} + sx^{n-4} - \&c. = 0$, cujus una radix α multo major ſit quam quæcunque alia, & erit ea radix $p - p^{-1}q + p^{-2}r \left.\begin{matrix}-s\\-q^2\end{matrix}\right\}p^{-3} + \&c.$ cujus ſeriei lex eadem erit ac ea, quæ traditur pro inveniendâ ſummâ $\alpha^m + \beta^m + \gamma^m + \delta^m + \&c.$ ubi $\alpha, \beta, \gamma, \delta, \&c.$ ſunt radices datæ æquationis vel in meis miſcell. analyt. vel poſtea in medit. algeb. ſi modo pro m ſcribatur 1, & termini in infinitum progrediantur; i. e. coefficiens generalis termini $p^{m-l}q^{\alpha}r^{\beta}s^{\gamma}\,\&c.$ ubi $l = 2\alpha + 3\beta + 4\gamma + \&c.$ erit $\pm m\,.$

$$\frac{m-l+1\,.\,m-l+2\,.\,m-l+3\,.\,m-l+4\ldots m-l+\alpha+\beta+\gamma+\&c.-1}{1\,.\,2\,.\,3\,..\,\alpha\times1\,.\,2\,.\,3\,..\,\beta\times1\,.\,2\,.\,3\,..\,\gamma\times\&c.};$$

ſignum affixum erit + vel —, ſi l ſit impar numerus, prout $\alpha + \beta + \gamma + \&c.$ ſit impar vel par numerus, &c.; ſed in hoc caſu $m = 1$, ergo erit prædicta coefficiens $\pm \dfrac{(2-l)\,.\,(3-l)\ldots(\alpha+\beta+\gamma+\&c.-l)}{1\,.\,2\,.\,3\,..\,\alpha\times1\,.\,2\,.\,3\,..\,\beta\times1\,.\,2\,.\,3\,..\,\gamma\times\&c.}$.

2. Sit æquatio $P - Qx + Rx^2 - Sx^3 + Tx^4 - \&c. = 0$, cujus una radix π ſit multo minor quam quæcunque alia: ſcribantur $\frac{1}{x} = v$, $\frac{Q}{P} = p$, $\frac{R}{P} = q$, $\frac{S}{P} = r$, $\frac{T}{P} = s$, &c. & reſultat æquatio $v^n - pv^{n-1} + qv^{n-2} - rv^{n-3} + \&c. = 0$, cujus maxima radix $\frac{1}{\pi}$ erit $p -$

$\left.\begin{matrix} p^{-1}q + p^{-2}r - p^{-3}s \\ -\; q^2 \end{matrix}\right\} + \&c.$ ut antea; ejus vero reciproca, i. e. radix quæſita (π) erit $p^{-1} + p^{-3}q - p^{-4}r + \left.\begin{matrix} s \\ +2q^2 \end{matrix}\right\} p^{-5} - \&c.$ ſeries prius tradita eadem evadet ac hæc, ſi modo in eâ pro m ſcribatur -1 & termini in infinitum progrediantur: hic animadvertendum eſt ultimo duos primos terminos recte oſtendere duplum figurarum numerum, tres vero primos terminos triplum figurarum numerum præbere, &c.

2. 1. Sit æquatio $x^n - px^{n-1} + qx^{n-2} + \&c. = 0$, cujus radix α multo major ſit quam quæcunque alia, tum erit $\alpha^\lambda = p^\lambda - \lambda p^{\lambda-2}q + \left.\begin{matrix} \lambda p^{\lambda-3}r - \lambda p^{\lambda-4}s \\ +\lambda.\frac{\lambda-3}{2}p^{\lambda-4}q^2 \end{matrix}\right\} + \&c.$ ſi in prædictâ ſerie pro m ſcribatur λ, & termini in infinitum progrediantur, tum eadem evadet ac ſeries prius tradita.

2. Sit æquatio $P - Qx + Rx^2 - \&c. = 0$, cujus radix π multo minor ſit quam quæcunque alia in eâ contenta, transformetur hæc æquatio in alteram $v^n - pv^{n-1} + qv^{n-2} - \&c. = 0$ cujus radix $v = \frac{1}{x}$; & erit $\frac{1}{\pi^\lambda} = p^\lambda - \lambda p^{\lambda-2} + \&c.$ ut antea, & $\pi^\lambda = p^{-\lambda} + \lambda p^{-\lambda-2}q - \left.\begin{matrix} \lambda p^{-\lambda-3}r + \lambda p^{-\lambda-4}s \\ -\lambda.-\frac{\lambda+3}{2}p^{-\lambda-4}q^2 \end{matrix}\right\} - \&c.$ quæ ſeries eadem erit ac ſeries reſultans e ſcribendo $-\lambda$ pro m in ſerie generali prius traditâ.

Cor. 1. Termini omnium ſerierum prædictarum, in quibus ſolummodo continentur coefficientes p & q, erunt æquales quantitati $\left(\frac{p + \sqrt{(p^2 - 4q)}}{2}\right)^{\pm\lambda}$; termini vero in quibus continentur coefficientes p, q & r erunt æquales quantitati $H^{\pm\lambda}$, ſi modo H ſit maxima radix cubicæ æquationis $x^3 - px^2 + qx - r = 0$; & ſic deinceps.

Cor. 2. Sint $\frac{p^n - p \cdot p^{n-1} + qp^{n-2} - rp^{n-3} + \&c.}{np^{n-1} - (n-1)p \cdot p^{n-2} + (n-2)qp^{n-3} - \&c.} = \alpha,$

$$\frac{\pi^n - np\pi^{n-1} + (n-1)q\pi^{n-2} - (n-2)r\pi^{n-3} + \&c.}{n\pi^{n-1} - (n-1)q\pi^{n-2} + (n-2)q\pi^{n-3} - \&c.} = \beta,$$

$$\frac{\varrho^n - np\varrho^{n-1} + (n-1)q\varrho^{n-2} - (n-2)r\varrho^{n-3} + \&c.}{n\varrho^{n-1} - (n-1)p\varrho^{n-2} + (n-2)q\varrho^{n-3} - \&c.} = \gamma,$$

$$\frac{\sigma^n - np\sigma^{n-1} + (n-1)q\sigma^{n-2} - (n-2)r\sigma^{n-3} + \&c.}{n\sigma^{n-1} - (n-1)p\sigma^{n-2} + (n-2)q\sigma^{n-3} - \&c.} = \delta, \&c.$$

in infinitum, ubi $p - \alpha = \pi$, $p - \alpha - \beta = \varrho$, $p - \alpha - \beta - \gamma = \sigma$, &c. tum quantitas ultimo reſultans, i. e. ad infinitam diſtantiam poſita, ſi modo reducatur in terminos ſecundum dimenſiones quantitatum $p, q, r, s,$ &c. progredientes, evadet $p - p^{-1}q + p^{-2}r \left.\begin{matrix} - p^{-3}s \\ - q^2 \end{matrix}\right\} + \&c.$ ſeries prius tradita.

Ex principiis in medit. algebr. ulterius promotis invenire liceat radicem, quæ multo minor ſit quam maxima radix; at multo major quam quæcunque alia; & ſic deinceps; ſed omnia hæc e principiis ſubſequentibus per ſe perſpicuis reſolutionem recipiant.

PROB. IX.

1. *Sit æquatio* $x^n - px^{n-1} + qx^{n-2} - rx^{n-3} \ldots \pm gx^{n-m+1} \mp hx^{n-m} \pm ix^{n-m-1} \mp kx^{n-m-2} \pm \&c. = 0$, *cujus radix* α *ſit multo major quam quæcunque alia* ($\beta, \gamma, \delta,$ &c.) *in datâ æquatione contenta, invenire radicem* α.

Per vulgarem algebram innoteſcunt $p = \alpha + (\beta + \gamma + \delta + \&c.)$, $q = \alpha(\beta + \gamma + \delta + \&c.) + (\beta\gamma + \beta\delta + \gamma\delta + \&c.)$, $r = \alpha(\beta\gamma + \beta\delta + \gamma\delta + \&c.) + (\beta\gamma\delta + \beta\gamma\varepsilon + \delta\beta\varepsilon + \&c.)$, $s = \alpha(\beta\gamma\delta + \&c.) + (\beta\gamma\delta\varepsilon + \&c.)$ &c. unde erit $\alpha = p - (\beta + \gamma + \delta + \&c.)$, ſed $\beta + \gamma + \delta + \&c. = \frac{q}{p} + \frac{(\beta\gamma + \beta\delta + \gamma\delta + \&c.) + (\beta^2 + \gamma^2 + \delta^2 + \&c.)}{p}$, at $(\beta^2 + \gamma^2 + \delta^2 + \&c.) + 2(\beta\gamma + \beta\delta + \gamma\delta + \&c.) =$

$$\frac{q^2 + 2\alpha(S^3 + 2S^2S^1 + 3S^1S^1S^1) + S^4 + 4S^3S^1 + 5S^2S^2 + 10S^2S^1S^1 + 18S^1S^1S^1S^1}{p^2},$$

(ubi deſigno $S^3 = \beta^3 + \gamma^3 + \delta^3 + \&c.$ $S^4 = \beta^4 + \gamma^4 + \delta^4 + \&c.$ $S^2S^1 = \beta^2\gamma + \beta^2\delta + \gamma^2\delta + \&c.$ $S^1S^1S^1 = \beta\gamma\delta + \beta\gamma\varepsilon + \beta\delta\varepsilon + \&c.$ $S^3S^1 = \beta^3\gamma + \beta^3\delta + \gamma^3\beta + \gamma^3\delta + \&c.$ $S^2S^2 = \beta^2\gamma^2 + \beta^2\delta^2 + \gamma^2\delta^2 + \&c.$ $S^2S^1S^1 = \beta^2\gamma\delta + \gamma^2\beta\delta + \delta^2\beta\gamma + \&c.$ quæ fere eadem eſt notatio, quâ uſus fui in miſcell. analyt. anno 1762 edit.) & $\beta\gamma + \beta\delta + \gamma\delta + \&c. = \frac{r + S^2S^1 + 2S^1S^1S^1}{p}$; & ſic deinceps; unde conſtat $\alpha = p - \frac{q}{p} - \frac{q^2}{p^3} + \frac{r}{p^2} - \&c.$ eædem ſeriei, quæ prius inventa fuit.

2. Sit δ radix prædictæ æquationis, quæ multo minor ſit quam m radices datæ æquationis, multo vero major quam reliquæ $n-m-1$; tum ex principiis hic traditis deduci poteſt $\delta = \frac{i}{h} - \left(\frac{k}{i} - \frac{gi^2}{h^3}\right) + \&c.$ termini, in quibus ſolummodo continentur coefficientes terminorum (x^{n-1}, x^{n-2}, x^{n-3}, ... x^{n-m-1}) obſervant legem in caſ. 1. prob. præced. traditam, cæterique legem haud multum diverſam habent, mutatis indicibus de affirmativis in negativos, & mutato denominatore de h & ejus poteſtatibus in i & ejus poteſtates.

Ex iiſdem principiis etiam detegi poſſunt approximationes ad duas vel plures radices datæ æquationis, quæ fere ſunt inter ſe æquales; etiamque eadem applicari poſſunt principia ad æquationes duas incognitas quantitates habentes, ita ut una exprimatur in terminis alterius; etiamque ad quaſcunque æquationes, quæ reduci poſſunt ad infinitas algebraicas æquationes.

Exhinc etiam conſtabunt leges, quas obſervant infinitæ ſeries radices æquationum algebraicarum exprimentes.

PROB.

PROB. X.

Datâ algebraicâ æquatione $x^n - px^{n-1} + qx^{n-2} - rx^{n-3} + \&c. = 0$, *cujus radix eſt* x, *invenire valorem quantitatis* x.

1mo. Sint omnes radices $\alpha, \beta, \gamma, \delta, \varepsilon$, &c. datæ æquationis poſſibiles; ſint vero α multo major quam β, β quam γ, γ quam δ, &c. & inveniatur per algebr. meditat. ſumma $\alpha^{2m} + \beta^{2m} + \gamma^{2m} + \delta^{2m} + \&c. = A^{2m}$; & α erit ad A in æquali vel majore ratione quam habet $1 : \sqrt[2m]{(n)}$. Facile conſtat ex ſupponendo omnes radices inter ſe eſſe æquales.

2do. Inveniatur ſumma rectangulorum $\alpha^{2m}\beta^{2m} + \alpha^{2m}\gamma^{2m} + \beta^{2m}\gamma^{2m} + \alpha^{2m}\delta^{2m} + \beta^{2m}\delta^{2m} + \gamma^{2m}\delta^{2m} + \alpha^{2m}\varepsilon^{2m} + \&c. = A'^{2m}$ per prædictas meditationes; & $\alpha\beta$ erit ad A' in æquali vel majore ratione quam $1 : \sqrt[2m]{\left(n \cdot \frac{n-1}{2}\right)}$.

3^{tio}. Ex quam plurimis directis functionibus ſingularum radicum inveniri poteſt maxima radix (α); & ex iis & quam plurimis directis functionibus productorum ſub quibuſque duabus radicibus in directam functionem ſingularum radicum erui poteſt radix β, quæ major eſt quam omnes radices præter α; & ſic progredi licet ad invenienda contenta $\alpha\beta\gamma$, $\alpha\beta\gamma\delta$, &c. etiamque quaſcunque algebraicas earundem radicum functiones.

Cor. 1. Iiſdem poſitis, $\sqrt[2n]{(\alpha^{2n} + \beta^{2n} + \gamma^{2n} + \delta^{2n} + \&c.)}$ nunquam differt a maxima radice α per quintam ejus partem; ni ſit $n = 3$, in quo caſu vix differt per majorem partem: & $\sqrt[4n]{(\alpha^{4n} + \beta^{4n} + \gamma^{4n} + \&c.)}$ nunquam differt a maximâ radice α per decuplam ejus partem, &c. hic haud ſupponitur $n = 2$.

Cor. 2. Limites, inter quos conſiſtit vera radix, inveniri poſſunt ſupponendo 1mo omnes vel quaſdam radices eſſe æquales, & 2do omnes præter quemdam numerum radicum evaneſcere.

Cor. 3. Sit æquatio $x^n - px^{n-1} + qx^{n-2} - \&c. = 0$, cujus omnes radices $\alpha, \beta, \gamma, \delta$, &c. ſint poſſibiles, quarum α major ſit quam β, β

quam γ, γ quam δ, &c.; invenire quantitatem, quæ habet ad radicem α minorem quam datam rationem, viz. $r:1$, ubi r major eſt quam 1: inveniantur logarithmi quantitatum n & r, qui dicantur ν & ϱ, & par numerus major quam $\frac{\nu}{\varrho}$ dicatur σ; tum erit $(\alpha^{\sigma}+\beta^{\sigma}+\gamma^{\sigma}+\delta^{\sigma}+\&c.)^{\frac{1}{\sigma}}$ quantitas quæſita. Hæc principia etiam applicari poſſunt ad inveniendas approximationes intra datos limites conſtitutas ad radices β, γ, &c., vel ad minimam radicem.

Cor. 4. Hinc e prob.   primo meditat. algebr. conſequitur lex, quam obſervat reverſio ſeriei $x=ay-by^2+cy^3-dy^4\ldots y^m$, i. e. ſeries exinde deducta $y=Ax+Bx^2+Cx^3+Dx^4+\&c.$ Inveniatur enim æquatio, cujus radices ſunt reciprocæ datæ æquationis radicum, quæ erit $v^m-\frac{a}{x}v^{m-1}+\frac{b}{x}v^{m-2}-\frac{c}{x}v^{m-3}+\frac{d}{x}v^{m-4}-\&c.=0$ ubi $v=\frac{1}{y}$; ſint $\alpha, \beta, \gamma, \delta, \ldots \pi$ radices hujuſce æquationis, ubi α ſit maxima radix & π minima, maximus igitur erit valor radicis $y=\frac{1}{\pi}$; tum $\sqrt[2n]{(\alpha^{2n}+\beta^{2n}+\gamma^{2n}+\&c.)}$ erit maxima radix α, ſi modo n ſit infinitus affirmativus numerus, etiamque $\sqrt[2n]{(\alpha^{-2n}+\beta^{-2n}+\gamma^{-2n}+\&c.)}$ erit minima radix π, & conſequenter maxima radix quantitatis $y=\frac{1}{\pi}$; ſed quoniam radices, ut conſtat ex obſervatione ad problema prædictum annexâ, eandem prorſus obſervant legem ac earum poteſtates; ſeries, quæ exprimit radicem $\pi=\sqrt[2n]{(\alpha^{-2n}+\beta^{-2n}+\gamma^{-2n}+\&c.)}$ eandem prorſus obſervat legem ac ſeries pro ſumma $(\alpha^{-1}+\beta^{-1}+\gamma^{-1}+\&c.)$ cum radix ſit $\frac{-2n}{2n}=-1$, & conſequenter ſeries erit per prob.   prædict. $p^{-1}+p^{-1-2}q-p^{-1-3}r+\begin{matrix}p^{-1-4}s\\+2p^{-1-4}q^2\end{matrix}-\&c.$ ſi modo æquatio ſit $z^m-pz^{m-1}+qz^{m-2}-\&c.=0$; hoc autem in loco erunt reſpective $p = \frac{a}{x}$, $q = \frac{b}{x}$, $r = \frac{c}{x}$, &c. in caſu propoſito requiratur, ut termini progrediantur ſecundum dimenſiones quantitatis x, unde in terminis $p^{-1-\mu} q^{\alpha} r^{\beta} s^{\gamma} \times$ &c. infra ſe poſitis, qui in eandem poteſtatem quantitatis x ducuntur, quantitas $-1-\mu+\alpha+\beta+\gamma+$ &c. $=$ $-1-\alpha-2\beta-3\gamma-$ &c. eandem conficiet ſummam, quæ erit ſucceſſive -1, -2, -3, -4, -5, &c. ubi $\mu = 2\alpha+3\beta+4\gamma$ $+$ &c. & conſequenter ſeries per prædictum problema invenietur $y = \frac{x}{a} + \frac{bx^2}{a^3} + \frac{2b^2-ca}{a^5}x^3 +$ &c. unde e prædicto problemate conſtat lex, quam obſervat hæc ſeries; exinde enim conſtat coefficientem termini $p^{-1-\mu} q^{\alpha} r^{\beta} s^{\gamma}$, &c. eſſe $\pm \frac{\mu \times (\mu-1)(\mu-2)(\mu-3)\ldots(\mu-\alpha-\beta-\gamma-\&c.+2)}{1.2.3..\alpha \times 1.2.3..\beta \times 1.2.3..\gamma \times \&c.}$: ſignum affixum erit $+$ vel $-$ prout $2\alpha + 3\beta + 4\gamma +$ &c. ſit par vel impar numerus; ſi $\alpha+\beta+\gamma+$ &c. $=1$, i. e. una litera α vel β vel γ, &c. ſit 1, cæteræ vero 0; tum coefficiens prædicta erit 1.

Et ſic, ſi modo deficiant quidam datæ ſeriei termini vel ay vel by^2, &c. facile deduci poteſt lex, quam obſervat ſeries valorem radicis π exprimens.

3. Summa $\alpha^{4m} + \beta^{4m} + \gamma^{4m} +$ &c. per ſummam $\alpha^{2m} + \beta^{2m} + \gamma^{2m} +$ &c. diviſa erit approximatio ad α^{2m}. Et ſic de ſummis quantitatum rationalium & irrationalium; reducantur rationales vel irrationales quantitates in ſeries, quarum primi termini ſint maximi, & exinde ſumma e ſingulis valoribus rationalium vel irrationalium quantitatum prope æqualis erit prædictis terminis; &c.

4. Habeat prædicta æquatio radices impoſſibiles, & e præcedente methodo haud generaliter inveſtigari poteſt maxima radix: pendet enim ex hoc, utrum impoſſibilis radix major ſit quam reliquæ radices, necne; i. e. impoſſibilis vel poſſibilis pars radicis impoſſibilis multo major ſit quam reliquæ radices, necne; quâ inventâ, i. e. datâ

approxima-

approximatione ad impoſſibilem vel poſſibilem partem radicis deduci poteſt approximatio ad alteram partem.

In quibuſcunque datis æquationibus ſuperiorum dimenſionum, quarum radices haud dantur, maxime probabile eſt multo majorem eſſe numerum impoſſibilium quam poſſibilium radicum, & conſequenter caſus non detegendi poſſibilem radicem per hanc regulam multo majorem eſſe quam caſus detegendi. Transformetur data æquatio in alteram, cujus radices ſint reciprocæ radicum datâ æquatione contentarum, & minima radix datæ æquationis vel negativa vel affirmativa præbet maximam reſultantis æquationis radicem; de eâ conſimiliter ratiocinari liceat.

5. Sæpe approximationes ad valores radicum inveſtigari poſſunt e comparandis diverſis æquationis terminis inter ſe, cum quidam termini æquationis multo majores eſſe ſupponantur quam reliqui.

Et conſimili methodo inveſtigari poſſunt approximationes ad quaſcunque algebraicas functiones datæ æquationis radicum: per meditat. algebr. enim inveniri poteſt aggregatum e ſingulis valoribus datæ functionis, ex ſingulis eorum quadratis, cubis, quadratoquadratis, quadratis ex uno quocunque valore in quadrata e ſingulis reliquis ductis, &c. unde per hoc problema inveniri poteſt approximatio ad valorem vel diverſos valores datæ functionis; vel quod idem eſt, ſemper deduci poteſt approximatio prædicta ex transformatione datæ æquationis in alteram, cujus radices ſunt data functio datæ æquationis radicum.

PROB. XI.

Conceſſâ methodo detegendi radices $\sqrt[2n]{}(-1)$, *invenire radices* n *poteſtatis impoſſibilis quantitatis* $a + b\sqrt{}(-1)$.

1. Sit *a* multo major quam *b*, tum ex vulgari extractione radicis *n* poteſtatis ſemper inveniri poteſt radix quæſita; ſi vero *a* ſit negativa quantitas, ducatur data quantitas $-a+b\sqrt{}(-1)$ in -1, & reſultat $(a-b\sqrt{}(-1))$; per prædictam & conceſſam methodos inveniantur radices

$\sqrt[m]{}(a-$

$\sqrt[n]{(a-b\sqrt{(-1)})}$ & $\sqrt[n]{(-1)}$, & e dividendo has radices per ſeſe conſequitur radix quæſita.

2. Sit b multo major quam a, ducatur data quantitas in $-\sqrt{(-1)}$, & reſultat $(b-a\sqrt{(-1)})$; ſed radices $\sqrt[n]{(b-a\sqrt{(-1)})}$ & $\sqrt[n]{(-\sqrt{(-1)})}=\sqrt[2n]{(-1)}$ e prædictis methodis erui poſſunt, ergo radix quæſita.

3. Si vero a & b ſint prope inter ſe æquales, tum forſan ab extractione $2n$ radicis ejus quadrati $a^2-b^2+2ab\sqrt{(-1)}$ in $-\sqrt{(-1)}$ $=2ab+(b^2-a^2)\sqrt{(-1)}$; vel ab extractione radicum ex aliis poteſtatibus erui poteſt radix quæſita; aliter vero in hoc caſu fingatur $\alpha\times(1\pm\sqrt{(-1)})$ radix quæſita, & conſequenter erit $(1-n.\frac{n-1}{2}+n.\frac{n-1}{2}.\frac{n-2}{3}.\frac{n-3}{4}-n.\frac{n-1}{2}.\frac{n-2}{3}.\frac{n-3}{4}.\frac{n-4}{5}.\frac{n-5}{6}+\&c.)\,\alpha^n=M\alpha^n=a'$, & erit $\alpha=\left(\frac{d'}{M}\right)^{\frac{1}{n}}=\beta$ prope; ſcribatur in æquatione $x^n-a-b\sqrt{(-1)}=0$ pro radice quæſitâ $\beta+\pi+(\beta'+\rho)\sqrt{(-1)}$, ubi β' quantitas aſſumpta paululum differt a quantitate β; & ex maximis reſultantis æquationis terminis, in quibus invenitur $\sqrt{(-1)}$, necne, ſeorſim nihilo æqualibus eſſe ſuppoſitis; reſultant duæ æquationes, e quibus deduci poſſunt π & ρ; & ſic deinceps.

Hoc in loco viſum eſt ſubſequentia ſubjicere hanc rem de impoſſibilibus radicibus leviter perſtringentia, viz. 1. erit $(a+\sqrt{(-a^2)})^{4n}+(a-\sqrt{(-a^2)})^{4n}=2\times(-4)^n a^{4n}$, $(a+\sqrt{(-a^2)})^{4n+2}+(a-\sqrt{(-a^2)})^{4n+2}=0$, & $(a+\sqrt{(-a^2)})^{2n+1}+(a-\sqrt{(-a^2)})^{2n+1}=\pm 2^{n+1}a^{2n+1}$, ubi litera n integrum denotat numerum; & ſignum quantitati $2^{n+1}a^{2n+1}$ affixum eſt affirmativum necne, prout $2n+1$ vel æquat $8m+1$, vel $8m+7$; necne; ubi m eſt integer numerus.

2. Quantitas $n.\frac{n-1}{2}\ldots\frac{n-r}{r+1}-n.n.\frac{n-1}{2}.\frac{n-2}{3}\ldots\frac{n-r+1}{r}+n.\frac{n-1}{2}.n.\frac{n-1}{2}.\frac{n-r+2}{r-1}-\ldots\pm n.\frac{n-1}{2}.\frac{n-2}{3}\ldots\frac{n-r}{r+1}$

æquat $\pm n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3} \ldots \frac{n-\frac{r-1}{2}}{\frac{r+1}{2}}$, vel o; erit $= o$, cum r ſit par numerus: ſignum affirmativum erit affixum, cum $r = 4s + 3$; ſin aliter negativum; ubi s eſt integer numerus.

PROB. XII.

Datâ algebraicâ æquatione relationem inter x & y *exprimente; invenire legem, quam obſervat ſeries deſignans quantitatem* y *in ſimplicibus terminis quantitatis* x; *ſi modo dentur leges, quam obſervant quantitates hujuſce generis ad quaſcunque poteſtates elevatæ vel in ſeſe ductæ.*

Ita reducatur data algebraica æquatio relationem inter x & y exprimens, ut π functio quantitatis x multo magis appropinquet ad unam quam ad ullam aliam radicem vel valorem quantitatis y; ſcribatur $v - \pi$ pro y in datâ æquatione & ſit æquatio reſultans $\alpha - \beta v + \gamma v^2 - d v^3 + \&c. = o$; deinde fingatur $w = \frac{1}{v}$, & erit $\alpha w^n - \beta w^{n-1} + \gamma w^{n-2} - \delta w^{n-3} + \&c. = a$, tum erit $\frac{\beta}{\alpha}$ approximatio ad radicem w, nunc per prob. 1. meditationum algebraicarum infinita ſeries, quæ erit ſumma poteſtatis vel potius radix $\left(\frac{1}{n}\right)$ infinitæ poteſtatis (n) e ſingulis radicibus, i. e. pro m ſcribatur 1; & pro $\frac{\beta}{\alpha}, \frac{\gamma}{\alpha}, \frac{\delta}{\alpha}$, &c. ſcribantur reſpective p, q, r, s, &c. erit $p - p^{1-2} q + p^{1-3} r - p^{1-4} s + \&c.$; coefficiens termini $p^{1-\mu} q^{\alpha} r^{\beta} s^{\gamma}$, &c. erit $\pm \frac{(\mu-2) \times (\mu-3) \times (\mu-4) \ldots (\mu-\alpha-\beta-\gamma-\&c.)}{1.2.3\ldots\alpha \times 1.2\ldots\beta \times 1.2\ldots\gamma \times \&c.}$, ſignum affixum erit + vel —, prout $\mu + \alpha + \beta + \gamma + \&c.$ ſit par vel impar numerus; & reſultantis infinitæ ſeriei collocentur termini ſecundum dimenſiones quantitatis x, vel potius ſecundum dimenſiones perparvæ quantitatis, & erit ſeries quæ exprimit unam radicem vel valorem quantitatis y.

Cor. 1. Ex primo, binis, ternis vel ſaltem ex n, qui ſit numerus dimenſionum datæ æquationis, maximis terminis, pro approximatione ad valorem quantitatis y inventis ſemper acquiri poteſt quantitas π prædicta.

Cor. 2. Si requiratur radix prædicta in terminis ſecundum legem $ax^r \times (x \pm \underset{\cdot}{x})^r + bx^{r+1} \times (x \pm \underset{\cdot}{x})^{r+1} + \&c. + a'x^{r+s} \times (x \pm \underset{\cdot}{x})^{r+s} + b'x^{r+s+1} \times (x \pm \underset{\cdot}{x})^{r+s+1} + \&c.$ progredientibus; reducatur ſeries, quæ exprimit radicem in terminis ejuſdem formulæ, & exinde facile deduci poteſt per hoc problema ſeries quæſita; & ſic inveniri poteſt radix prædicta in terminis progredientibus ſecundum alias leges in hoc libro traditas; aliter e terminis maximis continuo deduci poſſunt approximationes quæſitæ.

THEOR. XV.

Sit æquatio $1 - (\alpha + \beta + \gamma + \delta + \&c.)\frac{1}{x} + (\alpha\beta + \alpha\gamma + \beta\gamma + \&c.)\frac{1}{x^2} - (\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta + \&c.)\frac{1}{x^3} + (\alpha\beta\gamma\delta + \alpha\beta\gamma\epsilon + \&c.)\frac{1}{x^4} - \&c. = A$; i. e. ſit $A = 0$ æquatio, cujus radices ſint $\alpha, \beta, \gamma, \delta, \epsilon$, &c.; tum erit $A^n = 1 - n(\alpha + \beta + \gamma + \delta + \&c.)\frac{1}{x} + (n . \frac{n-1}{2}(\alpha^2 + \beta^2 + \gamma^2 + \delta^2 + \&c.) + n \times n(\alpha\beta + \alpha\gamma + \beta\gamma + \alpha\delta + \&c.))\frac{1}{x^2} - ((n . \frac{n-1}{2} . \frac{n-2}{3}(\alpha^3 + \beta^3 \pm \chi^3 \pm \delta^3 + \&c.) + n . n . \frac{n-1}{2}(\alpha^2\beta + \alpha^2\gamma + \beta^2\alpha + \beta^2\gamma + \&c.) + n . n . n(\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta + \&c.))\frac{1}{x^3} + \&c.$: in genere ſit $\alpha^m\beta^r\gamma^s\delta^t$, &c. quodcunque productum in coefficiente ad terminum (x^{-b}) hujuſe ſeriei annexã, tum erit $m + r + s + t + \&c. = b$; cuncta etiam producta, in quibus $m + r + s + t + \&c. = b$ ſemper prædicto termino x^{-b} ſunt adjungenda; coefficiens prædicti termini $\alpha^m \beta^r \gamma^s \delta^t$, &c. erit $n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3} \ldots \frac{n-m+1}{m} \times n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3} \ldots \frac{n-r+1}{r} \times n \cdot \frac{n-1}{2} \ldots \frac{n-s+1}{s} \times n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3} \ldots \frac{n-t+1}{t} \times \&c.$

Cor. 1. Sit $n = 1$, & omnes prædictæ coefficientes erunt 1, i. e. dividatur unitas per A, & coefficientes e ſingulis productis ($\alpha^m \beta^r \gamma^s \delta^t$, &c.) in terminis reſultantibus erunt 1.

Cor. 2. Sit α maxima radix, β multo major quam quæcunque alia, & ſic de reliquis (γ, δ, &c.), tum ita reducantur hæ coefficientes, ut exterminentur omnes termini, in quibus invenitur α ſolummodo; & reſultent termini, e quibus deduci poteſt propinquus valor quantitatis β; deinde exterminentur omnes termini in quibus ſolummodo inveniuntur α & β, vel termini maximi in quibus inveniuntur α & β, & erui poteſt propinquus valor quantitatis γ; & ſic deinceps: e comparandis terminis reſultantibus quam plurimarum functionum datæ æquationis erui poſſunt quam proximi valores radicum datæ æquationis.

Cor. 3. Eadem principia etiam applicari poſſunt ad æquationes irrationales quantitates involventes.

THEOR. XVI.

1. Sit æquatio $x^n - p x^{n-1} + q x^{n-2} - r x^{n-3} + \&c. = 0$, ſtatuatur $\frac{1}{x^n - p x^{n-1} + \&c.} = x^{-n} + p x^{-n-1} \ldots a x^m + b x^{m-1} + c x^{m-2} + d x^{m-3} + e x^{m-4} + f x^{m-5} + \&c.$ tum, ſi modo omnes radices (α, β, γ, δ, &c.) datæ æquationis ſint poſſibiles, & m ſit permagna quantitas, erit $\frac{b}{a} = (\alpha)$

maximæ radici datæ æquationis quam proxime: maximam radicem volo vel affirmativam vel negativam.

2. Et $\frac{bd-c^2}{ad-bc}$ erit quam proxime (β) maxima radix ex omnibus reliquis præter prædictam α.

3. Si vero numerator haud ſit unitas, ſed quantitas $bx^{\mu} + kx^{\mu-1} + lx^{\mu-2} + \&c.$ i. e. ſtatuatur $\frac{bx^{\mu} + kx^{\mu-1} + \&c.}{x^n - px^{n-1} + \&c.} = bx^{\mu-n} + (k+bp)x^{\mu-n-1} \ldots ax^m + bx^{m-1} + cx^{m-2} + \&c.$ & ex eâdem prorſus methodo inveniri poſſunt proximi valores radicum α & β, &c.

4. Si vero maxima affirmativa & negativa radix ſint quam proxime inter ſe æquales; in quo caſu ſit $n-m$ par numerus, & erit $\sqrt{\left(\frac{c}{a}\right)}$ vel $\sqrt{\left(\frac{d}{b}\right)}$ maxima radix (α) datæ æquationis prope; & $\left(\frac{d}{c}\right)$ maxima ex omnibus reliquis prope.

5. Sit $(x^n - px^{n-1} + qx^{n-2} - \&c.)^{-1} = x^{-n} + px^{-n-1} \ldots + ax^{-n-m} + bx^{-n-m-1} \ldots + dx^{-n-r} + ex^{-n-r-1} \ldots + gx^{-n-s} + hx^{-n-s-1} \ldots + kx^{-n-t} + lx^{-n-t-1} \ldots + \&c.$; tum erit $\overline{\frac{d}{a}}\Big|^{\frac{1}{r-m}} = \alpha$ maxima radix prædicta datæ æquationis; ſi modo m & r ſint permagnæ quantitates, & omnes radices ſint poſſibiles.

Et ex iiſdem principiis ulterius promotis quam plurimæ aliæ regulæ pro his inveniendis, etiamque conſimiles regulæ pro reliquis radicibus inveniendis deduci poſſunt.

6. Sit fractio $\frac{A}{B \times C}$ ad minimos terminos reducta, ubi B ſit rationalis functio quantitatis x, C vero irrationalis nullum habens rationalem factorem; reducatur fractio $\frac{A}{BC}$ in ſimplices terminos ſecundum dimenſiones quantitatis x progredientes, viz. $\alpha x^{\pi} + \beta x^{\pi+\rho} + \gamma x^{\pi+2\rho}$

$\gamma x^{\pi+2\rho} \ldots, a x^{\pi} + b x^{\pi+\rho} + c x^{\pi+2\rho} + \&c.$, & erit $\sqrt[\rho]{\left(\frac{a}{b}\right)}$ vel $\sqrt[\rho]{\left(\frac{b}{a}\right)}$ maxima radix æquationis $B \times C = 0$ prope: ſecunda radix, quæ minor erit quam prædicta, major vero quam ſingulæ reliquæ, erit $\sqrt[\rho]{\left(\frac{bd - c^2}{ad - bc}\right)}$, &c. prope; vel per ſingulas regulas prius traditas deduci poteſt.

Et ſic de reliquis radicibus detegendis.

Hæc principia ulterius etiam promovere liceat.

Si vero impoſſibiles radices $\pi + \sqrt{(-\rho^2)}$ in datâ æquatione contineantur, hæ regulæ ſemper detegent radicem quæſitam α vel β, cum radix quæſita major ſit quam $\pi + \rho$ vel $\pi - \rho$.

Ex hâc methodo ſæpe conſtabit approximatio ad maximam impoſſibilem radicem; & ex his regulis plerumque deduci poſſunt approximationes vel ad maximas poſſibiles vel impoſſibiles radices.

PROB. XIII.

Datâ algebraicâ æquatione $x^n - p x^{n-1} + q x^{n-2} - r x^{n-3} + \&c. = 0$, *invenire ejus radicis* (α) *valorem.*

Aſſumatur a quantitas ab radice α datæ æquationis parum diſcrepans; pro radice x datæ æquationis ſcribatur $a + b$, & reſultat æquatio $x^n - p x^{n-1} + q x^{n-2} - \&c. = 0 = (a^n - p a^{n-1} + q a^{n-2} - \&c.) + (n a^{n-1} - (n-1) p a^{n-2} + (n-2) q a^{n-3} - \&c.) b + (n . \frac{n-1}{2} a^{n-2} - (n-1) \times \frac{n-2}{2} p a^{n-3} + (n-2) \times \frac{n-3}{2} q a^{n-4} - \&c.) b^2 + \&c.$ fingatur quantitas b multo minor quam quantitas a, & neglectis quantitatibus in quibus inveniuntur ullæ dimenſiones quantitatis b, (ſimplice exceptâ) ob earum parvitatem reſultat ſimplex æquatio $a^n - p a^{n-1} + q a^{n-2} - \&c. + (n a^{n-1} - (n-1) p a^{n-2} + (n-2) q a^{n-3} - \&c.) b = 0$, unde b erit prope $= - \frac{a^n - p a^{n-1} + q a^{n-2} - \&c.}{n a^{n-1} - (n-1) p a^{n-2} + (n-2) q a^{n-3} - \&c.} = \pi$;

$= \pi$; in reſultante æquatione pro quantitate b ſcribatur $\pi + c$, & per eandem methodum repetitam inveniatur valor quantitatis c prope; & ſic deinceps.

2. Eædem omninò erunt approximationes inventæ, ſi modo in datâ æquatione 1^mo^ ut in præcedente caſu ſcribatur $a + b$ pro x, deinde per prædictam methodum inveniatur π valor quantitatis b prope; tum in datâ æquatione pro x ſcribatur $a + \pi + c$, & per methodum prius traditam inveniatur ρ valor quantitatis c prope; deinde ſcribatur in datâ æquatione pro x quantitas $a + \pi + \rho + d$ & per prædictam methodum inveniatur σ valor quantitatis d prope; & ſic deinceps. Hinc valor incognitæ quantitatis x per hanc methodum inventus erit

$$a - \frac{a^n - p a^{n-1} + q a^{n-2} - \&c.}{n a^{n-1} - (n-1) p a^{n-2} + \&c.} - \frac{(a+\pi)^n - p(a+\pi)^{n-1} + q(a+\pi)^{n-2} - \&c.}{n(a+\pi)^{n-1} - (n-1)p(a+\pi)^{n-2} + (n-2)q(a+\pi)^{n-3} - \&c.} - \frac{(a+\pi+\rho)^n - p(a+\pi+\rho)^{n-1} + q(a+\pi+\rho)^{n-2} - \&c.}{n(a+\pi+\rho)^{n-1} - (n-1)p(a+\pi+\rho)^{n-2} + (n-2)q(a+\pi+\rho)^{n-3} - \&c.} + \&c.$$

ubi litera a denotat proximum valorem quantitatis x datum, literæ vero π, ρ, σ, &c. reſpectivos valores præcedentium fractionum.

Valores quantitatum π, ρ, σ, τ, &c. per præcedentem methodum, i. e. per continuas ſubſtitutiones in reſultantibus haud in datis æquationibus, inventi, erunt iidem ac ii ex hâc methodo deducti.

3. Datâ æquatione $x^n - p x^{n-1} + q x^{n-2} - r x^{n-3} + \&c. = 0$, ſupponatur $x = a + e$, ubi e parvam habet rationem ad a; & ſcribatur in datâ æquatione pro x ejus valor $a + e$, & reſultat æquatio

$$a^n - p a^{n-1} + q a^{n-2} - \&c. + (n a^{n-1} - (n-1) p a^{n-2} + (n-2) q a^{n-3} - \&c.)\, e + \left(n \,.\, \frac{n-1}{2} a^{n-2} - (n-1) \times \frac{n-2}{2} p a^{n-3} + \&c.\right) e^2 + \&c. = P + \mathcal{Q} e + R e^2 + S e^3 + \&c. = 0,$$

unde ſit $P + \mathcal{Q} e$ prope $= 0$, & exinde e prope $= -\frac{P}{\mathcal{Q}}$, ſed erit $e = -\frac{P}{\mathcal{Q} + R e + S e^2 + \&c.}$ prope $= -$

$$\frac{P}{Q - \frac{RP}{Q} + \frac{SP^2}{Q^2} - \&c.} = - \frac{PQ^{n-1}}{Q^n - Q^{n-2}RP + Q^{n-3}SP^2 - \&c.}$$

prope $= - \frac{PQ}{Q^2 - RP}$.

4. Sit reſultans æquatio eadem ac in præcedente exemplo $P + Qe + Re^2 + Se^3 + Te^4 + \&c. = 0$, abjiciantur omnes termini ob parvitatem eorum, in quibus inveniuntur plures quam duæ dimenſiones quantitatis e, & reſultat æquatio $P + Qe + Re^2 = 0$, unde $e = \surd\left(\frac{Q^2 - 4RP}{4R^2}\right) - \frac{Q}{2R}$ prope.

Et ſimiliter abjiciantur omnes præter terminos, in quibus inveniuntur plures quam tres vel quatuor dimenſiones, &c. ex æquatione reſultante inveniatur valor quantitatis e, & invenitur plerumque propior approximatio ad valorem quantitatis e.

Cor. . Datâ algebraicâ æquatione $x^n - px^{n-1} + qx^{n-2} - \&c. = 0$, cujus radices ſint reſpective α, β, γ, δ, &c. i. e. ſit α minor quam β, β quam γ, &c. ſint etiam radices æquationis $nx^{n-1} - (n-1)px^{n-2} + (n-2)qx^{n-3} - \&c. = 0$ reſpective π, ρ, σ, τ, &c. tum cognitum eſt π, ρ, σ, τ, &c. limites eſſe, inter quos conſiſtunt datæ æquationis radices β, γ, δ, &c.

Aſſumantur vel π vel ρ vel σ vel τ, &c. tanquam approximatio ad datæ æquationis radicem x, ſcribatur in datâ æquatione pro x ejus valor aſſumptus $\pi + z$, & per methodum prius traditam invenietur ejus approximatio $- \frac{\pi^n - p\pi^{n-1} + q\pi^{n-2} - \&c.}{n\pi^{n-1} - (n-1)p\pi^{n-2} + (n-2)q\pi^{n-3} - \&c.}$; ſed quoniam π eſt radix æquationis $n\pi^{n-1} - (n-1)p\pi^{n-2} + \&c. = 0$, erit approximatio inventa quantitas infinita, & conſequenter ſeries ex hâc methodo reſultans infinite divergit; ergo convergentia ſeriei prædictæ haud pendet e ratione, quam habet quantitas aſſumpta pro radice α ad radicem ipſam; ſed ex hoc, quod quantitas aſſumpta pro radice multo propior ſit ad radicem quæſitam, quam ad ullam aliam datæ æquationis radicem.

THEOR.

THEOR. XVII.

Sit æquatio $x^n - px^{n-1} + qx^{n-2} -$ &c. $= 0$, cujus radices ſint α, β, γ, δ, &c. aſſumatur quantitas a pro radice α, & transformetur data æquatio in alteram, cujus radices ſunt minores vel majores quam radices datæ æquationis per quantitatem a, i. e. ſint $\alpha - a$, $\beta - a$, $\gamma - a$, &c. ſint $a : \alpha :: 1 : 1 + h$, $a : \beta :: 1 : 1 + k$, $a : \gamma :: 1 : 1 + l$, $a : \delta :: 1 : 1 + m$, &c. & ſi a multo propior ſit ad radicem α quam ad β vel γ vel δ, &c. tum h multo minor erit quam k vel l vel m, &c. & radices æquationis transformatæ erunt reſpective ha, ka, la, ma, &c. ergo $h \times k \times l \times m \times n' \times$ &c. $a^n = \pm(a^n - pa^{n-1} + qa^{n-2} -$ &c.$) = Qa^n$ contentum ſub ſingulis transformatæ æquationis radicibus; & ($klmn'$ &c. $+ hklm$ &c. $+ hkmn'$ &c. $+ hlmn'$ &c. $+$ &c.) $a^{n-1} = \pm(na^{n-1} - (n-1)pa^{n-2} + (n-2)qa^{n-3} -$ &c.$) = Pa^{n-1}$ aggregatum e ſingulis contentis ſub $n-1$ radicibus transformatæ æquationis; ergo approximatio per vulgares regulas inventa

$$-\frac{a^n - pa^{n-1} + qa^{n-2} - \&c.}{na^{n-1} - (n-1)pa^{n-2} + (n-2)qa^{n-3} - \&c.}$$

$$= \frac{(a-\alpha) \times (a-\beta) \times (a-\gamma) \times (a-\delta) \times \&c.}{(a-\alpha) \times (a-\beta) \times \&c. + (a-\beta) \times (a-\gamma) \times \&c. + (a-\gamma) \times (a-\delta) \times \&c. + \&c.}$$

: ad quantitatem quæſitam $\alpha - a = ha :: klmn'$ &c. $\left(\frac{Q}{h}\right) : klmn'$ &c. $+ hklm$ &c. $+ hlmn'$ &c. $+$ &c. (P).

2. Si vero quantitas aſſumpta multo propior ſit ad duas radices α & β datæ æquationis quam ad reliquas, i. e. ſi duæ radices transformatæ æquationis, quæ ſit $v^n \ldots Tv^4 - Sv^3 + Rv^2 - Qv + P = 0$, ſint multo minores quam reliquæ; tum radices æquationis $v^2 - \frac{Q}{R}v + \frac{P}{R} = 0$ erunt quam proxime illæ radices quæſitæ; ſi vero tres, tum radices æquationis $v^3 - \frac{R}{S}v^2 + \frac{Q}{S}v - \frac{P}{S} = 0$ erunt quam proxime

radices

radices quæſitæ; & ſic deinceps. Et facile methodo conſimili, i. e. pro coefficientibus P, Q, R, S, &c. ſcribendo earum valores in terminis radicum α, β, γ, &c. inveniri poſſunt rationes, quas habent radices inventæ per has regulas ad veras radices quæſitas.

Ab extractione radicum conſtare poſſunt limites quantitatis aſſumendæ pro primâ approximatione, ita ut ſeries convergat; ſed æque has approximationes ac datæ æquationis radices inveſtigare difficile eſt.

FIG. 5. 3. Aliter: ſit æquatio $x^n - px^{n-1} + qx^{n-2} -$ &c. $= 0$, aſſumatur $x^n - px^{n-1} + qx^{n-2} -$ &c. $= y$, ubi x ſit abſciſſa datæ curvæ, y vero ordinata: curvam vero ſecet abſciſſa Ap in tot (n) punctis, $(h, k, l, m, n, o,$ &c.$)$ quot dimenſiones abſciſſæ x inveniuntur in datâ æquatione, & ſint fp, eq, dr, cs, bt, &c. ejus (n) ordinatæ, quæ reſpective ſint maximæ; tum Ah, Ak, Al, Am, An, Ao, &c. erunt reſpectivæ radices datæ æquationis $x^n - px^{n-1} + qx^{n-2} -$ &c. $= 0$; & At, As, Ar, Aq, Ap, &c. erunt reſpectivæ radices æquationis $nx^{n-1} - (n-1)px^{n-2} + (n-2)qx^{n-3} -$ &c. $= 0$; ſi vero requiratur radix Am datæ æquationis $x^n - px^{n-1} + qx^{n-2} -$ &c. $= 0$, aſſumatur quantitas $Aa = a$ pro quantitate approximante ad radicem Am, ſcribatur $a + \pi$ pro radice x in datâ æquatione, & per regulam prius traditam invenietur approximatio π prope

$$= -\frac{a^n - pa^{n-1} + qa^{n-2} - \text{\&c.}}{na^{n-1} - (n-1)pa^{n-2} + (n-2)qa^{n-3} - \text{\&c.}} = am \times \frac{ah \times ak \times al \times an \times ao \times \text{\&c.}}{n \times ar \times as \times at \times aq \times ap \times \text{\&c.}};$$

unde approximatio inventa erit ad radicem quæſitam $:: \pm\, ah \times ak \times al \times$ &c. $: n \times ar \times as \times at \times$ &c.

Cor. . Hinc approximatio per hanc methodum inventa erit ad veram radicem quæſitam ultimo in ratione $mh \times mk \times ml \times mn \times mo \times$ &c. $: n \times mt \times ms \times mr \times mq \times mp \times$ &c. . . . i. e. in ratione compoſitâ e rationibus $mh : mt$, $mk : ms$, $ml : mr$, &c. & $1 : n$; ſit vero hæc ratio, quæ erit data $A : B$, & α radix quæſita; erit approximatio $\frac{A\alpha}{B}$, & differentia inter eam & veram radicem erit $\frac{B-A}{B}\alpha$;

ergo ultimo diverſæ approximationes per hanc methodum inventæ, videntur eſſe prope $\frac{A\alpha}{B}$, $\frac{A}{B} \times \frac{B-A}{B}\alpha$, $\frac{A}{B} \times \frac{(B-A)^2}{B^2}\alpha$, $\frac{A}{B} \times \frac{(B-A)^3}{B^3}\alpha$; &c. i. e. prope quantitates in geometricâ progreſſione, quamvis non revera ſunt; nam prædicta ratio $A : B$ ultimo erit ratio æqualitatis.

Sint vero duo ſucceſſivi termini ſeriei reſpective r & s, & erit ſumma ſeriei geometricæ $\frac{r^2}{r-s}$; hæc vero methodus haud in ultimâ approximatione tam celeriter converget, quam præcedens: ſi aſſumatur ſeries terminorum, quorum denominatores ſecundum poteſtates quantitatis $r-s$ progrediuntur, reſultabit ſeries in quibuſdam caſibus celeriter convergens.

Si vero impoſſibiles radices in datâ æquatione contineantur, tum ſæpe regula ſupra tradita approximationes magis convergentes deducet.

4. Datâ æquatione $x^n - p x^{n-1} + q x^{n-2} - \&c. = 0$, cujus radices ſint reſpective α, β, γ, δ, &c. i. e. ſint α major quam β, β quam γ, &c. pro x ſcribatur $a + \pi$, ubi a major eſt quam maxima radix α, tum invenietur π prope $= -\frac{a^n - p a^{n-1} + q a^{n-2} - \&c.}{n a^{n-1} - (n-1) p a^{n-2} + \&c.} =$

$$-\frac{(a-\alpha) \times (a-\beta) \times (a-\gamma) \times (a-\delta) \times \&c.}{(a-\alpha) \times (a-\beta) \times (a-\gamma) \times \&c. + (a-\beta) \times (a-\gamma) \times (a-\delta) \times \&c. + \&c.};$$

quicunque autem ſit valor quantitatis a, ſemper approximabit valor ad ſingulas datæ æquationis radices, i. e. $a + \pi$ propior erit ad ſingulas datæ æquationis radices quam a; & ſic deinceps; approximatio inventa erit ad radicem quæſitam vel in ratione $1 : n$ vel in majore ratione. Eadem etiam affirmari poſſunt de quantitate a, quæ minor eſt quam minima radix.

Si vero impoſſibiles radices in datâ æquatione contineantur, tum haud dici poteſt, annon ulla quantitas aſſumpta major quam maxima vel minor quam minima radix, ſemper ad radicem maximam vel minimam appropinquabit.

1. Sit æquatio $(x-p)^n = 0$, cujus omnes radices ſint æquales; aſſumatur a pro primâ approximatione, ſcribatur $a+\pi$ pro x in datâ æquatione, & erit prima approximatio π prope $= -\frac{(a-p)^n}{n(a-p)^{n-1}}$ $= -\frac{a-p}{n}$; ergo, ſi modo λ ſit vera diſtantia quantitatis a aſſumptæ a radice quæſitâ, erunt $\frac{n-1}{n}\lambda$ diſtantia ſecundæ approximationis a vera radice, $\frac{(n-1)^2}{n^2}\lambda$ diſtantia tertiæ a vera radice, $\frac{(n-1)^3}{n^3}\lambda$ diſtantia quartæ, & ſic deinceps.

2. In quâcunque æquatione algebraicâ $x^n - px^{n-1} + \&c. = 0$ ſint m radices prope inter ſe & quantitati a æquales; & e ſubſtitutione $a+\pi$ pro incognitâ quantitate x in datâ algebraicâ æquatione, ubi a ſit α prope, reſultabit æquatio $P + Q\pi + R\pi^2 + S\pi^3 + \&c. = 0$; ſi vero aſſumatur $m+1$ primorum hujuſce æquationis terminorum ſumma nihilo æqualis, i. e. $P + Q\pi + R\pi^2 \ldots H\pi^m = 0$, tum m radices æquationis erunt prope m radices æquationis $P + Q\pi + R\pi^2 + \&c. = 0$, quæ ſupponuntur inter ſe prope æquales.

In hoc caſu plerumque haud detegi poteſt approximatio e ſupponendo ſummam duarum vel trium vel quatuor, &c. . . . vel m terminorum nihilo æqualem, i. e. vel $P + Q\pi = 0$ vel $P + Q\pi + R\pi^2 = 0$, &c. ad m terminos.

5. Sit æquatio $P - Qe + Re^2 - \&c. = 0$, cujus radices ſint α, β, γ, &c. & cujus una radix (α) perparvam habeat rationem ad quamcunque aliam; tum erunt $P = \alpha\beta\gamma\delta$ &c. $Q = \alpha\beta\gamma$ &c. $+ \alpha\beta\delta$ &c. $+ \alpha\gamma\delta$ &c. + &c. $+ \beta\gamma\delta$ &c. $R = \alpha\beta$ &c. $+ \alpha\gamma$ &c. $+ \alpha\delta$ &c. + &c. $+ \beta\gamma$ &c. $+ \beta\delta$ &c. $+ \gamma\delta$ &c. + &c. i. e. Q erit prope $= \beta\gamma\delta$ + &c. $R = \beta\gamma$ &c. $+ \beta\delta$ &c. $+ \gamma\delta$ &c. + &c. &c. ſi vero ſtatuatur $P - Qe$ prope $= \alpha \times \beta\gamma\delta \times \&c. - \beta\gamma\delta\&c. \times e = 0$, erit $e = \frac{P}{Q}$ prope

=

$= \frac{\alpha\beta\gamma\delta\,\&c.}{\beta\gamma\delta\,\&c.} + \&c. = \alpha$ prope: vel adhuc propior erit ejus valor $\alpha -$ $\alpha^2 \times \left(\frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta} + \&c.\right) + \alpha^3 \left(\frac{1}{\beta^2} + \frac{1}{\gamma^2} + \frac{1}{\delta^2} + \&c. + \frac{1}{\beta\gamma} + \frac{1}{\beta\delta}\right.$ $\left. + \frac{1}{\gamma\delta} + \&c.\right) - \&c.$; & ſic deinceps.

2. Nunc ſtatuatur $P - Qe + Re^2$ prope $= (\beta\gamma\,\&c. + \beta\delta\,\&c. + \gamma\delta\,\&c.)\, e^2 - \beta\gamma\delta\,\&c.\, e + \alpha\beta\gamma\delta\,\&c. = 0$, tum ab reſolutione æquationis conſequetur radix $= \alpha + \alpha^3 \left(\frac{1}{\beta\gamma} + \frac{1}{\beta\delta} + \frac{1}{\gamma\delta} + \&c.\right)$ &c. & generaliter ſtatuantur m termini prædictæ æquationis $P + Qe + Re^2 + Se^3 + \&c. \ldots e^{m-1} = 0$, & inveniri poteſt ejus radix $= \alpha \pm \alpha^m \left(\frac{1}{\beta\gamma\delta\,\&c.} + \frac{1}{\beta\delta\varepsilon\,\&c.} + \frac{1}{\gamma\delta\varepsilon\,\&c.} + \&c.\right)$ ubi in ſingulis factoribus $\frac{1}{\beta\gamma\delta\,\&c.}$, $\frac{1}{\beta\delta\varepsilon\,\&c.}$, &c. tot contineantur diverſæ literæ β, γ, δ, &c. quot unitates in $m - 1$; & per $\frac{1}{\beta\gamma\delta\,\&c.} + \frac{1}{\beta\delta\varepsilon\,\&c.} + \&c.$ deſigno aggregatum e ſingulis quantitatibus prædicti generis. Signum affixum $\pm$ erit affirmativum vel negativum, prout $m - 1$ eſt par vel impar numerus.

6. Sit æquatio $0 = t - sz + rz^2 - qz^3 + pz^4 \ldots lz^m \pm kz^{m+1} \mp hz^{m+2} \pm \&c.$, in quâ una radix α ſit perparva reſpectu habito ad reliquas; & conſtat, quodſi æquatio $0 = t - sz$, i. e. $z = \frac{t}{s}$ præbeat valorem quantitatis α ad numerum π figurarum verum; tum ultimo reſolutio æquationis $0 = t - sz + rz^2 - qz^3 \ldots lz^m$ præbebit valorem prædictæ radicis α uſque ad numerum $m \times \pi$ figurarum verum.

7. Sit æquatio $0 = t - sz + rz^2 - qz^3 \ldots lz^m - kz^{m+1} + \&c.$, ducatur hæc æquatio in $0 = a - bz + cz^2 - dz^3 \ldots fz^m - gz^{m+1} + \&c.$, ita ut 1^mo^ evadat æquatio hujuſce formulæ $0 = T - Sz \ldots Kz^{m+1} + \&c.$ in quâ deficiunt termini $z^2, z^3, \ldots z^m$; tum plerumque,

rumque, ſi $\frac{t}{s}$ ſit valor quantitatis z ad π figuras verus, ultimo erit $\frac{T}{S}$ valor prædictæ quantitatis z uſque ad $m \times \pi$ figuras verus; 2^{do} evadat æquatio formulæ $0 = T - Sz + Rz^2 - Qz^3 \ldots Lz^m - Kz^{m+1} + \&c.$ ita vero conſtituta, ut innoteſcant radices æquationis reſultantis ex hypotheſi quod $(m+1)$ primi termini prædictæ æquationis nihilo fiant æquales, i. e. $0 = T - Sz + Rz^2 - Qz^3 \ldots Lz^m$; tum, ſi $\frac{t}{s}$ ſit valor quantitatis z ad π figuras verus, erit ultimo radix hujuſce æquationis valor prædictæ radicis ad $m \times \pi$ figuras verus.

Eadem principia etiam ad æquationes, in quibus duæ vel plures continentur incognitæ quantitates, applicari poſſunt.

7. Sit æquatio $y^n - py^{n-1} + qy^{n-2} - ry^{n-3} + \&c. = 0$, cujus una radix α multo major ſit quam quæcunque alia; & ſi requiratur ſeries, quæ ſecundum quantitates e, f, g, &c. reciproce progrediatur; ſcribatur $p + \frac{1}{e}$ vel $a + \frac{1}{e}$ pro y in datâ æquatione, ubi a eſt α prope, & reſultet æquatio $e^n - Pe^{n-1} + Qe^{n-2} - \&c. = 0$; in hac æquatione pro e ſcribatur $P + \frac{1}{f}$, & reſultat æquatio $f^n - P'f^{n-1} + \&c. = 0$; deinde in hâc æquatione pro f ſcribatur $P' + \frac{1}{g}$, & ſic deinceps; unde $y = p + \frac{1}{e} = p + \frac{f}{Pf+1} = p + \frac{P'g+1}{PP'g+P+g} = \frac{hP''P' + P' + h}{hP''P'P + PP' + hP + hP'' + 1} = \&c.$ Lex hujuſce ſeriei e lege ſeriei in theor. 45. medit. algebr. traditâ facile conſtabit.

Cor. 1. Approximationes ex hâc methodo inventæ mutatis mutandis eædem erunt ac approximationes prius deductæ.

Cor. 2. Si vero duæ radices ſint reſpective multo majores quam reliquæ, tum e tribus primis terminis nihilo æqualibus eſſe ſuppoſitis, viz. æquatione $y^2 - py + q = 0$ conſtabit approximatio ad utramque radicem. Et ſic de continuis approximationibus ad plures majores radices inveniendis.

Ex approximationibus ad duas vel plures majores radices datis, & princip. ſubſ. facile deduci poſſunt aliæ ad prædictas radices magis appropinquantes.

8. Hæc principia etiam ad æquationem duas vel plures variabiles quantitates habentem, vel ad plures æquationes applicari poſſunt.

Sit æquatio $x^n - px^{n-1} + qx^{n-2} - \&c. = o$, & ſi $-\frac{\alpha^n - p\alpha^{n-1} + q\alpha^{n-2} - \&c.}{n\alpha^{n-1} - (n-1)p\alpha^{n-2} + (n-2)q\alpha^{n-3} - \&c.} = \beta$, & $\frac{(\alpha+\beta)^n - p(\alpha+\beta)^{n-1} + q(\alpha+\beta)^{n-2} - \&c.}{n(\alpha+\beta)^{n-1} - (n-1)p(\alpha+\beta)^{n-2} + \&c.} = -\beta$; vel ſi prior fractio ſit l, & proximæ approximationes per eandem methodum inventæ ſint reſpective m, n, o, &c. & tandem inveniatur approximatio $= -l - m - n - o - \&c.$ tum ſeries conſtans e diverſis approximationibus nec convergens nec divergens dici poteſt.

Et conſimilia etiam ad æquationes diverſi generis, & plures æquationes plures incognitas quantitates habentes applicari poſſunt.

9. Sit data æquatio $x^n - px^{n-1} + qx^{n-2} \ldots tx^{n\mp m} + \&c. = o$, aſſumatur a pro approximatione ad minimam radicem, ſcribatur $a \mp v$ pro x in datâ æquatione & reſultet æquatio $v^n - Pv^{n-1} + Qv^{n-2} \ldots Tv^{n-m} + \&c. = o$ ubi T diverſum habeat ſignum a quantitate t, & n haud $\mp m$; tum plerumque quantitates inventæ haud accedent ad minimam radicem.

10. Si terminorum datæ æquationis ſigna ſemel ſolummodo vel progrediantur de + in +, vel − in −; vel mutabuntur de + in −, vel − in +; tum aſſumatur quæcunque negativa quantitas in uno caſu, affirmativa vero in altero, major vero quam negativa vel affirmativa radix, pro approximatione ad radicem prædictam, & ſemper convergent approximationes per prædictam methodum inventæ.

THEOR. XVIII.

Sit quadratica æquatio $x^2 - (a+s)x + as = o$, cujus radices ſunt a & s, & prima approximatio per methodum hic traditam inventa erit $\frac{as}{a+s} = s - \frac{s^2}{a} + \frac{s^3}{a^2} - \frac{s^4}{a^3} +$ &c. per eandem methodum inveniantur ſucceſſivæ approximationes, & approximatio ad $n-1$ diſtantiam a primâ erit $\frac{s^b}{a^{b-1}} - \frac{s^{b+1}}{a^b} + \frac{s^{3b}}{a^{3b-1}} - \frac{s^{3b+1}}{a^{3b}} + \frac{s^{5b}}{s^{3b+1}} - \frac{s^{5b+1}}{a^{5b}} + \frac{s^{7b}}{a^{7b-1}} - \frac{s^{7b+1}}{a^{7b}} +$ &c. ubi $b = 2^{n-1}$.

Cor. 1. Hinc conſtat, quod hæ approximationes ultimo convergent in rationibus fractionis $\left(\frac{s^b}{a^{b-1}}\right)$ inter ſe, & conſequenter approximationes ſic inventæ ultimo vergent in ratione majori quam quâvis geometricâ progreſſione; idem etiam de approximationibus æquationum ſuperiorum ordinum vel quorumcunque diverſorum generum vel plurium æquationum, &c. ſic inventis affirmari poteſt.

Cor. 2. Sit æquatio $x^n - px^{n-1} + qx^{n-2} - rx^{n-3} +$ &c. $= o$, cujus radices ſint $\alpha, \beta, \gamma, \delta,$ &c. σ; ubi σ eſt radix quæſita, fingantur $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta} +$ &c. $= P$, $\frac{1}{\alpha\beta} + \frac{1}{\alpha\gamma} + \frac{1}{\beta\gamma} + \frac{1}{\alpha\delta} +$ &c. $= \mathcal{Q}$, $\frac{1}{\alpha\beta\gamma} + \frac{1}{\alpha\beta\delta} +$ &c. $= R$, $\frac{1}{\alpha\beta\gamma\delta} +$ &c. $= S$, &c. in quibus quantitatibus haud continetur radix σ; & prima approximatio per prædictam methodum inventa erit $\frac{\sigma}{1+P\sigma} = \sigma - P\sigma^2 + P^2\sigma^3 -$ &c. ſecunda vero erit fractio, cujus numerator erit $\frac{P\sigma^2}{(1+P\sigma)^2} + \frac{P\mathcal{Q}\sigma^4}{(1+P\sigma)^3} -$

$PR\sigma^5$

$\frac{PR\sigma^5}{(1+P\sigma)^4} + \frac{PS\sigma^6}{(1+P\sigma)^5} -$ &c. denominator vero $\frac{1+P^2\sigma^2}{1+P\sigma} - \frac{2PQ\sigma^3 - Q\sigma^2}{(1+P\sigma)^2} + \frac{3PR\sigma^4 - R\sigma^3}{(1+P\sigma)^3} - \frac{4PS\sigma^5 - S\sigma^4}{(1+P\sigma)^4} +$ &c.

PROB. XIV.

Sit data æquatio $x^n - mx^{n-1} + lx^{n-2} - kx^{n-3} \ldots . dx^3 - cx^2 + bx - a = 0$, *& datis propinquis valoribus ad unam vel duas vel plures ejus radices, invenire valores ad prædictas radices magis appropinquantes.*

1. Sit β quantitas ab unâ ſolummodo radice haud longe diſtans, & ſcribatur $(x^{n-1} - Px^{n-2} + Qx^{n-3} \ldots Sx^3 - Tx^2 + Vx + W)(x - \beta - e) = x^n - mx^{n-1} + lx^{n-2} +$ &c. . . $+ dx^3 - cx^2 + bx - a = 0$; inde reſultant æquationes $P = m - \beta - e$, $Q = l - (\beta + e)P = l - m(\beta + e) + (\beta + e)^2$, & ſic $R = k - (\beta + e)l + (\beta + e)^2 m - (\beta + e)^3$, & ultimo reſultat æquatio $(\beta + e)^n - m(\beta + e)^{n-1} + l(\beta + e)^{n-2} - k(\beta + e)^{n-3} +$ &c. $= 0$, ſed per hypotheſin e eſt perparva reſpectu habito ad β, ergo ultimo erit $\beta^n - m\beta^{n-1} + l\beta^{n-2} -$ &c. $= -(n\beta^{n-1} - (n-1)m\beta^{n-2} + (n-2)l\beta^{n-3} -$ &c.$)\,e$, & exinde conſtat approximatio e: aliter vero ab ultimis terminis incipiendum eſt, & facile ad eandem concluſionem pervenire liceat.

2. Sint a, b, c, d, &c. approximationes ad r radices datæ æquationis $x^n - mx^{n-1} + lx^{n-2} -$ &c. $= 0$, invenire alias ad r radices magis appropinquantes. Supponantur α, β, γ, δ, ε, &c. perparvæ quantitates, quæ ſint approximationes quæſitæ; aſſumatur æquatio $(x^{n-r} - Px^{n-r-1} + Qx^{n-r-2} - Rx^{n-r-3} +$ &c.$)\,((x - a - \alpha) \times (x - b - \beta) \times (x - c - \gamma) \times (x - d - \delta) \times$ &c.$) = x^n - mx^{n-1} + lx^{n-2} - kx^{n-3} +$ &c; $= 0$, hujuſce æquationis fiant correſpondentes termini inter ſe æquales (omnibus quantitatibus ſemper neglectis, vel pro nihilo habitis, in quibus plures quam una dimenſio quantitatum α, β, γ, δ, &c. continentur), & ita reducantur æquationes reſultantes, ut exterminentur incognitæ quantitates P, Q, R, S, &c. & exorientur r ſimplices

æquationes totidem incognitas quantitates $\alpha, \beta, \gamma, \delta,$ &c. habentes, e quibus inveniri poſſunt prædictæ quantitates.

2. 2. Sit æquatio $x^n - px^{n-1} + qx^{n-2} - rx^{n-3} + sx^{n-4} - tx^{n-5} +$ &c. $= o$, cujus ſint $a, b, c, d,$ &c. approximationes ad n radices $\alpha, \beta, \gamma, \delta,$ &c. nunc fingantur diverſæ radices reſpective $\alpha + \pi$, $b + \rho$, $c + \sigma$, $d + \tau$, &c. ducantur hæ quantitates in ſeſe, & erit contentum $(x - a - \pi) \times (x - b - \rho) \times (x - c - \sigma) \times$ &c. $= x^n - ((a + b + c +$ &c.$) + (\pi + \rho + \sigma + \tau +$ &c.$))\, x^{n-1} + ((ab + ac + bc +$ &c.$) + (b + c + d +$ &c.$)\, \pi + (a + c + d +$ &c.$)\, \rho + (a + b + d +$ &c.$)\, \sigma +$ &c.$)\, x^{n-2} - ((abc + abd + bcd +$ &c.$) + (bc + bd + cd +$ &c.$)\, \pi +$ &c.$)\, x^{n-3} +$ &c. $= o$, dicantur ſumma e ſingulis quantitatibus $(a + b + c + d,$ &c.$)$ productorum e ſingulis binis in ſe ductis $(ab + ac + bc +$ &c.$)$, contentorum e ſingulis ternis, quatuor, &c. reſpective $P, Q, R, S, T,$ &c. deinde ſupponantur $\pi + \rho + \sigma + \tau +$ &c. $= p - P = \alpha$, $a\pi + b\rho + c\sigma + d\tau +$ &c. $= (p - P) \times P - q + Q = P\alpha - q + Q = \beta$, $a^2\pi + b^2\rho + c^2\sigma +$ &c. $= P\beta - Q\alpha + r - R = \gamma$, $a^3\pi + b^3\rho + c^3\sigma +$ &c. $= P\gamma - Q\beta + R\alpha - s + S = \delta$, $a^4\pi + b^4\rho + c^4\sigma +$ &c. $= P\delta - Q\gamma + R\beta - S\alpha + t - T$, & ſic deinceps (unde cuidam inſpicienti facile conſtat lex, quam obſervant termini $\alpha, \beta, \gamma, \delta,$ &c.); inveniantur n ſimplices æquationes hujuſce generis, & ex quibuſcunque n earum ſimplicibus æquationibus erui poſſunt approximationes ad ſingulas radices quæſitæ $\pi, \rho, \sigma, \tau,$ &c.

Cor. 1. E principiis prius traditis conſtabit, ut quantitates hâc methodo inventæ eo magis approximent, quo minus diſtant approximationes datæ ab unâ radice quam ab reliquis; ſi vero una approximatio data minus diſtet a duabus radicibus quam a reliquis, tum ad inveſtigandam quantitatem magis adhuc appropinquantem utendum eſt quadraticâ æquatione; & ſic deinceps.

Cor. 2. Rationes, quibuſcum vergunt ad radices quæſitas approximationes inventæ, facile deduci poſſunt ſcribendo pro ſingulis quantitatibus in approximationibus prædictis contentis earum functiones datæ æquationis radicum.

Omnia hæc etiam ad æquationes, quæ haud ſunt algebraicæ, applicari poſſunt.

PROB. XIX.

Sit quæcunque æquatio $A = o$, ubi litera A deſignat quamcunque functionem quantitatis x; fingatur x variabilis, $\dot{x}$ vero conſtans, & inveniatur $\frac{\dot{A}}{\dot{x}} = p$, $\frac{\ddot{A}}{\dot{x}^2} = q$, $\frac{\dddot{A}}{\dot{x}^3} = r$, &c. detur propinquus valor quantitatis x in datâ æquatione, qui dicatur a; ſcribatur a pro x in quantitatibus A, p, q, r, s, &c. & pro quantitatibus reſultantibus ſcribantur P, Q, R, S, T, &c. reſpective; deinde aſſumatur æquatio $P - Qe + \frac{1}{2}Re^2 - \frac{1}{2 \cdot 3}Se^3 + \frac{1}{2 \cdot 3 \cdot 4}Te^4 -$ &c. in infinitum $= o$, ſcribatur etiam $a - e$ pro x in datâ æquatione $A = o$, & reſultet æquatio $A' = o$, cujus radix e eadem erit ac ea in æquatione $P - Qe + \frac{1}{2}Re^2 -$ &c. $= o$ contenta; nunc fingatur $P - Qe = o$, & ſit α radix æquationis $A' = o$, quæ ſit perparva, & æquationis $P - Qe = o$ radix exprimetur per ſeriem ſubſequentis formulæ, i. e. $e = \frac{P}{Q} = \alpha + m\alpha^2 +$ &c. nunc aſſumatur $P - Qe + \frac{1}{2}Re^2 = o$, & ſic exprimetur valor ejus radicis $e = \alpha + m\alpha^3 +$ &c. & univerſaliter aſſumatur aggregatum m terminorum præcedentis æquationis $P - Qe + \frac{1}{2}Re^2 - \frac{1}{6}Se^3 +$ &c. . . . $e^{m-1} = o$, & per ſeriem hujuſce formulæ $e = \alpha + m'\alpha^m +$ &c. exprimetur radix æquationis prædictæ $P - Qe + \frac{1}{2}Re^2 - \frac{1}{6}Se^3 +$ &c. . . $= o$.

Ex. Sit quantitas exponentialis x^x & augeatur x per e, tum evadet $(x^x) = (x + e)^{x+e} = x^x + x^x(\log. x + 1)e + \frac{1}{2}x^x(\log. x^2 + 2\log. x + x^x + x^{x-1})e^2 +$ &c. x^x nunquam evadit $= o$.

Cor. . Si duæ datæ æquationis radices ſint prope inter ſe & quantitati a æquales, tum aſſumatur $P - Qe + \frac{1}{2}Re^2 = o$, & inveniatur propior valor quantitatis e, & ſic ex aggregato m terminorum $P - Qe + \frac{1}{2}Re^2 -$ &c. $= o$ nihilo æquali aſſumpto, deduci poteſt propior valor quantitatis e, cum m radices datæ æquationis ſint prope inter ſe & datæ quantitati a æquales.

THEOR. XX.

Datis duabus æquationibus $Hy^n + (a + bx)y^{n-1} + \&c. = 0$ & $Ky^m + (A + Bx)y^{m-1} + \&c. = 0$ duas incognitas quantitates x & y habentibus, ſint correſpondentes radices incognitarum quantitatum (x & y) reſpective $\alpha, \beta, \gamma, \delta$, &c. & π, ρ, σ, τ, &c. invenire ex approximatione correſpondentes valores (α & π) incognitarum quantitatum (x & y).

Neceſſe eſt, ut prius dentur k & l ad prædictos correſpondentes valores (α & π) multo magis appropinquantes, quam ad ullam aliam e correſpondentibus incognitarum quantitatum (x & y) radicibus (β, γ, δ, &c. & ρ, σ, τ, &c.). In datis æquationibus pro quantitatibus (x & y) ſcribantur reſpective $z - k$ & $v - l$, & ſint æquationes reſultantes $\Gamma + (\Delta z + \mathrm{E}v) + (\zeta z^2 + \mathrm{H}zv + \Theta v^2) + \&c. = 0$ & $(\Pi + \mathrm{P}z + \Sigma v) + (\mathrm{T}z^2 + \Psi zv + \Phi v^2) + \&c. = 0$; & e conſtitutione coefficientium colligi poſſunt $\Gamma + \Delta z + \mathrm{E}v$ prope $= 0$, & $\Pi + \mathrm{P}z + \Sigma v$ prope $= 0$, & exinde approximatio ad z erit $\frac{\mathrm{E}\Pi - \Sigma\Gamma}{\Sigma\Delta - \mathrm{EP}}$, & approximatio ad v erit $\frac{\Delta\Pi - \mathrm{P}\Gamma}{\mathrm{PE} - \Delta\Sigma}$.

2. Si vero ſumantur plures termini vel ex unâ vel ſingulis duabus æquationibus, quorum aggregatum nihilo fiat æquale, i. e. $\Gamma + (\Delta z + \mathrm{E}v)$ prope $= 0$ & $\Pi + (\mathrm{P}z + \Sigma v) + (\mathrm{T}z^2 + \Psi zv + \Phi v^2)$ prope $= 0$; vel $\Pi + \mathrm{P}z + \Sigma v = 0$ & $\Gamma + (\Delta z + \mathrm{E}v) + \zeta z^2 + \mathrm{H}zv + \theta v^2 = 0$; vel $\Gamma + (\Delta z + \mathrm{E}v) + (\zeta z^2 + \mathrm{H}zv + \Theta v^2) = 0$ & $\Pi + (\mathrm{P}z + \Sigma v) + (\mathrm{T}z^2 + \Psi zv + \Phi v^2) = 0$; &c. ita vero ut omnes termini vel nullius & unius; vel nullius, unius & duarum; &c. dimenſionum contineantur in ſingulis duobus aggregatis, quæ nihilo fiant æqualia, & de convergentiâ approximationum ad quantitates

titates z & v ex his æquationibus inventarum dijudicare liceat e theor. præced. & medit. algebr.

3. Si vero k & l multo magis approximent ad duas correſpondentes radices incognitarum quantitatum (x & y) reſpective quam ad ullas alias, & pro x & y ſubſtituantur reſpective $z-k$ & $v-l$ in datis æquationibus, & e conſtitutione coefficientium colligi poſſunt vel $\Gamma+\Delta z+\mathrm{E}v$ prope $=0$ & $\Pi+(\mathrm{P}z+\Sigma v)+(\mathrm{T}z^2+\Psi zv+\Phi v^2)$ prope $=0$; vel $\Pi+\mathrm{P}z+\Sigma v$ prope $=0$, & $\Gamma+(\Delta z+\mathrm{E}v)+(\zeta z^2+\mathrm{H}zv+\Theta v^2)$ prope $=0$; e quibus æquationibus inveniri poſſunt approximationes ad z & v: & ſic progredi licet, cum k & l ad tres vel plures correſpondentes incognitarum quantitatum radices multo magis approximent quam ad reliquas.

4. Sint duæ æquationes duas incognitas quantitates x & y habentes; pro x & y in datis æquationibus ſcribantur reſpective $b+e$ & $m+\sigma$, & reſultent æquationes $A+Be+C\sigma+D=0$ & $P+Qe+R\sigma+S=0$, ubi literæ D & S in ſingulis terminis plures habent dimenſiones perparvarum quantitatum e & σ quam unam; & P, Q, R; A, B & C, habent vero nullas; tum approximationes ad quantitates e & σ per methodum hic traditam inventæ erunt reſpective $e=\frac{RA-CP}{CQ-RB}$, & $\sigma=\frac{QA-BP}{RB-QC}$; ſi vero $CQ=RB$, tum approximationes ſic inventæ erunt infinitæ.

Hinc ſint duæ datæ æquationes $\alpha=0$ & $\pi=0$, ubi α & π ſunt functiones quantitatum x & y; inveniantur earum fluxiones, viz. $\dot{\alpha}=a\dot{x}+b\dot{y}$ & $\dot{\pi}=p\dot{x}+q\dot{y}$; aſſumantur duæ æquationes $ab+bm=0$ & $pb+qm=0$, & ex iis inveniantur quantitatum x & y valores (b & m) qui erunt limites inter valores quantitatum x & y in datis æquationibus contenti, ſcribantur $b+e$ & $m+\sigma$ reſpective pro x & y in datis æquationibus, & approximationes per methodum prius traditam inventæ erunt valde magnæ; ni duo valores quantitatum x & y in datis æquationibus contenti ſint reſpective b & m prope.

Et ſic de limitibus inventis ex fingendo $a=0$ & $p=0$, &c.

THEOR. XXI.

Sint duæ vel plures n invariabiles quantitates (x, y, z, &c.) in datâ æquatione contentæ, quarum ſint a, b, c, &c. reſpectivi valores prope; ſcribantur $a+\alpha$, $b+\beta$, $c+\gamma$, &c. pro earum reſpectivis valoribus in datâ æquatione; & reſultet æquatio $A+B\alpha+C\beta+D\gamma+$ &c. $+K=0$, ubi A, B, C, D, &c. denotant quantitates in reſultante æquatione contentas, in quibus haud continetur α vel β vel γ vel &c. K, K', K'', &c. vero ſint quantitates, in quarum ſingulis terminis continentur plures quam una dimenſiones quantitatum α vel β vel γ &c.

Et ſic ſcribantur $a+\alpha'$, $b+\beta'$, $c+\gamma'$, &c. pro earum reſpectivis valoribus (x, y, z, &c.) in datâ æquatione & reſultet æquatio $A+B\alpha'+C\beta'+D\gamma'+$ &c. $+K'=0$; & ſic ſcribantur $a+\alpha''$, $b+\beta''$, $c+\gamma''$, &c. pro earum reſpectivis valoribus (x, y, z, &c.) in datâ æquatione, & reſultet $A+B\alpha''+C\beta''+D\gamma''+$ &c. $+K''=0$; & ſic deinceps: unde ex n ſubſtitutionibus prædicti generis ſi modo rejiciantur quantitates K, K', K'', &c. reſultabunt n ſimplices æquationes $A+B\alpha+C\beta+D\gamma+$ &c. $=0$, $A'+B\alpha'+C\beta'+D\gamma'+$ &c. $=0$, $A''+B\alpha''+C\beta''+D\gamma''+$ &c. $=0$ totidem incognitas quantitates B, C, D, &c. habentes, e quibus detegi poſſunt quantitates ipſæ B, C, D, &c.

2. Si vero duo ſint valores quantitatis x prope inter ſe & valori a æquales, qui correſpondent uni valori (b, c, d, &c.) e ſingulis reliquis variabilibus quantitatibus; tum hæc erit formula æquationis quæſitæ, viz. $P\pi^2+q\pi+r\rho+s\sigma+t\tau+$ &c. $+q'\pi\rho+r'\pi\sigma+$ &c. $+A=0$, ubi $a+\pi$, $b+\rho$, $c+\sigma$, $d+\tau$, &c. ſunt correſpondentes valores e ſingulis variabilibus x, y, z, &c. in datâ æquatione contentis; P, P', P'', q, r, s, t, &c. q', r', &c. A ſunt coefficientes quæſitæ: ſi vero duo ſint valores quantitatum x & y prope inter ſe & quantitatibus a & b reſpective æquales, qui correſpondent uni valori (c, d, &c.) e ſingulis reliquis (z, v, &c.) tum erit formula æquationis quæſitæ $P\pi^2+P'\pi\rho+P''\rho^2+q\pi+r\rho+s\sigma+t\tau+$ &c. $+s'\pi\sigma+s''\rho\sigma$ $+$ &c.

Exhinc conſtat formula æquationis, in quâ plures valores plurium variabilium quantitatum ſint prope inter ſe æquales.

Ea, quæ hic tradita fuerunt, facile ad plures æquationes plures variabiles quantitates habentes applicari poſſunt.

Convergentiæ harum æquationum ex principiis prius traditis dijudicandæ ſunt.

PROB. XV.

Datis infinitis æquationibus duas vel plures incognitas quantitates habentibus, invenire unam ex iis in terminis reliquarum.

Primo ita transformetur data æquatio, ut incognitæ quantitates in eâ contentæ evadant perparvæ; ſi requiratur ſeries ſecundum dimenſiones prædictarum quantitatum aſcendens: ſin requiratur deſcendens ſecundum dimenſiones quarundam quantitatum; tum ita transformetur data æquatio, ut evadant quantitates permagnæ; & perfici poteſt hoc problema ex iiſdem principiis, quæ tradita fuere de finitis æquationibus.

Si prima approximatio incognitæ quantitatis y, cujus infiniti in datâ æquatione continentur termini, involvat quantitatem, quæ haud in ſe continet ullas incognitas quantitates; tum plerumque reſolutionem infinitæ æquationis vel ſummationem infinitæ ſeriei exiget ſingulus terminus quæſitus. E. g. Sit æquatio $1 + a - \frac{a^3}{2 \cdot 3} + \frac{a^5}{2 \cdot 3 \cdot 4 \cdot 5} - \&c. = l + \frac{l^2}{1 \cdot 2} + \frac{l^3}{1 \cdot 2 \cdot 3} + \frac{l^4}{1 \cdot 2 \cdot 3 \cdot 4} + \&c.$ invenire l in terminis quantitatis a; aſſumatur $l = e$ prope & a perparva quantitas, ſcribatur $e + p$ pro l, & reſultat $1 = e + \frac{e^2}{2} + \frac{e^3}{2 \cdot 3} + \frac{e^4}{2 \cdot 3 \cdot 4} + \&c.$ & exinde $e = \log. 2$; etiamque $(1 + e + \frac{e^2}{2} + \frac{e^3}{2 \cdot 3} + \&c.)$

$+$ &c.) $p = 2p = a$ prope; ſcribatur $e + \frac{a}{2} + q$ pro l in datâ æquatione & invenietur valor quantitatis $q = -\frac{a^2}{8}$ prope; & ſic deinceps, unde valor quantitatis $l = \log. 2 + \frac{a}{2} - \frac{a^2}{8} - \frac{a^3}{24}$, &c.

Cor. . Omnis infinita ſeries exorta ex hâc reſolutione erit $e + \frac{e^2}{2} + \frac{e^3}{1.2.3} +$ &c. & haud difficilis erit inveſtigatio ex datâ vel datis infinitis æquationibus, an finitus vel infinitus numerus infinitarum ſerierum in reſolutione prædictâ exorietur.

PROB. XVI.

1. *Sit* y *ſumma datæ ſeriei ſecundum dimenſiones quantitatis* x *progredientis,* v *vero ſumma ſeriei ſecundum dimenſiones quantitatis* z *progredientis, & detur ſeries ſecundum dimenſiones quantitatum* z *&* x *progrediens, cujus ſumma ſit* w; *invenire quantitatem* w *in ſerie ſecundum dimenſiones quantitatum* y *&* v *progrediente. Primo ita reducantur omnes hæ quantitates per methodos prius traditas, ut evadant perparvæ quantitates, quarum dimenſiones in ſerie quæſitâ aſcendunt: vel permagnæ, quarum dimenſiones in prædictâ ſerie deſcendunt; tum pro* x *&* z *& earum functionibus in ſerie* w *ſcribantur earum valores e functionibus* y *&* v *deducti, & perficitur problema.*

Ex. 1. Datis logarithmis y & v duarum quantitatum $1 + x$ & $1 + z$, invenire logarithmum quantitatis $1 + x + z$: erunt $y = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 -$ &c. $v = z - \frac{1}{2}z^2 + \frac{1}{3}z^3 -$ &c. $w = (x + z) - \frac{1}{2}(x + z)^2 + \frac{1}{3}(x + z)^3 -$ &c. reducantur hæ æquationes ita ut inveniatur w in terminis ſecundum dimenſiones quantitatis y & v

progredientibus, & reſultabit

$$\begin{array}{l} w = y + v - yv + y\left(\tfrac{1}{2}v^2 - \dfrac{1}{2\cdot3}v^3 + \dfrac{1}{2\cdot3\cdot4}v^4 - \&c.\right) - \tfrac{3}{4}y^2v^2 + y^2\left(\dfrac{7}{12}v^3 - \&c.\right) - \&c. \\ \qquad\qquad + v\left(\tfrac{1}{2}y^2 - \dfrac{1}{2\cdot3}y^3 + \dfrac{1}{2\cdot3\cdot4}y^4 - \&c.\right) \qquad\qquad + v^2\left(\dfrac{7}{12}y^3 - \&c.\right) \end{array}$$

Ex. 2. Sit $y = t - \frac{t^3}{3} + \frac{t^5}{5} - \&c.$, $v = T - \frac{T^3}{3} + \frac{T^5}{5} - \&c.$, & $w = T + t - \frac{(T+t)^3}{3} + \frac{(T+t)^5}{5} - \&c.$; tum erit $w = y + v - yv\,(y + v) + \frac{1}{3}vy\,(5\,(v^3 + y^3) + 7\,(vy^2 + yv^2)) - \&c.$, quæ ſeries facile deduci poteſt ex aſſumendo ſeriem $w = a(y+v) + b(y^2v + v^2y) + (c\,(y^4v + v^4y) + d\,(y^3v^2 + v^3y^2)) + (e\,(y^6v + v^6y) + f\,(y^5v^2 + v^5y^2) + g\,(v^4y^3 + y^4v^3)) + \&c.$; & in hâc ſerie ſcribendo pro v & y earum datos valores; deinde ex æquando reſultantem ſeriem datæ ſeriei $T + t - \frac{(T+t)^3}{3} + \&c.$ erui poſſunt coefficientes a, b, c, d, &c.

PROB. XVII.

Sit p *conſtans quantitas, invenire formulam ſeriei* (A) p + aqx + (br + cq²) x² + (ds + eqr) x³ + *&c. in infinitum progredientis, ita ut ejus ſingulæ poteſtates* m *&* n, *& conſequenter radices eandem prorſus obſervent legem, i. e. ſi in ſerie, quæ exprimit* m *poteſtatem ſeriei* A, *pro* m *ſcribatur* n, *reſultet ſeries, quæ exprimet* n *poteſtatem ſeriei* A.

Aſſumantur duæ ſeries n & m poteſtates exprimentes, quæ eandem prorſus obſervant legem, & conſequenter erunt hujuſce formulæ

$$p^m + mp^{m-1}qx + \left.\begin{matrix} mrp^{m-2} \\ +(a'm^2 + bm)q^2p^{m-2} \end{matrix}\right\}x^2 + \left.\begin{matrix} m\,.\,sp^{m-3} \\ +(a'm^2 + bm)qrp^{m-3} \end{matrix}\right\}x^3 \begin{matrix} + \&c.\&c. \\ + \&c. \end{matrix}$$

ſcribatur n pro m in hâc ſerie & reſultat $p^n + np^{n-1}qx + \left.\begin{array}{r} nrp^{n-2} \\ +(a'n^2+bn)q^2p^{n-2} \end{array}\right\} x^2$ $+ \left.\begin{array}{r} nsp^{n-3} \\ +(a'n^2+bn)qrp^{n-3} \end{array}\right\} x^3 +$ &c. Ducantur hæ duæ ſeries in ſeſe & ſeriem reſultare oportet $p^{n+m} + (n+m)p^{n+m-1}qx + \left.\begin{array}{r} (n+m)rp^{n+m-2} \\ +(a'(n+m)^2+b(n+m)q^2)p^{n+m-2} \end{array}\right\} x^2$ $+$ &c. ut vero hæc ſeries eadem fiat ac reſultans, coefficiens omnis $+$ &c.

termini, in quo continentur duæ literæ q & r, q & s, &c. vel r & s, &c. & generaliter Q & R, erit $((Am^2+Bm)+(An^2+Bn)+2mn)$ $QR = A(m+n)^2 + B(m+n)$, unde $A = 1$, & B quæcunque quantitas ad libitum aſſumenda; ſit R idem ac Q, tum erit $A = \frac{1}{2}$.

Coefficiens omnis termini, in quo continentur tres literæ $Q \times R \times S$, erit $(Am^3 + Hm^2 + Cm)$; unde e præcedenti multiplicatione reſultabit $A(m^3+n^3) + H(m^2+n^2) + C(m+n) + (m^2+bm)n + (n^2+bn)m + (m^2+b'm)n + (n^2+b'n)m + (m^2+b''m)n + (n^2+b''n)m = A(m+n)^3 + H(m+n)^2 + C(m+n)$ unde $A = 1$, $H = b + b' + b''$, C vero quæcunque quantitas ad libitum aſſumenda: in hoc caſu per m^2+bm, $m^2+b'm$, & $m^2+b''m$ volo coefficientes quantitatum QR, QS & RS in poteſtate m reſpective; coefficiens vero omnis termini, in quo continentur quatuor literæ Q, R, S, T erit $(m^4 + Km^3 + Lm^2 + Mm)$, ubi $3K = H + H' + H'' + H''' + b + c + d + e + f + g$, $L = bc + de + fg + C + C' + C'' + C'''$, exiſtentibus $m^3 + Hm^2 + Cm$, $m^3 + H'm^2 + C'm$; $m^3 + H''m^2 + C''m$, $m^3 + H'''m^2 + C'''m$; & $m^2 + bm$, $m^2 + cm$, $m^2 + dm$, $m^2 + em$, $m^2 + fm$, $m^2 + gm$ reſpective coefficientibus quantitatum QRS, QRT, QST, RST; & QR, ST; QS, RT; QT, RS; M vero erit quæcunque quantitas ad libitum aſſumenda: & ſic inveniri poſſunt coefficientes quantitatum, in quibus plures continentur literæ; etiamque inveniri poſſunt innumeræ ſeries diverſi generis, quarum poteſtates vel functiones vel eandem obſervant legem, vel quamcunque aliam legem.

In theoremate ſequente per $\left(\frac{\overset{n+m+r+\&c.}{\dot{P}}}{\dot{x}^{n}\dot{y}^{m}\dot{z}^{r}\ \&c.}\right)$ eadem, quæ in prob. 67. libri primi, deſignetur quantitas.

THEOR. XXII.

1. Sit quantitas P, quæ duas variabiles quantitates x & y continet, quarum incrementa ſint e & o: novus valor quantitatis $(P) = P + \left(\left(\frac{\dot{P}}{\dot{x}}\right)e + \left(\frac{\dot{P}}{\dot{y}}\right)o\right) + \left(\frac{1}{2}\left(\frac{\ddot{P}}{\dot{x}^{2}}\right)e^{2} + \frac{1}{2}\left(\frac{\ddot{P}}{\dot{y}^{2}}\right)o^{2} + \left(\frac{\ddot{P}}{\dot{x}\dot{y}}\right)e\,o\right) + \left(\frac{1}{6}\left(\frac{\dddot{P}}{\dot{x}^{3}}\right)e^{3} + \frac{1}{2}\left(\frac{\dddot{P}}{\dot{x}^{2}\dot{y}}\right)e^{2}o + \frac{1}{2}\left(\frac{\dddot{P}}{\dot{x}\dot{y}^{2}}\right)e\,o^{2} + \frac{1}{6}\left(\frac{\dddot{P}}{\dot{y}^{3}}\right)o^{3}\right) +$ &c. in infinitum, ubi coefficiens termini $\left(\frac{\overset{n+m}{\dot{P}}}{\dot{x}^{n}\dot{y}^{m}}\right)e^{n}o^{m}$ ſemper erit $\frac{1}{1.2.3..n} \times \frac{1}{1.2.3..m}$.

2. Si vero dentur tres (x, y, z) variabiles quantitates in quantitate P, quarum incrementa ſint e, o & i; evadet novus valor quantitatis $(P) = P + \left(\frac{\dot{P}}{\dot{x}}\right)e + \left(\frac{\dot{P}}{\dot{y}}\right)o + \left(\frac{\dot{P}}{\dot{z}}\right)i +$ &c.; tum erit coefficiens termini $\left(\left(\frac{\overset{n+m+r}{P}}{\dot{x}^{n}\dot{y}^{m}\dot{z}^{r}}\right)e^{n}o^{m}i^{r}\right) = \frac{1}{1.2.3...n} \times \frac{1}{1.2.3...m} \times \frac{1}{1.2.3..r}$.

3. Si dentur quatuor vel plures $(x, y, z, v,$ &c.$)$ variabiles quantitates in quantitate, quarum incrementa ſint $e, o, i, k,$ &c, tum coefficiens termini $\left(\frac{\overset{n+m+r+s+\&c.}{P}}{\dot{x}^{n}\dot{y}^{m}\dot{z}^{r}\dot{v}^{s}\ \&c.}\right)e^{n}o^{m}i^{r}k^{s}$ &c. in novo valore erit $= \frac{1}{1.2.3..n} \times \frac{1}{1.2.3..m} \times \frac{1}{1.2.3..r} \times \frac{1}{1.2.3..s}$, &c.: in hoc novo valore cunctæ inſuper adjungendæ ſunt quantitates prædicti generis, ubi indices n, m, r & s integros reſpective denotant numeros.

Et ſic deinceps.

Cor. 1. Sint duæ æquationes $A = o$ & $B = o$, quæ ſint reſpective functiones quantitatum x & y; inveniantur fluxiones quantitatum A & B ex hypotheſi quod x ſolummodo ſit variabilis & $\dot{x}$ ſit conſtans, quæ ſint reſpective $\left(\frac{\dot{A}}{\dot{x}}\right) = p$, $\left(\frac{\ddot{A}}{\dot{x}^2}\right) = p'$, $\left(\frac{\dddot{A}}{\dot{x}^3}\right) = p''$, &c. $\left(\frac{\dot{B}}{\dot{x}}\right) = q$, $\left(\frac{\ddot{B}}{\dot{x}^2}\right) = q'$, $\left(\frac{\dddot{B}}{\dot{x}^3}\right) = q''$, &c. deinde inveniantur fluxiones quantitatum A & B ex hypotheſi, quod y ſolummodo ſit variabilis & $\dot{y}$ conſtans, & ſint reſpective $\left(\frac{\dot{A}}{\dot{y}}\right) = r$, $\left(\frac{\ddot{A}}{\dot{y}^2}\right) = r'$, $\left(\frac{\dddot{A}}{\dot{y}^3}\right) = r''$, &c. $\left(\frac{\dot{B}}{\dot{y}}\right) = s$, $\left(\frac{\ddot{B}}{\dot{y}^2}\right) = s'$, $\left(\frac{\dddot{B}}{\dot{y}^3}\right) = s''$, &c. ultimo ſupponantur quantitates x & y variabiles, $\dot{x}$ vero & $\dot{y}$ conſtantes, & ſint $\left(\frac{\ddot{A}}{\dot{x}\dot{y}}\right) = n$, &c. $\left(\frac{\ddot{B}}{\dot{x}\dot{y}}\right) = m$, &c. aſſumantur k & l proximi valores quantitatum x & y reſpective, ſcribantur hi valores pro x & y reſpective in quantitatibus A, B; p, p', p'', &c. q, q', q'', &c. r, r', r'', &c. s, s', s'', &c. n, m, &c. & ſint quantitates reſultantes a, b; P, P', P'', &c. Q, Q', Q'', &c. R, R', R'', &c. S, S', S'', &c. N, M, &c. tum aſſumantur æquationes $o = a + pe + r\acute{o} + \frac{1}{2}p'e^2 + n\acute{o}e + \frac{1}{2}r'\acute{o}^2 +$ &c. & $o = b + qe + s\acute{o} + \frac{1}{2}q'e^2 + m\acute{o}e + \frac{1}{2}s'\acute{o}^2 +$ &c. ſcribantur $k + e$ & $l + \acute{o}$ reſpective pro x & y in datis æquationibus $A = o$ & $B = o$, & reſultent æquationes $A' = o$ & $B' = o$, quarum radices e & $\acute{o}$ erunt eædem ac radices e & $\acute{o}$ ex aſſumptis æquationibus deductæ. Nunc ſupponantur $o = a + pe + r\acute{o}$ & $o = b + qe + s\acute{o}$, & ex his æquationibus conſequuntur $e = \frac{sa - rb}{rq - sp}$ & $\acute{o} = \frac{qa - pb}{ps - qr}$: hinc conſtat, quod in omnibus caſibus approximationes ex hâc methodo deductæ pendent ex hoc, utrum quantitates k & l aſſumptæ pro radicibus ipſis ſint multo propiores ad radices quæſitas, quam ad reliquas incognitarum (x & y) radices, necne.

Cor. 2. Si vel duæ vel plures radices quæſitæ datarum æquationum ſint prope inter ſe æquales, tum plures termini ex æquationibus aſſumptis retineantur: quod etiam, cum radices quæſitæ haud ſint prope æquales, propiorem approximationem deducet.

Eadem principia etiam applicari poſſunt ad plures (m) æquationes plures m incognitas quantitates habentes.

Cor. 3. Sint duæ prædictæ æquationes $\alpha = 0$ & $\pi = 0$, pro x & y in his æquationibus ſcribantur reſpective $h + k\sqrt{(-1)} + a\sqrt{(-1)} + e$ & $m + n\sqrt{(-1)} + o + u\sqrt{(-1)}$, ubi $h + k\sqrt{(-1)}$ & $m + n\sqrt{(-1)}$ ſunt quantitates multo propiores ad unum valorem quantitatum x & y quam ad reliquos; ſcribantur hi valores pro x & y in datis æquationibus, & reſultent $A + Be + Co' + D + \sqrt{(-1)}(La + Mu + N)$ prope $= 0$, $P + Qe + Ro' + S + \sqrt{(-1)}(Ta + Hu + K)$ prope $= 0$, ubi P, Q, R, S, T, H, K; A, B, C, D, L, M & N ſunt cognitæ quantitates; D vero & S in ſingulis terminis plures habent dimenſiones quantitatum e, o, a & u quam unam; reliquæ vero habeant nullas: fingantur $A + Be + Co' = 0$, $P + Qe + Ro' = 0$, $La + Mu + N = 0$ & $Ta + Hu + K = 0$, ex hiſce æquationibus inveniri poſſunt approximationes ad valores quantitatum perparvarum a, e, o' & u.

Si duo vel plures valores reſultantium quantitatum a, e, o', u, &c. ſint inter ſe æquales, tum e principiis prius traditis erui poſſunt novæ approximationes.

Ea, quæ prius tradita fuere de convergentia poſſibilium radicum, ad convergentiam impoſſibilium radicum æque applicari poſſunt.

Ex. 1. Ex datâ fluente fluxionis $(1 - ax^2)^{-\frac{1}{2}}\dot{x}$ invenire fluentem fluxionis $(1 - (a + b)x^2)^{-\frac{1}{2}}\dot{x}$, ſi modo b ſit perparva quantitas. Inveniatur incrementum quantitatis $(1 - (a + b)x^2)^{-\frac{1}{2}}$ ex hypotheſi quod a ſolummodo ſit variabilis & b ſit ejus incrementum, quod erit $(1 - ax^2)^{-\frac{1}{2}} + \frac{bx^2}{2(1 - ax^2)^{\frac{3}{2}}} + \frac{1 \cdot 3\, b^2 x^4}{2^2 \cdot 2\,(1 - ax^2)^{\frac{5}{2}}} +$

$\frac{1 . 3 . 5 b^3 x^6}{2^3 . 2 . 3 . (1 - a x^2)^{\frac{7}{2}}} + \&c.$ ducatur hæc ſeries in $\dot{x}$ & inveniantur fluxionum reſultantium fluentes, & perficitur exemplum.

Ex. 2. Sit fluxio $(a + b x^n + c x^{2n} \ldots b x^{ln})^m \dot{x}$, augeantur coefficientes a, b, c, d, &c. per perparva incrementa α, β, γ, δ, &c. & reſultat fluxio $(a + b x^n + c x^{2n} \ldots b x^{ln})^m \dot{x} + m (\alpha + \beta x^n + \gamma x^{2n} + \&c.) \times (a + b x^n + c x^{2n} + \&c.)^{m-1} \dot{x} + \frac{1}{2} m . (m - 1) (\alpha + b x^n + \gamma x^{2n} + \&c.)^2 \times (a + b x^n + c x^{2n} + \&c.)^{m-2} \dot{x} + \&c.$ ſed ex datis independentibus fluentibus l fluxionum formulæ $(a + b x^n + c x^{2n} \ldots b x^{ln})^{m \pm \pi} x^{\rho n} \dot{x}$, ubi π & ρ ſunt integri numeri, acquiri poſſunt omnium ejuſdem formulæ fluxionum fluentes, & conſequenter fluentes e ſingulis terminis prædictæ fluxionis.

Ex. 3. Sit fluxio $(a + b x + c x^2 + d x^3)^m \dot{x} = (a + b x)^m \dot{x} + m (a + b x)^{m-1} \times (c x^2 + d x^3) \dot{x} + m \times \frac{m - 1}{2} (a + b x)^{m-2} \times (c x^2 + d x^3)^2 \dot{x} + \&c.$, ubi c & d ſunt parvæ quantitates; tum erit ſeries exprimens fluentem datæ fluxionis $\frac{1}{(m + 1) b} (a + b x)^{m+1} + \left(\frac{m}{b^3} \times \left(\frac{(a + b x)^{m+2}}{m + 2} - \frac{2 a (a + b x)^{m+1}}{m + 1} + \frac{a^2 (a + b x)^m}{m}\right) \times c\right) + \left(\frac{m}{b^4} \left(\frac{(a + b x)^{m+3}}{m + 3} - \frac{3 a (a + b x)^{m+2}}{m + 2} + \frac{3 a^2 (a + b x)^{m+1}}{m + 1} + \frac{a^3 (a + b x)^m}{m}\right) \times d\right) + (m . \frac{m - 1}{2} \frac{1}{b^5} \times \left(\frac{(a + b x)^{m+4}}{m + 4} - \frac{4 a (a + b x)^{m+3}}{m + 3} + \frac{6 a^2 (a + b x)^{m+2}}{m + 2} - \frac{4 a^3 (a + b x)^{m+1}}{m + 1} + \frac{a^4 (a + b x)^m}{m}\right) c^2) + \&c.$

Ex. 4. Sit fluxio $(a + b x + c x^2 + d x^3 + e x^4)^m \dot{x} = (a + b x + c x^2)^m \dot{x} + m (a + b x + c x^2)^{m-1} (d x^3 + e x^4) \dot{x} + m . \frac{m - 1}{2} (a + b x + c x^2)^{m-2} (d x^3 + e x^4)^2 \dot{x} + \&c.$; ubi d & e ſunt parvæ quantitates: inveniantur fluentes harum fluxionum ſucceſſive, & reſultat ſeries quæſita.

Ex. 5. Sit fluxio $(a^3 + d' + (3a^2b + b')x + (3ab^2 + c')x^2 + (b^3 + d')x^3)^m \dot{x}$; ubi d', b', c', &c. ſunt parvæ quantitates; tum erit fluxio $(a + bx)^{3m}\dot{x} + m(a + bx)^{3\times(m-1)}(d' + b'x + c'x^2 + d'x^3)\dot{x} + m.\frac{m-1}{2}(a + bx)^{3\times(m-2)}(a' + b'x + c'x^2 + d'x^3)^2\dot{x} +$ &c., cujus fluens erit $\frac{1}{(3m+1)b}(a + bx)^{3m+1} +$ &c. &c.

Ex. 6. Erit $\int.\frac{\dot{x}}{x+e} = \log.(x + e) = \log.x + \frac{e}{x} - \frac{e^2}{2x^2} + \frac{e^3}{3x^3} -$ &c., & $\int.\frac{\dot{x}}{x-e} = \log.(x - e) = \log.x - \frac{e}{x} - \frac{e^2}{2x^2} - \frac{e^3}{3x^3} - \frac{e^4}{4x^4} -$ &c., & erit $\int.\frac{\dot{x}}{a^2 - (x+e)^2} = \int.\frac{\dot{x}}{a^2 - x^2} + \frac{e}{a^2 - x^2} +$ &c.

Ex. 7. Sit N numerus, cujus log. eſt v; tum erit numerus, cujus log. eſt $(v \pm e) = N \pm Ne + \frac{Ne^2}{2} \pm \frac{Ne^3}{2.3} + \frac{Ne^4}{2.3.4} + \frac{Ne^5}{2.3.4.5} +$ &c.

Ex. 8. Sit radius $= 1$ & A arcus, cujus ſinus ſit S & coſinus C, & $A \pm e$ arcus qui multum haud differt ab arcu A, i. e. ſit e perparva quantitas; tum erit ſinus arcus $(A \pm e) = S \pm Ce - \frac{1}{2}Se^2 \mp \frac{1}{2.3}Ce^3 + \frac{1}{2.3.4}Se^4 \pm$ &c. in infinitum $= S(1 - \frac{1}{2}e^2 + \frac{1}{2.3.4}e^4 -$ &c.$) \pm C(e - \frac{1}{2.3}e^3 + \frac{1}{2.3.4.5}e^5 -$ &c.$) = S \times$ coſ. arc. $e \pm C \times$ ſin. arc. e: & coſinus prædicti arcus $(A \pm e) = C \pm Se - \frac{1}{2.3}Ce^2 \mp \frac{1}{1.2.3.4.5}Se^3 + \frac{1}{2.3.4.5.6.7}Ce^4 \pm$ &c. in infinitum $= C(1 - \frac{1}{1.2}e^2 + \frac{1}{2.3.4}e^4 -$ &c.$) \pm S(e - \frac{1}{2.3}e^3 + \frac{1}{2.3.4.5}e^5 +$ &c.$) = \pm S \times$ ſin. arc. $e + C \times$ coſ. arc. e.

Cor. 1. Sit æquatio $y = A - \frac{A^3}{2\cdot 3} + \frac{A^5}{2\cdot 3\cdot 4\cdot 5} - \&c.$ vel $y = 1 - \frac{A^2}{2} + \frac{A^4}{1\cdot 2\cdot 3\cdot 4} - \&c.$, vel magis generaliter $y = E\left(A - \frac{A^3}{2\cdot 3} + \&c.\right) + F\left(1 - \frac{A^2}{2} + \frac{A^4}{2\cdot 3\cdot 4} - \&c.\right)$, ubi E & F ſunt quantitates ad libitum aſſumendæ; tum erit fluens generalis fluxionalis æquationis $\ddot{y} + y\dot{z}^2 = o$, ubi $z = A$; unde generalis fluens hujuſce fluxionalis æquationis erit $y = E \times$ ſin. arc. $z + F \times$ coſ. ejuſdem arc. z.

Cor. 2. Erit $2S \times C =$ ſin. dupli arcus A, ergo $2SC = 2\left(A - \frac{A^3}{2\cdot 3} + \frac{A^5}{2\cdot 3\cdot 4\cdot 5} - \&c.\right) \times \left(1 - \frac{A^2}{2} + \frac{A^4}{1\cdot 2\cdot 3\cdot 4} - \&c.\right) = 2A - \frac{8A^3}{2\cdot 3} + \frac{32A^5}{2\cdot 3\cdot 4\cdot 5} - \frac{128A^7}{2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7} + \&c. - \&c.$ Et ſic ex vulgari trigonometriâ detegi poſſunt multæ conſimiles propoſitiones.

8.2. Et ſimiliter ſit z log. quantitatis $x + \surd(1 + x^2)$, tum erit $x = z + \frac{z^3}{2\cdot 3} + \frac{z^5}{2\cdot 3\cdot 4\cdot 5} + \&c.$ ſit z log. quantitatis $x' + \surd(x'^2 - 1)$, tum erit $x' = 1 + \frac{z^2}{2} + \frac{z^4}{2\cdot 3\cdot 4} + \frac{z^6}{2\cdot 3\cdot 4\cdot 5\cdot 6} + \&c.$

Cor. Augeatur z per e, tum evadet $v = z + e + \frac{(z+e)^3}{2\cdot 3} + \frac{(z+e)^5}{2\cdot 3\cdot 4\cdot 5} + \&c. = x + ex' + \frac{1}{2}e^2x + \frac{1}{2\cdot 3}e^3x' + \frac{1}{2\cdot 3\cdot 4}e^4x + \&c. = x\left(1 + \frac{1}{1\cdot 2}e^2 + \frac{1}{2\cdot 3\cdot 4}e^4 + \frac{1}{2\cdot 3\cdot 4\cdot 5\cdot 6}e^6 + \&c.\right) + x'\left(e + \frac{1}{2\cdot 3}e^3 + \frac{1}{2\cdot 3\cdot 4\cdot 5}e^5 + \&c.\right) = x \times$ valorem quantitatis x' cum $z = e + x' \times$ valorem quantitatis x cum $z = e$.

Ad eundem modum inveniri poteſt $v' = 1 + \frac{(z+e)^2}{1\cdot 2} + \frac{(z+e)^4}{1\cdot 2\cdot 3\cdot 4}$

+

$+\frac{(z+e)^6}{1.2.3.4.5.6} + \&c. = x' + e \times x + \frac{e^2}{1.2} x' + \frac{e^3}{1.2.3} x + \&c. = x'(1 + \frac{e^2}{2} + \frac{e^4}{2.3.4} + \frac{e^6}{2.3.4.5.6} + \&c.) + x(e + \frac{e^3}{1.2.3} + \frac{e^5}{1.2.3.4.5} + \&c.) = x' \times$ valorem quantitatis x' cum $z = e + x \times$ valorem quantitatis x cum $z = e$; &c.

Cor. 1. Hinc generalis fluens fluxionalis æquationis $\ddot{y} = y\dot{z}^2$ erit $Ex + Fx' = y$, ubi $z = \log. x + \sqrt{(x^2+1)} = \log. x' + \sqrt{(x'^2 - 1)}$.

Cor. 2. Erit $2(z + \frac{z^3}{2.3} + \frac{z^5}{2.3.4.5} + \&c.) \times (1 + \frac{z^2}{2} + \frac{z^4}{2.3.4} + \&c.) = 2z + \frac{2^3 z^3}{2.3} + \frac{2^5 z^5}{2.3.4.5} + \&c.$ Et ſic de omnibus propoſitionibus iis conſimilibus ex vulgari trigonometriâ deductis.

Ex. 9. Sit x tangens arcus A; tum erit arcus, cujus tangens eſt $(x \pm e) = A \pm \frac{e}{1+x^2} \pm \&c.$

Ex. 10. Sit y ſinus, & erit correſpondens arcus $= \int. \frac{\dot{y}}{\sqrt{(1-y^2)}}$; & arcus, cujus ſinus eſt $y + e$, erit $\int. \frac{\dot{y}}{\sqrt{(1-(y+e)^2)}} = \int. \frac{\dot{y}}{\sqrt{(1-y^2)}} + \frac{e}{\sqrt{(1-y^2)}} + \&c.$

Ex. 11. Fluxio hyperbolici arcus (A) poteſt eſſe $\left(\frac{a^2+x^4}{x^4}\right)^{\frac{1}{2}} \dot{x}$, ubi $xy = a$, & x eſt abſciſſa & y ordinata hyperbolæ; ſit etiam $xy = a + e$, tum erit fluxio ejus hyperbolici arcus $= \left(\frac{x^4+(a+e)^2}{x^4}\right)^{\frac{1}{2}} \dot{x} = \left(\frac{x^4+a^2}{x^4}\right)^{\frac{1}{2}} \dot{x} + \frac{e \times (x^4+a^2)^{-\frac{1}{2}} \times a}{x^2} \times \dot{x} + (\frac{1}{2} \frac{(x^4+a^2)^{-\frac{1}{2}}}{x^2} - \frac{1}{2} \frac{(x^4+a^2)^{-1\frac{1}{2}}}{x^2} a^2) e^2 \dot{x} + \&c.$, cujus fluens erit $A + e \times \&c. + \&c.$

Ex. 12. Sit fluxio arcus elliptici vel hyperbolici $= \frac{\surd(t^4+(c^2\pm t^2)x^2)}{t\surd(t^2\pm x^2)}$ $\dot{x} = \dot{A}$, ubi c & t ſunt conjugatæ diametri & x abſciſſa: augeantur diametri t & c per perparvas quantitates a & e, & evadet fluxio $\dot{A} = \frac{\surd((t+a)^4+((c+e)^2\pm(t+a)^2)x^2)}{(t+a)\surd((t+a)^2\pm x^2)} \times \dot{x} = \frac{\surd(t^4+(c^2\pm t^2)x^2)}{t\surd(t^2\pm x^2)}$ $\dot{x}\,(\dot{A}) + \frac{(2t^3a+(ce\pm ta)x^2)t\times(t^2\pm x^2)-(2t^2\pm x^2)a+(t^4+}{(t^4+(c^2\pm t^2))^{\frac{1}{2}}\times t^2\times}$ $\frac{(c^2\pm t^3)x^2)}{(t^2\pm x^2)^{1\frac{1}{2}}}$ + &c., cujus fluens erit arcus hyperbolæ vel ellipſeos.

Ex. 13. Sit fluxio $\frac{\surd(1-cx^2)\dot{x}}{\surd(1-x^2)}$, ubi c ſit perparva quantitas; tum erit data fluxio (ſi modo pro $\surd(1-x^3)$ ſcribatur (P)) $= \frac{\dot{x}}{P} - \frac{cx^2\dot{x}}{2P} - \frac{c^2x^4\dot{x}}{8P} - \frac{c^3x^6\dot{x}}{16P}$ − &c. cujus fluentes reperiantur $A - \frac{c}{2}\times\frac{1.A-xP}{2} - \frac{c^2}{8}\times\frac{3B-x^3P}{4} - \frac{c^3}{16}\times\frac{5C-x^5P}{6} - \frac{5c^4}{128}\times\frac{7D-x^7P}{8}$ − &c., ubi $A = \int.\frac{\dot{x}}{P}$, $B = \frac{1.A-xP}{2}$, $C = \frac{3B-x^3P}{3}$, &c.

Hinc facile deduci poteſt peripheria ellipſeos, quæ haud multum differt a circulo.

Hinc e principiis prius traditis, ſi duæ fluxiones haud multum vel in variabilibus vel in invariabilibus quantitatibus iis contentis inter ſe differant, & fluente alterius fluxionis datâ, inveniri poteſt approximatio ad alteram.

Si inter fluxiones, quarum fluentes quæruntur, nulli dentur termini, qui vel nihil vel infiniti evadant; tum plerumque ſeries reſultantes convergent; ſin aliter vero non.

Eadem principia cum aliis prius traditis applicari poſſunt ad æquationes,

tiones, in quibus quantitates vel variabiles vel invariabiles paulullum diſtent a quantitatibus correſpondentibus in datis æquationibus; datæ enim æquationes præbent primas approximationes; in æquationibus ſupra dictis ex primis vero approximationibus inventis facile per principia prius tradita erui poſſunt approximationes propiores; & ſic deinceps.

THEOR. XXIII.

Datâ æquatione $o = p - qx + rx^2 - sx^3 +$ &c. ubi $p, q, r, s,$ &c. ſunt invariabiles quantitates; ſit $x = \alpha$ prope, ſcribatur $\alpha \times \pi$ pro x in datâ æquatione & invenietur $\pi = \dfrac{p - r\alpha^2 + 2s\alpha^3 - 3t\alpha^4 + \&c.}{q\alpha - 2r\alpha^2 + 3s\alpha^3 - 4t\alpha^4 + \&c.}$ prope; & ſic deinceps: ſint vero $\alpha, \beta, \gamma, \delta, \varepsilon,$ &c. ſucceſſivæ approximationes e prob. 13. deductæ, i. e. ſit $y = \alpha + \beta + \gamma + \delta + \varepsilon +$ &c. tum per præcedentem methodum invenietur $y = \alpha \times \dfrac{\alpha + \beta}{\alpha} \times \dfrac{\alpha + \beta + \gamma}{\alpha + \beta} \times \dfrac{\alpha + \beta + \gamma + \delta}{\alpha + \beta + \gamma} \times$ &c. quæ reſolutio etiam ad algebraicas plures variabiles quantitates habentes, ad fluxionales, incrementiales, &c. æquationes applicari poteſt.

PROB. XVIII.

Datâ quantitate π, *in quâ continentur quæcunque invariabiles quantitates* a, b, c, d, *&c.; invenire ejus valorem, cum* a *nihilo evadat æqualis.*

Ita reducatur data quantitas π, ut nec denominator nec numerator habeat denominatorem, & conſequenter in quantitate reſultante ρ haud contineatur negativa poteſtas vel radix: in quantitate ρ pro a ſcribatur $a + o$; & reducantur & numerator & denominator in duas ſeries ſecundum dimenſiones quantitatis x progredientes, quæ ſint reſpective $fo^{\lambda} + ko^{\mu} +$ &c. & $f'o^{\lambda'} + k'o^{\mu'} +$ &c.; tum ſi λ major ſit quam λ', erit $\pi = o$; ſi $\lambda = \lambda'$, erit $\pi = \dfrac{f}{f'}$; ſi autem λ minor ſit quam λ', tum erit π infinita quantitas. Aliter:

Ex datâ æquatione $y = \pi$, ubi y eſt quantitats aſſumpta haud in quantitate π contenta; inveniatur valor quantitatis $a = \sigma$; deinde ex æquatione $\sigma = 0$ inveniatur valor quantitatis $y = \tau$, & τ erit quantitas quæſita.

Et ſic inveniri poteſt valor quantitatis π, cum plures quantitates a, b, c, &c. fiant nihilo æquales.

THEOR. XXIV.

1. Summa vel differentia quarumcunque duarum vel plurium infinitarum ſerierum convergentium, vel earum functio rationalis & integralis, erit etiam convergens ſeries.

2. Sint duæ diverſæ ſeries reguraliter divergentes ſubſequentis vero formulæ $a + dr + d''r^2 + d'''r^3 + d''''r^4 +$ &c. in infinitum, tum plerumque erit earum ſumma vel differentia, &c. ſeries divergens; facile conſtant e differentiis inter correſpondentes terminos duarum ſerierum per principia prius tradita caſus, in quibus ſeries convergit.

3. Summa vel differentia duarum ſerierum infinitarum, quarum una eſt divergens, altera vero convergens, ſemper erit divergens.

4. Ducantur duæ convergentes ſeries in ſeſe, & ſemper reſultabit ſeries convergens, ut prius aſſeritur.

5. Ducatur divergens ſeries in convergentem, & æquatio reſultans poteſt eſſe vel divergens vel convergens; poteſt eſſe convergens, cum ſumma ſeriei convergentis in divergentem ductæ nihilo ſit æqualis.

6. Ducatur divergens ſeries in divergentem, & æquatio reſultans poteſt eſſe vel divergens vel convergens.

7. Dividatur vel convergens vel divergens ſeries per convergentem vel divergentem, & ſeries reſultans poteſt eſſe vel divergens vel convergens; dividatur convergens ſeries per alteram nihilo æqualem, & ſeries reſultans erit divergens; ſin aliter non.

THEOR.

THEOR. XXV.

1. Sit quantitas $(a \pm x)^m$, ubi m haud ſit affirmativus numerus, & ſeries $a^m \pm m a^{m-1} x + m \,.\, \frac{m-1}{2} a^{m-2} x^2 \pm \&c. = (a \pm x)^m$ converget, cum a major ſit quam x, diverget autem cum minor ſit.

1.2. Sit quantitas $(x \pm a)^m$, & ſeries $x^m \pm m x^{m-1} a + m \,.\, \frac{m-1}{2} x^{m-2} a^2 \pm \&c.$ erit convergens, cum x major ſit quam a, &c.

Cum x prope $= a$, tum nec hæc nec illa ſeries celeriter converget.

1.3. Sit $x = a$ & m affirmativa quantitas, tum ſeries $(a-a)^m = a^m \times$ $\left(1 - m + \frac{m-1}{2} A - \frac{m-2}{3} B + \frac{m-3}{4} C \ldots \pm \frac{m-k}{k+1} H \pm \frac{k+1-m}{k+2} I \left(1 + \frac{k+2-m}{k+3} + \frac{k+3-m}{k+4} P + \frac{k+4-m}{k+5} Q + \&c.\right.\right.$ in infinitum$\left.\left.\right)\right) = 0$; ubi literæ $A, B, C, \ldots H$ & I; etiamque P, Q, &c. præcedentes terminos reſpective denotant, & $k+2$ major eſt quam m; ultimo convergit.

Sit m negativa quantitas, & ſeries ultimo diverget.

1.4. Series $(a+x)^m = a^m + m a x^{m-1} x + \&c.$ converget, cum m ſit affirmativa vel negativa & minor quam 1 & $x = a$.

2. Sit m vel quantitas affirmativa vel negativa & minor quam 2; tum ſeries $\int . (a+x)^m \dot{x} = a^m x + \frac{m}{2} a^{m-1} x^2 + \frac{m \,.\, (m-1)}{2 \,.\, 3} a^{m-2} x^3 + \frac{m(m-1)(m-2)}{2 \,.\, 3 \,.\, 4} a^{m-3} x^4 + \&c.$, cum $x = a$, ſemper convergit: ſin aliter non.

3. Sit $\int . (a+x)^m \dot{x} = a^m x + \frac{m}{2} a^{m-1} x^2 + \frac{m(m-1)}{2 \,.\, 3} a^{m-2} x^3 + \&c.$; hæc ſeries ſemper convergit, ſi $x = -a$, & m ſit affirmativa vel negativa & minor quam 1.

4. Sit ſeries $\int . (x \pm a)^m \dot{x} = \frac{x^{m+1}}{m+1} \pm a x^m + \frac{m}{2} a^2 x^{m-1} \pm \&c.$; hæc ſeries converget, cum x major ſit quam a.

4. 2. Sit $x=a$, & ſeries $\int.(x-a)^m\dot{x}=\frac{x^{m+1}}{m+1}-ax^m+\frac{m}{2}a^2x^{m-1}-\&c.$ converget, cum m ſit affirmativa quantitas, vel negativa & minor quam -1: ſit $x=a$, & ſeries $\int.(x+a)^m\dot{x}=\frac{x^{m+1}}{m+1}+ax^m+\frac{m}{2}a^2x^{m-1}+\&c.$ converget, cum m vel ſit affirmativa vel negativa & minor quam 2.

THEOR. XXVI.

Sit quæcunque rationalis fluxio $\frac{\dot{x}}{x^n-px^{n-1}+qx^{n-2}-\&c.}$; inveniatur ejus fluens in ſerie ſecundum dimenſiones quantitatis x progrediente, quæ ſit $Ax+Bx^2+Cx^3+\&c.$ Sint vero $\alpha, \beta, \gamma, \delta$, &c. ſucceſſivæ radices æquationis $x^n-px^{n-1}+qx^{n-2}-\&c.=0$, i. e. α minor ſit quam β, β quam γ, &c. per regulas prius traditas inveniatur $\frac{\dot{x}}{x^n-px^{n-1}+\&c.}=\frac{a\dot{x}}{\alpha-x}+\frac{b\dot{x}}{\beta-x}+\frac{c\dot{x}}{\gamma-x}+\&c.$ reducantur hæ fluxiones in ſeries ſecundum dimenſiones quantitatis x progredientes, quarum inveniantur fluentes & conſtat e præcedentibus, quod ſeries exinde orta $Ax+Bx^2+Cx^3+\&c.$ haud converget, ni x inter $+\alpha$ & $-\alpha$, inter β & $-\beta$, inter γ & $-\gamma$, &c. interponatur; vel $x=-\alpha$, cum plures valores quantitatis x ſint $\pm\alpha$, tum in quibuſdam caſibus prius traditis converget ſeries; ergo neceſſe eſt ut duo valores π & ρ quantitatis x, inter quos requiratur fluens, inter $+\alpha$ & $-\alpha$, $+\beta$ & $-\beta$, $+\gamma$ & $-\gamma$, &c. interponerentur, vel $x=-\alpha$ in quibuſdam caſibus; aliter ſeries erit divergens.

Conſtat, quod convergentia ſeriei $Ax+Bx^2+\&c.$ dijudicanda eſt e convergentiâ iſtius ſeriei, quæ inter omnes conſimiles ſeries fluentes fluxionum $\frac{a\dot{x}}{\alpha-x}$, $\frac{b\dot{x}}{\beta-x}$, $\frac{c\dot{x}}{\gamma-x}$, &c. reſpective exprimentes, minime convergit: i. e. ex ſerie $\int.\frac{a\dot{x}}{\alpha-x}=\frac{ax}{\alpha}+\frac{ax^2}{2\alpha^2}+\frac{ax^3}{3\alpha^3}+\&c.$

2. Nunc transformetur data fluxio, ita ut ejus fluens ab quocunque alio valore c quantitatis x inter γ & δ duas proxime ſucceſſivas radices poſito incipiat, i. e. ſcribatur $z+c$ pro x in datâ fluxione; & conſtat, quod ſi unus valor c quantitatis x inter radices quaſcunque γ & δ interponatur, & ſit differentia d inter γ & c minor quam differentia inter c & δ, & ſi modo transformetur data fluxio $\frac{\dot{x}}{x^n - px^{n-1} + \&c.}$, ita ut fluens incipiat a puncto c, i. e. in datâ fluxione pro x & $\dot{x}$ ſcribantur $z+c$ & $\dot{z}$, fluxionis $\frac{\dot{z}}{z^n - Pz^{n-1} + Qz^{n-2} - \&c.}$ reſultantis fluentem inter duos valores f & g, qui inter duos valores $\gamma - c$ & $c - \gamma$ quantitatis z reſpective continentur per ſeriem convergentem $az + bz^2 + cz^3 + \&c.$ exprimi poſſe; vel quod idem eſt fluentem fluxionis $\frac{\dot{x}}{x^n - px^{n-1} + \&c.}$, quæ inter duos valores $c \pm f$ & $c \pm g$ quantitatis x continetur, per differentiam inter duas convergentes ſeries $af + bf^2 + cf^3 + \&c.$ & $ag + bg^2 + cg^3 + \&c.$ exprimi.

3. Si vero $c = \frac{\gamma + \delta}{2}$, tum quæcunque fluens fluxionis $\frac{\dot{x}}{x^n - px^{n-1} + \&c.}$, inter duos valores γ & δ quantitatis x e prædictâ ſerie $az + bz^2 + cz^3 + \&c.$ inveſtigari poteſt. Hinc ex eâdem ſerie $az + bz^2 + cz^3 + \&c.$ per præcedentem methodum derivatâ haud deduci poteſt fluens fluxionis $\frac{\dot{x}}{x^n - px^{n-1} + \&c.}$ inter duos valores π & ρ quantitatis x contenta, ni π & ρ inter duas ſibi ipſis proximas radices quantitatis x conſtituantur.

4. Sit $\frac{P}{(\alpha - x)(\beta - x)(\gamma - x)\&c.} = \frac{a'}{\alpha - x} + \frac{b'}{\beta - x} + \frac{c'}{\gamma - x} + \&c.$ $= A + Bx + Cx^2 + Dx^3 + \&c.$ & ſeries $A + Bx + Cx^2 + Dx^3 + \&c.$ convergit; ſi $\alpha, \beta, \gamma,$ &c. ſint majores quam x: ſint duo valores a & b quantitatis x inter duos diviſores vel radices β & α ſibi ipſis

proximas poſiti; ſcribatur $z - \pi$ pro x in datâ æquatione $\frac{P}{(\alpha - x)(\beta - x)\ \&c.}$, & reſultet $\frac{Q}{(\lambda - z)(\mu - z)(\nu - z)} = H + Lz + Mz^2$ &c. quæ converget, cum z minor ſit quam λ & μ & ν, &c. ſin major ſit quam λ vel μ vel ν, &c.; tum ſemper diverget: nunc fingatur π major quam a, minor vero quam b, & quantitates $\pi - a$ & $b - \pi$ inter minimam affirmativam radicem $\lambda = \alpha - \pi$ & minimam negativam $\mu = \pi - \beta$ poſitæ, tum hæc ſeries $H + Lz + Mz^2 +$ &c. quæ invenit aream inter duos valores a & b quantitatis x, maxime celeriter converget, cum $\pi = \frac{a + b}{2}$, ut prius docetur; & exinde $z = \pi - a$ & $b - \pi$ reſpective.

5. Cum vero ſeries exprimens fluentem fluxionis $\frac{P}{(\alpha - x)(\beta - x)}\frac{\dot{x}}{(\gamma - x)\ \&c.} = (H + Lz + Mz^2 + \&c.)\,\dot{z}$ per hanc methodum deducta haud ſatis celeriter convergat, tum interpolandi ſunt plures n termini. Invenire autem tales valores quantitatis z, ut ſeries per hanc methodum deductæ maxime celeriter convergant, pro x ſcribantur ſucceſſive n quantitates $z - \pi$, $z - \rho$, $z - \sigma$, $z - \tau$, &c. & differentiæ quantitatum π, ρ, σ, τ, &c. & radicis α (ſi modo hæ differentiæ inter α & π, α & ρ, &c. ſint reſpective minores quam reſpectivæ differentiæ inter quantitates π, ρ, &c. & β) erunt in geometricâ progreſſione; ſi vero differentiæ inter reliquas quantitates σ, τ, &c. & β ſint minores quam differentiæ prædictæ inter α & σ, τ, &c. reſpective, tum differentiæ prædictæ inter β & σ, τ, &c. erunt reſpective in eâdem geometricâ progreſſione.

6. Convergentiæ duarum ſerierum $x + \frac{x^2}{2a} + \frac{x^3}{3a^2} + \frac{x^4}{4a^3} + \&c. = \int.\frac{a\dot{x}}{a - x}$, & $z + \frac{z^2}{2b} + \frac{z^3}{3b^2} + \&c. = \int.\frac{b\dot{z}}{b - z}$, & ſimiliter duarum ſe-

rierum $x - \frac{x^2}{2a} + \frac{x^3}{3a^2} - \&c. = \int . \frac{a\dot{x}}{a+x}$, & $z - \frac{z^2}{2b} + \frac{z^3}{3b^2} - \&c. = \int . \frac{b\dot{z}}{b-z}$; erunt inter ſe æquales, cum $\frac{x}{a} = \frac{z}{b}$.

FIG. α. 7. In prædictis ſeriebus interpolandis, ut ſeries evadant maxime convergentes, neceſſe eſt ſeries prædictas ſucceſſivas æqualiter convergere: ſint tres ſucceſſivæ radices α, β & γ reſpective $A\alpha$, $A\beta$ & $A\gamma$; & requiratur fluens inter prædictos valores a & b contenta, qui 1^mo^ inter puncta α & β interjaceant: transformetur abſciſſa x, quæ incipit a puncto A, ita ut incipiat a punctis π, ρ, σ, τ, &c., & inveniantur fluentes inter puncta a & π, π & c; c & ρ, ρ & d; d & σ, σ & e; &c.; tum ut convergentiæ evadant inter ſe æquales, cum puncta a & b multo propius ad punctum β quam ad punctum α accedant, erunt $\frac{\pi a}{\pi\beta} = \frac{\pi c}{\pi\beta} = \frac{\rho c}{\rho\beta} = \frac{\rho d}{\rho\beta} = \frac{\sigma d}{\sigma\beta} = \frac{\sigma e}{\sigma\beta} = \frac{\tau e}{\tau\beta} = \&c.$, unde $a\pi = c\pi$, $c\rho = \rho d$, $\sigma d = \sigma e$, &c.; etiamque $\beta\pi$, $\beta\rho$, $\beta\sigma$, &c. erunt quantitates in geometricâ progreſſione; & ſic deinceps.

Sit b ultimum punctum diviſionis, i.e. punctum diviſionis, cum abſciſſa incipiat a puncto β; tum erit prædicta quotiens $\frac{\pi a}{\pi\beta} = \&c. = \frac{\beta b}{\gamma\beta}$ vel $\frac{\beta b}{\alpha\beta}$, prout $\gamma\beta$ minor ſit quam $\alpha\beta$, necne: in hoc caſu punctum β inter puncta a & b ponitur.

Si autem π propius ſit ad punctum α quam ad punctum β, & ρ propius ad punctum β quam ad punctum α; tum erit $\frac{a\pi}{\alpha\pi} = \frac{\pi c}{\alpha\pi} = \frac{\rho c}{\rho\beta} = \frac{\rho d}{\rho\beta} = \frac{\sigma d}{\sigma\beta} = \&c.$; cum ſeries prædictæ maxime celeriter convergant.

8. Si vero fractio $\frac{P}{(x-\alpha)\,(x-\beta)\,(x-\gamma)\ \&c.} = \frac{a'}{x-\alpha} + \frac{b'}{x-\beta} + \&c.$

&c. $= Ax^{-n} + Bx^{-n-1} +$ &c. reducatur ad terminos ſecundum reciprocas dimenſiones quantitatis x progredientes, tum ſeries erit convergens, ſi modo x ſit major quam maxima radix vel affirmativa vel negativa ($\alpha, \beta, \gamma, \delta,$ &c.); aliter non.

9. Si vero quæcunque radices ſint impoſſibiles, e. g. ſit $\alpha = c + d\sqrt{(-1)}$, tum ſi in ſerie cujus termini ſecundum reciprocas dimenſiones quantitatis x progrediuntur, x ſemper major ſit quam ulla poſſibilis radix, & quam $c + d$ & $c - d$; vel magis generaliter ſi x^n infinite major ſit quam $(c + d\sqrt{(-1)})^n + (c - d\sqrt{(-1)})^n$ (cum n ſit infinita quantitas), & x major quam quæcunque poſſibilis radix, tum ſeries exinde ortæ erunt convergentes: ſi vero in ſeriebus, quarum termini ſecundum dimenſiones quantitatis x aſcendunt, quantitas x ſit minor quam quæcunque poſſibilis radix etiamque quam $c + d$ & $c - d$ & $e + f$ & $e - f$, ubi $c \pm d\sqrt{(-1)}$ & $e \pm f\sqrt{(-1)}$, &c. ſint impoſſibiles radices, ſeries per methodum prius traditam deduci poſſunt convergentes; vel magis generaliter ſi in aſcendente ſerie x^n ſemper infinite minor ſit quam $\frac{(c^2 + d^2)^n}{(c + d\sqrt{(-1)})^n + (c - d\sqrt{(-1)})^n}$; cum n ſit quantitas infinite magna, &c., & x minor quam quæcunque poſſibilis radix, tum ſeries reſultans ſemper converget.

10. Si vero quantitas reducenda ad terminos progredientes ſecundum dimenſiones quantitatis x ſit quæcunque algebraica irrationalis fractio; fingantur irrationales quantitates reſpective & denominator nihilo æquales, & inveniantur radices æquationum exinde reſultantium, & ſi valor quantitatis x major ſit quam quæcunque poſſibilis vel impoſſibilis radix æquationum reſultantium, i. e. quam diviſor cujuſcunque irrationalis quantitatis, &c. tum deſcendens ſeries ſemper converget; ſi vero valores a & b quantitatis x inter duas proximas radices vel diviſores prædictos ponantur, tum per præcedentem transformationem inveniri poſſunt duæ aſcendentes ſeries, quæ inveniunt fluentem datæ quantitatis in $\dot{x}$ ductæ inter prædictos valores a & b quantitatis x contentam, & quæ ſemper convergent.

Et ſic per transformationem prius traditam interpolari poſſunt inter duos valores a & b quantitatis x quicunque alii, qui præbeant ſeries magis celeriter convergentes, e quibus conſequitur fluens quæſita.

Infinitæ ſunt aliæ transformationes, quæ ſeries magis convergentes præbeant.

Conſimilia iis, quæ in hoc loco & in prob. præced. traduntur, ad ſeries ſecundum legem in prob. præced. traditam, & infinitas alias facile applicari poſſunt; earum convergentiæ enim ex iiſdem principiis dijudicandæ ſunt.

11. Series ſecundum reciprocas poteſtates quantitatis x magis frequenter convergent, quam ſeries ſecundum directas poteſtates progredientes, e. g. reciprocæ ſeries ex algebraicis quantitatibus prius deductæ ſemper convergunt, cum x ſit major quam maxima affirmativa & negativa radix, i. e. quam ullus diviſor datæ quantitatis, uſque ad ejus infinitam magnitudinem.

12. Hinc ad detegendam finitam fluentem fluxionis $\frac{\dot{x}}{x^n - px^{n-1} + qx^{n-2} - \&c.}$ inter quoſcunque duos valores a & b quantitatis x, inter quos ponuntur diverſæ radices $\alpha, \beta, \gamma,$ &c. æquationis $x^n - px^{n-1} + qx^{n-2} - \&c. = o$, per præcedentem methodum ſaltem requiruntur $(2n)$ diverſæ ſeries, quæ e radicibus (x) æquationis $x^n - px^{n-1} + qx^{n-2} - \&c. = o$ datis facile acquiri poſſunt.

In hoc caſu ita transformetur data fluxio, ut fluentes a prædictis radicibus $\alpha, \beta, \gamma,$ &c. incipiant, vel ab iis terminentur.

13. Sit fluxio $\frac{(Ax^m + Bx^{m-1} + Cx^{m-2} + \&c.)\dot{x}}{x^n - px^{n-1} + qx^{n-2} - \&c.}$, ubi n ſit integer numerus, cujus fluens ſit ſeries $ax^r + bx^s + cx^t + \&c.$ terminorum ſecundum dimenſiones quantitatis x progredientium, & ſi quantitates $Ax^m + Bx^{m-1} + Cx^{m-2} + \&c.$ & $x^n - px^{n-1} + qx^{n-2} - \&c.$ nullum habeant communem diviſorem; tum omnia, quæ prius de ſerie, fluentem fluxionis $\frac{\dot{x}}{x^n - px^{n-1} + qx^{n-2} - \&c.}$ deſignante, dicta fuerint,

rint, æque ad ſeriem prædictam $ax^r + bx^s + cx^t +$ &c. applicari poſſunt.

14. Sit fluxio $(a + bx + cx^2 + dx^3 \,..\, x^n)^m \times (A + Bx^r + Cx^s + \&c.) \times \dot{x}$, ubi m haud ſit integer affirmativus numerus, & quantitates $a + bx + cx^2 + dx^3 \,..\, x^n = 0$, & $A + Bx^r + Cx^s + \&c. = 0$ nullas habeant radices inter ſe æquales: ſint radices datæ quantitatis $a + bx + cx^2 \,..\, x^n = 0$ reſpective $\alpha, \beta, \gamma, \delta$, &c. fluens vero datæ fluxionis per ſeriem $Px^h + Qx^k + Rx^l +$ &c. ſecundum dimenſiones quantitatis x progredientem exprimatur; tum hæc ſeries ſemper invenietur convergens, cum x minor ſit quam α & β & γ, &c. ſi vero major ſit, quam α vel β vel γ, &c. tum divergit ſeries.

15. Si vero ſcribantur $z + e$ & $\dot{z}$ pro x & $\dot{x}$ in datâ fluxione, & inveniatur ſeries ſecundum dimenſiones quantitatis z progrediens, viz. $P'z^{h'} + Q'z^{k'} +$ &c. quæ ſit fluens reſultantis fluxionis; in hâc ſerie pro z ſubſtituantur duo diverſi valores π & ρ quantitatis z, quibus correſpondent duo valores $\pi + e$ & $\rho + e$ quantitatis x, & ſi hi duo valores $\pi + e$ & $\rho + e$ etiamque e inter duas proxime ſucceſſivas, e. g. γ & δ prædictæ æquationis radices ponantur; & π & ρ minores ſunt quam $\pm(e - \gamma)$ vel $\pm(e - \delta)$, tum convergentes erunt duæ ſeries reſultantes.

16. Hinc etiam ad detegendam finitam fluentem fluxionis $(a + bx + cx^2 + \&c.)^m \times (A + Bx^r + \&c.)\dot{x}$ inter duos quoſcunque quantitatis x valores contentam per præcedentem methodum requiruntur ſaltem $2n$ diverſæ ſeries, quæ e radicibus æquationis $a + bx + cx^2 + dx^3 + \&c. \,..\, x^n = 0$ datis facile deduci poſſunt; $n + 2$ enim requiruntur diverſi valores prædictæ quantitatis e.

17. Eodem modo ſit fluxio $(a + bx + cx^2 + \&c.)^m (d + ex + fx^2 + \&c.)^n \times (h + kx + lx^2 + \&c.)^p$ &c. $\times (Ax^r + Bx^s + \&c.)\dot{x}$, ubi quantitates $a + bx + cx^2 +$ &c. $d + ex + fx^2 +$ &c. $h + kx +$ &c. &c. nullum habeant diviſorem inter ſe æqualem vel communem, & ſint α, β, γ, &c. λ, μ, ν, &c. π, ρ, σ, &c. &c. reſpective radices æquationum $a + bx + cx^2 + \&c. = 0$, $d + ex + fx^2 + \&c. = 0$, $h + kx + lx^2 + \&c. = 0$, &c. & ſic ſcribantur $z + e$ & $\dot{z}$ pro x & $\dot{x}$ in datâ fluxione,

&

& e fluxione reſultante inveniatur ſeries ſecundum dimenſiones quantitatis z progrediens, quæ ſit $Pz^{\Gamma} + \mathcal{Q}z^{\Delta} +$ &c. in hâc ſerie pro z ſcribantur duo valores $\daleth$ & v, quibus correſpondent duo valores $\daleth + e$ & $v + e$ quantitatis x: ſi duo valores $\daleth + e$ & $v + e$ conſtituantur inter duos proxime ſucceſſivos valores λ & μ radicum α, β, γ, &c. λ, μ, ν, &c. π, ρ, σ, &c. &c. & $\daleth$ & v ſint minores quam $\mp (e - \lambda)$ vel $\pm (e - \mu)$, tum convergent ſeries reſultantes ex hâc ſubſtitutione.

18. Hinc ſi numerus radicum α, β, γ, &c. λ, μ, ν, &c. π, ρ, &c. &c. ſit ν, tum ex 2ν diverſis ſeriebus proprie inſtitutis prædicti generis datis, quæ incipiunt a reſpectivis radicibus α, β, γ, &c. vel ad eas terminantur, acquiri poteſt fluens datæ fluxionis inter quoſcunque duos ϕ & χ quantitatis x valores contenta, ni fluens prædicta ſit infinite magna. Cum $T + e$ vel $v + e$ æqualis ſit valori radicum prædictarum, tum e principiis prius traditis in omnibus præcedentibus caſibus erui poteſt, utrum ſeries reſultans ſit convergens, necne.

Sed hic animadvertendum eſt quod ſi in omnibus præcedentibus caſibus valor quantitatis x aſſumptus ſuperet maximam radicem prædictarum æquationum, tum recurrendum eſt ad ſeriem ſecundum reciprocas poteſtates quantitatis x progredientem, & ſemper converget ſeries.

In pleriſque caſibus præſtat per methodum prius traditam plures ſeries aſſumere, quæ correſpondent pluribus valoribus quantitatis x inter ϕ & χ poſitis.

19. Si vero impoſſibiles radices in prædictis æquationibus contineantur, quæ ſint reſpective $x + a + \surd(-b^2)$ & $x + a - \surd(-b^2)$, & ſi modo dici poſſit cum converget ſeries $Ax^r + Bx^s + Cx^t +$ &c. $= \frac{1}{x^2 + 2ax + a^2 + b^2}$, quod ſemper evadet ut prius dicitur in ſeriebus ſecundum dimenſiones quantitatis x progredientibus, cum $a - b$ major ſit quam x; in ſeriebus autem ſecundum reciprocas ejus dimenſiones progredientibus, cum x major ſit quam $a + b$, tum etiam dici

poteſt, cum fluentes fluxionum prius traditarum etiam convergent; & magis generaliter, ſi $\frac{(a+\surd(-b^2))^n+(a-\surd(-b^2))^n}{2}$, ubi n ſit infinita quantitas, infinite minor ſit quam x^n, tum converget reciproca ſeries prædicta; ſi x^n infinite minor ſit quam $\frac{(a^2+b^2)^n}{(a+\surd(-b^2))^n+(a-\surd(-b^2))^n}$, tum aſcendens ſeries ſemper converget; cum autem majores ſint hæ quantitates, tum ſemper divergunt ſeries prædictæ.

20. Sit fluxio $P^{-r}\times Q^{-m}\times R\dot{x}$, cujus fluens requiritur, ubi literæ P, Q & R deſignant quaſcunque algebraicas functiones literæ x^n nullos diviſores communes habentes, r & m fractiones quaſcunque; aſſumatur $P^{-r+1}\times Q^{-m+1}\times(A+Bx^n+Cx^{2n}+Dx^{3n}+\&c.)$ pro fluente quæſitâ, & ſi hæc ſeries $A+Bx^n+Cx^{2n}+Dx^{3n}+\&c.$ in infinitum progrediatur, tum hæc ſeries ſemper converget, cum ſeries ad ſimplices terminos ex diviſione, extractione radicum, &c. reducta convergat.

21. Sit irrationalis quantitas duplicis formulæ $(a+bx+cx^2+\&c.+(e+fx+gx^2+\&c.)^m)^r=P$; ubi m & r ſunt fractionales vel negativi indices; reducatur hæc ſeries ad terminos ſecundum dimenſiones quantitatis x progredientes, viz. $Ax^t+Bx^{t+s}+\&c.$: ſint α, β, γ, δ, &c. radices æquationis $e+fx+gx^2+\&c.=0$, quarum α minor ſit quam β, β quam γ, γ quam δ, &c., deinde inveniantur radices (x) æquationis $a+bx+cx^2+\&c.+(e+fx+gx^2+\&c.)^m$ $\frac{1}{m}$ $=0$ (iiſdem valoribus radicis $\surd$ uſitatis, qui in datâ ſerie uſurpantur) quæ ſint π, ρ, σ, τ, &c., quarum π minor ſit quam ρ, ρ quam σ, σ quam τ, &c.; tum, ſi x minor ſit quam minima radix α, β, γ, &c.; π, ρ, σ, &c.; ſeries aſcendens ſemper erit convergens: ſi major ſit quam maxima α, β, γ, &c.; π, ρ, σ, &c.; ſeries reciproca, i. e. deſcendens ſemper erit convergens.

Eadem etiam applicari poſſunt ad quaſcunque algebraicas irrationales quantitates.

Conſimilia etiam vere affirmari poſſunt de convergentiâ harum algebraicarum quantitatum iis, quæ prius tradita fuere de convergentiâ ſerierum prius traditarum.

Conſimilia etiam affirmari poſſunt (mutatis mutandis) de convergentiâ ſerierum ſecundum dimenſiones quantitatis x progredientium, quæ exprimunt fluentes omnium algebraicarum quantitatum, e. g. $\int . P\dot{x}$, iis, quæ prius tradita fuere de convergentiâ ſerierum, quæ ſunt fluentes fluxionum prius traditarum.

2. 2. Hinc facile deduci poſſunt infinitæ divergentes ſeries, quæ quodammodo dici poſſunt æquales finitis quantitatibus; ſit fluens contenta inter valores a & b quantitatis x finita quantitas; vel, ſi in datâ functione pro x ſcribantur a & b reſpective, reſultent functiones, quæ ſint finitæ quantitates; deinde in datâ fluente vel functione pro x ſcribatur $z+e$, &c. & fluens vel functio reſultet, quæ in ſeriem ſecundum dimenſiones quantitatis z expanſa præbeat divergentem ſeriem, cum $z = a - e$ vel $b - e$, &c.; tum reſultans divergens ſeries, vel diverſæ reſultantes ſeries, earum ſumma, differentia, & rationalis & integralis functio quodammodo dici poteſt æqualis datæ finitæ quantitati ex datâ functione facile deducendæ.

Et ſic de pluribus hujuſcemodi functionibus.

Et idem mutatis mutandis affirmari poteſt de incrementialibus quantitatibus ſimiliter reductis.

2. 3. Sit quæcunque fluxio $A\dot{x}$, ubi A ſit functio quantitatis x; in quantitate A pro x ſcribantur reſpective a, $a + \frac{1}{n}b$, $a + \frac{2}{n}b$, $a + \frac{3}{n}b$ ... $a + \frac{n-1}{n}b$, $a + b$, ubi n eſt integer numerus, & ſint quantitates reſultantes reſpective $A, B, C, D, \ldots\ldots P, Q, R$; tum fluens fluxionis datæ inter valores a & b quantitatis x contenta, inter duas quantitates $(A + B + C \ldots P + Q)\left(\frac{a-b}{n}\right)$, & $(B + C + D \ldots P + Q + R)$

$\mathcal{Q}+R)\left(\frac{a-b}{n}\right)$ verſatur, ſi nullus valor quantitatis x in æquatione $\frac{A}{x}=0$ inter duos prædictos valores a & b contineatur.

THEOR. XXVII.

1. Sit data quantitas A functio incognitæ quantitatis x, cujus minima radix in æquationibus per methodum in præcedente problemate traditam deductis ſit α; deinde transformetur hæc quantitas A in alteram (B), cujus incognita quantitas ſit z functio quantitatis x, & inveniatur minima radix æquationum ex quantitate B per prædictam methodum reſultantium, quæ ſit β; reducantur quantitates A & B ad ſeries ſecundum dimenſiones quantitatum x & z aſcendentes, & ſint a & π correſpondentes valores quantitatum x & z, tum ſerierum convergentiæ erunt majores vel minores prout rationes $\alpha : a$ & $\beta : \pi$ ſint majores vel minores: ſint α' & β' maximæ radices prædictarum æquationum, & reducantur quantitates A & B ad ſeries ſecundum dimenſiones quantitatum x & z deſcendentes, tum convergentiæ ſerierum reſultantium erunt minores vel majores prout rationes $\alpha' : a$ & $\beta' : \pi$ ſunt majores vel minores.

2. Sit data fluxio $A\dot{x}$, ubi A eſt prædicta functio quantitatis x; & reducatur quantitas ad ſeriem ſecundum dimenſiones quantitatis x vel aſcendentem vel deſcendentem, & inveniatur ejus fluens; transformetur data fluxio $A\dot{x}$ in alteram $B\dot{z}$, cujus variabilis z eſt quæcunque functio variabilis x; reducantur duæ fluxiones $A\dot{x}$ & $B\dot{z}$ ad ſeries ſecundum dimenſiones quantitatum x & z reſpective aſcendentes vel deſcendentes; & ſint a & b valores quantitatis x, inter quos continetur fluens quæſita, etiamque π & ρ eorum correſpondentes valores quantitatis z: tum convergentiæ ſerierum reſultantium erunt majores vel minores prout rationes α': ad minorem quantitatem a vel b, vel β' ad minorem π vel ρ, in deſcendentibus ſeriebus; vel major a vel $b : \alpha$, vel major π vel $\rho : \beta$ in aſcendentibus ſeriebus, ſint minores vel majores.

Facile conſtant e præced. theor.

Ex. 1. Fluens fluxionis $\int . \frac{\dot{x}}{1+x} = 1 - \frac{1}{2}x + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \&c.$ vel $= \frac{x}{1+x} - \frac{x^2}{2(1+x)^2} + \frac{x^3}{3(1+x)^3} - \frac{x^4}{4(1+x)^4} + \frac{x^5}{5(1+x)^5} -$ &c.; hæc poſterior ſeries, quæ progreditur ſecundum dimenſiones quantitatis $\frac{x}{1+x}$ (modo x ſit affirmativa quantitas) ſemper celerius converget quam prior; nam $\frac{x}{1+x} = v$ minor erit quam x: ſit $x = 1$, tum ſeries erunt $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \&c.$ & $\frac{1}{2} - \frac{1}{2 \cdot 4} + \frac{1}{3 \times 8} - \frac{1}{4 \times 16} + \frac{1}{5 \times 32} - \frac{1}{6 \times 64} + \&c.$; hæc ſeries multo celerius converget quam prior, nam in priori ſerie ſingulus terminus erit ad ejus ſucceſſivum prope in ratione æqualitatis, in poſteriori in ratione $2 : 1$ prope: aliter, ſit $\frac{x}{1+x} = v$ & exinde $\frac{\dot{x}}{1+x} = \frac{\dot{v}}{1-v}$; radix æquationis $1 + x = 0$ eſt -1, & radix æquationis $1 - v = 0$ eſt 1; cum autem requiratur fluens inter duos valores 0 & $\frac{1}{2}$ quantitatis v, qui valoribus 0 & 1 quantitatis x correſpondent: minima vel unica radix prioris æquationis $1 + x = 0$ erit ad quantitatem $x :: \pm 1 : 1$; at unica radix æquationis $1 - v = 0$ erit ad quantitatem v (in majore quam præcedente ratione) $:: 1 : \frac{1}{2}$; ergo ſeries $1 - \frac{1}{2 \cdot 2^2} + \frac{1}{3 \cdot 2^3} - \&c.$ magis celeriter coverget, quam correſpondens ſeries $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \&c.$

Ex. 2. Sit ſeries $\int . \frac{\dot{x}}{1+x} = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \&c.$; pro x & $\dot{x}$ ſcribantur $\frac{a+bv}{c+dv}$ & $\frac{bc-da}{(c+dv)^2}\dot{v}$ in fluxione $\frac{\dot{x}}{1+x}$, & reſultat $\frac{(bc-da)\dot{v}}{(c+dv)(c+a+(d+b)v)}$: cujus fluens reducta ad ſeriem evadet $(bc-da)\left(\frac{1}{c.(c+a)}v - \frac{2cd+cb+ad}{c^2(c+a)^2} \times \frac{v^2}{2} + \&c.\right)$; hujuſce ſeriei quicunque terminus ad infinitam diſtantiam erit ad ejus ſucceſſivum quam proxime in minore duarum ſubſequentium ratione, viz. $c : dv$ & $c + a : (d + b)\,v$: ſi requiratur fluens fluxionis inter duos valores α & β quantitatis x contenta, & conſequenter inter duos correſpondentes valores $\frac{c\alpha - a}{b - d\alpha}$ & $\frac{c\beta - a}{b - d\beta}$ quantitatis v; tum convergentia feriei $x - \frac{1}{2}x^2 + \frac{1}{3}x^3 -$ &c. i. e. ſerierum $\alpha - \frac{1}{2}\alpha^2 + \frac{1}{3}\alpha^3 -$ &c. & $\beta - \frac{1}{2}\beta^2 + \frac{1}{3}\beta^3 -$ &c. major vel minor erit, prout minor ratio $1 : \alpha$ vel $1 : \beta$ major vel minor ſit; & convergentia ſeriei $\frac{1}{c\,(c + a)}\,v - \frac{2cd + cb + ad}{c^2\,(c + a)^2} \times \frac{v^2}{2} +$ &c. major vel minor erit prout minor ratio $c : \frac{c\alpha - a}{b - d\alpha} \times d$, $c : \frac{c\beta - a}{b - d\beta} \times d$; $a + c : (d + b) \times \frac{c\alpha - a}{b - d\alpha}$ vel $a + c : (d + b) \times \frac{c\alpha - a}{b - d\alpha}$ major vel minor ſit: convergentia totius dijudicanda eſt ex minori convergentiâ.

Hoc conſtat ex theoremate præcedente; nam radix denominatoris $1 + x = 0$ in priori caſu eſt -1; in poſteriori caſu radices denominatoris ſunt $= -\frac{c}{d}$ & $= -\frac{d + b}{c + a}$; & rationes radicum ad v in poſteriori caſu & radicis -1 ad x in priori erunt præcedentes.

Exhinc facile deduci poſſunt valores quantitatum a, b, c & d; in quibus prædictæ rationes evadunt maximæ, & conſequenter ſeries convergunt maxime.

THEOR. XXVIII.

Sit P functio quantitatis x; reducatur in duas ſeries, quarum altera $ax^r + bx^{n+r} + cx^{2n+r} +$ &c. $= A$ ſecundum dimenſiones quantitatis x^n aſcendit, altera vero $a'x^{r'} + bx^{r'-n} +$ &c. ſecundum dimenſiones ejuſdem quantitatis x^n deſcendit; & ſi una ſeries A covergat cum $x = \alpha$, tum altera B ſemper divergit cum $x = \alpha$: ni cum $x = \beta$, ubi ſeries (A) exinde reſultans ſit ultima ſeries convergens, vel prima quæ non convergit; etiamque ſit ſeries B prima convergens, vel ultima quæ non convergit: & in utroque caſu eadem ſit ſeries.

Ex. 1. Sit $\int . \frac{\dot{x}}{1+x^2} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \&c. = -\frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} + \&c.$; tum ſi una ſeries convergit, altera neceſſario diverget; ni $x = 1$, in quo caſu utraque ſeries eadem evadit, at una eſt alterius negativa, viz. una eſt $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \&c.$, altera vero $-1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} - \&c.$

Cor. Series, quæ eſt ſumma duarum prædictarum ſerierum $A \pm B$ ſemper divergit, ni utraque ſeries eadem evadat, $A = B$; i. e. in exemplo ſeries $x - x^{-1} - \left(\frac{x^3 - x^{-3}}{3}\right) + \left(\frac{x^5 - x^{-5}}{5}\right) - \&c.$ ſemper divergit, ni $x = 1$.

THEOR. XXIX.

Reducatur functio data quantitatis x ad minimos terminos, ita ut quantitates in numeratore & denominatore contentæ nullum habeant denominatorem: fingatur denominator $\mathfrak{Q} = 0$, & omnis irrationalis quantitas vel in numeratore vel in denominatore contenta nihilo æqualis, & ſit α minima radix affirmativa vel negativa, at non $= 0$ prædictarum reſultantium æquationum; deinde reducatur data functio ad ſeriem ſecundum dimenſiones quantitatis x aſcendentem, tum hæc ſeries ſemper converget, ſi quantitas x contineatur inter α & $-\alpha$, quodſi x major ſit quam α, tum prædicta ſeries diverget.

Si hæc ſeries in $\dot{x}$ ducatur, tum ſeries denotans fluentem inter duos valores a & b quantitatis (x) contentam ſemper converget, cum a & b inter α & $-\alpha$ interponantur.

2. Reducatur prædicta functio ad ſeriem ſecundum reciprocas dimenſiones quantitatis x deſcendentem; & ſit λ maxima radix prædictarum reſultantium æquationum, & requiratur fluens inter duos valores a & b quantitatis x contenta; ſeries ſemper converget, cum

utræque a & b ſint majores quam λ; ſi una ſit minor quam λ, tum ſemper diverget ſeries prædicta.

3. Cum $x = \alpha$ in priori caſu, vel $= \lambda$ in poſteriori; tum in quibuſdam caſibus evadet convergens ſeries, in quibuſdam non; qui caſus facile conſtant e principiis prius traditis.

4. Si quædam radices ſint impoſſibiles, e. g. ſint $a + b\sqrt{(-1)}$ & $a - b\sqrt{(-1)}$; tum aſcendens ſeries ſemper converget, cum x^{-n} infinite major ſit quam $\frac{2\left(a^n - n \,.\, \frac{n-1}{2} a^{n-2}b^2 + n \,.\, \frac{n-1}{2} \,.\, \frac{n-2}{3} \,.\, \frac{n-3}{4} a^{n-4}b^4 - \&c.\right)}{(a^2 + b^2)^n}$, ubi n eſt quantitas infinite magna; & deſcendens ſeries ſemper converget, cum x^n infinite major ſit quam $2\left(a^n - n \,.\, \frac{n-1}{2} a^{n-2}b^2 + n \,.\, \frac{n-1}{2} \,.\, \frac{n-2}{3} \,.\, \frac{n-3}{4} a^{n-4}b^4 + \&c.\right)$.

Ea, quæ prius tradita fuerunt de interpolatione ſerierum, hìc etiam applicari poſſunt.

Cor. Exhinc deduci poſſunt infinitæ convergentes ſeries, quarum ſummæ innoteſcunt; nam aſſumatur quæcunque functio quantitatis x, &c. pro ſummâ, & reducatur ea ad ſeriem ſecundum dimenſiones quantitatis x, &c. progredientem; per theorema hìc datum ita aſſumatur valor quantitatis x, ut ſeries evadat convergens; & conſit coroll.

PROB. XIX.

Transformare datam algebraicam quantitatem in alteram, cujus termini progrediuntur ſecundum quantitates formulæ $a\,x^r\,(x \pm 1)^r$.

Erunt $\pm x^n = x^n \times (x \pm 1)^n - n x^{n+1} \times (x \pm 1)^{n+1} + n \,.\, \frac{n+3}{2} x^{n+2} \times (x \pm 1)^{n+2} - n \,.\, \frac{n+4}{2} \,.\, \frac{n+5}{3} x^{n+3} \times (x \pm 1)^{n+3} + n \,.\, \frac{n+5}{2} \,.\, \frac{n+6}{3} \,.\, n+7$

$\frac{n+7}{4} x^{n+4} \times (x \pm 1)^{n+4}$ — &c. & $x^{-n} = x^{-\frac{1}{2}n} \times (x \pm 1)^{-\frac{1}{2}n} \pm \frac{n}{2} \times x^{\frac{-n-1}{2}} \times (x \pm 1)^{\frac{-n-1}{2}} + \frac{n^2}{8} x^{\frac{-n-2}{2}} \times (x \pm 1)^{\frac{-n-2}{2}} \pm \frac{n \times (n+1) \times (n-1)}{2 \cdot 4 \cdot 6} x^{\frac{-n-3}{2}} \times (x \pm 1)^{\frac{-n-3}{2}}$ — &c.

Scribantur hæ quantitates pro ſuis valoribus in datâ algebraicâ quantitate, & perficitur problema.

Ex vulgaribus vel novis, &c. modis multiplicandi, dividendi & radices extrahendi reduci poteſt algebraica quantitas ad ſeriem infinitam ſecundum prædictam legem progredientem.

Ex methodis haud diſſimilibus transformari poſſunt datæ algebraicæ quantitates in terminos ſecundum dimenſiones quantitatum hujuſce formulæ $x^r \times (x \pm \underset{\cdot}{x})^r \times (x \pm 2\underset{\cdot}{x})^r \ldots (x \pm n\underset{\cdot}{x})^r$, vel ſecundum infinitas alias progredientes.

THEOR. XXX.

Incrementum geometricæ ſeriei $ax + a^2x^2 + a^3x^3 +$ &c. erit $= \frac{a(x+\underset{\cdot}{x})}{1-ax-a\dot{x}} - \frac{ax}{1-ax} = \left(\frac{a\underset{\cdot}{x}}{(1-ax)^2} + \frac{a^2\underset{\cdot}{x}^2}{(1-ax)^3} + \frac{a^3\underset{\cdot}{x}^3}{(1-ax)^4} + \&c.\right) = \frac{a\underset{\cdot}{x}}{(1-a\underset{\cdot}{x})} + \left(\frac{a}{(1-a\underset{\cdot}{x})^2} - a\right)x + \left(\frac{a^2}{(1-a\underset{\cdot}{x})^3} - a^2\right)x^2 + \left(\frac{a^3}{(1-a\underset{\cdot}{x})^4} - a^3\right)x^3 + \&c.$

THEOR. XXXI.

1. Sit ſeries $A + Bx + Cx^3 + Dx^5 + Ex^7 +$ &c. in infinitum; pro ejus integrali primi ordinis, cum x creſcat uniformiter, aſſumi poteſt ſeries $ax + bx \times (x - \underset{\cdot}{x}) + cx^2(x - \underset{\cdot}{x})^2 + dx^3(x - \underset{\cdot}{x})^3 +$ &c. cujus incrementum erit $a\underset{\cdot}{x} + bx(x + \underset{\cdot}{x} - (x - \underset{\cdot}{x})) + cx^2((x + \underset{\cdot}{x})^2 - (x - \underset{\cdot}{x})^2) + \&c. = a\underset{\cdot}{x} + 2bx\underset{\cdot}{x} + (4c + 2d\underset{\cdot}{x}^2)x^3\underset{\cdot}{x} + (6d + 8e\underset{\cdot}{x}^3)$

$8e\underset{\cdot}{x}^3)x^5\underset{\cdot}{x}+$ &c. unde $a=\frac{A}{\underset{\cdot}{x}}$, $B=2b\underset{\cdot}{x}$, &c. vel ſeries $ax+bx.$ $(x-\underset{\cdot}{x})+cx^3\times(x-\underset{\cdot}{x})^3+dx^5.(x-\underset{\cdot}{x})^5+$ &c. cujus incrementum erit $a\underset{\cdot}{x}+2b\underset{\cdot}{x}x+2c\underset{\cdot}{x}^3x^3+(2d\underset{\cdot}{x}^4+6c)\underset{\cdot}{x}x^5+$ &c. vel ſeries $ax+bx^2+cx^2(x-\underset{\cdot}{x})^2+dx^4(x-\underset{\cdot}{x})^4+$ &c. cujus incrementum erit $a\underset{\cdot}{x}+b\underset{\cdot}{x}^2+2b\underset{\cdot}{x}x+4c\underset{\cdot}{x}x^3+8d\underset{\cdot}{x}x^5+$ &c. & ex æquatis correſpondentibus harum & datæ ſeriei $(A+Bx+Cx^3+$ &c.) terminis inveſtigari poſſunt coefficientes $a, b, c, d,$ &c.

2. Sit ſeries infinita $A+Bx+Cx^2+Dx^3+$ &c. aſſumatur pro ejus integrali primi ordinis, cum x creſcat uniformiter, ſeries $ax+bx\times(x-\underset{\cdot}{x}).(x-2\underset{\cdot}{x})+cx^2\times(x-x)^2\times(x-2\underset{\cdot}{x})^2+dx^3.(x-\underset{\cdot}{x})^3.(x-2\underset{\cdot}{x})^3+$ &c. cujus incrementum erit $a\underset{\cdot}{x}+3b\underset{\cdot}{x}\times x\times(x-\underset{\cdot}{x})-c(3\underset{\cdot}{x}^2-6x\underset{\cdot}{x})x^2\times(x-\underset{\cdot}{x})^2+$ &c. $=a\underset{\cdot}{x}-3b\underset{\cdot}{x}^2\times x+(3b\underset{\cdot}{x}-3c\underset{\cdot}{x}^4)x^2+$ &c. & ex æquatis hujuſce & datæ ſeriei correſpondentibus coefficientibus erui poſſunt quantitates $a, b, c, d,$ &c.

Et ſic infinitæ diverſæ ſeries ſecundum dimenſiones quantitatis x progredientes facile deduci poſſunt, quæ datam ſeriem deſignant.

2. 2. Si vero integralis n ordinis prædictæ ſeriei $A+Bx+Cx^2+$ &c. requiratur: pro integrali quæſitâ aſſumatur ſeries $ax+bx\times(x\pm\underset{\cdot}{x})\times(x\pm2\underset{\cdot}{x})\ldots(x\pm n\underset{\cdot}{x})+cx^2\times(x\pm\underset{\cdot}{x})^2\times(x\pm2\underset{\cdot}{x})^2\ldots(x\pm n\underset{\cdot}{x})^2+dx^3\times(x\pm\underset{\cdot}{x})^3\times(x\pm2\underset{\cdot}{x})^3$ &c. cujus incrementum n ordinis inveniatur in terminis ſecundum dimenſiones quantitatis x progredientibus, & reſultantis & datæ ſeriei termini correſpondentes inter ſe æquentur; & exinde deduci poſſunt coefficientes $a, b, c, d,$ &c.

3. Seriem infinitam $A+Bx^{-n-1}+Cx^{-n-2}+Dx^{-n-3}+$ &c. in alteram transformare, cujus termini ſecundum reciprocas dimenſiones quantitatis x progrediuntur, & cujus integralis cujuſcunque n ordinis exprimi etiam poteſt in terminis ſecundum eandem legem progredientibus.

Aſſumatur ſeries $a+\frac{b}{x}+\frac{c}{x\times(x+\underset{\cdot}{x})}+\frac{d}{x\times(x+\underset{\cdot}{x})\times(x+2\underset{\cdot}{x})}+\frac{e}{x\times(x+\underset{\cdot}{x})\times(x+2\underset{\cdot}{x})\times(x+3\underset{\cdot}{x})}+$ &c. & inveniatur incrementum n ordinis, & deinde æquentur termini datæ & reſultantis æquationis correſpondentes, & ex æquationibus reſultantibus erui poſſunt coefficientes a, b, c, &c.

4. Sit ſeries $Ax^{s} + Bx^{s+t} + Cx^{s+2t} + Dx^{s+3t} +$ &c. cujus integralis requiritur; pro integrali primi ordinis aſſumatur $ax^{s} \times (x \pm \underset{\cdot}{x})^{s} + bx^{s+1} \times (x \pm \underset{\cdot}{x})^{s+1} + cx^{s+2} \times (x \pm \underset{\cdot}{x})^{s+2} +$ &c. &c. $+ a'x^{s+t} \times (x \pm \underset{\cdot}{x})^{s+t} + b'x^{s+t+1} \times (x \pm \underset{\cdot}{x})^{s+t+1} + c'x^{s+t+2} \times (x \pm \underset{\cdot}{x})^{s+t+2} +$ &c. $+ a''x^{s+2t} \times (x \pm \underset{\cdot}{x})^{s+2t} + b''x^{s+2t+1} \times (x \pm \underset{\cdot}{x})^{s+2t+1} +$ &c. $+$ &c. inveniatur ejus incrementum, cujus termini fiant reſpective æquales correſpondentibus datæ ſeriei terminis, & exinde erui poſſunt cofficientes a, b, c, &c. a', b', c', &c. a'', b'', c'', &c. &c.

Si requiratur integralis n ordinis prædictæ ſeriei, aſſumatur pro integrali quæſitâ quantitas $ax^{s} \times (x \pm \underset{\cdot}{x})^{s} \times (x \pm 2\underset{\cdot}{x})^{s} \,..\, (x \pm n\underset{\cdot}{x})^{s} + bx^{s+1} \times (x \pm \underset{\cdot}{x})^{s+1} \times (x \pm 2\underset{\cdot}{x})^{s+1} \,..\, (x \pm n\underset{\cdot}{x})^{s+1} + cx^{s+2} \times (x \pm \underset{\cdot}{x})^{s+2} \times (x \pm 2\underset{\cdot}{x})^{s+2} \,..\, (x \pm n\underset{\cdot}{x})^{s+2} +$ &c. $+ a'x^{s+t} \times (x \pm \underset{\cdot}{x})^{s+t} \times (x \pm 2\underset{\cdot}{x})^{s+t} \,..\, (x \pm n\underset{\cdot}{x})^{s+t} + b'x^{s+t+1} \times (x \pm \underset{\cdot}{x})^{s+t+1} \times (x \pm 2\underset{\cdot}{x})^{s+t+1} \,..\, (x \pm n\underset{\cdot}{x})^{s+t+1} +$ &c. $+ a''x^{s+2t} \times (x \pm \underset{\cdot}{x})^{s+2t} \times (x \pm 2\underset{\cdot}{x})^{s+2t} \,...\, (x \pm n\underset{\cdot}{x})^{s+2t} +$ &c. $+$ &c. inveniatur ejus incrementum n ordinis, deinde ejus & datæ ſeriei correſpondentes termini fiant inter ſe æquales, & exinde erui poſſunt cofficientes a, b, c, &c. a', b', c', &c. a'', b'', c'', &c.

5. In aſſumptis quantitatibus hujuſce formulæ $ax^{r} \times (x - \underset{\cdot}{x})^{r} \times (x - 2\underset{\cdot}{x})^{r} \times \,...\, (x - n\underset{\cdot}{x})^{r}$, ſi r ſit fractio, cujus denominator ſit par numerus, & impoſſibiles quantitates irrepant in ſeriem quæſitam; tum impoſſibiles quantitates evitandi gratiâ aſſumantur quantitates hujuſce formulæ vel $(-x)^{r} \times (\underset{\cdot}{x} - x)^{r} \times (2\underset{\cdot}{x} - x)^{r} \,...\, (n\underset{\cdot}{x} - x)^{r}$ vel $x^{r} \times (x + \underset{\cdot}{x})^{r} + (x + 2\underset{\cdot}{x})^{r} \,...\, (x + n\underset{\cdot}{x})^{r}$, cujus incrementum eſt affirmativum.

Hæc methodus uſui inſervit, cum duo valores quantitatis, inter quos requiritur integralis, ſint perparvæ quantitates.

6. Sit ſeries $Ax^{-s} + Bx^{-s-t} + Cx^{-s-2t} +$ &c. cujus integralis requiritur in terminis ſecundum reciprocas dimenſiones quantitatis x progredientibus: aſſumatur pro integrali primi ordinis quantitas $ax^{-\frac{1}{2}s} \times (x \pm \underset{\cdot}{x})^{-\frac{1}{2}s} + bx^{-\frac{1}{2}s-\frac{1}{2}} \times (x \pm \underset{\cdot}{x})^{-\frac{1}{2}s-\frac{1}{2}} + cx^{-\frac{1}{2}s-1} \times$

$(x \pm \dot{x})^{-\frac{1}{2}s-1} + \&c. + a'x^{-\frac{1}{2}s-\frac{1}{2}t} \times (x \pm \dot{x})^{-\frac{1}{2}s-\frac{1}{2}t} + b'x^{-\frac{1}{2}s-\frac{1}{2}t-\frac{1}{2}} \times (x \pm \dot{x})^{-\frac{1}{2}s-\frac{1}{2}t-\frac{1}{2}} + c'x^{-\frac{1}{2}s-\frac{1}{2}t-1} \times (x \pm \dot{x})^{-\frac{1}{2}s-\frac{1}{2}t-1} + \&c. + a''x^{-\frac{1}{2}s-t} \times (x \pm \dot{x})^{-\frac{1}{2}s-t} + \&c. + \&c.$ cujus inveniatur incrementum primi ordinis, & ex terminis datæ & reſultantis ſeriei correſpondentibus inter ſe æqualibus eſſe ſuppoſitis, erui poſſunt coefficientes a, b, c, &c. a', b', c', &c. a'', b'', c'', &c. &c.

6. 2. Si modo requiratur integralis n ordinis ſeriei prius traditæ $Ax^{-s} + Bx^{-s-t} + Cx^{-s-2t} + \&c.$ & cujus termini ſecundum dimenſiones quantitatis x progrediuntur: pro integrali quæſitâ aſſumatur quantitas $ax^{-\frac{s}{n}} \times (x \pm \dot{x})^{-\frac{s}{n}} \times (x \pm 2\dot{x})^{-\frac{s}{n}} \ldots (x \pm (n-1)\dot{x})^{-\frac{s}{n}} + bx^{-\frac{s-1}{n}} \times (x \pm \dot{x})^{-\frac{s-1}{n}} \times (x \pm 2\dot{x})^{-\frac{s-1}{n}} \ldots (x \pm (n-1)\dot{x})^{-\frac{s-1}{n}} + cx^{-\frac{s-2}{n}} \times (x \pm \dot{x})^{-\frac{s-2}{n}} \ldots (x \pm (n-1)\dot{x})^{-\frac{s-2}{n}} + \&c. + a'x^{\frac{-s-t}{n}} \times (x \pm \dot{x})^{\frac{-s-t}{n}} \times (x \pm 2\dot{x})^{\frac{-s-t}{n}} \ldots (x \pm (n-1)\dot{x})^{\frac{-s-t}{n}} + b'x^{\frac{-s-t-1}{n}} \times (x \pm \dot{x})^{\frac{-s-t-1}{n}} \times \ldots (x \pm (n-1)\dot{x})^{\frac{-s-t-1}{n}} + \&c. + a''x^{\frac{-s-2t}{n}} \times (x \pm \dot{x})^{\frac{-s-2t}{n}} \times (x \pm 2\dot{x})^{\frac{-s-2t}{n}} \ldots (x \pm (n-1)\dot{x})^{\frac{-s-2t}{n}} + \&c.$ cujus inveniatur incrementum, quod reducatur ad terminos ſecundum reciprocas dimenſiones quantitatis x progredientes, deinde e correſpondentibus datæ & reſultantis ſeriei reſpective inter ſe æquatis erui poſſunt coefficientes a, b, c, &c. a', b', c', &c. a'', b'', c'', &c.

7. Sit incrementum $P^n \times Q^m \times \&c. \times R$, cujus integralis requiritur; ubi P, Q, R, &c. ſunt functiones quantitatis x, n vero & m quæcunque fractiones; ita reducatur hoc incrementum, ut in eo nulli contineantur diviſores; qui ſint ſucceſſivæ integrales; i. e. ne ſit $T \times T'$ diviſor quantitatis $P^n \times Q^m \times \&c.$ ubi T eſt ſucceſſiva integralis quantitatis T'; reducatur datum incrementum ad hanc formulam $T \times T' \times T'' \times T''' \ldots \overset{m-1}{T'} \times \overset{m}{T'} \times A^n \times B^m \times C^s \times \&c.$ pro integrali quæſitâ (ſi requiratur ſeries aſcendens) aſſumatur ſeries $T \times T' \times T'' \times T''' \ldots \times \overset{m-1}{T'} \times A^n \times A'^n \times B^m \times B'^m \times C^s \times C'^s \times \&c. \; (a + bx^r \times (x \pm \dot{x})^r + cx^{r+1} (x \pm \dot{x})^{r+1} + \&c. + ex^{2r} \times (x \pm \dot{x})^{2r} + fx^{2r+1} (x \pm \dot{x})^{2r+1} + \&c. +$

$lx^{3r} \times (x \pm \underset{\cdot}{x})^{3r} +$ &c.) cujus inveniatur incrementum, quod fiat æquale dato incremento, & exinde deduci poſſunt coefficientes *a*, *b*, *c*, &c. *e*, *f*, &c. *l*, &c.

Si vero requiratur ſeries deſcendens, tum ſeriei præcedentis $a + bx^{r} \times (x \pm \underset{\cdot}{x})^{r} + cx^{r+1} \times (x \pm \underset{\cdot}{x})^{r+1} +$ &c. &c. loco ſubſtituatur vel $a + bx^{-\frac{r}{2}} \times (x \pm \underset{\cdot}{x})^{-\frac{r}{2}} + cx^{-\frac{r}{2}-\frac{1}{2}} \times (x \pm \underset{\cdot}{x})^{-\frac{r}{2}-\frac{1}{2}} +$ &c. $+ ex^{-2r} \times (x \pm \underset{\cdot}{x})^{-2r} + fx^{-2r-\frac{1}{2}} \times (x \pm \underset{\cdot}{x})^{-2r-\frac{1}{2}} +$ &c. vel infinitæ conſimiles ſubſtitutiones.

Facile conſtat, quod infinitæ aliæ ſeries hujuſmodi pro integrali quæſitâ aſſumi poſſunt.

Ex transformatione harum ſerierum in alias, quarum variabiles quantitates quandam habeant relationem ad variabiles quantitates datarum ſerierum, deduci poſſunt ſeries ſecundum dimenſiones aliarum quantitatum progredientes.

Hæc principia promovere liceat etiam ad inveniendas ſeries ſecundum datam legem progredientes, & quarum infinitæ aliæ functiones præter fluentes & integrales ſecundum eandem legem progrediuntur.

Ut ſeries hujuſce generis evadant convergentes vel magis celeriter convergentes, præſtat interpolare plures inter duos datos valores $(z + a\underset{\cdot}{z}$ & $z + b\underset{\cdot}{z})$ quantitatis *x*, i. e. ſcribere pro *x* diverſos valores $z + e\underset{\cdot}{z}$, $z' + e'\underset{\cdot}{z}$, &c., ubi *e*, *e'*, &c. ſunt integri numeri inter duos valores *a* & *b* poſiti.

Interpolatio hujuſce generis, & ſerierum exinde reſultantium convergentia e principiis in prob. traditis dijudicari poſſunt.

Eadem principia etiam applicari poſſunt ad ſeries ex incrementialibus æquationibus reſultantes.

PROB. XX.

Transformare datam quantitatem in terminos ſecundum datam legem progredientes: hi termini reducantur ad ſimplices terminos ſecundum dimenſiones quantitatis x *progredientes, reducatur etiam data quantitas ad terminos ſimplices ſecundum dimenſiones quantitatis* x *progredientes, & fiant* *coefficientes ſingulorum terminorum hujuſce progreſſus reſpective æquales coefficientibus correſpondentium terminorum e priore reductione deductorum.*

Ex. 1. Sit quantitas $(a+bx)^m$; invenire ſeriem, cujus lex ſit $(e+fx)^2 + (gx+hx^2)^2 + (kx^2+lx^3)^2 +$ &c. $= (a+bx)^m$; reducantur data quantitas & termini ſeriei prædictæ ad ſeries, quarum termini ſecundum dimenſiones quantitatis x progrediuntur, & evadunt $a^m + ma^{m-1}bx + m \cdot \frac{m-1}{2} a^{m-2}b^2x^2 +$ &c. & $e^2 + 2efx + (f^2+g^2)x^2 + 2ghx^3 +$ &c. & ex æquatis correſpondentibus terminis duarum reſultantium ſerierum, viz. $a^m = e^2$; $ma^{m-1}b = 2ef$; $m \cdot \frac{m-1}{2} a^{m-2}b^2 = f^2 + g^2$, &c. conſequentur $e = a^{\frac{m}{2}}$, $f = \frac{ma^{m-1}b}{2e}$, $g = \sqrt{(m \cdot \frac{m-1}{2} a^{m-2}b^2 - f^2)}$, &c.

Ex. 2. Ex eodem modo deduci poteſt $\frac{a}{z-\alpha} = \frac{a}{z} + \frac{a\alpha}{z.(z+1)} + \frac{a \times \alpha(\alpha+1)}{z.(z+1).(z+2)} + \frac{a\alpha \times (\alpha+1) \times (\alpha+2)}{z.(z+1).(z+2).(z+3)} +$ &c.

Cor. 1. Si α vel æqualis vel major ſit quam z, tum hæc ſeries haud converget; ſin aliter paululum magis celeriter hæc ſeries converget quam ſeries $\frac{a}{z} + \frac{a\alpha}{z^2} + \frac{a\alpha^2}{z^3} +$ &c.

Cor. 2. Si vero incrementum quantitatis z ſit 1, tum integralis ſeriei $\left(\frac{a}{z} + \frac{a\alpha}{z.(z+1)} + \frac{a\alpha \times (\alpha+1)}{z.(z+1).(z+2)} + \text{\&c.}\right)$ erit $\boxed{\frac{a}{z}} + \frac{a\alpha}{z} + \frac{a\alpha \times (\alpha+1)}{2z.(z+1)} +$ &c. hæc vero ſeries converget, ſi z major ſit quam α.

Hinc facile conſtat omnia, quæ traduntur de convergentiâ fluentium datarum fluxionum æque de convergentiâ integralium conſimilium incrementorum affirmari poſſe.

Facile e præcedentibus propoſitionibus deduci poſſunt convergentiæ ſerierum in hoc problemate traditarum; reducantur enim hæ ſeries ad alias ſecundum dimenſiones quantitatis x progredientes, & ex convergentiis ſerierum reſultantium plerumque facile erui poſſunt convergentiæ ſerierum prædictarum.

PROB. XXI.

Datis æquationibus algebraicis, invenire proximos valores unius incognitæ quantitatis x *terminis vero alterius* y.

E comparandis terminis datæ æquationis inter ſe ſæpe conſtat proximus valor unius incognitæ quantitatis y.

Fere vero præſtat terminos datarum æquationum, qui maximi ſint, ex hypotheſi datâ quod x vel ſit perparva vel permagna quantitas, inter ſe æquales eſſe ſupponere, neglectis omnibus reliquis terminis, qui minores ſint, & ex æquationibus reſultantibus deducere proximos valores incognitæ quantitatis quæſitæ y.

In datâ æquatione algebraicâ ſcribatur $(a+p)$ quantitatis y proximus valor a per incognitam quantitatem p auctus pro y, & e terminis reſultantis æquationis, qui ex hypotheſi datâ maximi reſultant, inter ſe æquales eſſe ſuppoſitis, neglectis iis, qui minores ſint, deduci poteſt proximus valor (b) quantitatis p; deinde vel in datâ æquatione pro y ſcribatur $a+b+q$, vel quod idem erit in reſultante æquatione pro p ſcribatur $p+q$, & conſimili methodo inveniatur proximus valor quantitatis q; & ſic deinceps; e. g. ſit æquatio $y^3+a^2y-2a^3+axy-x^3=0$ ubi x ſit perparva quantitas; ſupponatur $x=0$ & termini igitur qui maximi inveniuntur, erunt $y^3+a^2y-2a^3=0$, unde $y=a$ prope; ſcribatur $a+p$ pro y in datâ æquatione $y^3+a^2y-2a^3+axy-x^3=0$, & reſultat æquatio $p^3+3ap^2+(4a^2+ax)p+a^2x-x^3=0$; termini autem, qui maximi ſint in hâc æquatione, erunt $4a^2p+a^2x=0$, neglectis omnibus, qui minores ſint; unde $p=-\frac{x}{4}$ prope; ſcribatur igitur pro p in reſultante æquatione

$-\frac{x}{4}+q$, & inveniri poſſit $q=\frac{x^2}{64a}$ prope; & ſic deinceps; unde $y=$ $a-\frac{x}{4}+\frac{x^2}{64a}-\frac{131x^3}{512a^2}+$ &c.

2. Sit vero æquatio $y^3+axy+x^2y-a^3-2x^3=0$ data, x vero permagna quantitas; termini maximi, qui ex hâc hypotheſi in eâ continentur, erunt $y^3+x^2y-2x^3=0$, unde $y=x$ prope; ſcribatur igitur $x+p$ pro y in datâ æquatione, & reſultat æquatio $p^3+3xp^2+(4x^2+ax)p+ax^2-a^3=0$, in quâ termini, qui maximi e prædictâ hypotheſi inveniuntur, erunt $4x^2p+ax^2=0$, unde $p=-\frac{a}{4}$ prope; ſcribatur igitur in reſultante æquatione $-\frac{a}{4}+q$ pro p, & e terminis maximis inveniri poteſt $q=\frac{a^2}{64x}$ prope; & ſic deinceps; unde $y=x-\frac{a}{4}+\frac{a^2}{64x}+\frac{131a^3}{512x^2}+$ &c.

Cor. 1. Facile conſtat, quod e diviſione termini, in quem haud ingreditur q vel r, &c. per coefficientem termini, in quo continetur una ſolummodo poteſtas quantitatis p vel q vel r, &c. ſucceſſive, duplo plures termini, uno dempto, valoris quantitatis y in terminis quantitatis x detegentur.

Cor. 2. Sit b proximus valor quantitatis y in æquatione $P=0$, ſcribatur $b+p$ pro y in datâ æquatione $P=0$, & reſultet æquatio $a+bp+cp^2+dp^3+$ &c. $=0$, ubi a, b, c, d, &c. ſunt functiones ipſius x, tum aſſumantur vel $a+bp=0$, vel $a+bp+cp^2=0$, &c. & e poſterioribus aſſumptionibus conſequentur propiores valores quantitatis y, quam e prioribus, ut facile conſtat e theor. 17.

Cor. 3. Convergentia ſeriei: 1^{mo} pendet ex hoc, quanto propior ſit valor aſſumptus pro quantitate y ad unam radicem y quam ad reliquas; tum etiam, ſi modo æquatio aſſumpta ſit $a+bp=0$, pendet e rae ratione, quam habent quantitates aſſumptæ pro $\frac{b}{a}$ ad quantitatem ipſam $\frac{b}{a}$, &c.

Cor. 4. Si n ſint radices quantitatis y prope inter ſe & radici quæſitæ æquales, tum aſſumendi ſunt $(n+1)$ termini æquationis $o = a + bp + cp^2 \,..\, p^n$, e quibus detegi poteſt propior valor ſingularum n radicum a quantitate aſſumptâ haud multum diſtantium.

3. 1. Si vero x haud longe diſtet a quantitate b, tum ſcribatur quantitas b pro x in datâ æquatione, & ex æquatione reſultante $(P = o)$ inveniatur quantitas y, quæ multo magis approximat ad unam radicem quam ad reliquas, quæ ſit f: in datâ æquatione pro x ſcribatur $b + z$; & pro y ſcribatur $p + f$: fiant termini reſultantis æquationis, qui ex hâc hypotheſi maximi evadant, nihilo æquales; & exinde ſequitur valor quantitatis p prope; unde ſequitur $p + f$ valor quantitatis y prope; & ſic redintegratâ operatione deduci poteſt quantitas ad valorem quantitatis y magis appropinquans; & ſic deinceps: ſi inveniatur radix æquationis $P = o$ accurate; tum hæc methodus evadet eadem ac duo præcedentes: & hic (mutatis mutandis) applicari poſſunt omnia, quæ ad præcedentes methodos applicantur.

Ex. Sit æquatio $y^2 - 5xy + 4x^2 - x = o$, ubi $x = 2$ prope; pro x ſcribatur $2 + z$ & reſultat $y^2 - (10 + 5z)y + 14 + 15z + 4z^2 = o$: ſi $z = o$, tum reſultat $y^2 - 10y + 14 = o$, unde $y = 8$ prope: & conſequenter 8 vel 2 ſunt quantitates, quæ multo magis approximant ad unam radicem quantitatis y quam ad reliquas; e. g. aſſumatur 8 quantitas approximans ad radicem quæſitam: in reſultante æquatione pro y ſcribatur $8 + p$, & reſultat $64 + 16p + p^2 - 80 - 10p - 40z - 5pz + 14 + 15z + 4z^2 = o$; collocentur maximi termini hujuſce æquationis primi, iis vero proximi ſecundi, & ſic deinceps; i. e. $(A = o)$ $(64 - 80 + 14 - 40z + 15z + 16p - 10p) + (p^2 - 5pz + 4z^2) = (-2 - 25z + 6p) + (p^2 - 5pz + 4z^2) = o$: aſſumantur termini, qui ſunt maximi nihilo æquales, i. e. $- 2$

$-25z+6p=o$, & exinde reſultabit valor quantitatis $p=\frac{1}{3}+4\frac{1}{6}z$ prope: in æquatione $A=o$ pro p ſcribatur $\frac{1}{3}+4\frac{1}{6}z+q$; & ex terminis æquationis reſultantis (qui maximi ſunt) inveniatur valor quantitatis q prope, viz. $a'+b'z+c'z^2$; & ſic deinceps: & exinde reſultabit ſeries quæ exprimit valorem quantitatis y, viz. $y=(8+\frac{1}{3}+a'+\&c.)+(4\frac{1}{6}+b'+\&c.)z+(c'+\&c.)z^2+\&c.=(8+\frac{1}{3}+a'+\&c.)+(4\frac{1}{6}+b'+\&c.)(x-a)+(c'+\&c.)(x-a)^2+\&c.$

Eadem principia, &c. ad fluxionales, &c. æquationes applicari poſſunt.

3. 2. Ex methodis haud diſſimilibus inveniri poteſt quantitas y per ſeriem quantitatum, quæ ſunt functiones alterius quantitatis x in datâ æquatione contentæ, ſecundum datam legem progredientium, e. g. ſit ſeries quantitatum per legem $A+Bx\times(x-1)+Cx^2\times(x-1)^2+Dx^3\times(x-1)^3+\&c.$ progredientium, & per methodos haud diſſimiles deduci poſſunt coefficientes A, B, C, &c. hæ vero coefficientes ex ſerie, cujus termini ſecundum quamcunque aliam datam legem progrediuntur, etiam inveſtigari poſſunt.

4. Sit æquatio algebraica $y^n-py^{n-1}+qy^{n-2}-\&c.=o$, ubi p, q, &c. ſunt functiones quantitatis x, inveniatur y in ejus x terminis, i. e. ſit $y=a'x^r+b'x^{r+s}+c'x^{r+2s}+d'x^{r+3s}+\&c.$ & convergentia hujuſce ſeriei pendet ex hoc, nempe quanto minus diſtant approximationes ſucceſſivæ ab unâ radice quam ab reliquis, &c. ut prius docetur: in fluentibus, integralibus, &c. exinde deducendis, neceſſe eſt ut dentur duo, &c. valores a & b quantitatis x, inter quos continentur fluentes, integrales, &c. ut vero hæ ſeries convergant, quantitates a & b haud longe diſtent ab eâdem radice vel ex eâdem vel oppoſitis radicis partibus; ſi vero una litera a haud multum diſtet ab correſpondente radice æquationis $ny^{n-1}-(n-1)py^{n-2}+(n-2)qy^{n-3}-\&c.=o$, vel duæ literæ ex oppoſitis partibus ejuſdem radicis ponantur, tum ſeries prædictæ haud convergent: in caſibus quando ſeries prædicta vel non omnino vel non ſatis convergat, præſtet ut transformetur data æquatio in eâ pro x ſcribendo $z-\alpha$, $z-\beta$, $z-\gamma$, $z-\delta$, &c. reſpective, ubi α, β, γ, δ, &c. λ inter valores a & b ponantur, ita qui-

dem ut ſeries reſultans e primâ æquatione, derivatâ a ſubſtitutione $z - \alpha$ pro x in datâ æquatione, cum $z = \alpha - a$ vel $\alpha - c$, converget; & ſimiliter ſint ſeries reſultantes e ſubſtitutionibus $\beta - c$ & $\beta - d$ pro z in ſecundâ, $\gamma - d$ & $\gamma - e$ in tertiâ, & ſic deinceps uſque ad ultimum $\lambda - b$, ſemper convergentes; tum ex iis erui poteſt fluens vel integralis quæſita.

Hic animadvertendum eſt, quod ſi omnes hæ ſeries evadant convergentes, neceſſario nonnullæ abſciſſæ (z) incipiant a ſingulis radicibus (x) inter a & b poſitis, vel ad eas terminentur.

Si a & b ex oppoſitis partibus ejuſdem radicis ponantur; tum haud convergent duæ ſeries reſultantes e ſcribendo a & b pro x, ni abſciſſa incipiat a radice inter a & b poſitâ.

5. Datis unâ duabus vel pluribus (n) æquationibus $(n+1)$ incognitas quantitates habentibus, inveniantur valores quantitatis x, cum y evadat vel infinita vel $=0$, quorum ſit minimus α & maximus λ; tum ſeries aſcendens ſemper converget, cum x minor ſit quam α; & ſeries deſcendens converget, cum x major ſit quam λ, &c.

Eadem principia etiam ad fluxionales & incrementiales æquationes applicari poſſunt.

6. Si vero quædam quantitates vel p vel q vel r, &c. ſint functiones, quæ habent functionem quantitatis x in denominatore; fiat prædicta functio nihilo æqualis, & haud converget ſeries aſcendens $y = a'x^{r} + b'x^{r+s} + c'x^{r+2s} +$ &c. vel ſeries exprimens fluentem fluxionis $y\dot{x}$, &c. ſi x major ſit quam ulla radix prædictæ æquationis; ſi vero ſit ſeries deſcendens, tum haud converget, ni x major ſit quam quæcunque radix prædictæ æquationis.

Inveniantur igitur valores $(\pi, \rho, \sigma,$ &c.) quantitatis x inter a & b poſiti, in quibus y vel evadat infinita vel nihilo æqualis; & prædictæ abſciſſæ (z) vel ab valoribus $(\pi, \rho, \sigma,$ &c.) incipiant, vel ad eos terminentur; præſtat ut ab iis incipiant.

7. Sit P prædicta functio, & in quibuſdam caſibus pro quantitate quæſitâ y aſſumatur quantitas formulæ $P \times (fx^{r} + gx^{r+s} + hx^{r+2s} +$ &c.)

&c.) ſubſtituatur hæc quantitas reducta ad infinitam ſeriem pro y in datâ æquatione, & ex æquatis correſpondentibus terminis inter ſe deduci poſſunt coefficientes f, g, h, &c. In quibuſdam etiam caſibus aſſumatur quantitas prædictæ formulæ cum alterâ ſerie $a'x^r + b'x^{r+s}$ + &c. pro y, vel cum pluribus quantitatibus prædictarum formularum & ſerie $d'x^r + b'x^{r+s}$ + &c. & exinde erui poſſunt ſeries magis celeriter convergentes. Irrationales compoſitæ quantitates vel ex diviſione, extractione radicum, &c. ad ſimplices terminos reducendæ ſunt, quotieſcunque id exigat pro approximatione inveniendâ opus.

8. Cum vero ratio, quam habeat data approximatio ad radicem quæſitam haud permultum exſuperet rationes, quas habet data approximatio ad unam, duas vel plures alias radices; tum ope quadraticæ, cubicæ, &c. æquationis quærendæ ſunt novæ approximationes ad duas, tres vel plures radices datæ æquationis; unde conſtat ratio, quare in his caſibus ſi differentiæ indicum & ſimplicium & compoſitarum quantitatum ad ſimplices reductarum vel ſint s vel habeant maximum communem denominatorem s; pro y aſſumenda eſt ſeries hujuſce formulæ $ax^r + bx^{r+\frac{1}{2}s} + cx^{r+s} + dx^{r+1\frac{1}{2}s} + ex^{r+2s}$ + &c. vel $ax^r + bx^{r+\frac{1}{3}s} + cx^{r+\frac{2}{3}s} + dx^{r+s} + ex^{r+1\frac{1}{3}s}$ + &c. &c.

9. Si vero dentur quantitates ad duas vel plures hujuſce formulæ radices æquationis maxime appropinquantes; tum e principiis prius traditis erui poſſunt novæ approximationes ad radices prædictas propius accedentes.

PROB. XXII.

1. *Contineantur tres* (y, x, z) *incognitæ quantitates in datâ æquatione, invenire* y *in terminis quantitatum* x *&* z.

Primo reducantur duæ quantitates x & z ad alias x' & z', ut vel evadant perparvæ vel permagnæ, vel altera perparva & altera vero permagna, prout requiratur ſeries aſcendens vel deſcendens ſecundum dimenſiones quantitatum x' & z'; deinde fingatur altera x' prope in ratione directâ vel inversâ cujuſdam poteſtatis m quantitatis z'; & ex æquatis terminis inter ſe, qui maximi reſultent ex hâc hypotheſi, inveniatur (a) valor quantitatis y prope, ſcribatur quantitas $a + \pi$ pro y in datâ æquatione, & ex eâdem methodo erui poteſt valor quantitatis π prope; & ſic deinceps.

Hoc prob. etiam reſolvi poteſt ex aliis principiis in prob. præced. datis.

2. Si vero contineantur plures quam tres (y, x, z) incognitæ quantitates in datâ æquatione, per proceſſum omnino eundem erui poteſt y in ſerie e terminis quantitatum x', z', &c. conſtante.

3. Si vero detur fluxionalis vel incrementialis, &c. æquatio; ex principiis in hoc problemate, &c. traditis erui poteſt valor quantitatis y in ſerie ſecundum dimenſiones reliquarum progrediente, ſi modo y in terminis reliquarum exprimi poſſit.

Si dentur plures n vel algebraicæ vel fluxionales vel incrementiales æquationes, $n + 2$ vel plures variabiles quantitates habentes; tum ex principiis, quæ in hoc problemate, &c. prius tradita fuere, erui poteſt altera y in ſerie ſecundum dimenſiones reliquarum progrediente, ſi modo y in terminis reliquarum exprimi poſſit.

Hoc problema etiam reſolutionem accipere poteſt ex aſſumptâ ſerie progrediente ſecundum dimenſiones reliquarum quantitatum, i.e. principiis, iis in prob. 21. contentis, analogis; etiamque ex aſſumptis quibuſdam reliquarum quantitatum dimenſionibus inter ſe æqualibus.

PROB. XXIII.

1. *Datis duabus vel pluribus* (n) *algebraicis vel fluxionalibus vel incrementialibus æquationibus* n + 1 *variabiles quantitates* (y, x, z, *&c.*) *habentibus; invenire unam* (y) *ex iis in terminis reliquarum* (x, z, *&c.*)

Ita reducantur hæ æquationes, ſcribendo in iis pro reliquis x, z, &c. reſpective $x' + a$, $z' + b$, &c. ut x' & z' &c. perparvæ vel permagnæ evadant, prout requiratur ſeries aſcendens vel deſcendens ſecundum earum dimenſiones; & ſimiliter de earum fluxionibus, incrementis, &c.: deinde ex æquatis terminis ſingularum æquationum, qui maximi ſunt, reſpective inter ſe, erui poſſunt α, β, γ, &c. prope valores quantitatis y reſpectivi in terminis quantitatum x', z', &c. ſcribantur hi valores inventi per variabiles quantitates incognitas aucti, i. e. $x'' + \alpha$, $z'' + \beta$, &c. pro y, & pro $\dot{y}$, $\ddot{y}$, &c. $\underset{\cdot}{y}$, $\underset{\cdot\cdot}{y}$, &c. earum valores $\dot{x}'' + \dot{\alpha}$, $\dot{z}'' + \dot{\beta}$, $\ddot{x}'' + \ddot{\alpha}$, $\dot{z}'' + \ddot{\beta}$, &c. $\underset{\cdot}{x}'' + \underset{\cdot}{\alpha}$, $\underset{\cdot}{z}'' + \underset{\cdot}{\beta}$, $\underset{\cdot\cdot}{x}''' + \underset{\cdot\cdot}{\beta}$, &c. reſpective in datis æquationibus, & ex æquationibus reſultantibus facile detegi poſſunt valores prope quantitatum x'', z'', &c. reſpective, & ſic deinceps; vel aliter e principiis prius traditis erui poſſunt formulæ ſerierum, quæ exprimunt quantitatem y terminis vero quantitatum x', z', &c. quæ ſint $y = A'z'^{k} + B'z'^{k+l} + C'z'^{k+2l} +$ &c. &c. vel pro incrementis primi ordinis $y = Ax'^{r} \times (x+1)'^{r} +$ &c. &c. &c. ſcribantur hæ quantitates pro ſuis valoribus in datis æquationibus, & ex ſingulis terminis reſultantis æquationis nihilo æqualibus eſſe ſuppoſitis erui poſſunt coefficientes A, B, C, &c. A', B', C', &c. &c.

Convergentiæ ſerierum e pluribus æquationibus deductarum vel plures variabiles quantitates habentium e principiis prius traditis de convergentiâ ſeriei ex unâ æquatione erutæ, facile dijudicari poſſunt.

2. Dentur æquationes involventes variabiles quantitates x, y, &c. & earum fluxiones, incrementa, &c. ſint etiam z & v datæ functiones quantitatum x, y, &c. invenire v in terminis ſecundum dimenſiones quantitates z progredientibus: hoc perficitur vel e reductione plurium æquationum in pauciores, ita ut exterminentur omnes incognitæ quantitates præter z & v; vel ex aſſumptis ſeriebus pro ſingulis incognitis quantitatibus ſecundum dimenſiones quantitatis z progredientibus, & ex earum ſubſtitutionibus in datis æquationibus pro ſuis valoribus, & terminorum correſpondentium coefficientibus nihilo æqualibus eſſe ſuppoſitis, erui poſſunt indices & coefficientes ſerierum aſſumptarum quæſitæ.

THEOR. XXXII.

Sit fluxionalis æquatio relationem inter variabiles x & y & earum fluxiones exprimens, ubi $\dot{x}$ ſit conſtans; pro y ſcribatur Ax^{n} & pro $\dot{y}$,

$\ddot{y}$, &c. ſcribantur reſpective $nAx^{n-1}\dot{x}$, $n.(n-1)Ax^{n-2}\dot{x}^2$, &c. in datâ æquatione, ubi A & n ſunt quæcunque invariabiles quantitates aſſumendæ; & e terminis reſultantis æquationis, qui inveniuntur maximi ex hypotheſi datâ, quod x ſit perparva vel permagna quantitas, inter ſe æquatis, deduci poteſt quantitas Ax^n proximus valor quantitatis y; tum in datâ æquatione pro y ſcribatur Ax^n+p, & pro $\dot{y}$, $\ddot{y}$, &c. reſpective $nAx^{n-1}\dot{x}+\dot{p}$, $n.(n-1)Ax^{n-2}\dot{x}^2+\ddot{p}$, &c. & e terminis maximis reſultantis æquationis inveniatur proximus valor quantitatis p, qui ſit Bx^m; in æquatione reſultante pro p ſcribatur ejus valor Bx^m+q, & pro $\dot{p}$, $\ddot{p}$, &c. reſpective $mBx^{m-1}\dot{x}+\dot{q}$, $m.(m-1)Bx^{m-2}\dot{x}^2+\ddot{q}$, &c. & conſimili methodo inveniatur proximus valor quantitatis q, & ſic deinceps; unde tandem innoteſcet $y=Ax^n+Bx^m+$ &c. prope.

Si $x=a$ prope, tum ſcribatur in datâ æquatione $a+z$ vel $a+\frac{1}{z}$ pro x, & ex methodo hìc traditâ inveniatur y in terminis quantitatis z.

2. Sit r ordo fluxionalis æquationis, & in ſerie quæ exprimit valorem quantitatis y in terminis quantitatis x, i. e. $y=Ax^n+Bx^m+$ &c. ſemper tot invenientur invariabiles quantitates ad libitum aſſumendæ, quot ſit ordo prædictus, ſi modo ſeries ab initio terminorum incipiat, i.c. recte inſtituatur. e. g. Sit æquatio $\dot{y}=\dot{x}+y\dot{x}-3x\dot{x}+x^2\dot{x}+yx\dot{x}$, invenire quantitatem y in ſerie ſecundum dimenſiones quantitatis x progrediente; ſi aſſumatur $y=x$ prope pro proximo valore quantitatis y, tum haud incipit ab initio recte inſtituto ſeries; fingatur $y=A+bx+p$, ſcribatur hæc quantitas pro y & $b\dot{x}+\dot{p}$ pro $\dot{y}$ in datâ æquatione, & reſultat $b\dot{x}+\dot{p}=\dot{x}-3x\dot{x}+A\dot{x}+bx\dot{x}+p\dot{x}+x^2\dot{x}+Ax\dot{x}+bx^2\dot{x}+px\dot{x}$; termini autem, qui ex hypotheſi aſcendentis ſeriei maximi ſint, inveniantur $b\dot{x}=\dot{x}+A\dot{x}$, unde $b=1+A$; & æquatio reſultans $\dot{p}=(A+b-3)x\dot{x}+(x^2+p+bx^2+px)\dot{x}$; hujus reſultantis æquationis termini maximi erunt $\dot{p}=(A+b-3)x\dot{x}$, unde $p=\frac{1}{2}(A+b-3)x^2$ prope, & conſequenter $y=A+(A+1)x+(A-1)x^2+$ &c. ubi A eſt quantitas ad libitum aſſumenda.

3. Si vero prædicta quantitas in fluente inventâ haud contineatur,

tum fluens inventa erit valor particularis, qui nonnunquam in generali fluente fluxionalis æquationis, & ſic integrali, &c. haud continetur. Idem etiam verum eſt de algebraicis æquationibus ut conſtat e ſubſequen. exemp.

Sit æquatio $y = \surd(a^2 + bx + cx^2) = a + \frac{bx}{2a} + \left(\frac{c}{2a} - \frac{b^2}{8a^3}\right)x^2$ $+$ &c. In hâc ſerie ſemper continetur quantitas generalis a; nunc ſit $a = 0$, & reſultat æquatio particularis $y = \surd(bx + cx^2) = b^{\frac{1}{2}}x^{\frac{1}{2}}$ $+ \frac{cx^{1\frac{1}{2}}}{2b^{\frac{1}{2}}} +$ &c. quæ ſeries particularis in priori generali haud continetur; eveniant enim quidam particulares caſus, in quibus generalis reſolutio fallit; quibus quidem caſibus generalis reſolutio præbet divergentem ſeriem, vel ejus quidam numeratores nihilo evadunt æquales.

II. Datâ incrementiali æquatione relationem inter variabiles x, y & earum incrementa relationem exprimente; invenire ejus integrale, i. e. variabilem y in terminis ſecundum dimenſiones quantitatis x progredientibus.

1$^{\text{mo}}$. Sit x permagna quantitas reſpectu habito ad reliquos æquationis terminos; tum abjectis omnibus terminis, qui haud maximi reſultant ex hâc ſuppoſitione, e terminis exinde reſultantibus inveniatur per methodum in fluxionalibus, &c. æquationibus reſolvendis traditam proximus valor quantitatis y terminis vero quantitatis x, qui ſit Ax^{-n}, deinde ſcribatur $\frac{\pm A}{x^{\frac{1}{2}n} \times (x \pm \underset{\cdot}{x})^{\frac{1}{2}n}} + p$ pro y in datâ æquatione, & ex æquatione reſultante abjectis omnibus terminis qui haud maximi reſultant e prædictâ hypotheſi inveniatur proximus valor quantitatis p, qui ſit Ax^{-m}; tum pro p ſcribatur $\frac{\pm A}{x^{\frac{1}{2}m} \times (x \pm \underset{\cdot}{x})^{\frac{1}{2}m}} + q$, & ſic deinceps; unde tandem reſultabit valor quantitatis y quæſitus.

2$^{\text{do}}$. Si vero x perparva ſit reſpectu habito ad quoſdam æquationis terminos, tum abjectis omnibus terminis, qui haud maximi reſultant ex hâc hypotheſi, e reliquis deducatur proximus valor quantitatis y, qui ſit Ax^n; deinde ſcribatur pro y in datâ æquatione $Ax^n \times (x \pm \dot{x})^n + p$, & ex æquatione reſultante inveniatur per conſimilem methodum proximus valor quantitatis p; & ſic repetitâ operatione tandem reſultabit integralis quæſita.

Et ſic e principiis prius traditis erui poteſt integralis ex aſſumendo quantitatem formulæ $Az^n + \&c. + a + \frac{b}{z} + \frac{c}{z\,.\,z+\dot{z}} + \frac{d}{z\,.\,z+\dot{z}\,.\,z+2\dot{z}} + \&c.$ pro integrali, ſi n ſit integer numerus, & ſubſtituendo hanc quantitatem pro ejus valore in datâ æquatione, ex æquatis correſpondentium terminorum coefficientibus erui poſſunt quantitates A, &c. n, b, c, &c.

III. Aliter: Datâ fluxionali æquatione relationem inter variabiles x, y, & earum fluxiones exprimente, invenire quantitatem y in terminis ſeriei progredientis ſecundum dimenſiones quantitatis z.

Aſſumatur $y = Ax^n$ pro primo ſeriei termino, quâ quantitate pro ejus valore y in datâ æquatione ſubſtitutâ, & ejus fluxionibus $nAx^{n-1}\dot{x}$, $n\,.\,(n-1)\,Ax^{n-2}\dot{x}^2$, &c. pro $\dot{y}$, $\ddot{y}$, &c. fiant omnes termini qui habent maximas vel minimas dimenſiones quantitatis x, prout ſeries requiritur aſcendens vel deſcendens, inter ſe æquales; & ex æquatione reſultante conſtabit primus terminus ſeriei quæſitæ Ax^n; & pro reliquis indicibus ſcribe quantitatem Ax^n & ejus fluxiones pro ſuis valoribus in datâ æquatione, ſubtrahe minimum indicem reſultantis æquationis terminorum in ſerie aſcendente vel maximum in ſerie deſcendente de ſingulis reliquis, & omnia reſidua ſibi ipſis & omnibus aliis addantur, & ſic de reliquis; & e quantitatibus reſultantibus conſtabunt differentiæ indicum, i. e. quantitates r, s, t, &c.

Scribatur igitur ſeries $Ax^n + Bx^{n+r} + Cx^{n+s} + Dx^{n+t} + \&c.$ in datâ æquatione pro y, & ejus ſucceſſivæ fluxiones reſpective pro $\dot{y}$, $\ddot{y}$, &c. & ex æquatis correſpondentibus terminis reſultantis æquationis, in quibus eædem inveniuntur dimenſiones quantitatis x, nihilo reſpective; reſultabunt coefficientes B, C, D, &c. quæſitæ.

Si vero duo vel tres vel plures valores quantitates Ax^n ſint inter ſe æquales, dividantur prædictæ differentiæ per duo, tres, &c. & reſultabunt differentiæ quæſitæ, &c. i. e. ſeries erunt hujuſmodi $Ax^n + Bx^{n+\frac{1}{2}r} + Cx^{n+r} + Dx^{n+\frac{3}{2}r} +$ &c. vel $Ax^n + Bx^{n+\frac{1}{3}r} + Cx^{n+\frac{2}{3}r} +$ &c.

Si primus valor quantitatis y inventus ſit imaginarius, tum forſan e ſcribendo $v + a$ & $z + b$ pro y & x in datâ æquatione reſpective, ita aſſumi poſſunt invariabiles quantitates a & b, ut haud evadat primus valor quantitatis y imaginarius; & ejus approximationes e methodis prius traditis deduci poſſunt.

Hic animadvertendum eſt hanc methodum reſolutionis ſemper exigere, ut quantitates v & z, vel v vel z, ſint perparvæ vel permagnæ, prout requiritur ſeries aſcendens vel deſcendens.

Per perparvas vel permagnas quantitates intelligo quantitates ſic deductas; ſit ſeries deducta aſcendens $y = a'z^{\lambda} + b'z^{\mu} + c'x^{\nu} +$ &c. $= P$; & inveniatur (π) valor quantitatis z, cum P evadat divergens ſeries; tum neceſſe eſt, ut z non major ſit quam $\pm\pi$, aliter ſeries diverget: ſit P ſeries deſcendens, & ρ minimus valor quantitatis z, cum evadat divergens, tum neceſſe eſt, ut z non minor ſit $\pm\rho$, aliter ſeries diverget: aliter per perparvas intelligo quantitates multo magis approximantes ad unam quam ad reliquas radices.

In fluente fluxionalis æquationis n ordinis n continentur invariabiles quantitates pro conditione problematis aſſumendæ; unde aſſumi poſſunt (n) correſpondentes valores ſingularum quantitatum x, $\dot{x}$, y, $\dot{y}$, $\ddot{y}$, &c. Et ſic de incrementialibus, &c. æquationibus.

PROB. XXIV.

Datâ fluxionali æquatione n *ordinis relationem inter quantitates* x, y, $\frac{\dot{y}}{\dot{x}}$, $\frac{\ddot{y}}{\dot{x}^2}$, . . . $\frac{\overset{n-1}{\dot{y}}}{\dot{x}^{n-1}}$, $\frac{\overset{n}{\dot{y}}}{\dot{x}^n}$ *exprimente, quarum* a, a′, b, c, d, e, . . . A, B, C, D, E, F *ſint*

E, F *ſint correſpondentes valores, & a quæſitis valoribus haud longe diſtantes; invenire quantitates ad quæſitos valores magis appropinquantes.*

1. Reducatur æquatio ſcribendo in eâ pro $\frac{\overset{n}{\dot y}}{\dot x^n}$ ejus valorem $F + d'(x-a)^\lambda$, & exinde pro $\frac{\overset{n-1}{\dot y}}{\dot x^{n-1}}$, ejus valorem vel fluentem $E + F(x-a) + \frac{d'}{\lambda+1}(x-a)^{\lambda+1}$, & pro $\frac{\overset{n-2}{\dot y}}{\dot x^{n-2}}$ ejus fluentem $D + E(x-a) + \frac{1}{2}F(x-a)^2 + \frac{d'}{(\lambda+1)(\lambda+2)}(x-a)^{\lambda+2}$, & ſic deinceps; & ex æquatis terminis qui maximi reſultant ex hypotheſi quod $x-a$ ſit perparva quantitas deduci poſſunt valores quantitatum a & λ; & ſic deinceps, uſque donec inveniatur formula æquationis variabilem y in terminis variabilis x exprimentis; & deinde, vel per continuas hujuſce generis approximationes, vel per ſubſtitutionem ſeriei formulæ prædictæ pro y, & ejus fluxionum pro $\dot y, \ddot y, \dddot y$, &c. & ex correſpondentibus datæ & reſultantis æquationis terminis inter ſe & nihilo reſpective æquatis, erui poteſt ſeries quæſita.

Aliter: pro x ſcribatur $z+a$, & pro $\frac{\overset{n}{\dot y}}{\dot x^n}$, $\frac{\overset{n-1}{\dot y}}{\dot x^{n-1}}$, $\frac{\overset{n-2}{\dot y}}{\dot x^{n-2}}$, $\frac{\overset{n-3}{\dot y}}{\dot x^{n-3}}$, &c. ſcribantur reſpective $F + d'z^\lambda$, $E + Fz + \frac{d'}{\lambda+1}z^{\lambda+1}$, $D + Ez + \frac{1}{2}Fz^2 + \frac{d'}{(\lambda+1)(\lambda+2)}z^{\lambda+2}$, &c. & per præcedentem methodum progrediendum eſt.

Hæc methodus etiam ad incrementiales æquationes applicari poteſt.

Si vero integrales vel fluentiales quantitates in datis incrementialibus vel fluentialibus æquationibus contineantur, tum ex principiis hic datis, & iis de fluentibus, &c. fluentialium fluxionum, &c. inveſtigandis prius traditis, erui poteſt ſeries valorem quantitatis y exprimens.

2. Summa vel differentia duarum ſerierum (A' & B'), quarum una A' exprimit y in terminis ſecundum dimenſiones quantitates x aſcendentibus;

dentibus; altera vero y in terminis ſecundum dimenſiones quantitatis x deſcendentibus erit data quantitas; at ſi deſcendens ſeries converget, tum aſcendens pene in omnibus caſibus diverget; & vice versâ ſi aſcendens ſeries convergat, tum deſcendens ſeries in prædictis caſibus etiam diverget.

3. Series, quæ exprimunt vel reſolutiones fluxionalium vel incrementialium, &c. æquationum, vel fluentes &c. fluxionum, &c. quæ progrediuntur ſecundum dimenſiones quantitatis variabilis x, debent eſſe convergentes, haud quidem ſolummodo cum α ſit datus valor quantitatis x, ſed etiam cum $x = \beta$; in utriſque autem his caſibus ni α prope $= \beta$ plerumque haud converget ſeries, ut facile conſtari poteſt ex iis, quæ prius data fuerunt: ſi non convergant utræque ſeries, ſcribatur $z + e$ pro x in datâ æquatione, & ex reſultante æquatione inveniatur ſeries ſecundum dimenſiones quantitatis (z) $x - e$ progrediens, quæ convergit cum z vel $= \alpha - e$ vel $= a$; in hâc reſultante æquatione pro z ſcribatur $z' + e'$, & ex novâ reſultante æquatione inveniatur ſeries, quæ convergit, cum z' vel $= a + e'$ vel a'; deinde pro z' in æquatione ultimo derivatâ ſcribatur $z'' + e''$, & ex æquatione reſultante inveniatur ſeries quæ convergit cum z'' vel $= a' + e''$ vel $= a''$; & ſic deinceps; uſque donec inveniatur ſeries, quæ convergit, cum z'^{m} vel $= a'^{m} + e'^{m+1}$ vel $\beta - \alpha + e + e' + e'' \,..\, + e'^{m+1}$. Sit vero ſumma duarum ſerierum, quæ prius reſultabant ex hypotheſi quod $z = \alpha - e$ & a, etiamque cum $z' = a + e'$ & a', tertio cum $z'' = a' + e''$ & a'', & ſic deinceps; reſpective π, ρ, σ, τ, &c. tum erit ſumma ſeriei inter duos valores (α & β) quantitatis x contenta $= \pi + \rho + \sigma +$ &c.

Cor. . Sit æquatio algebraica relationem inter variabiles x & y deſignans, inveniatur y vel $\int . y\dot{x}$, &c. per ſeriem ſecundum dimenſiones quantitatis x progredientem; ſcribatur $z + e$ pro x in data æquatione, & ex æquatione reſultante inveniatur y vel $\int . y\dot{x}$, &c. in ſerie ſecundum dimenſiones quantitatis z progrediente, quæ ſit convergens; & ſic inveniantur aliæ ſeries convergentes, quæ correſpondeant diverſis valoribus quantitatis e, qui ſint reſpective e, e', e'', &c. & aſſumi poſſunt valores e, e', e'', &c. ita ut e ſeriebus reſultantibus numero finitis deduci poteſt ſumma ſerierum vel fluentis inter quoſcunque duos valores α & β quantitatis x contentæ.

Hic obſervandum eſt, ut quo plures recte aſſumantur diverſi valores e, e', e'', &c. inter duos datos valores α & β quantitatis x, eo magis celeriter convergent ſeries reſultantes; etiamque animadvertendum eſt, ſi ullus valor quantitatis y inter valores α & β quantitatis x contentus evadat vel infinite magnus vel $= o$; utraſque ſeries reſultantes ex ſubſtitutione valorum α & β pro x non evadere convergentes: in interpolandis ſeriebus neceſſe eſt, ut valores quantitatis (x) vel incipiant vel terminentur in punctis, in quibus quantitas y evadat vel $= o$ vel infinita quantitas.

Ea, quæ de approximationibus fluentium fluxionum in infinitum progredientium, æque ad has fluxionales æquationes applicari poſſunt.

4. Priuſquam vero finis huic parti de infinitis ſeriebus, quæ exprimunt valores variabilium (x & y) in datis fluxionalibus, &c. æquationibus contentarum imponatur, quædam animadvertenda ſunt de quibuſdam difficultatibus, quibus premitur methodus prius tradita prædictos valores inveniendi: e. g. in æquatione fluxionali primi ordinis, ſæpe ſi modo a ſit proximus valor quantitatis y & ſcribatur $a + p$ pro primâ approximatione, in æquationes ſubſequentes vel $\frac{\dot p}{p} = b x^r \dot x$ vel $p^r \dot p = b \frac{\dot x}{x}$ incidamus: ſi modo ſcribatur Ax^n pro p & $nAx^{n-1}\dot x$ pro $\dot p$ in datâ æquatione tanquam approximatio, tum manifeſto apparet, quod fallet hæc methodus; ergo vel ſcribendæ ſunt in datâ æquatione $\alpha + v$ & $\dot v$ pro p & $\dot p$ in priori caſu, & $\alpha' + z$ & $\dot z$ pro x & $\dot x$ in poſteriori; & deinde per methodos prius traditas progrediendum eſt; vel in priori caſu aſſumenda eſt $p = D e^{b'x^{r+1}}$ prope approximatio, ubi D eſt quantitas ad libitum aſſumenda: ſubſtituantur hæc quantitas per alteram aucta & ejus fluxio pro p & $\dot p$ in datâ æquatione, & exinde per methodos prius traditas progredi liceat; poſtea enim ullâ urgetur difficultate opus: in poſteriori inveniatur x in terminis quantitatis p, &c.; aliter $p = ((r+1)\, b \times \log. x + D)^{\frac{1}{r+1}}$ prope; & ſic deinceps.

5. Hinc conſtat in fluxionali æquatione primi ordinis incidere liceat in fluxionalem homogeneam æquationem primi ordinis; in fluxionali ſecundi, tertii, &c. ordinis æquatione in quamcunque homogeneam fluxionalem æquationem ſecundi, tertii, &c. ordinis.

1. In fluxionali æquatione ſecundi ordinis ſint termini maximi $a\ddot{y}^b\dot{y}^s y^t + \&c. = bx^r\dot{x}^{s+2b}$; ex hâc æquatione inveniatur fluens, & exinde prima approximatio: aliter ſcribatur Ax^n pro y in datâ æquatione, & reſultat $n^{s+b}(n-1)^b aA^{t+s+b} \times x^{(t+s+b)n-s-2b}\dot{x}^{s+2b} + \&c. = bx^r\dot{x}^{s+2b}$, ſi vero vel $t+s+b=0$ vel $r+s+2b=0$ vel $n=1$, tum haud ex prædictâ methodo deduci poteſt reſolutio; in priori enim caſu $A^{t+s+b=0} = 1$, quicunque ſit valor quantitatis A, ergo ad libitum aſſumi poteſt A; ſi autem non ſint $n^{s+b}(n-1)^b a = b$ & $r+s+2b=0$; tum non aſſumi poteſt Ax^n pro approximatione: in ſecundo caſu $n=0$, cum $r+s+t$ haud nihilo ſit æqualis, & conſequenter $n^{s+b}=0$, ni $s+b=0$; & iterum non aſſumi poteſt Ax^n pro approximatione: idem etiam vero eſt, cum $n-1=0$, nam in eo caſu $(n-1)^b=0$: in his caſibus vel ſcribendæ ſunt quantitates $\alpha+v$ & $\beta+z$ & earum fluxiones $\dot{v}$, $\ddot{v}$, &c. & $\dot{z}$ pro y, x, $\dot{y}$, $\ddot{y}$, &c. & $\dot{x}$ in datâ æquatione, & ex æquatione reſultante deducenda eſt æquatio: vel ſubſtitutio in caſu primo uſurpanda eſt, in quâ ita aſſumi poſſunt coefficientes F, E, D, &c., a', &c., ut per hanc methodum ſemper detegi poteſt ſeries; nulla enim poſtea occurrit quantitas, quæ exigit fluentem fluxionis, i. e. ſeries incipit a termino, poſt quem nulla quantitas ad libitum aſſumi poteſt: vel in priori caſu pro y aſſumatur Ae^{x^n}, & exinde pro $\dot{y}$ & $\ddot{y}$ reſpective $nAx^{n-1}\dot{x}e^{x^n}$, & $n.(n-1).Ax^{n-2}\dot{x}^2e^{x^n} + n^2Ax^{2n-2}\dot{x}^2e^{x^n}$; ſi vero $s=-2b$, fallet etiam hæc methodus. Cum vero y haud facile erui poſſit e quantitate x, forſan præſtet transformare datam æquationem, ita ut fluat uniformiter y; deinde invenire x in ſerie ſecundum dimenſiones quantitatis y progrediente, tum ex reverſione ſerierum deduci poteſt y in ſerie ſecundum dimenſiones quantitatis x progrediente.

2. Sint maximi termini $\overset{m}{\dot{v}}{}^{\pi} . \overset{m-1}{\dot{v}}{}^{\alpha} . \overset{m-2}{\dot{v}}{}^{\beta} . \overset{m-3}{\dot{v}}{}^{\gamma} \times \&c. = ax^r\dot{x}^{m\pi+(m-1)\alpha+(m-2)\beta+\&c.}$;

deinde inveniantur radices, i. e. quantitas v in terminis quantitatis x æquationis reſultantis a ſupponendo omnes terminos, qui maximi ſunt, ſimul adjunctos nihilo æquales eſſe pro primis approximationibus; ſi irrationales quantitates in datâ vel datis æquationibus contineantur, tum ſemper rejiciendi ſunt termini, qui haud maximi ſunt, in prædictâ æquatione contenti: vel aliter; ſcribatur Ax^n pro z in datâ æquatione, & ſi vel $\pi + \alpha + \beta + \gamma +$ &c. $= 0$ vel $+ n = (\pi + \alpha + \beta + \gamma +$ &c.$)$ $(m) - \alpha - 2\beta - 3\gamma -$ &c. vel $n = 1, 2, \ldots m$; &c. fallit hæc methodus: in multis hiſce caſibus erui poteſt approximatio ſcribendo ae^{x^n} pro y, & ejus fluxiones pro $\dot{y}$, $\ddot{y}$, &c. reſpective, & ex æquatis terminis erui poteſt approximatio quæſita, &c.: vel hìc applicari poſſunt omnia in præcedente exemplo tradita.

6. Si hæc poſterior methodus fallat, tum ita transformetur æquatio ut fluat uniformiter y, & inveniatur x in ſerie ſecundum dimenſiones quantitatis y progrediente; & denique ſi hæc methodus fallat, tum per methodum prius traditum ſcribantur $v + \alpha$ & $z + \beta$ & earum fluxiones pro x & y & earum fluxionibus reſpective in datâ æquatione, ubi v & z ſunt perparvæ quantitates; vel diverſis aliis modis transformentur variabiles datæ æquationis quantitates in alias, quæ aſſignabilem habent relationem ad variabiles datæ æquationis quantitates, &c.

In quibuſdam caſibus, cum utræque prædictæ quantitates nihilo evadant æquales, ex earum coefficientibus nihilo æqualibus eſſe ſuppoſitis reſultabunt indices quæſiti.

Omnia hæc æque ad incrementiales æquationes applicari poſſunt.

7. In hujuſmodi reſolutionibus omnium algebraicarum, fluxionalium, incrementialium, &c. æquationum primo reducendæ ſunt æquationes, ita ut earum variabiles quantitates evadant perparvæ vel permagnæ, prout in ſerie quæſitâ earum dimenſiones aſcendunt vel deſcendunt.

8. Pleraque ex iis, quæ hìc traduntur de ſeriebus ex algebraicis, fluxionalibus & incrementialibus æquationibus deductis æque ad ſeries ex infinitis aliis æquationibus derivatas applicari poſſunt.

9. Si vero incrementialis ſit æquatio, tum formula ſeriei aſſumendæ ſit $y = Ax^{s}(x \pm \underset{\cdot}{x})^{r} + Bx^{t}(x \pm \underset{\cdot}{x})^{t} +$ &c. ſi ſit incrementialis æquatio primi ordinis; vel $y = Ax^{s} \times (x \pm \underset{\cdot}{x})^{s} \times (x \pm 2\underset{\cdot}{x})^{t} + Bx^{t} \times (x \pm \underset{\cdot}{x})^{t} \times (x \pm 2\underset{\cdot}{x})^{t} +$ &c. ſi modo ſit incrementialis æquatio ſecundi ordinis; & ſic deinceps: vel $y = Ax^{s} \times (x \pm \underset{\cdot}{x})^{s} \times (x \pm b) + Bx^{t} \times (x \pm \dot{x})^{t} \times (x \pm c)$ pro integrali incrementialis æquationis primi ordinis aſſumi poteſt, & ſic de infinitis aliis.

Hic autem animadvertendum eſt, quod tot erunt quantitates invariabiles ad libitum aſſumendæ in ſerie reſultante ab initio, quot ſit ordo incrementialis æquationis, ut conſtat e præcedentibus.

Hæc methodus etiam extendi poteſt ad duas vel plures algebraicas, fluxionales, incrementiales, &c. æquationes tres vel plures variabiles quantitates involventes.

10. Hæ methodi etiam applicari poſſunt ad æquationes ex infinitis terminis, & earum fluxionibus, &c. conſtantes. E. g. Sit æquatio algebraica $x = ay + by^2 + cy^3 + dy^4 +$ &c. & inveniri poteſt $y = \frac{x}{a} - \frac{b}{a^3}x^2 + \frac{2b^2 - ac}{a^5}x^3 +$ &c. & ſic ſit $x = ay + by^3 + cy^5 +$ &c. & inveniri poteſt $y = \frac{x}{a} - \frac{b}{a^4}x^3 + \frac{3b^2 - ac}{a^7}x^5 +$ &c.

2. Sit $ay + by^2 + cy^3 +$ &c. $= \alpha x + \beta x^2 + \gamma x^3 +$ &c. aſſumatur $y = \frac{\alpha}{a}x + lx^2 + mx^3 + nx^4 +$ &c. quæ erit ſeries formulæ quæſitæ, ſcribatur hæc quantitas pro y in datâ æquatione & reſultat $\alpha x + \left(al + b\frac{a^2}{\alpha^2}\right)x^2 + \left(\frac{c\alpha^3}{a^3} + \frac{2l\alpha b}{a} + am\right)x^3 +$ &c. $= \alpha x + \beta x^2 + \gamma x^3 +$ &c. unde $al + \frac{b\alpha^2}{a^2} = \beta$ & conſequenter $l = \frac{\beta a^2 - b\alpha^2}{a^3}$, & $\frac{c\alpha^3}{a^3} + \frac{2l\alpha b}{a} + am = \gamma$, &c.

Si vero dentur æquationes relationes inter quantitates hujuſce formulæ $1 \times 2 \times 3 \, .. \, z$, earum fluxiones. &c. in quibus numerus factorum continuo augetur, deſignantes; tum e continuis ſubſtitutionibus, approximationibus, interpolationibus, &c. inveniri poteſt una quantitas in terminis alterius.

Converti poſſunt algebraicæ, fluxionales, &c. æquationes in ſeries, quarum termini ſecundum prædictas formulas progrediuntur.

THEOR. XXXIII.

Cum y ſit infinita quantitas, tum ejus fluxiones ($\dot{y}$, $\ddot{y}$, $\dddot{y}$, &c.) quorumcunque ordinum erunt infinitæ quantitates, i. e. ſit y quæcunque functio quantitatis x, & ſi y fiat infinita quantitas, cum x maneat finita, tum ſemper $y : x$; $\dot{y} : \dot{x}$; $\ddot{y} : \dot{x}^2$; $\dddot{y} : \dot{x}^3$; &c. erunt rationes infinite magnæ, quarum ſingula ſubſequens infinite major erit quam præcedens.

Idem etiam de incrementis integralium affirmari poteſt.

PROB. XXV.

Datâ fluxionali æquatione P = o *variabiles quantitates* x, y *& earum fluxiones habente, ubi* x *&* y *ſunt abſciſſæ & correſpondentes ordinatæ ad curvam; invenire fluxionalem æquationem ejus aſymptotos generaliter deſignantem.*

1^mo^. Fingatur y infinita quantitas, & ex hâc hypotheſi ſint A termini qui maximi reſultant, B vero termini qui ſint maximi ex omnibus reliquis terminis; 2^do^. fingatur x infinita quantitas, y vero finita, deinde ſint termini, qui ex hâc hypotheſi evadant maximi vel proxime ſuperiores, reſpective C & D; tum æquationes $A + B = o$ & $C + D = o$ deſignabunt curvas eaſdem habentes aſymptotos, quas habet data curva.

THEOR. XXXIV.

Datâ præcedente fluxionali æquatione ($P = o$) n ordinis, & literis A, B, C & D eaſdem quantitates ac in præcedente problemate denotantibus;

tantibus; æquationis $A + B = 0$ inveniatur generalis fluens $K = 0$, & ſi modo invariabiles quantitates ad libitum aſſumendæ eædem ſint in æquatione $K = 0$ ac in fluente fluxionalis æquationis $P = 0$, i. e. ſi modo a, b, c, d, &c. correſpondentes dati valores quantitatum $x, y, \dot{x}$, &c. in unâ, ſint etiam correſpondentes valores prædictarum quantitatum in alterâ $K = 0$; & exinde deducantur valores quantitatis x, cum y evadat infinita, qui ſint $\alpha, \beta, \gamma, \delta$, &c. & ſi valor π vel ρ quantitatis x, inter quos requiritur fluens, ſit major quam radix α vel β vel γ vel δ, &c. tum plerumque haud converget ſeries ſecundum dimenſiones quantitatis x aſcendens; ſi vero minor ſit π vel ρ quam ulla radix α vel β vel γ vel δ, &c. tum plerumque haud converget ſeries ſecundum dimenſiones quantitatis x deſcendens.

Si vero ita transformetur data æquatio, ut abſciſſa z ab alio puncto incipiat, tum e principiis prius traditis pro algebr. &c. æquationibus facile conſtat, ut haud convergent ſeries ſecundum dimenſiones quantitatis z progredientes, ni π & ρ inter duas proximas radices β & γ quantitatis x conſtituantur.

THEOR. XXXV.

1. Sit prædicta fluxionalis æquatio $P = 0$, & ex eâ inveniatur æquatio, cujus radices ſint limites inter radices, &c. datæ æquationis, i. e. fingatur y ſolummodo variabilis, & inveniatur fluxio datæ æquationis, & ſi valor quantitatis x pro radice y aſſumptus ſit radix inventæ æquationis, tum infinite divergit approximatio deducta; ejus convergentia pendet ex hoc, nempe quo magis approximatio inventa ad unam radicem accedat quam ad reliquas.

2. Ut ſeries ex æquationibus hujuſce generis reſultantes celeriter convergant, neceſſe eſt plerumque interpolare plures ſeries per methodos conſimiles iis pro algebraicis æquationibus prius traditis; ſed cavendum eſt, ne diverſis correctionibus utamur, i. e. ut correſpondentes valores variabilium & earum fluxionum ſemper conveniant.

Omnia hæc etiam ad incrementiales æquationes applicari poſſunt.

3. Haud raro in hiſce æquationibus quantitas y evadet impoſſibilis quantitas, & in hoc caſu ſeries nunquam converget, ni pro primâ approximatione aſſumatur impoſſibilis quantitas, cujus poſſibilis pars ab poſſibili, impoſſibilis vero ab impoſſibili quantitatis parte haud longe diſtet: hoc in caſu valor quantitatis y nunquam exprimi poteſt per ſeriem ſecundum dimenſiones quantitatis x progredientem, ni x ſit impoſſibilis quantitas.

4. Sit $y = A$, ubi A ſit ſeries ſecundum dimenſiones quantitatis x progrediens, deinde detegatur ſeries $y = B$, quæ exprimit y in terminis novæ quantitatis z, ubi z ſit data functio quantitatis x vel functio quantitatum x & y; tum in multis caſibus e datâ functione prædictâ & caſibus, in quibus convergit ſeries A, & principiis hoc in capite traditis erui poſſunt caſus, in quibus convergit ſeries $y = B$.

PROB. XXVI.

Datis algebraicis, fluxionalibus, vel incrementialibus, &c. æquationibus, quantitates x *&* y, *&c. involventibus; invenire valorem quantitatis* y *per terminos quoſcunque irrationales quantitatis* x *ſecundum datam legem progredientes, ſi modo is per tales terminos exprimi poſſit.*

Primo e terminis, qui maximi ſunt, datarum æquationum inveniatur primus terminus (α) datæ formulæ: ſcribatur $\alpha + p$ pro y in datis æquationibus, & e terminis reſultantium æquationum, qui maximi ſunt, inveniatur propinquus valor quantitatis p datæ formulæ, & ſic iteratis operationibus tandem invenietur ſeries quæſita: vel aſſumatur ſeries generaliter expreſſa in terminis ſecundum datam formulam progredientibus, ſcribatur hæc ſeries pro y in datâ æquatione & ejus fluxiones, &c. pro $\dot{y}$, $\ddot{y}$, &c. & e coefficientibus correſpondentium terminorum reſultantis æquationis nihilo æqualibus eſſe ſuppoſitis erui poteſt ſeries quæſita.

Ex. 1. Sit æquatio $\dot{y} = (a + bx^m)^{\mu - 1} x^{\pi} \dot{x}$, invenire quantitatem y per ſeriem conſtantem e terminis formulæ $(a + bx^m)^{\mu} z^t$; aſſumatur

matur $y = (a + bx^m)^\mu (Ax^s + Bx^{s+m} + Cx^{s+2m} + \&c.)$ ſcribatur hic valor pro y & ejus fluxiones pro $\dot{y}$, $\ddot{y}$, &c. in datâ æquatione, & fiant correſpondentes termini reſultantis æquationis nihilo reſpective æquales, & ex æquationibus exinde deductis conſequentur $s = \pi + 1$, $A = \frac{1}{as}$, $B = -\frac{\mu m b A + s b A}{(s+m)a}$, &c. vel aſſumatur $(a + bx^m)^\mu \times \frac{1}{\pi + 1 . a} x^{\pi+1} - p$ pro fluente quæſitâ, & erit $\dot{p} = \frac{\mu m + s}{\pi + 1 . a} b x^{m+\pi} (a + bx^m)^{\mu-1} \dot{x}$, deinde aſſumatur pro p quantitas $\frac{\mu m + s}{(\pi + 1)(m + \pi + 1) a^2} b x^{m+\pi+1} (a + bx^m)^\mu + q$ & erit $\dot{q} = \frac{(\mu m + s) \times (\mu m + m + s) b^2}{(\pi + 1)(m + \pi + 1)} \times x^{2m+\pi} (a + bx^m)^{\mu-1} \dot{x}$, & ſic deinceps; unde facile conſtat lex ſeriei reſultantis.

Ex. 2. Sit $Z = -\log. z$, invenire integralem incrementi $v = \frac{\underset{\cdot}{z}}{z}$; nunc fingatur integralis incrementi $\left(\frac{\underset{\cdot}{z}}{z}\right)$ eſſe $Z + \frac{a}{z} + \frac{b\underset{\cdot}{z}}{z . z + \underset{\cdot}{z}} + \frac{c\underset{\cdot}{z}^2}{z . z + \underset{\cdot}{z} . z + 2\underset{\cdot}{z}} + \&c.$ cujus incrementum invenietur $\underset{\cdot}{Z} + \frac{a\underset{\cdot}{z}}{z . z + \underset{\cdot}{z}} + \frac{2b\underset{\cdot}{z}^2}{z . z + \underset{\cdot}{z} . z + 2\underset{\cdot}{z}} + \frac{3c\underset{\cdot}{z}^3}{z . z + \underset{\cdot}{z} . z + 2\underset{\cdot}{z} . z + 3\underset{\cdot}{z}} + \&c. = \frac{\underset{\cdot}{z}}{z}$; ſed $\underset{\cdot}{Z}$ erit per prob. 2. lib. 2^di^. $= \frac{\underset{\cdot}{z}}{z} - \frac{\underset{\cdot}{z}^2}{2z^2} + \frac{\underset{\cdot}{z}^3}{3z^3} - \frac{\underset{\cdot}{z}^4}{4z^4} + \&c. = \frac{\underset{\cdot}{z}}{z} - \frac{\underset{\cdot}{z}^2}{2z . z + \underset{\cdot}{z}} - \frac{\underset{\cdot}{z}^3}{6z . z + \underset{\cdot}{z} . z + 2\underset{\cdot}{z}} - \frac{\underset{\cdot}{z}^4}{4z . z + \underset{\cdot}{z} . z + 2\underset{\cdot}{z} . z + 3\underset{\cdot}{z}} - \&c.$ ſcribatur hæc quantitas pro ejus valore in æquatione prædictâ $\underset{\cdot}{Z} + \frac{a\underset{\cdot}{z}}{z . z + \underset{\cdot}{z}} + \&c. = \frac{\underset{\cdot}{z}}{z}$, unde $a = \frac{1}{2}$, $2b = \frac{1}{6}$, $3c = \frac{1}{4}$, &c. & conſequenter integralis quæſita erit $v = Z + \frac{1}{2z} + \frac{\underset{\cdot}{z}}{12z . z + \underset{\cdot}{z}} +$

$\underset{\cdot}{z}^2$

$\frac{\underset{\cdot}{z}^2}{12z\,.\,z+\underset{\cdot}{z}\,.\,z+2\underset{\cdot}{z}}+\frac{19\underset{\cdot}{z}^3}{30z\,.\,z+\underset{\cdot}{z}\,.\,z+2\underset{\cdot}{z}\,.\,z+3\underset{\cdot}{z}}-$ &c. In hâc integrali deducendâ ſupponitur $\underset{\cdot}{z}$ invariabilis quantitas.

Cor. . In detegendis integralibus incrementorum ſæpe occurret incrementum $\frac{\underset{\cdot}{z}}{z}$, cujus integralis ſupra datur.

2. Si vero requiratur quæcunque functio (v) quantitatum y & x, vel fluens (v) cujuſcunque fluxionis e quantitatibus y & x & earum fluxionibus compoſitæ, &c. per ſeriem cujus termini progrediuntur vel ſecundum dimenſiones quantitatis x, vel ſecundum quamcunque aliam legem, cujus termini ad ſummam continuo convergunt; inveniatur proximus valor quantitatis v, qui ſit a, deinde pro v ſcribatur $a+p$, & ex æquationibus reſultantibus inveniatur proximus valor quantitatis p, & ſic deinceps uſque donec tandem invenietur valor quantitatis v quæſitus.

Hæ æquationes reduci poſſunt ad unam, quæ exprimit relationem inter variabiles x & v.

3. Sint vero duæ vel plures æquationes tres vel plures variabiles quantitates (x', y', z', &c.) habentes; 1mo. ita reducendæ ſunt datæ æquationes per methodos prius traditas; ut evadant earum variabiles (x, y, z, &c.) perparvæ vel permagnæ, prout requiritur ſeries aſcendens vel deſcendens; deinde fiant termini utrarumque æquationum, qui ſint ex hypotheſi, quod x vel ſit perparva vel permagna quantitas, maximi, inter ſe æquales; iis, qui minores ſint, abjectis, & exinde deducantur proximi valores quantitatum y, z, &c. in terminis quantitatis x, qui ſint reſpective a, α, &c. ſcribantur $a+p$, $\alpha+\pi$, &c. in datis æquationibus reſpective pro y, z, &c. & e terminis, qui maximi ſint in reſultantibus æquationibus, inveniantur proximi valores quantitatum p, π, &c. & ſic deinceps; unde tandem eruentur valores quantitatum y, z, &c. correſpondentes.

Aliter: Per præcedentem methodum inveniantur proximi valores quantitatum variabilium x', y', z', &c. in datis æquationibus contentarum & transformentur datæ æquationes, ita ut earum variabiles x,

y, z, &c.

y, z, &c. evadant perparvæ vel permagnæ, tum ſint $y = Ax^{n}$ prope, $z = ax^{m}$ prope, &c.; ſcribantur hi proximi valores Ax^{n}, ax^{r}, &c. pro y, z, &c. in datis æquationibus, & e differentiâ indicum reſultantium ex hâc ſubſtitutione colligi poſſunt formulæ ſerierum $y = Ax^{n} + Bx^{n+m} + Cx^{n+2m} +$ &c. & $z = ax^{r} + bx^{r+s} + cx^{r+2s} +$ &c. &c. ſcribantur hæ quantitates pro ſuis valoribus in datis æquationibus, & ex æquatis correſpondentibus terminis reſultantium æquationum conſequuntur coefficientes A, B, C, &c. a, b, c, &c.

Si vero duo proximi valores quantitatis ſint Ax^{n}, tum aſſumenda eſt ſeries formulæ $Ax^{n} + Bx^{n+\frac{1}{2}m} + Cx^{n+m} +$ &c. $= y$; ſi tres, tum ſeries formulæ $Ax^{n} + Bx^{n+\frac{1}{3}m} + Cx^{n+\frac{2}{3}m} + Dx^{n+m} +$ &c. aſſumenda eſt; & ſic deinceps.

4. Eadem methodus etiam applicari poteſt ad fluxionales, integrales, &c. æquationes. e. g. Sint fluxionales æquationes (l, r, s, &c.) ordinum datæ, & ita transformentur, ut evadant variabiles perparvæ vel permagnæ; in iis pro z & y ſcribantur αx^{m} & Ax^{n}, &c. reſpective, & per prædictas methodos progrediendum eſt; tum plerumque in ſingulis reſultantibus ſeriebus continebuntur ($l + r + s +$ &c.) variabiles quantitates ad libitum aſſumendæ: aliter; in prædictis æquationibus pro $\frac{\overset{l}{\dot{y}}}{\dot{x}^{l}}$, $\frac{\overset{l-1}{\dot{y}}}{\dot{x}^{l-1}}$, $\frac{\overset{l-2}{\dot{y}}}{\dot{x}^{l-2}}$, &c.; $\frac{\overset{l}{\dot{z}}}{\dot{x}^{l}}$, $\frac{\overset{l-1}{\dot{z}}}{\dot{x}^{l-1}}$, $\frac{\overset{l-2}{\dot{z}}}{\dot{x}^{l-2}}$, &c. &c.; ubi l major eſt quam r vel s, &c.; ſcribantur reſpective $a + bx^{\lambda}$, $c + ax + \frac{b}{\lambda + 1} x^{\lambda+1}$, $d + cx + \frac{a}{2} x^{2} + \frac{b}{(\lambda + 1)(\lambda + 2)} x^{\lambda+2}$, &c.; $a' + b'x^{\mu}$, $c' + a'x + \frac{b'}{\mu + 1} x'^{\mu+1}$, $d' + c'x + \frac{a'}{2} x^{2} + \frac{b'}{(\mu + 1)(\mu + 2)} x^{\mu+2}$, &c. &c. reſpective; & exinde deduci poſſunt quantitates y, z, &c. in terminis quantitatis (x), quæ fluit uniformiter: ſed hìc animadvertendum eſt, quod nunquam plures quam $l + r + s +$ &c. poſſunt eſſe invariabiles quantitates ad libitum aſſumendæ.

THEOR. XXXVI.

Ex iiſdem æquationibus vel ad ſummas ſerierum vel ad fluentes fluxionum deduci poſſunt diverſæ incrementiales vel fluxionales, &c. æquationes, ut e ſequentibus conſtabit; quæ autem omnes diverſæ æquationes, ſi generaliter corrigantur, ſemper eaſdem præbent ſeries.

Ex. 1. Sit æquatio $y^2 = a^2 + bx + \frac{x^4}{c^2}$, ex hâc æquatione erui poteſt valor quantitatis y in terminis ſecundum dimenſiones quantitatis x progredientibus $y = a + \frac{bx}{2a} - \frac{b^2x^2}{8a^3} + \frac{b^3x^3}{16a^3} -$ &c. ſit $a = 0$ & erit ſeries inventa $y = b^{\frac{1}{2}}x^{\frac{1}{2}} + \frac{1}{2}\frac{x^{\frac{7}{2}}}{b^{\frac{1}{2}}c^2} -$ &c.

Inveniatur fluxio datæ æquationis, & erit $2y\dot{y} = b\dot{x} + \frac{4x^3\dot{x}}{c^2}$, unde $y = b^{\frac{1}{2}}x^{\frac{1}{2}} + \frac{1}{2}\frac{x^{\frac{7}{2}}}{b^{\frac{1}{2}}c^2}$ &c. ſed inveniatur hujus æquationis generalis fluens & aſſumenda eſt quantitas $y^2 = a^2$ prope, pro y ſcribatur $a + p$ in æquatione $2y\dot{y} = b\dot{x} + \frac{4x^3\dot{x}}{c^2}$, & reſultat $2(a + p)\dot{p} = b\dot{x} + \frac{4x^3\dot{x}}{c^2}$, & e terminis, qui maximi ſint, inter ſe æqualibus eſſe ſuppoſitis invenietur $2a\dot{p} = b\dot{x}$ prope, unde $p = \frac{bx}{2a}$ prope, & exinde reſultat prior ſeries $y = a + \frac{bx}{2a} +$ &c.

Dividatur data æquatio $y^2 = a^2 + bx + \frac{x^4}{c^2}$ per x, & reſultat $\frac{y^2}{x} = \frac{a^2}{x} + b + \frac{x^3}{c^2}$, cujus fluxio erit $\frac{2y\dot{y}}{x} - \frac{y^2\dot{x}}{x^2} = \frac{3x^2\dot{x}}{c^2} - \frac{a^2\dot{x}}{x^2}$: termini hujuſce æquationis, qui maximi inveniuntur, erunt $\frac{y^2\dot{x}}{x^2} = \frac{a^2\dot{x}}{x^2}$; unde $y^2 = a^2$ prope, & $y = \pm a$ prope, ſcribatur $a + p$ pro y in fluxione

$2y\dot{y}$

$\frac{2y\dot{y}}{x} - \frac{y^2\dot{x}}{x^2} = \frac{3x^2\dot{x}}{c^2} - \frac{a^2\dot{x}}{x^2}$, & reſultat $\frac{2 \times (a+p)\dot{p}}{x} - \frac{(2ap+p^2)\dot{x}}{x^2} = \frac{3x^2\dot{x}}{c^2}$; maximi termini hujus æquationis ſunt $\frac{2a\dot{p}}{x} - \frac{2ap\dot{x}}{x^2}$, fiant nihilo æquales, & reſultat $\frac{\dot{p}}{p} = \frac{\dot{x}}{x}$, unde $p = \frac{bx}{2a}$ prope, & æquatio fiet eadem ac prior $y = a + \frac{bx}{2a} +$ &c. Et ſic deinceps.

Cor. 1. Aſſumatur fluens $A + B = o$, ubi A & B ſunt quæcunque functiones literarum x & y, & a invariabilis quantitas generaliter aſſumpta, inveniatur quantitas y ex hâc æquatione in terminis ſecundum dimenſiones quantitatis x progredientibus, i. e. ſit $y = Ax^n + Bx^{n \pm m} + Cx^{n \pm 2m} \dots Ex^{n \pm rm} +$ &c. ubi E ſit terminus in quem primum ingreditur litera a; inveniatur fluxio præcedentis fluentis $\dot{\pi} = o$ in quâ haud continetur a, & ex hâc æquatione per methodos prius datas inveniatur y in terminis quantitatis x, quæ erit $y = Ax^n$ $Bx^{n \pm m} + Cx^{n \pm 2m} +$ &c. uſque ad terminum $Dx^{n \pm (r-1)m} + Ex^{n \pm rm} +$ &c. ubi per prædictam methodum ſi p ſit proxima approximatio vel quod idem eſt ſi in datâ æquatione $\dot{\pi} = o$ ſcribatur $Ax^n + Bx^{n \pm m} + Cx^{n \pm 2m} \dots Dx^{n \pm (r-1)m} + p$ pro y, reſultabit æquatio $\frac{\dot{p}}{p} = \frac{(n \pm rm)\dot{x}}{x}$ prope, cujus fluens erit $p = lx^{n \pm rm}$ prope, ubi l denotat quamcunque quantitatem ad libitum aſſumendam; unde hæc ſeries ſemper eadem evadet ac præcedens.

Hinc facile deduci poſſunt infinitæ æquationes, quarum ſeries generales, quæ exprimunt y in terminis quantitatis x, erunt eædem.

Hæc principia facile applicari poſſunt ad fluxionales æquationes ſuperiorum ordinum, vel ad fluxionales æquationes plures variabiles quantitates & earum fluxiones habentes.

Cor. 2. Series, quæ exprimunt generalem valorem quantitatis y in terminis vero ſecundum dimenſiones quantitatis x progredientibus, eædem inveniuntur e diverſis æquationibus $\overset{n}{\dot{A}} + \overset{n}{\dot{B}} = o$, $\overset{n-1}{\dot{A}} + \overset{n-1}{\dot{B}} + a\dot{x}^{n-1} = o$, $\overset{n-2}{\dot{A}} + \overset{n-2}{\dot{B}} + ax\dot{x}^{n-2} + b\dot{x}^{n-2} = o$, &c. uſque ad $\dot{A} + \dot{B} \pm ax^{n-1}\dot{x} + bx^{n-2}\dot{x} + cx^{n-3}\dot{x} + \text{\&c.} \ldots\ldots l\dot{x} = o$, & $A + B + ax^{n} + bx^{n-1} + cx^{n-2} \ldots l' = o$, ubi literæ $a, b, c, \ldots l$ & l' quaſcunque invariabiles denotant quantitates.

Duæ ſeries, quæ exprimunt prædictum generalem valorem, eædem inveniuntur e duabus æquationibus $\pi = o$ & $L \times \pi = o$, ubi π eſt fluxionalis quantitas ſuperioris ordinis quam quantitas L.

PROB. XXVII.

Ex datis relationibus inter valores x *&* z *incognitæ quantitatis* x *& inter ſummas duarum ſerierum reſultantium; invenire coefficientes ipſius ſeriei.*

Sit ſumma ſeriei hujuſce formulæ $ax^{m} + bx^{m+r} + cx^{m+2r} + \text{\&c.} = \mathcal{Q}$; inveniatur valor (R) ejuſdem ſeriei, cum x evadat (z) quæcunque data functio quantitatis x, i. e. $az^{m} + bz^{m+r} + cz^{m+2r} + \text{\&c.} = R = Ax^{m'} + Bx^{m'+r'} + Cx^{m'+2r'} + \text{\&c.} + A'x^{b} + B'x^{b+s} + C'x^{b+2s} + \text{\&c.}$; tum ex quantitate $\mathcal{Q} = ax^{m} + bx^{m+r} + \text{\&c.}$ & relatione inter ſummas inveniatur $R = a'x^{m'} + b'x^{m'+r'} + c'x^{m'+2r'} + \text{\&c.}$; & ex æquatis correſpondentibus terminis reſultant $A = a'$, $B = b'$, $C = c'$, &c., & exinde deduci poſſunt coefficientes a, b, c, &c. quæſitæ.

Ex. Cum x evadat $2x = z$, tum ſeries $ax^{m} + bx^{m+r} + cx^{m+2r} + \text{\&c.}$ evadat $(ax^{m} + bx^{m+r} + cx^{m+2r} + \text{\&c.})^{2}$; in datâ ſerie pro x ſcribatur $2x$, & reſultat $a2^{m}x^{m} + b \times 2^{m+r}x^{m+r} + c \times 2^{m+2r}x^{m+2r} + \text{\&c.} = (ax^{m} + bx^{m+r} + \text{\&c.})^{2} = a^{2}x^{2m} + 2abx^{2m+r} + (2ac + b^{2})x^{2m+2r} + \text{\&c.}$; & ex æquatis correſpondentibus terminis reſultant $a2^{m} \times x^{m} = a^{2}x^{2m}$, & conſequenter $m = 2m$; & exinde $m = o$ & $2^{m}a = 2^{o}a = a = a^{2}$; unde $a = 1$; & $2abx^{2m+r} = 2abx^{r} = b \times 2^{m+r}x^{r}$, unde $2^{r} = 2$ & exinde $r = 1$; & $c \times 2^{2r} \times x^{2r} = c \times 2^{2} \times x^{2} = (2ac + b^{2})x^{2r} = (2c + b^{2})x^{2r}$, unde $4c = 2c + b^{2}$, & exinde $c = \frac{b^{2}}{2}$; & ſic deinceps; &

tandem reſultat series $a x^m + b x^{m+r} + c x^{m+2r} + \&c. = 1 + b x + \frac{b^2}{2} x^2 + \frac{b^3}{2 . 3} x^3 + \frac{b^4}{2 . 3 . 4} x^5 + \&c.$; quæ ex logarithmo invenit numerum.

Ex. 2. Sit prædicta ſeries $a + b x^r + c x^{2r} + d x^{3r} + \&c.$; cum vero x evadat $2 x = z$, tum ſeries $a + b x^r + \&c.$ evadat $(a + b x^r + c x^2 + \&c.)^s = a^s + s a^{s-1} b x^r + s . \frac{s-1}{2} a^{s-2} b^2 x^{2r} + \&c. = a + b \times 2^r x^r$

$$+ s \quad a^{s-1} c x^{2r}$$

$+ c \times 2^{2r} x^{2r} + d \times 2^{3r} \times x^{3r} + \&c.$, unde $a = a^s$ & conſequenter $a = 1$; & $2^r b = s b$ & exinde $2^r = s$ & $r = \frac{\log. s}{\log. 2}$; & $s . \frac{s-1}{2} a^{s-2} b^2 + s a^{s-1} c = 2^{2r} \times c$ & $s . \frac{s-1}{2} b^2 = (2^{2r} - s) c = (s^2 - s) c$, unde $c = \frac{1}{2} b^2$: etiamque $s . \frac{s-1}{2} . \frac{s-2}{3} b^3 + s . (s-1) b c + s d = 2^{3r} d = s^3 d$ & conſequenter $s . \frac{s-1}{2} . \frac{s-2}{3} b^3 + s . (s-1) . b . \frac{b^2}{2} = (s^3 - s) d$ & $s . \frac{s-1}{2} . \frac{s+1}{3} b^3 = (s^3 - s) d$ & $d = \frac{b^3}{1 . 2 . 3}$: & ſic $e = \frac{b^4}{1 . 2 . 3 . 4}$, &c; & ſeries $a + b x^r + \&c. = 1 + \frac{b}{1} x^r + \frac{b^2}{1 . 2} x^{2r} + \frac{b^3}{1 . 2 . 3} x^{3r} + \frac{b^4}{1 . 2 . 3 . 4} x^{4r} + \&c.$, ubi $r = \frac{\log. s}{\log. 2}$, quæ erit 1, cum $s = 2$; & -1 cum $s = \frac{1}{2}$.

Ex. Sit quantitas $a x + b x^2 + c x^3 + \&c. = l$, quæ ſit logarithmus quantitatis $1 + x$, at, cum numerus $(1 + x)$ evadat $z = (1 + x)^2$; tum logar. l evadat duplus; pro x ſcribatur igitur $2 x + x^2$ in prædictâ ſerie $a x + b x^2 + c x^3 + \&c.$, & reſultabit $a (2 x + x^2) + b (4 x^2 + 4 x^3 + x^4) + c (2 x + x^2)^3 + \&c. = 2 a x + (a + 4 b) x^2 + (4 b + 8 c) x^3 + (b + 12 c + 16 d) x^4 + \&c. = 2 l = 2 a x + 2 b x^2 + 2 c x^3 + 2 d x^4 + \&c.$: ex æquatis correſpondentibus terminis reſultant $2a = 2a$; $2b = a + 4b$, unde $b = -\frac{a}{2}$; & $4b + 8c = 2c$, & conſequenter $c = -\frac{b}{3}$; &c.

Et ſic progredi liceat ad plura & magis generalia exempla; etiamque ex reſolutionibus inter plures ſucceſſivos terminos ad ſeriem deducendam.

2. Sint V & W datæ functiones quantitatis x, i. e. $V = X$ & $W = X'$ invenire V in terminis quantitatis W.

Reducantur duæ æquationes $V = X$ & $W = X'$ ad unam, ita ut exterminetur x, & reſultat æquatio quæſita.

Aliter: Sit $V = ax^{m} + bx^{m+n} + cx^{m+2n} +$ &c. & $W = d'x^{r} + b'x^{r+s} + c'x^{r+2s} +$ &c.; tum erit $V = AW^{\frac{m}{r}} + BW^{\frac{m}{r}+t} + CW^{\frac{m}{r}+2t} +$ &c.; in hâc æquatione pro V & W ſcribantur earum valores $ax^{m} +$ &c., $d'x^{r} +$ &c.; & æquentur correſpondentes termini æquationis reſultantis, & exinde facile erui poſſunt coefficientes a, b, c, &c., d', b', &c.

Si $r = o$, tum pro $W - d$ ſcribatur W'; & evadet $V = A'W'^{\frac{m}{s}} +$ &c.

Si exponentiales, fluentiales, &c. quantitates in prædictis æquationibus $V = ax^{m} +$ &c. & $W = ax^{r} +$ &c. contineantur; tum plerumque ita reducendæ ſunt exponentiales, fluentiales, &c. quantitates, ut progrediantur ſecundum dimenſiones perparvæ quantitatis o, & deinde per methodum hìc traditam acquiri poteſt problematis reſolutio.

PROB. XXVIII.

Reducere datam ſeriem ad factores.

Sit ſeries $ax^{m} + bx^{m+n} + cx^{m+2n} +$ &c.: aſſumantur factores, qui ad ſeries reducti præbent ſeries ejuſdem formulæ ac data ſeries, & qui in ſeſe continuo ducti continent tot ſaltem incognitas & independentes coefficientes, quot reſultant termini deſtruendi.

Ex. Sit quantitas $a + bx + cx^2 + \&c.$; & erit æqualis contento e ſingulis factoribus $(e + fx)^m \times (e' + f'x)^{m'} \times (e'' + f''x)^{m''} \times \&c.$, quibus factoribus $(e+fx)^m$, $(e'+f'x)^{m'}$, &c. ad ſeries $e^m + me^{m-1}fx + \&c.$, $e'^{m'} + m'e'^{m'-1}f'x + \&c.$ ejuſdem ac datæ ſeriei formulæ reductis; plures continentur incognitæ & independentes coefficientes e, e', e'', &c.; f, f', f'', &c., &c., m, m', m'', &c. quam termini datæ ſeriei æquandi; i. e. æquentur correſpondentes termini & reſultant æquationes $e^m \times e'^{m'} \times e''^{m''} \times \&c. = a$, $me^{m-1}e'^{m'}e''^{m''} \&c.\, f + m'e'^{m'-1}e^m e''^{m''} \&c.\, f' + \&c. = b$, & ſic deinceps: hìc plures involvuntur incognitæ & independentes coefficientes quam reſultantes æquationes, & conſequenter facile erui poſſunt coefficientes, quæ ſolutionem problematis præbent.

Cor. Sint factores $(ex^r + fx^s + gx^t + \&c.)\,(e'x^{r'} + f'x^{s'} + g'x^{t'} + \&c.)\,(e''x^{r''} + f''x^{s''} + g''x^{t''} + \&c.)$ &c. $= ax^m + bx^{m+n} + \&c.$, ubi $r + r' + r'' + \&c. = m$, & $s - r$, &c. $= n$; $e \times e' \times e'' \times \&c. = a$, &c.

Eadem principia etiam applicari poſſunt ad quantitates in quibus incognitæ quantitates continentur.

PROB. XXIX.

Datâ æquatione quantitatem v involvente; ubi v ſit quæcunque quantitas, quæ e datâ quantitate x deduci poteſt; invenire valorem quantitatis x.

1. Primo inveniatur a proximus valor quantitatis x, ſcribatur quantitas a pro x in datâ æquatione, & ſit quantitas reſultans A; deinde ſcribatur $a + \pi$ (ubi π ſit perparva quantitas) pro x in datâ æquatione, & ſit quantitas reſultans B; tum erit $A - B : A :: \pi : \dfrac{A\pi}{A-B}$, unde $a + \dfrac{A\pi}{A-B}$ erit propior approximatio ad quantitatem x; & ſic redintegratâ operatione propior adhùc conſtabit valor quantitatis x.

2. Si vero duo valores quantitatis x ſint prope quantitati a & inter

ter ſe æquales; tum ſcribendæ ſunt pro x in datâ æquatione reſpective a, $a + \pi$, $a + 2\pi$; & ſint quantitates reſultantes reſpective A, B, C; erunt duæ radices quadraticæ æquationis $\frac{1}{2}(A - 2B + C)e^2 - \frac{1}{2}(C - 4B + 3A)\pi e + A\pi^2 = 0$, quæ ſint α & β, propiores approximationes ad duos valores quantitatis x prædictos, i. e. $a + \alpha$ & $a + \beta$ erunt prope duo valores quantitatis x: ſi autem α multo minor ſit quam β, tum erit $\alpha = \dfrac{2A\pi}{C - 4B + 3A}$ prope.

Cor. 1. Sit æquatio $x^n - px^{n-1} + qx^{n-2} - \&c. = 0$: in hâc æquatione pro x ſcribantur reſpective a & $a + \pi$, & reſultant quantitates $a^n - pa^{n-1} + qa^{n-2} - \&c. = A$, & $A + (na^{n-1} - (n-1)pa^{n-2} + (n-2)qa^{n-3} - \&c.)\pi + \&c. = B$, & conſequenter $B - A = (na^{n-1} - (n-1)pa^{n-2} + (n-2)qa^{n-3} - \&c.)\pi$ prope: $a^n - pa^{n-1} + qa^{n-2} - \&c. = A :: \pi : \dfrac{a^n - pa^{n-1} + qa^{n-2} - \&c.}{na^{n-1} - (n-1)pa^{n-1} + (n-2)qa^{n-2} - \&c.} = e$; & exinde $a - e$ erit propior valor quantitatis x.

Cor. 2. Sint vero n valores quantitatis x prope inter ſe & quantitati a æquales, ſcribantur pro x in datâ æquatione reſpective a, $a + \pi$, $a + 2\pi$, $a + 3\pi$, . . . $a + (n - 1)\pi$, & ſint quantitates reſultantes reſpective A, B, C, D, E, &c. & $d = A - B$, $d' = A - 2B + C$, $d'' = A - 3B + 3C - D$, $d''' = A - 4B + 6C - 4D + E$, ... $d'^{m-1} = A - mB + m.\frac{m-1}{2}C - m.\frac{m-1}{2}.\frac{m-2}{3}D + \&c.$ & ſint α, β, γ, δ, &c. n radices (e) æquationis $A\pi^n - d\pi^{n-1}e + \frac{1}{2}d'\pi^{n-2}e \times (e - \pi) - \frac{1}{2 \cdot 3}d''\pi^{n-3}e \times (e - \pi) \times (e - 2\pi) + \frac{1}{2 \cdot 3 \cdot 4}d'''\pi^{n-4}e \times (e - \pi) \times (e - 2\pi) \times (e - 3\pi) - \&c. = 0$, tum erunt $a + \alpha$, $a + \beta$, $a + \gamma$, $a + \delta$, &c. reſpective approximationes ad n valores quantitatis x prædictos.

Si vero ſcribantur pro x in datâ æquatione (n) quantitates a, $a + \pi$, $a + \rho$, $a + \sigma$, $a + \tau$, &c.; & ſint (n) quantitates reſultantes A, B, C, D, E, &c.; & ſint (n) radices æquationis $A - \left(\frac{A}{\pi} - \frac{B}{\pi}\right)e$

$$+\left(\frac{A}{\pi\rho}+\frac{B}{\pi(\pi-\rho)}+\frac{C}{\rho(\rho-\pi)}\right)e\times(e-\pi)-\left(\frac{A}{\pi\rho\sigma}-\frac{B}{\pi(\pi-\rho)(\pi-\sigma)}-\frac{C}{\rho(\rho-\pi)(\rho-\sigma)}-\frac{D}{\sigma(\sigma-\pi)(\sigma-\rho)}\right)e\times(e-\pi)\times(e-\rho)+\left(\frac{A}{\pi\rho\sigma\tau}+\frac{B}{\pi(\pi-\rho)(\pi-\sigma)(\pi-\tau)}+\frac{C}{\rho(\rho-\pi)(\rho-\sigma)(\rho-\tau)}+\frac{D}{\sigma(\sigma-\pi)(\sigma-\rho)(\sigma-\tau)}+\frac{E}{\tau(\tau-\pi)(\tau-\rho)(\tau-\sigma)}\right)e\times(e-\pi)\times(e-\rho)\times(e-\sigma)+\&c. = 0$$

reſpective α, β, γ, δ, &c., &c.; tum erunt $a+\alpha$, $a+\beta$, $a+\gamma$, $a+\delta$, &c. approximationes ad n radices datæ æquationis, quæ ſunt prope inter ſe æquales. Aliter: ſint (n) radices prædictæ α, β, γ, δ, &c. radices (e) æquationis

$$\frac{(e-\pi)\times(e-\rho)\times(e-\sigma)\times(e-\tau)\ \&c.}{\pm\pi\rho\sigma\tau\ \&c.}\times A+\frac{e\times(e-\rho)\times(e-\sigma)\times(e-\tau)\times\&c.}{\pi\times(\pi-\rho)\times(\pi-\sigma)\times(\pi-\tau)\times\&c.}\times B+\frac{e\times(e-\pi)\times(e-\sigma)\times(e-\tau)\times\&c.}{\rho\times(\rho-\pi)\times(\rho-\sigma)\times(\rho-\tau)\times\&c.}\times C+\frac{e\times(e-\pi)\times(e-\sigma)\times(e-\tau)\times\&c.}{\sigma\times(\sigma-\pi)\times(\sigma-\rho)\times(\sigma-\tau)\times\&c.}\times D+\frac{e\times(e-\pi)\times(e-\rho)\times(e-\sigma)\times\&c.}{\tau\times(\tau-\pi)\times(\tau-\rho)\times(\tau-\sigma)\times\&c.}\times E+\&c.=0,$$

tum erunt $a+\alpha$, $a+\beta$, $a+\gamma$, $a+\delta$, &c. approximationes ad (n) prædictas radices.

Lex hujuſce ſeriei facile conſtat ex obſervatis ipſius terminis.

Si vero a, $a+\pi$, $a+\rho$, $a+\sigma$, &c. ſint multo propiores ad unam radicem datæ æquationis quam ad reliquas; tum (n) radices prædictæ æquationis, cujus radix eſt e, erunt reſpective $f-\pi$, $f-\rho$, $f-\sigma$, $f-\tau$, &c. prope, & radix quæſita $a+f$ prope.

Cor. Hinc $1-\frac{\pi}{\pi}=0$, $1-\frac{\rho}{\pi}+\frac{\rho\times(\rho-\pi)}{\rho\pi}=0$, $1-\frac{\sigma}{\pi}+\frac{\sigma\times(\sigma-\pi)}{\rho\pi}-\frac{\sigma\times(\sigma-\pi)\times(\sigma-\rho)}{\rho\pi\sigma}=0$, $1-\frac{\tau}{\pi}+\frac{\tau\times(\tau-\pi)}{\pi\rho}-\frac{\tau\times(\tau-\pi)\times(\tau-\rho)}{\pi\rho\sigma}+\frac{\tau.(\tau-\pi).(\tau-\rho).(\tau-\sigma)}{\pi\rho\sigma\tau}=0$, & ſic deinceps; etiamque $\frac{\rho}{\pi}+$

$\rho\,\times$

$\frac{\rho\times(\rho-\pi)}{\pi(\pi-\rho)}=0$, $\frac{\sigma}{\pi}+\frac{\sigma\times(\sigma-\pi)}{\pi(\pi-\rho)}+\frac{\sigma.(\sigma-\pi)\times(\sigma-\rho)}{\pi(\pi-\rho)(\pi-\sigma)}=0$, $\frac{\tau}{\pi}+\frac{\tau(\tau-\pi)}{\pi(\pi-\rho)}+\frac{\tau(\tau-\pi)(\tau-\rho)}{\pi(\pi-\rho)(\pi-\sigma)}+\frac{\tau.(\tau-\pi).(\tau-\rho).(\tau-\sigma)}{\pi.(\pi-\rho).(\pi-\sigma).(\pi-\tau)}=0$, &c. & $\frac{\sigma\times(\sigma-\pi)}{\rho(\rho-\pi)}+\frac{\sigma(\sigma-\pi)(\sigma-\rho)}{\rho(\rho-\pi)(\rho-\sigma)}=0$, $\frac{\tau\times(\tau-\pi)}{\rho(\rho-\pi)}+\frac{\tau(\tau-\pi)(\tau-\rho)}{\rho(\rho-\pi)(\rho-\sigma)}+\frac{\tau(\tau-\pi)(\tau-\rho)(\tau-\sigma)}{\rho(\rho-\pi)(\rho-\sigma)(\rho-\tau)}=0$, &c., & $\frac{\tau(\tau-\pi)(\tau-\rho)}{\sigma(\sigma-\pi)(\sigma-\rho)}+\frac{\tau(\tau-\pi)(\tau-\rho)(\tau-\sigma)}{\sigma(\sigma-\pi)(\sigma-\rho)(\sigma-\tau)}=0$, &c.

Principia hìc tradita applicari poſſunt ad inveniendam ſeriem, quæ exprimit quantitatem v terminis vero ſecundum dimenſiones quantitatis z progredientibus ex datâ æquatione relationem inter x & y exprimente, ubi literæ z & v reſpective denotant quaſcunque functiones vel algebraicas vel fluentiales, integrales, &c. literarum x & y.

2. Datis m æquationibus m incognitas quantitates x, y, z, &c. habentibus, ſint vero α, β, γ, &c. proximi valores incognitarum quantitatum x, y, &c. reſpective inter ſe correſpondentes; aſſumantur $n+1$ diverſi valores quantitatis x, viz. α, $\alpha+\pi$, $\alpha+\pi'$, $\alpha+\pi''$, &c. &c. & ſic aſſumantur $n+1$ diverſi correſpondentes valores quantitatis y qui ſint reſpective β, $\beta+\rho$, $\beta+\rho'$, $\beta+\rho''$, &c. & ſic de reliquis, ubi $\pi, \pi' \pi''$, &c. ρ, ρ', ρ'', &c. ſunt perparvæ quantitates; ſcribantur α, β, &c. $\alpha+\pi$, $\beta+\rho$, &c. $\alpha+\pi'$, $\beta+\rho'$, &c. &c. pro ſuis valoribus x, y, &c. in datis æquationibus, & reſultent quantitates A, B, C, &c. reſpective in unâ æquatione; P, Q, R, &c. in ſecundâ, &c. & reſultabunt e primâ æquatione n ſimplices æquationes $a\pi+b\rho+$ &c. $=B-A$, $a\pi'+b\rho'+$ &c. $=C-A$, $a\pi''+b\rho''+$ &c. $=D-A$, &c. e ſecundâ vero æquatione n ſimplices æquationes $h\pi+k\rho+$ &c. $=Q-P$, $h\pi'+k\rho'+$ &c. $=R-P$, $h\pi''+k\rho''+$ &c. $=S-P$, &c. ex his $2n$, &c. æquationibus inveſtigari poſſunt coefficientes a, b, &c. h, k, &c. ultimo aſſumantur m æquationes $A+ae+bi+$ &c. $=0$, $P+he+ki+$ &c. $=0$; &c. e quibus deduci poſſunt

valores quantitatum $e, i,$ &c. & erunt $\alpha + e, \beta + i,$ &c. propiores valores quantitatum $x, y,$ &c.

E. g. Sit $m = 2$; e duabus ſimplicibus æquationibus $a\pi + b\rho + A - B = o$ & $a\pi' + b\rho' + A - C = o$ deduci poſſunt coefficientes a & b, & ex datis etiam æquationibus $k\pi + k\rho + P - Q = o$, & $h\pi' + k\rho' + P - R = o$ erui poſſunt coefficientes h & k; quibus datis fingantur $ae + bi + A = o$, $he + ki + P = o$, e quibus eruantur valores quantitatum e & i, & erunt $\alpha + e$ & $\beta + i$ propiores valores quantitatum x & y.

In his ſubſtitutionibus cavendum eſt, ne æquationes reſultantes a ſe invicem ſint dependentes.

In his caſibus ſæpe præſtat aſſumere $\alpha = o$, $\beta = c$, $\gamma = o$, &c. cum x evadat α, tum Γ evadat A; etiamque cum x evadat $\alpha + \pi$, tum Γ evadat B; exinde cum x evadat $\alpha + \rho$, tum Γ evadet $A + \frac{B - A}{\pi}\rho$ prope, ſi modo π & ρ ſint perparvæ quantitates.

2. Cum n quantitatites $x, y,$ &c. evadant reſpective $\alpha, \beta,$ &c.; tum Γ evadat A; etiamque cum prædictæ quantitates $x, y,$ &c. evadant $\alpha + \pi, \beta + \rho,$ &c., tum Γ evadat B; & ſimiliter cum $x, y,$ &c. evadant $\alpha + \pi', \alpha + \pi'',$ &c.; $\beta + \rho', \beta + \rho'',$ &c.; tum Γ evadat reſpective $C, D, E,$ &c.

Aſſumantur (n) ſimplices æquationes $a\pi + b\rho +$ &c. $= B - A$, $a\pi' + b\rho' +$ &c. $= C - A$, $a\pi'' + b\rho'' +$ &c. $= D - A$, $a\pi''' + b\rho''' +$ &c. $= E - A$, &c.; ex his æquationibus inveniantur (n) incognitæ quantitates $a, b, c, d,$ &c.; tum erit $\Gamma - A = a\lambda + b\mu +$ &c.; cum $x, y,$ &c. evadant reſpective $\alpha + \lambda, \beta + \mu,$ &c.

2. 2. Si vero duo vel plures valores quantitatum $x, y,$ &c. ſint $\alpha, \beta,$ &c. prope; tum e pluribus aſſumptis reſpondentibus valoribus quantitatum $x, y,$ &c. pro ſuis valoribus in datis æquationibus ſubſtitutis; & ex principiis iiſdem, quæ in hoc prob. & in meditat. algebr. traduntur, facile conſequi poſſunt valores ad veros valores magis appropinquantes.

Et ſic de n æquationibus plures $n + m$ incognitas quantitates habentibus.

Convergentiæ harum ſerierum pendent omnino ex iiſdem principiis ac convergentiæ ſerierum prius traditarum.

Hìc obſervandum eſt, ſi quantitates reſultantes prædictæ haud creſcant vel decreſcant ultimo in eâdem ratione ac iſtæ minimæ differentiæ quantitatum ipſarum a veris valoribus, tum ex hâc methodo haud deduci poteſt reſolutio; ex hypotheſi ultimarum rationum inter prædictas differentias & quantitates reſultantes ſequitur reſolutio.

THEOR. XXXVII.

Sit $y = \int . X\dot{x}$, ubi X eſt functio quantitatis x; & $y = b$ cum $x - a$; & ſi valor quantitatis x parum exſuperet a, tum per prob. 5. libri primi erit $y = Xx - \frac{1}{2}Px^2 + \frac{1}{6}\mathcal{Q}^3 - \frac{1}{24}R^4 +$ &c. ubi $P\dot{x} = \dot{X}$, $\mathcal{Q}\dot{x} = \dot{P}$, $R\dot{x} = \dot{\mathcal{Q}}$, &c. ſed pro x ſcribatur $z - a$, & invenietur $y = b + X(z - a) - \frac{1}{2}P(z - a)^2 + \frac{1}{6}\mathcal{Q}(z - a)^3 -$ &c.

Cor. . Præſtabit per intervalla procedere tribuendo ipſi x ſucceſſive valores a, a', a'', a''', &c. & pro ſingulis valores quantitatum X, P, $\mathcal{Q}$, &c. convenientes computando, & exinde aggregatum e ſingulis valoribus ſeriei $Xx - \frac{1}{2}Px^2 + \frac{1}{6}\mathcal{Q}^3 -$ &c. detegendo, quod erit y fluens quæſita.

THEOR. XXXVIII.

Sint a & b ſimul valores quantitatum x & y reſpective; & ſit y functio quantitatis x; & 1^mo^. cum x fiat $x - n\dot{x}$; tum y evaderet $y - n\dot{y} + n . \frac{n+1}{2}\ddot{y} - \frac{n . (n+1) . (n+2)}{2 . 3}\dddot{y} +$ &c. Ponatur $x - n\dot{x} = a$ & erit $n = \frac{x - a}{\dot{x}}$ pene infinitus numerus; cum vero $x = a$ per hypotheſin erit $y = b$, & conſequenter $y = b + (x - a)\frac{\dot{y}}{\dot{x}} - \frac{1}{2}\frac{\ddot{y}}{\dot{x}^2}(x - a)^2$

$a)^2 + \frac{1}{6}\frac{\dddot{y}}{\dot{x}^3}(x - a)^3 -$ &c. 2do. Fiat vero x jam $x + n\dot{x}$ & y evaderet $y + n\dot{y} + n \,.\, \frac{n-1}{2}\ddot{y} +$ &c. ſcribatur $\frac{x-a}{\dot{x}}$ pro n & fit $y = b +$ $\frac{\dot{y}}{\dot{x}}(x - a) + \frac{1}{2}\frac{\ddot{y}}{\dot{x}^2}(x - a)^2 + \frac{1}{6}\frac{\dddot{y}}{\dot{x}^3}(x - a)^3 +$ &c.

Cor. 1. Si progreſſus ab a ad x in intervalla æqualia ſecundum differentiam α diſpertiantur, & termini in ſingulis ſeriebus ultimo præcedentibus notentur per ${}'X, {}'P, {}'\mathcal{Q}, {}'R,$ &c. ubi $\dot{y} = X\dot{x}$, $\dot{X} = P\dot{x}$, $\dot{P} = \mathcal{Q}\dot{x}$, &c. & $A, A', A'', \ldots {}'X, X$ ſint reſpective ſucceſſivi valores quantitatis X ad ſingula intervalla; & $B, B', B'', \ldots {}'P, P$ ſint reſpective ſucceſſivi valores quantitatis P ad prædicta intervalla, & ſic deinceps; tum ex ſecundo caſu reſultant $y = b + \alpha(A + A' + A'' \ldots + {}'X) + \frac{1}{2}\alpha^2(B + B' + B'' \ldots + {}'P) + \frac{1}{6}\alpha^3(C + C' + C'' \ldots + {}'\mathcal{Q}) + \frac{1}{24}\alpha^4(D + D' + D'' \ldots + {}'R) +$ &c. Si inter limites & valorem priorem $y = b + \alpha(A' + A'' \ldots X) - \frac{1}{2}\alpha^2(B' + B'' \ldots + P) + \frac{1}{6}\alpha^3(C' + C'' \ldots + \mathcal{Q}) -$ &c. ſumatur medium arithmeticum prodibit ſeries adhuc magis convergens $y = b + \alpha(A' + A'' + A''' \ldots + {}'X) + \frac{1}{2}\alpha(A + X) + \frac{1}{4}\alpha^2(B - P) + \frac{1}{6}\alpha^3(C' + C'' \ldots + {}'\mathcal{Q}) + \frac{1}{12}\alpha^3(C + \mathcal{Q}) + \frac{1}{48}\alpha^4(D - R) +$ &c.

Convergentiæ harum ſerierum e principiis prius traditis dijudicari poſſunt.

Ex principiis prius traditis progredi liceat ad convergentiam ſerierum harum formularum deducendam, viz. $\sqrt[m]{(a - \sqrt[m]{(b - \sqrt[m]{(c - \&c.)})})}$ vel $\sqrt[m]{(a - \sqrt[n]{(b - \&c.)})}$ vel $\sqrt[m]{\left(a + \cfrac{b}{\sqrt[m]{\left(c + \cfrac{d}{\sqrt[m]{(e + \&c.)}}\right)}}\right)}$; & ſic de infinitis diverſorum generum ſeriebus & æquationibus.

THEOR.

THEOR. XXXIX.

1. Sit $x = \sqrt[n]{(a + \sqrt[n]{(a + \sqrt[n]{(a \text{ \&c.})})})}$; erit x vel radix æquationis $x^n - a - x = 0$, vel $(x^n - a)^n - a - x = 0$, vel $((x^n - a)^n - a)^n - a - x = 0$, & ſic deinceps; omnes enim hæ præcedentes æquationes erunt diviſores ſubſequentium.

2. Sit $x = \sqrt[n]{(a\sqrt[n]{(a\sqrt[n]{(a \text{ \&c.})})})}$; & erit x radix æquationis $x^{n-1} - a = 0$ vel $x^{n^2-1} - a^{n+1} = 0$ vel $x^{n^3-1} - a^{n^2+n+1} = 0$ & denique $x^{n^m-1} - a^{n^{m-1}+n^{m-2}+\&c.\ldots+1} = 0$, ubi m eſt integer numerus; omnes enim hæ præcedentes æquationes erunt etiam diviſores ſubſequentium.

Eadem principia etiam applicari poſſunt ad conſimiles ſeries, in quibus duæ vel plures continentur literæ vel radices, quæ eodem ordine recurrunt; etiamque ad ſeries ſubſequentes.

THEOR. XL.

1. Sit $x = \sqrt[m]{a} + \cfrac{b}{\sqrt[m]{a} + \cfrac{b}{\sqrt[m]{a} + \&c.}}$, & erit reſolutio æquationis

$x^{m+1} - ax = b$, &c.

2. Sit $x = \sqrt[m]{a} + \cfrac{b}{\sqrt[n]{c} + \cfrac{d}{\sqrt[m]{a} + \cfrac{b}{\&c.}}}$, & erit reſolutio æquationis

$(x^m - a)^n \times (cx + d) = b^n x$; &c.

Cor. . Sit cubica æquatio $x^3 - px^2 + qx - r = 0$, aſſumantur $m = 2$, n & $c = 1$, & reſultat æquatio $x^3 + dx^2 - (a + b)x - ad = 0$,

$= o$, unde aſſumptis $d = -p$, $a = \frac{r}{d} = \frac{-r}{p}$, & $b = -(a + q)$, reſultat reſolutio cubicæ æquationis.

3. Sit $x = \sqrt[n]{\frac{a}{b + \sqrt[n]{\frac{a}{b + \sqrt[n]{\frac{a}{b + \&c.}}}}}}$, unde $x^{n+1} + bx^n = a$, &c.

4. Sit $x = \sqrt[n]{a + \frac{b}{c + \frac{d}{\sqrt[n]{a + \frac{b}{c + \frac{d}{\sqrt[n]{a + \&c.}}}}}}}$, & erit æquatio exinde reſultans $cx^{n+1} + dx^n - acx - bx - da = o$, &c. Infinitæ ejuſmodi ſolutiones harum æquationum facile dari poſſunt.

Si modo dentur quæcunque quantitates, quæ iterum perpetuo occurrunt; tum ex iis facile deduci poſſunt æquationes, quarum reſolutiones per has methodos dantur.

Hujuſce generis quantitates in ſeſe multiplicari vel per ſe dividi poſſunt per vulgares methodos, ſed plerumque producta & quotientes reſultantes erunt quantitates maxime compoſitæ; ſed tædet de his plura adjicere.

Sit z diſtantia a primo ſeriei termino, & ſint a, b, c, d, &c. functiones quantitatis z, tum ex ratione ſucceſſivorum terminorum ad infinitam diſtantiam facile conſtat, annon ſeries ſit convergens.

SCHOLIUM.

Veteres ad areas curvilinearum figurarum approximabant, inſcribendo in illis vel deſcribendo circa eas rectilineas figuras.

In curvis haud quadrabilibus, ubi etiam ordinata ſit data functio abſciſſæ, methodi recentiorum vix aut ne vix ſuperiores cenſendæ ſunt; ut vero ſeries per has methodos inventæ reddantur celeriter convergentes, plurimæ interpolandæ ſunt plerumque intermediæ ſeries, æque ac inſcribendæ vel deſcribendæ rectilineæ figuræ in curvilineis figuris; & in multis caſibus rectilineæ figuræ magis convergent quam ſeries interpolandæ: reductio quantitatum ad terminos ſecundum dimenſiones quantitatis x progredientes ni in fluentibus detegendis perraro uſui inſervit; nam quantitates ipſæ plerumque ſine tali reductione majore facilitate deduci poſſunt; & reductio prædicta ad integrales aſcendentes, &c. detegendas non applicari poteſt. In fluentibus detegendis ut prædicitur plerumque plurimæ interpolandæ ſunt ſeries, aliter haud convergit ſeries.

LIBER IV.

DE SUMMATIONE SERIERUM, &c.

THEOR. I.

1. SINT ſummæ, i. e. integrales ſucceſſivæ s, s', s'', &c. & erunt earum incrementa, i. e. ſucceſſivi termini $s - s' = t$, $s' - s'' = t'$, &c. ubi per z ſemper deſignetur diſtantia termini dicti a primo ſeriei termino.

Ex. 1. Sint s & s' reſpective $z \times (z-1).(z-2)..(z-n)$ & $(z+1).z.(z-1)...(z-n+1)$; tum erit $t = (n+1).z.(z-1).(z-2)...(z-n+1)$, & conſequenter ſi terminus generalis ſeriei ſit $t = z.(z-1).(z-2)...(z-n+1)$, ſum erit ejus ſumma $s = \frac{1}{n+1} z.(z-1)...(z-n) + A$, ubi A ſit ſumma omnium terminorum ab initio ſeriei uſque ad terminum $z = n$.

Cor. Conſtat e prob. 6. lib. 2^{di}. omnem quantitatem hujuſce generis $Az^n + Bz^{n-1} + Cz^{n-2} +$ &c. ubi n ſit integer numerus, reſolvi poſſe in quantitates ſubſequentis formulæ $az.(z-1).(z-2)..(z-n+1) + bz.(z-1).(z-2)..(z-n+2) + cz.(z-1)...(z-n+3) + dz.(z-1)..(z-n+4) +$ &c. unde $A = a$, $b = B + n.\frac{n-1}{2}a$, &c. & ſumma ſeriei, cujus termini ſunt prædicti generis, detegi poteſt.

Ex. 2. Sint s & s' reſpective $\frac{1}{z.(z+1).(z+2)..(z+n-1)}$ &

(z+1)

$\frac{1}{(z+1).(z+2)..(z+n)}$, tum erit $s - \acute{s} = t = \frac{n}{z(z+1).(z+2)..(z+n)}$, & conſequenter ſi $t = \frac{1}{z.(z+1).(z+2)..(z+n)}$, tum erit $s = \frac{1}{nz.z+1.z+2\ldots z+n-1} + A$, ubi A ſit ſumma quorumcunque præcedentium terminorum.

Cor. 1. Conſtat e prob. prædictâ omnem quantitatem, quæ expandi poteſt in infinitas ſeries hujuſce generis $Az^{-n} + Bz^{-n-m} + Cz^{-n-2m} +$ &c. ubi n & m ſunt integri numeri affirmativi, & A, B, C, &c. invariabiles coefficientes, etiam ad infinitam ſeriem hujuſmodi $\frac{a}{z(z+1)(z+2)..(z+(n-1))} + \frac{b}{z(z+1)(z+2)(z+3)\ldots(z+n)} + \frac{c}{z(z+1)(z+2)(z+3)(z+4)..(z+n+1)} +$ &c. reduci poſſe.

E. g. Series $\frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \frac{1}{z^4} +$ &c. in infinitum invenietur $= \frac{1}{z} + \frac{1}{z.z+1} + \frac{1.2}{z.z+1.z+2} + \frac{1.2.3}{z.z+1.z+2.z+3} + \frac{1.2.3.4}{z.z+1.z+2.z+3.z+4} +$ &c. in infinitum. Sit $z = n$, & hæc ſeries ultimo convergit prope in eâdem ratione, quam ſeries $\frac{1}{1} + \frac{1}{2^n} + \frac{1}{3^n} + \frac{1}{4^n} +$ &c.; prior $\frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} +$ &c. ſemper ultimo convergit in majore ratione quam poſterior; ni z haud major ſit quam 1, in quo caſu earum ſumma erit infinita.

Cor. 2. Sit terminus generalis ſeriei $\frac{1}{z+a.z+b.z+c.z+d.\&c.}$, ubi a, b, c, d, &c. ſint integri numeri & inæquales, tum hic terminus ſemper reſolvi poteſt in plures, quorum integrales ex hoc exemplo deduci poſſunt.

Et ex dato termino generali erui poteſt ſeriei ſumma, & e theor. 2. lib. 2^{di}. corrigi poteſt ſumma acquiſita.

2. Ad hunc particularem caſum, ubi z incrementum diſtantiæ a primo ſeriei termino $= 1$, applicari poſſunt omnia, quæ prius tradita fuerunt de incrementialibus quantitatibus & æquationibus.

Ex. Sit generalis terminus $\frac{\alpha z^m + \beta z^{m-1} + \gamma z^{m-2} + \&c.}{z^n + a z^{n-1} + b z^{n-2} + \&c.}$, ubi $z^n + a z^{n-1} + b z^{n-2} + \&c. = z(z+1)(z+2)\ldots(z+n-1)$; & m numerus non major quam n; (ſi m major ſit quam n, tum terminus facile reduci poteſt e diviſione numeratoris per denominatorem ad integram quantitatem & propriam fractionem prædicti generis); invenire ſummam ſeriei, cujus generalis terminus eſt prædictus.

Statuatur $\frac{\alpha z^m + \beta z^{m-1} + \gamma z^{m-2} + \delta z^{m-3} + \&c.}{z(z+1)(z+2)(z+3)\ldots(z+n-1)} = p + \frac{q}{z} + \frac{r}{z.(z+1)} + \frac{s}{z(z+1)(z+2)} + \frac{t}{z . z+1 . z+2 . z+3} + \ldots + \frac{l}{z . z+1 . z+2 \ldots (z+n-1)}$; ſi $m = n$, tum $p = \alpha$, $q = \beta - ap$, $r = \gamma - pb - aq$, $s = \delta - pc - qb - ra + r$, $t = \varepsilon - pd - qc - ar(b-a+1) - s(a-3)$, &c.; ubi $z+1 . z+2 . z+3 \ldots z+n-1 = z^{n-1} + a z^{n-2} + b z^{n-3} + c z^{n-4} + d z^{n-5} + \&c.$, cujus ſeriei ſumma eſt $A + pz + \text{integ.} \left(\frac{q}{z}\right) + \frac{r}{z} + \frac{s}{2z . z+1} + \frac{t}{3z . z+1 . z+2} + \&c.$; ubi A eſt quantitas invariabilis pro conditione problematis aſſumenda.

Cor. 1. Summa hujuſce ſeriei inter quoſcunque duos a' & b' valores quantitatis z contenta erit $p(a' - b') + \text{integ.} \left(\frac{q}{z}\right) + \left(\frac{r}{a'} - \frac{r}{b'}\right) +$

2 a'.

$\left(\frac{s}{2a'.a'+1} - \frac{s}{2b'.b'+1}\right) + \&c.$; ſed integ. $\left(\frac{q}{z}\right) = \frac{q}{a'} + \frac{q}{a'+1} + \frac{q}{a'+2} \ldots \frac{q}{b'}$: erit infinita, ſi b ſit infinitus numerus; ſumma terminorum prædictorum ſemper erit finita quantitas, cum a' ſit finita quantitas & m minor quam $n-1$; ſin aliter non.

Cor. 2. Si q haud ſit $= o$, tum ſumma prædictorum terminorum in infinitum progredientium in finitis terminis non exprimi poteſt, ſin aliter ſemper exprimi poteſt: hinc, ſi m minor ſit quam $n-1$, tum ſeriei prædictæ ſumma finitis terminis ſemper exprimi poteſt; i. e. ſit b' infinitus numerus & $q = o$, ſumma ſeriei prædictæ $= \frac{r}{a'} + \frac{s}{2a'.a'+1} + \frac{t}{3a'.a'+1.a'+2} + \&c.$; quæ ſemper terminat.

2. Sit generalis terminus $\frac{\alpha z^m + \beta z^{m-1} + \&c.}{(z+e)(z+e+1)(z+e+2)\ldots(z+e+n-1)}$; tum eadem affirmari poſſunt de ſummâ ſeriei exinde reſultantis, quæ prius affirmabantur de ſerie, cujus generalis terminus eſt $\frac{\alpha z^m + \beta z^{m-1} + \&c.}{z(z+1)(z+2)\ldots(z+n-1)}$: terminus enim

$$\frac{\alpha z^m + \beta z^{m-1} + \&c.}{(z+e)(z+e+1)(z+e+2)\ldots(z+e+n-1)} = p + \frac{q}{z+e} \frac{r}{z+e.z+e+1} + \frac{s}{z+e.z+e+1.z+e+2} + \&c.$$

3. Sit terminus generalis $\frac{\alpha z^m + \beta z^{m-1} + \gamma z^{m-2} + \&c.}{z+a.z+b.z+c.z+d.\&c.}$, ubi $a-b$, $a-c$, $a-d$, &c. ſunt integri numeri, & m minor quam n per numerum majorem quam unitatem, & n eſt numerus factorum $z+a$, $z+b$, $z+c$, &c. tum ſeries ſemper integrari poteſt, &c.: ducatur enim numerator & denominator dati termini $\frac{\alpha z^m + \beta z^{m-1}}{z+a.z+b}$ +

$\left.\frac{+\gamma z^{m-2}+\&c.}{.z+c.z+d.\&c.}\right)$ reſpective in $(z+a+1)(z+a+2)(z+a+3)\ldots(z+b-1)\times(z+b+1)(z+b+2)(z+b+3)\ldots(z+c-1)\times(z+c+1)(z+c+2)(z+c+3)\ldots(z+d-1)\times(z+d+1)(z+d+2)$ &c.: & terminus generalis in terminum generalem formulæ prius traditæ tranſit.

Sit generalis terminus

$$\frac{\alpha z^m+\beta z^{m-1}+\gamma z^{m-2}+\&c.}{z.z+1.z+2\ldots(z+n-1)\times z+e.z+e+1.z+e+2\,..\,z+e+n'-1}=a+\left(\frac{b}{z}+\frac{b'}{z+e}\right)+\left(\frac{c}{z.z+1}+\frac{c'}{z+e.z+e+1}\right)+\left(\frac{d}{z.z+1.z+2}+\frac{d'}{z+e.z+e+1.z+e+2}\right)+\&c.,$$

ubi m eſt integer numerus non major quam $n+n'$: ejus integralis erit $A+az+$ integ. increm. $\left(\frac{b}{z}+\frac{b'}{z+e}\right)+\frac{c}{z}+\frac{c'}{z+e}+\frac{d}{2z.z+1}+\frac{d'}{2z+e.z+e+1}+$ &c.: exhinc conſtat integralem vel ſummam ſeriei cujus generalis terminus prius traditur detegi poſſe, ſi modo b & b' nihilo ſint reſpective æquales.

Ex iiſdem principiis conſtat integralem generalis termini, vel ſummam ſeriei, cujus terminus eſt

$$\frac{\alpha z^m+\beta z^{m-1}+\&c.}{z+e.z+e+1.z+e+2\ldots(z+e+n-1)\times z+e'.z+e'+1.z+e'+2\ldots(z+e'+n'-1)\times z+e''.z+e''+1.z+e''+2\ldots(z+e''+n''-1)\times\&c.}=a+\left(\frac{b}{z+e}+\frac{b'}{z+e'}+\frac{b''}{z+e''}+\&c.\right)+\left(\frac{c}{z+e.z+e+1}+\frac{c'}{z+e'.z+e'+1}+\frac{c''}{z+e''.z+e''+1}+\&c.\right)+\left(\frac{d}{z+e.z+e+1.z+e+2}+\&c.\right)+$$

&c.) + &c., ubi m eſt integer numerus $= n + n' + n'' +$ &c. ejus integralis vel ſumma erit $A + az +$ integ. increm. $\left(\frac{b}{z+e} + \frac{b'}{z+e'} + \frac{b''}{z+e''} + \&c.\right) + \left(\frac{c}{z+e} + \frac{c'}{z+e'} + \frac{c''}{z+e''} + \&c.\right) + \left(\frac{d}{2 . z+e . z+e+1} + \&c.\right) +$ &c.: exhinc conſtat integralem dati generalis termini finitis terminis detegi poſſe, ſi modo integralis incrementi $\left(\frac{b}{z+e} + \frac{b'}{z+e'} + \frac{b''}{z+e''} + \&c.\right)$ aſſignari poſſit; & conſequenter ſummam ſeriei, cujus generalis terminus prius traditur; & ſic deinceps.

Sit generalis terminus $\frac{\alpha z^m + \beta z^{m-1} + \gamma z^{m-2} + \&c.}{z+a . z+b . z+c . \&c. \times z+a' . z+b' . z+c' . z+d' . \&c. \times z+a'' . z+b'' . z+c'' . \&c.}$, ubi $b - a$, $c - a$, $d - a$, &c.; $b' - a'$, $c' - a'$, $d' - a'$, &c.; $b'' - a''$, $c'' - a''$, $d'' - a''$, &c., &c.; ſunt integri numeri, at non $a' - a$, $a'' - a$, $a''' - a'$, &c., & a minor eſt quam b, b quam c, c quam d, &c.; & a' quam b', b' quam c', &c.; &c.: ducantur & numerator & denominator dati termini in $z+a+1 . z+a+2 . z+a+3 \ldots z+b-1 \times z+b+1 . z+b+2 . z+b+3 \ldots z+c-1 \times z+c+1 . z+c+2 \ldots z+d-1 \times z+d+1 . z+d+2 \ldots \&c. \times z+a'+1 . z+a'+2 . z+a'+3 \ldots z+b'-1 \times z+b'+1 . z+b'+2 . z+b'+3 \ldots z+c'-1 \times z+c'+1 . z+c'+2 \ldots z+d'-1 \times z+d'+1 \ldots \&c. \times z+a''+1 . z+a''+2 . z+a''+3 \ldots z+b''-1 \times z+b''+1 . z+b''+2 \ldots z+c''-1 \times z+c''+1 . z+c''+2 \ldots z+d''-1 \times z+d''+1 . z+d''+2 \ldots \&c. \times \&c.$ & terminus datus in terminum formulæ prius traditæ tranſit.

Sit (n) numerus quantitatum $(a, a', a'', a''', \&c.)$; tum ex (n) independentibus integralibus vel ſummis, viz. $\frac{l}{z+a}$, $\frac{l'}{z+a'}$, $\frac{l''}{z+a''}$,

&c., deduci poſſunt integrales vel ſummæ omnium ſerierum prædictarum formularum.

4. Sit generalis terminus $\dfrac{\alpha z^{m} + \beta z^{m-1} + \gamma z^{m-2} + \&c.}{(z+e)^{b}(z+e+1)^{b}(z+e+2)^{b} \ldots (z+e+n-1)^{b}} = \dfrac{l}{z+e} + \dfrac{l'}{(z+e)^{2}} + \dfrac{l''}{(z+e)^{3}} \ldots + \dfrac{\overset{b-1}{l'}}{(z+e)^{b}}$

$$+ \frac{k}{z+e\,.\,z+e+1} + \frac{(2z+2e+1)k'}{(z+e)^{2}(z+e+1)^{2}} + \frac{(3z^{2}+3ze+e^{2}+3z+3e+1)k''}{(z+e)^{3}(z+e+1)^{3}}$$

$$+ \ldots \frac{((z+e+1)^{b}-(z+e)^{b})\overset{b-1}{k'}}{(z+e)^{b}(z+e+1)^{b}} + \frac{i}{z+e\,.\,z+e+1\,.\,z+e+2} + \frac{((z+e+2)^{2}-(z+e)^{2})i'}{(z+e)^{2}(z+e+1)^{2}(z+e+2)^{2}} + \frac{((z+e+2)^{3}-(z+e)^{3})i''}{(z+e)^{3}(z+e+1)^{3}(z+e+2)^{3}}$$

$$+ \ldots \frac{((z+e+2)^{b}-(z+e)^{b})\overset{b-1}{i'}}{(z+e)^{b}(z+e+1)^{b}(z+e+2)^{b}} + \frac{f}{z+e\,.\,z+e+1\,.\,z+e+2\,.\,z+e+3} + \frac{((z+e+3)^{2}-(z+e)^{2})f'}{(z+e)^{2}(z+e+1)^{2}(z+e+2)^{2}(z+e+3)^{2}}$$

$+$ &c.; tum ex b independentibus integralibus vel ſummis ſerierum harum formularum, e.g. $\dfrac{l}{z+e}$, $\dfrac{l'}{(z+e)^{2}}$, $\dfrac{l''}{(z+e)^{3}}$, .. $\dfrac{\overset{b-1}{l'}}{(z+e)^{b}}$ erui poſſunt ſummæ omnium aliorum incrementorum vel ſerierum earundem formularum: vel generaliter ſit terminus $\dfrac{\alpha z^{m} + \beta z^{m-1} + \gamma z^{m-2} + \&c.}{(z+a)^{b}(z+b)^{b'}(z+c)^{b''} \times \&c. \times (z+a')^{i}(z+b')^{i'}(z+c')^{i''} \times \&c. \times (z+a'')^{k}(z+b'')^{k'} \times \&c. \times \&c.}$; ubi quantitates a, a', a'', &c. ſunt reſpective minores quam b, c, &c.; b', c', &c.; b'', c'', &c.; &c.; & $b-a$, $c-a$, &c.; $b'-a'$, $c'-a'$, &c.; $b''-a''$, $c''-a''$, &c. ſunt integri numeri; at non $a'-a$ vel $a''-a$ vel $a''-a'$, &c. ſeries, cujus prædictus fuit generalis terminus, eadem erit ac ſeries, cujus generalis terminus eſt $\dfrac{\alpha' z^{m'} + \beta' z^{m'-1} +}{(z+a)^{\pi}(z+a+1)^{\pi}(z+}$

$$\frac{\beta' z^{m'-1} + \gamma' z^{m'-2} + \&c.}{a+z)^{\pi} \,..\, (z+b)^{\pi} (z+b+1)^{\pi} \,..\, (z+c)^{\pi}\,\&c. \times (z+d')^{\rho} \times (z+d'+1)^{\rho}}$$

$$\overline{..\,(z+b')^{\rho} \,..\, (z+c')^{\rho} \times \&c. \times (z+a'')^{\sigma} (z+a''+1)^{\sigma} \times \&c.} = \frac{\lambda}{z+a}$$

$$+ \frac{\lambda'}{(z+a)^2} + \frac{\lambda''}{(z+a)^3} \ldots + \frac{\overset{\pi-1}{\lambda'}}{(z+a)^{\pi}} + \frac{\chi}{z+d'} + \frac{\chi'}{(z+d')^2} +$$

$$\frac{\chi''}{(z+d')^3} \ldots + \frac{\overset{\rho-1}{\chi'}}{(z+d')^{\rho}} + \frac{\theta}{z+a''} + \frac{\theta'}{(z+a'')^2} + \&c. + \frac{\zeta}{(z+a)}$$

$$\overline{(z+a+1)} + \frac{((z+a+1)^2-(z+a)^2)\zeta'}{(z+a)^2(z+a+1)^2} \ldots + \frac{((z+a+1)^{\pi}-(z+a)^{\pi})\overset{\pi-1}{\zeta'}}{(z+a)^{\pi}(z+a+1)^{\pi}}$$

$+ \&c. + \&c.$; ubi π, ρ, σ, &c. reſpective ſunt maximi inter indices h, h', h'', &c.; i, i', i'', &c.; k, k', k'', &c.; &c.; qui omnes ſunt integri numeri: ſi enim numerator & denominator dati generalis termini ducantur reſpective in $(z+a)^{\pi-h} \times (z+a+1)^{\pi} \times (z+a+2)^{\pi} \ldots \times (z+b-1)^{\pi} \times (z+b)^{\pi-h'} \times (z+b+1)^{\pi} \times (z+b+2)^{\pi} \,..\, (z+c-1)^{\pi} \times (z+c)^{\pi-h''} \times (z+c+1)^{\pi} \times (z+c+2)^{\pi} \times \&c. \times (z+a')^{\rho-i} \times (z+d'+1)^{\rho} \times (z+d'+2)^{\rho} \ldots (z+b'-1)^{\rho} \times (z+b')^{\rho-i'} \times (z+b'+1)^{\rho} \times (z+b'+2)^{\rho} \,..\, (z+c'-1)^{\rho} \times (z+c')^{\rho-i''} \times (z+c+1)^{\rho} \times \&c. \times \&c.$; tum reſultat poſterior generalis terminus: hinc ex $\pi + \rho + \sigma + \&c.$ integralibus vel ſummis inter ſe independentibus ſerierum, quarum generales termini ſunt prædicti, viz. ex integralibus vel ſummis ſerierum, quarum termini ſunt $\frac{\lambda}{z+a}$, $\frac{\lambda'}{(z+a)^2}$, .. $\frac{\overset{\pi-1}{\lambda'}}{(z+a)^{\pi}}$; $\frac{\chi}{z+d'}$, $\frac{\chi'}{(z+d')^2}$, ... $\frac{\overset{\rho-1}{\chi'}}{(z+d')^{\rho}}$; $\frac{\theta}{z+a''}$, $\frac{\theta'}{(z+a'')^2}$, ... $\frac{\overset{\sigma-1}{\theta'}}{(z+a'')^{\sigma}}$, &c. deduci poſſunt integrales vel ſummæ omnium ſerierum, quarum generales termini ſunt prædicti.

Hæc magis generaliter per diverſas methodos prius tradita fuere in methodo incrementorum libro ſecundo contentorum.

Cor. . Hinc facile inveniri poſſunt innumeræ ſeries, quarum generalis ſumma innoteſcit. Aſſumatur enim quæcunque functio quantitatis z pro ſummâ s, & differentia inter duos ejus ſucceſſivos valores, i. e. inter s & s', ubi s' ſit valor, qui reſultat ſcribendo in datâ functione $z+1$ pro z, invenietur terminus quæſitus, i. e. $s - s' = t$.

2. Sit generalis terminus x^{rz+n} in $\frac{a}{z} + \frac{b}{z.z+1} + \frac{c}{z.z+1.z+2} + \frac{d}{z.z+1.z+2.z+3} +$ &c. erit ſumma æqualis x^{rz+n} in $\frac{a}{1-x^r.z} + \frac{b-Ax^r}{1-x^r.z.z+1} + \frac{c-2Bx^r}{1-x^r.z.z+1.z+2} + \frac{d-3Cx^r}{1-x^r.z.z+1.z+2.z+3}$, ubi quantitates A, B, C, D, &c. deſignant coefficientes terminorum præcedentes eos, in quibus reperiuntur; ſcilicet $A = \frac{a}{1-x^r}$, $B = \frac{b-Ax^r}{1-x^r}$, $C = \frac{c-2Bx^r}{1-x^r}$, &c. Fingatur enim $S = x^{rz+n}$ in $\frac{A}{z} + \frac{B}{z.z+1} + \frac{C}{z.z+1.z+2} +$ &c. deinde ſcribatur $S-T$ pro S & $z+1$ pro z, & reſultat $S-T = x^{rz+n+r}$ in $\frac{A}{z+1} + \frac{B}{z+1.z+2} + \frac{C}{z+1.z+2.z+3} +$ &c. hoc eſt $S-T = x^{rz+n}$ in $\frac{Ax^r}{z+1} + \frac{Bx^r}{z+1.z+2} + \frac{Cx^r}{z+1.z+2.z+3} +$ &c. quæ reducta ad formam ipſius S, evadit $S-T = x^{rz+n}$ in $\frac{Ax^r}{z} + \frac{Bx^r - Ax^r}{z.z+1} + \frac{Cx^r - 2Bx^r}{z.z+1.z+2} + \frac{Dx^r - 3Cx^r}{z.z+1.z+2.z+3} +$ &c. ſubducito valorem ipſius $S-T$ a valore ipſius S, & relinquetur terminus $T = x^{rz+n}$ in $\frac{A(1-x^r)}{z} + \frac{B(1-x^r)+Ax^r}{z.z+1} + \frac{C(1-x^r)+2Bx^r}{z.z+1.z+2} +$ &c. Hic denique valor ipſius T collatus cum

illo in propoſitione dat $A(1-x^{r})=a$, $B(1-x^{r})+Ax^{r}=b$, $C(1-x^{r})+2Bx^{r}=c$, &c.

Sit r negativa quantitas, & eadem erit lex ſeriei progreſſionis, ac ea cum r ſit affirmativa quantitas.

Ex. 2. Erit integralis incrementi $\left(\frac{A}{z^{n}}\right)=\frac{a}{z^{n-1}}+\frac{b}{z^{n}}+\frac{c}{z^{n+1}}+\frac{d}{z^{n+2}}+$ &c., ubi z denotat diſtantiam a primo ſeriei termino, & $a=\frac{A}{n-1}$, $b=\frac{(n-1)a}{2}$, $c=\frac{nb}{2}-\frac{n.n-1}{2.3}a$, $d=\frac{n+1}{2}c-\frac{(n+1).n}{2.3}b+\frac{n+1.n.n-1}{2.3.4}a$, $e=\frac{n+2}{2}d-\frac{n+2.n+1}{2}c+\frac{n+2.n+1.n}{2.3.4}b-\frac{n+2.n+1.n.n-1}{2.3.4.5}a$, &c.

Ex. 3. Sit generalis terminus x^{rz+m} in $\frac{A}{z^{n}}$, tum erit ejus integralis $x^{rz+m}\times\left(\frac{a}{z^{n}}+\frac{b}{z^{n+1}}+\frac{c}{z^{n+2}}+\frac{d}{z^{n+3}}+\text{\&c.}\right)$, ubi $a=\frac{A}{1-x^{r}}$, $b=-\frac{nx^{r}}{1-x^{r}}a$, $c=-\frac{1}{1-x^{r}}((n+1)b-(n+1).\frac{n}{2}a)x^{r}$, $d=-\frac{1}{1-x^{r}}((n+2)c-(n+2).\frac{n+1}{2}b+(n+2).\frac{n+1}{2}.\frac{n}{3}a)x^{r}$, $e=-\frac{1}{1-x^{r}}((n+3)d-(n+3).\frac{n+2}{2}c+(n+3).\frac{n+2}{2}.\frac{n+1}{3}b-(n+3).\frac{n+2}{2}.\frac{n+1}{3}.\frac{n}{2}a)x^{r}$, &c.

Cor. Sit generalis ſeriei terminus $\frac{A}{z^{n}}+\frac{B}{z^{n+1}}+\frac{C}{z^{n+2}}+\frac{D}{z^{n+3}}+$ &c., vel x^{rz+m} in $\frac{A}{z^{n}}+\frac{B}{z^{n+1}}+$ &c., tum e duobus præcedentibus exemplis erui poteſt ejus integralis in ſeriebus ſecundum dimenſiones quantitatis z progredientibus.

Hæc facile demonſtrari poſſunt e ſcribendo z & $z+1$ pro z in integrali aſſumptâ, & duarum quantitatum reſultantium differentiam inveniendo, quæ erit generalis terminus datus.

In omnibus hiſce caſibus, ſi aliquis factor contineatur in denominatore, qui non habet alium ab eo per integrum numerum diſtantem, tum non integrari poteſt data ſeries.

Et ſic de terminis diverſas exponentiales quantitates involventibus.

PROB. I.

1. *Datâ æquatione relationem exprimente inter ſucceſſivas ſummas* s, s', s'', *&c. datæ ſeriei, ejus ſucceſſivos terminos* t, t', t'', *&c. & quantitatem* z *diſtantiam a primo ſeriei termino, invenire æquationem inter ſummas ſucceſſivas &* z.

Pro terminis t, t', t'', &c. in datâ æquatione ſcribantur eorum valores $s - s'$, $s' - s''$, $s'' - s'''$, &c. & reſultat æquatio quæſita.

2. Datâ prædictâ æquatione, invenire æquationem relationem inter t, t', t'', &c. & z exprimentem: in datâ æquatione pro ſummis s', s'', s''', &c. ſcribantur $s - t$, $s - t - t'$, $s - t - t' - t''$, &c. & habebitur æquatio $A = 0$, in quâ ſolummodo continetur ſumma s; inveniatur incrementum hujuſce æquationis $(A = 0)$ quod ſit æquatio $B = 0$: aliter in æquatione $(A = 0)$ pro s ſcribatur $s - t$, & pro t, t', &c. ſcribantur t', t'', &c. & pro z, $z + 1$, &c.; & reſultat æquatio $(C = 0)$, in quâ etiam ſolummodo continetur ſumma s, reducantur duæ æquationes $A = 0$ & $C = 0$, vel quod idem eſt $A = 0$ & $B = 0$ in unam, ita ut exterminetur s, & reſultat æquatio quæſita.

Cor. . Hinc facile deduci poſſunt infinitæ ſeries, quarum ſummæ dantur; aſſumatur enim æquatio ad ſummas, & ex eâ deducatur æquatio ad terminos, e quâ erui poſſunt termini, i. e. ſeries quæſita.

Ex. 1. Sit æquatio $(z - n)s = (z - 1)s' = (z - 1) \times (s - t)$, unde $(n - 1)s = (z - 1)t$; pro s ſcribatur $s - t = s'$, & t' pro t, & $z + 1$ pro z, & reſultat æquatio $(n - 1)s = (n - 1)t + zt'$; ſubduc hanc æquationem de æquatione prædictâ $(n-1)s=(z-1)t$ & reſultat æquatio ad terminos $(z-n)t=zt'$.

3. Datam prædictam æquationem $(A=0)$ in alteram transformare, in quâ deficit z: inveniatur incrementum $A'=0$ datæ æquationis ex hypotheſi quod $\underset{.}{z}=1$; reducantur hæ duæ æquationes $A=0$ & $\underset{.}{A'}=0$ in unam, ita ut exterminetur z, & evadet æquatio quæſita.

4. Ex datis relationibus inter ſucceſſivos terminos, invenire relationes inter ſucceſſivos valores functionis quantitatis z diſtantiæ a primo ſeriei termino, quæ prædictos terminos denotant.

In datis relationibus pro t, t', t'', &c. ſcribantur reſpective $\phi:z$, $\phi:z+1$, $\phi:z+2$, &c., & reſultant æquationes quæſitæ.

THEOR. II.

Sint $s-s'=\underset{.}{s}=t$, $s'-s''=t'$ &c. $t-t'=\underset{..}{s}$, &c. tum erunt $s-2s'+s''=t-t'=\underset{..}{s}$, $s-3s'+3s''-s'''=t-2t'+t''=\underset{\because}{s}$, & in genere $s-ns'+n\cdot\frac{n-1}{2}s''-n\cdot\frac{n-1}{2}\cdot\frac{n-2}{3}s'''+\&c.=t-(n-1)t'+(n-1)\cdot\frac{n-2}{2}t''-\&c.=\underset{n}{\underset{.}{s}}=\underset{n-1}{\underset{.}{t}}$; etiamque $s-n\underset{.}{s}+n\cdot\frac{n-1}{2}\underset{..}{s}-n\cdot\frac{n-1}{2}\cdot\frac{n-2}{3}\underset{\because}{s}+\&c.=$ integ. increm. $(t)-nt+n\cdot\frac{n-1}{2}\underset{.}{t}-n\cdot\frac{n-1}{2}\cdot\frac{n-2}{3}\underset{..}{t}+\&c.=\overset{n}{s'}$ & $\overset{n}{t'}=t-n\underset{.}{t}+n\cdot\frac{n-1}{2}\underset{..}{t}-n\cdot\frac{n-1}{2}\cdot\frac{n-2}{3}\underset{\because}{t}+\&c.=s-(n+1)\underset{.}{s}+(n+1)\cdot\frac{n}{2}\underset{..}{s}-\&c.$

Cor. . Datam æquationem relationem inter ſucceſſivos terminos & ſummas exprimentem in incrementialem transformare; pro s, s', s'', &c. t, t', t'', &c. ſcribantur reſpective earum valores, & transformatur data æquatio in incrementialem, cujus integralis vel erit s vel t; & vice verſâ data incrementialis æquatio, cujus integralis eſt s, transformari poteſt in æquationem relationem inter ſucceſſivas ſummas, terminos, &c. deſignantem, ſcribendo in datâ æquatione pro s, $\underset{.}{s}$, $\underset{..}{s}$, &c. earum valores in hoc theoremate aſſignatos.

PROB. II.

Detur æquatio relationem inter ſucceſſivas ſeriei ſummas S, S', *&c. &* z *diſtantiam a primo ſeriei termino exprimens; invenire, utrum ſumma* S *ad infinitam diſtantiam ſit finita, necne.*

Inveniatur ex datâ æquatione ſeries deſcendens, quæ exprimit ſummam S terminis vero quantitatis z, i. e. ſit ſeries $az^m +$ &c. $+ A$ $Bz^{-r} +$ &c. ubi m ſit affirmativa quantitas, $-r$ vero negativa quantitas, tum ſeries ad infinitam diſtantiam erit infinite magna: ſit $m = 0$, & conſequenter ſeries $A + Bz^{-r} + Cz^{-s} +$ &c. ubi $-r$ & $-s$, &c. ſunt negativæ quantitates, tum ſumma ad infinitam diſtantiam erit finita, viz. $= A$; ſi vero ſit prædicta ſeries $Bz^{-r} + Cz^{-s} +$ &c. tum erit ſumma ad infinitam diſtantiam infinite parva.

Et ſic e datâ æquatione inter ſucceſſivos terminos relationem exprimente & principiis prius traditis detegi poteſt, utrum ſumma ſit finita, necne.

THEOR. III.

Sit data æquatio relationem inter ſucceſſivos ſeriei terminos (T, T', &c.) & ſummas, &c. exprimens $\alpha \times \beta \times \gamma \times$ &c. $= 0$; tum erunt $\alpha = 0$, $\beta = 0$, $\gamma = 0$, &c. reſpective diverſæ æquationes, quæ diverſas ſeries datæ æquationis denotant. E. g. Sit æquatio data $zT^2 + (z^2 + z + 1)TT' + (z^2 + z)T'^2 = 0$, ubi z ſit diſtantia a primo ſeriei termino; inveniantur datæ æquationis diviſores $zT + (z + 1)T' = 0$ & $T + zT' = 0$, qui diverſas ſeries datæ æquationis denotant; & ſic deinceps.

THEOR. IV.

Sit terminus $\overset{n}{T'}$ vel ſumma $\overset{n+1}{S'}$, cujus ordo n ſit maximus; tum in ſerie ejus ſummam S exprimente, aſſumi poſſunt quæcunque $n+1$ invariabiles quantitates ad libitum aſſumendæ, &c.

Hìc ad ſeries applicari poſſunt omnia, quæ prius tradita fuerint de pluribus (n) fluxionalibus vel incrementialibus æquationibus plures ($n+1$) variabiles quantitates habentibus, & earum correctionibus, &c.

THEOR. V.

1. Datâ æquatione ($A=o$) relationem inter ſucceſſivas ſummas, (S, S', &c.) terminos (T, T', &c.) & quantitatem z deſignante; in eâ ($A=o$) pro S, S', &c. T, T', &c. & z ſcribantur reſpective S', S'', &c. T', T'', &c. & $z+1$, & reſultet æquatio $B=o$; tum generalis ſeries, quæ ſit radix æquationis $rA+B=o$, ubi r ſit quæcunque invariabilis quantitas, haud eadem invenietur ac ſeries, quæ erit radix æquationis $B=o$.

2. Sit prædicta æquatio data $A=a$, & reſultans $B=b$; tum haud eadem erit radix generalis æquationis $BA=ba$, ac æquationis $B=b$.

Conſtant e theor. 9. libri ſecundi.

PROB. III.

Datâ æquatione A = o *relationem exprimente inter terminos* (T, T′, T″, *&c.*) *datæ ſeriei ſecundum dimenſiones quantitatis* x *progredientis, invenire æquationem relationem inter ſucceſſivos ejus fluxionis terminos* (t, t′, *&c.*) *deſignantem.*

1. In æquatione $A=o$ pro z diſtantiâ datæ ſeriei termini a primo ſcribatur $z+1$, & pro T & T' ſcribantur reſpective T' & T''; datæ æquationis $A=o$ & reſultantis $B=o$ inveniantur fluxiones $\alpha=o$ & $\pi=o$;

in æquationibus $\alpha = o$ & $\pi = o$ pro $\dot{T}$, $\dot{T}'$ & $\dot{T}''$ ſcribantur reſpective t, t' & t''; reducantur hæ æquationes ad unam, ita ut exterminentur quantitates T & T' & T'', & reſultat æquatio quæſita.

2. Si vero plures termini $T, T', T'', \ldots \overset{n-1}{T'}$ in datâ æquatione contineantur, tum inveniantur $n - 1$ ſucceſſivi valores datæ æquationis ſcribendo in eâ $z + 1$, $z + 2 \ldots z + n - 1$; $T', T'', \ldots \overset{n}{T'}$; $T'', T''', \ldots \overset{n+1}{T'}$; &c. ſucceſſive pro z, T, T', &c. & inveſtigentur eorum fluxiones, & exorientur n novæ æquationes; in his æquationibus pro $\dot{T}$, $\dot{T}'$, $\dot{T}''$, &c. ſcribantur reſpective t, t', t'', &c. & reducantur hæ $2n$ æquationes ad unam, ita ut exterminentur quantitates T, T', T'', &c. & reſultat æquatio quæſita.

Et ſic de inveniendis æquationibus relationem inter ſucceſſivos terminos ſerierum exprimentibus, quarum termini algebraicam vel fluxionalem vel incrementialem habeant relationem ad ſucceſſivos terminos, inter quos dantur æquationes relationes exprimentes.

PROB. IV.

Datis æquationibus relationem deſignantibus inter ſummas (S, S′, *&* s, s′) *in diverſis ſeriebus, & æquationibus ad terminos* (T, T′, *&c.*) *in alterutrâ; invenire æquationes ad terminos in alterâ.*

Scribantur $S - T$, $S - T - T'$, &c. $s - t$, $s - t - t'$, &c. pro S', S'', &c. s', s'', &c. reſpective in datis æquationibus; deinde in æquationibus reſultantibus progrediatur e relatione variabilium præcedente ad ſubſequentem, i. e. ſcribatur $S - T$ pro S & $s - t$ pro s; & T', T'', &c. t', t'', &c. reſpective pro T, T', &c. t, t', &c. &c. & ita reducantur per vulgarem algebram datæ & reſultantes æquationes, ut exterminentur S & s; deinde per methodos prius traditas ita reducantur hæ æquationes reſultantes & æquatio ad terminos in alterutrâ, ut exterminentur T, T', T'', &c. & reſultabit æquatio relationem inter t, t', &c. deſignans.

Et ex iiſdem principiis & datâ vel datis æquationibus relationes inter ſucceſſivas ſummas & terminos exprimentibus erui poſſunt æquationes relationes inter ſucceſſivas ſummas vel ſucceſſivos terminos ſolummodo exprimentes.

PROB. V.

Datis duabus ſeriebus, i. e. methodo inveniendi terminos reſpectivos e datâ eorum diſtantiâ z *a primo ſeriei termino; invenire relationem inter terminos correſpondentes duarum ſerierum.*

Scribantur pro correſpondentibus duarum ſerierum terminis reſpective T & t; & dentur duæ æquationes relationes inter z & T, & z & t reſpective exprimentes; reducantur hæ duæ æquationes ad unam, ita ut exterminetur z, & reſultat æquatio relationem inter T & t exprimens.

Ex. . Sint duæ ſeries reſpective $0 + 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} +$ &c. & $0 + \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} +$ &c. & conſequenter erunt $\frac{1}{z^2} = T$ & $\frac{1}{z \times z + 1} = t$; reducantur hæ æquationes, ita ut exterminetur z & reſultat æquatio $tT^{\frac{1}{2}} + Tt = 1$.

Cor. . Datâ æquatione relationem exprimente inter diſtantiam a primo ſeriei termino (z) & terminum ipſum (T), & datâ æquatione relationem exprimente inter correſpondentes duarum ſerierum terminos (T & t); invenire æquationem relationem inter z & t exprimentem; reducantur duæ æquationes relationem inter z & T, T & t exprimentes ad unam, ita ut exterminetur T, & fit corollarium.

Et ſic de relationibus inter terminos, &c. plurium æquationum deducendis.

PROB. VI.

Datis duabus ſeriebus; invenire, annon earum ſummæ datam habent inter ſe generalem relationem.

In datâ æquatione exprimente relationem inter S & s duas ſummas quæſitas, ſcribantur pro S & s reſpective earum ſucceſſivæ correſpondentes ſummæ $S - T$ & $s - t$, modo T & t ſint ſucceſſivi dati termini utriuſque ſeriei reſpective; ſcribantur etiam in datis ſeriebus pro S, $S - T$, & pro s, $s - t$, &c.; & ſi eadem adhuc reſultet relatio inter S & s; &c.; tum datur relatio inter prædictas ſummas; ſin aliter vero non.

PROB. VII.

Invenire ſeriem, cujus ſummæ ſucceſſivæ (s) *quamcunque habeant relationem ad ſummas ſucceſſivas* (S) *datæ ſeriei.*

In datâ æquatione relationem exprimente pro S ſcribatur ejus valor ſucceſſivus $S - T$, ſi modo T ſit proximus terminus datæ ſeriei; ex æquatione reſultante inveniatur proximus valor ſummæ ſeriei quæſitæ s, & differentia inter duas ſucceſſivas ſummas erit terminus ſeriei quæſitus.

THEOR. VI.

1. Summa (S) nullius ſeriei exprimi poteſt per algebraicam æquationem inter prædictam ſummam S & quantitatem x, ſecundum cujus dimenſiones progreditur ſeries, relationem deſignantem; ni dimenſiones quantitatis x vel earum differentiæ denotari poſſint per unam vel duas vel denique n finitum numerum arithmeticarum ſerierum; etiamque, ſi modo z denotet diſtantiam a primo datæ ſeriei termino, ni terminorum ipſorum coefficientes exprimi poſſint in rationalibus terminis literis z; etiamque, ni differentia inter dimenſiones quantitatis z in eorum numeratore & denominatore contentas, eadem ſit; ſi vero detur æquatio inter ſucceſſivos datæ ſeriei terminos relationem exprimens, ni differentia prædicta eadem manet.

2. Summa nullius ſeriei exprimi poteſt per fluxionalem æquationem relationem inter prædictam ſummam S & quantitatem x, ſecundum cujus dimenſiones progreditur ſeries, deſignantem; ni dimenſi-

ones quantitatis x vel earum differentiæ exprimi poſſint per unum vel duo vel denique n numerum arithmeticarum ſerierum; etiamque ni coefficientes exprimi poſſint in rationalibus terminis literæ z, & differentia inter dimenſiones quantitatis z in denominatore & numeratore contentas eadem maneat, vel uniformiter augeatur vel minuatur: eadem etiam affirmari poſſunt de ſeriebus, quarum relationes per algebraicas æquationes inter ſucceſſivos terminos deſignantur.

Hæc vero principia applicari poſſunt ad deducendam unius ſeriei ſummam ex alterâ, &c.

THEOR. VII.

In ſeriebus ex diviſione ortis eadem erit relatio inter terminos t, t', t'', &c. ac inter ſummas s, s', s'', &c. ſucceſſivas.

Sit enim æquatio data inter ſummas ſucceſſivas $ps + qs' + rs'' +$ &c. $= o$, & ex hac æquatione per problema 1. inveniri poteſt æquatio inter terminos ſucceſſivos, quæ erit $pt + qt' + rt'' +$ &c. $= o$: enim $s' + t = s$, $s'' + t' = s'$, $s''' + t'' = s''$, & ſic deinceps; unde $ps + qs'' + rs'' +$ &c. $= p(s' + t) + q(s'' + t') +$ &c. $= (ps' + qs'' + rs''' +$ &c. $= o) + pt + qt' + rt'' +$ &c. $= o$, & exinde $pt + qt' + rt'' +$ &c. $= o$.

THEOR. VIII.

Sit ſeries $\dfrac{A}{p + qx + rx^2 + sx^3 + \text{\&c.}} = a + bx + cx^2 + dx^3 \ldots \overset{\overline{a-3}}{T'}x^{n-3} + \overset{\overline{a-2}}{T'}x^{n-2} + \overset{\overline{a-1}}{T'}x^{n-1} + \overset{a}{T'}x^n +$ &c. tum erit ſemper ultimo $p\overset{a}{T'} + q\overset{\overline{a-1}}{T'} + r\overset{\overline{a-2}}{T'} + s\overset{\overline{a-3}}{T'} +$ &c. $= o$, ſi modo $A = h + kx + lx^2 +$ &c. ſit quantitas haud in infinitum pergens; conſtat e ducendo ſeriem $a + bx + cx^2 +$ &c. in $p + qx + rx^2 +$ &c. quod productum exinde ortum erit $pa + (pb + qa)x + (pc + qb + ra)x^2 +$ &c. $= A$.

THEOR.

THEOR. IX.

Sit ſeries $S = a + bx + cx^2 + dx^3 +$ &c. *in infinitum, cujus relatio inter ſucceſſivos terminos* t, t', t'', &c. *deſignetur per æquationem* $pt + qt' + rt'' + \sigma t''' +$ &c. $= o$ *ubi* p, q, r, σ, &c. *ſunt invariabiles quantitates; tum erit ſumma ſeriei*

$$S = \frac{a \times (qx^{n-1} + rx^{n-2} + sx^{n-3} + \tau x^{n-4} + \&c.) + b \times (rx^{n-1} + sx^{n-2} + \tau x^{n-3} + \&c.) + c \times (sx^{n-1} + \tau + \&c.) + d \times (\tau x^{n-1} + \&c.)}{px^n + qx^{n-1} + rx^{n-2} + sx^{n-3} + \&c.},$$

ubi n *ſit minor per unitatem quam numerus literarum* p, q, r, &c.

Cor. 1. Hinc datâ æquatione relationem inter *n* ſucceſſivos terminos deſignante; pro $(n-1)$ terminis aſſumi poſſunt quæcunque datæ quantitates; ergo in hoc caſu ita aſſumi poſſunt $n-1$ primi termini ut numerator ſit diviſor denominatoris, & conſequenter ſeries prædicta $a + bx + cx^2 +$ &c. $= \frac{1}{\alpha + x}$; & ſic aſſumi poſſunt $(n-1)$ primi termini, ut ſeries $a + bx + cx^2 +$ &c. (reſultans ex æquatione $pt + qt' + rt'' +$ &c. $= o$) $= \frac{\pi + \rho x}{\alpha' + \beta' x + \gamma' x^2}$, & ſic deinceps.

Cor, 2. Fingatur $p \times (x + \alpha) \times (x + \beta) \times (x + \gamma) \times$ &c. $= px^n + qx^{n-1} + rx^{n-2} +$ &c. & erunt $\frac{A}{x + \alpha}$, $\frac{B}{x + \beta}$, $\frac{C}{x + \gamma}$, &c., $\frac{Fx + G}{(x + \alpha) \times (x + \beta)}$, $\frac{Hx + I}{(x + \alpha) \times (x + \beta)}$, &c. particulares valores ſeriei per æquationem $pt + qt' + rt' +$ &c. $= o$ denotatæ.

Cor. 3. Ex ultimâ relatione terminorum, quæ ſit conſtans, i. e. prædicti generis, ſequitur methodus ſummandi ſeries quam proxime, in quibus relatio terminorum eſt variabilis: ſit æquatio $pt(z^2 + az + b) + qt'(z^2 + cz + d) = o$, & ultima relatio terminorum erit

$pt z^2 + qt' z^2 = o$, & exinde per prob. erit ſumma quæſita prope $= \frac{q}{q+p} t$.

Et ſic ſit ultima relatio, i. e. relatio ad infinitam diſtantiam $pt + qt' + rt'' = o$, & erit ſumma terminorum longe diſtantium $= \frac{(q+r)t + st'}{p+q+r}$, i. e. ubi z diſtantia a primo ſit permagnus numerus: & ex iiſdem principiis ulterius promotis corrigere liceat approximationem inventam. Conſtat e ſcribendo t & t' pro a & b, & 1 pro x in hujuſce problematis reſolutione.

PROB. VIII.

Æquationem relationem inter n *ſucceſſivos terminos* (t, t′, t″, t‴, *&c.*) *viz.* $pt + qt' + rt'' + st''' +$ &c. $= 0$ *exprimentem, cujus ſeries ſit* $a + bx + cx^2 + dx^3 +$ *&c. in plures dividere.*

Summa ſeriei prædictæ erit $\frac{a(qx^{n-1} + rx^{n-2} + sx^{n-3} + \&c.) + b(rx^{n-1} + sx^{n-2} + \&c.) + c(sx^{n-1} + \tau x^{n-2} + \&c.) + \&c.}{px^n + qx^{n-1} + rx^{n-2} + sx^{n-3} + \&c.}$; ſed e lem. cap. 2. lib. 1. dividi poteſt hæc fractio in n ſubſequentes $\frac{a}{x-\alpha} + \frac{b}{x-\beta} + \frac{c}{x-\gamma} +$ &c. ubi $p(x-\alpha) \times (x-\beta) \times (x-\gamma) \times \&c. = px^n + qx^{n-1} + \&c.$ & conſequenter data dividitur æquatio in alias, viz. $T - \alpha T' = o$, $T - \beta T' = o$, &c. & ſeries $a + bx + cx^2 +$ &c. dividitur in n ſeries formulæ $\left(\frac{a}{x-\alpha}\right) = -\frac{a}{\alpha} - \frac{ax}{\alpha^2} - \frac{ax^2}{\alpha^3} -$ &c.

2. Sit ultima relatio terminorum prædicta $pt + qt' + rt'' + \&c. = o$; dividi poteſt per præcedentem methodum ſeries ejus in n alias, quarum terminorum longe diſtantium ſumma æquat terminum ſeriei, cujus relatio datur: vel ob $(a + bx)^m = a^m + m\frac{a^m \times bx}{a+bx} + m$.

$$\frac{m+1}{2}\frac{a^m \times b^2x^2}{(a+bx)^2} + m\,.\,\frac{m+1}{2}\,.\,\frac{m+2}{3}\,.\,\frac{a^m \times b^3x^3}{(a+bx)^3} + \&c. = \frac{a^{m+1}}{a+bx} +$$

$$\frac{(m+1)\,a^{m+1} \times bx}{(a+bx)^2} + \frac{(m+1)\,.\,(m+2)}{2}\frac{a^{m+1}b^2x^2}{(a+bx)^3} + \frac{(m+1) \times m+2) \times}{2\,.}$$

$$\frac{(m+3)}{3}\frac{a^{m+1} \times b^3x^3}{(a+bx)^4} + \&c. = \frac{a^{m+n}}{(a+bx)^n} + \frac{(m+n)\,a^{m+n}bx}{(a+bx)^{n+1}} +$$

$$\frac{(m+n) \times (m+n+1)}{2} \times \frac{a^{m+n}b^2x^2}{(a+bx)^{n+2}} + \frac{(m+n) \times (m+n+1) \times (m}{2\quad .\quad 3}$$

$$\frac{+n+2)\,a^{m+n+1}b^3x^3}{(a+bx)^{n+3}} + \&c.$$ & quoniam ad infinitam diſtantiam eadem invenietur relatio inter ſucceſſivos terminos ſeriei $a + bx + cx^2 + dx^3 + \&c. = \frac{A + Bx + Cx^2 + \&c.}{(px^n + qx^{n-1} + rx^{n+2} + \&c.)^m}$, ubi numerator terminat, quicunque ſit valor indicis m; exinde dividi poteſt ſumma ſeriei

$$\frac{a(qx^{n-1} + rx^{n-2} + sx^{n-3} + \&c.) + b(rx^{n-1} + sx^{n-2} + \&c.) + c(sx^{n-1} + \&c.)}{px^n + qx^{n-1} + rx^{n-2} + \&c.}$$

multis modis in plures alias, &c.

Et ſic de pluribus conſimilibus methodis.

3. Convergentiæ ſerierum, quarum dantur æquationes relationes inter ſucceſſivas ſummas & terminos exprimentes, deduci poſſunt ex principiis de convergentiis incrementialium æquationum traditis, vel nonnunquam ex convergentiis terminorum ad infinitam diſtantiam poſitorum.

THEOR. X.

Sit T generalis ſeriei functio quantitatis z diſtantiæ a primo ſeriei termino, tum ejus ſumma per eandem methodum omnino inveſtiganda eſt ac integralis incrementi, quod eſt eadem functio quantitatis z, cujus incrementum eſt 1.

THEOR. XI.

Sit æquatio $A = o$ ad ſummas, n habens invariabiles quantitates (a, b, c, d, &c.) ad libitum aſſumendas; dantur n diverſæ æquationes relationes inter T & T', vel T' & T'', &c. exprimentes; dantur etiam $n \cdot \frac{n-1}{2}$ æquationes diverſæ relationes inter T, T' & T'', &c.; exprimentes; & ſic deinceps.

Inveniatur ſucceſſivus valor ($B = o$) æquationis ($A = o$); ſcribendo in eâ pro z, $z + 1$; pro S, S'; & ſic deinceps; ita reducantur duæ æquationes $A = o$ & $B = o$ ad unam, ut exterminentur quantitates a, b, c, d, &c. ſucceſſive; & reſultabunt n diverſæ æquationes $n - 1$ invariabiles quantitates ad libitum aſſumendas habentes; tum ita reducantur hæ reſultantes æquationes, ut exprimant relationes inter ſucceſſivos terminos T & T'', & invenientur n diverſæ æquationes prædictæ. Et ſic de reliquis.

THEOR. XII.

Datâ æquatione relationem inter ſucceſſivos terminos exprimente, & e prob. 18, lib. 2^{di}. conſtat ſemper dari multiplicatorem, qui in datam æquationem ductus producet æquationem, cujus ſummatio innoteſcit.

Cor. 1. Si detur æquatio $A =$ conſt. quæ ſit ſummatio generalis æquationis $B = o$.

Ex incremento æquationis $A =$ conſt. inveniatur æquatio ad terminos $C = o$; tum erit $\frac{C}{B}$ unus multiplicator, qui reddit æquationem $B = o$ ſummabilem.

Cor. 2. Si generalis ſummatio æquationis $aT^{n} + bT^{n-1} + cT^{n-2} \ldots fT^{2} + gT' + hT = o$ (ubi $T, T', \ldots T^{n-2}, T^{n-1}, T^{n}$ ſunt termini ſucceſſivi, & $a, b, c, \ldots f, g, h$ ſunt functiones ipſius z diſtantiæ a primo vel quocunque alio ſeriei termino) innoteſcat; tum innoteſcent diverſi multiplicatores, qui eam reddent ſummabilem; ſint π, ρ, σ, τ, &c. functiones quantitatis z; tum π, ρ, σ, τ, &c. erunt etiam multiplicatores, qui reddent æquationem $aT^{n} + bT^{n-1} + cT^{n-2} \ldots fT^{2} + gT' + hT = Z$ ſummabilem, ubi Z eſt quæcunque functio ipſius z.

Hìc applicari poſſunt omnia, quæ prius traduntur de transformatione, &c. incrementorum incrementialium æquationum, &c. facile enim mutari poteſt incrementialis æquatio in æquationem inter ſucceſſivos terminos, &c.

PROB. IX.

Invenire ſeries, quæ ſummationem recipiunt.

1. Aſſumatur ſeries quælibet unitate determinata (utrum ſit accurate ſummabilis necne, nihil refert, modo ejus termini ad nihilum perpetuo convergant); e quâ ſeriem eandem termino ſuo primo, vel terminis duobus primis, vel terminis tribus primis, &c. multatam ſubtrahe; inde illud fit, ut quod relinquitur vel æquale ſit termino primo ſeriei aſſumptæ, vel terminis ejus duobus primis, vel terminis tribus primis, & ſic in infinitum.

Hanc autem operandi rationem circa ſeries hâc ſubtractione comparatas iterare licet, unde novæ ſeries in infinitum exurgent, quæ omnes ſummabiles erunt.

Ex. 1. Sumatur ſeries $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} +$ &c. e quâ ſubtrahatur ipſamet demptis $n-1$ terminis ſuis primis, hoc eſt ſubtrahatur $\frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} +$ &c. & relinquitur $\frac{n-1}{n \times 1} + \frac{n-1}{2 \cdot (n+1)} + \frac{n-1}{3 \cdot n+2} + \frac{n-1}{4 \cdot n+3} + \frac{n-1}{5 \cdot n+4} +$ &c. $= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \ldots \frac{1}{n-1}$: rurſus a ſerie quam modo comparavimus, ſubtrahatur ipſamet demptis $m-1$ terminis ſuis primis, hoc eſt $\frac{n-1}{m \times (n+m-1)}$

+

$$+\frac{n-1}{(m+1)\,.\,(n+m)}+\frac{n-1}{(m+2)\,.\,(n+m+1)}+\text{\&c. \& relinquitur}$$

$$\frac{(n-1)\times(m(n+m-1)-n)}{n\times m\times(n+m-1)}+\frac{(n-1)\times((m+1)\times(n+m)-2\times(n+1))}{(n+1)\times(m+1)\times(n+m)}$$

$$+\frac{(n-1)\times((m+2)\,.\,(n+m+1)-3\times(n+2))}{(n+2)\times(m+2)\times(n+m+1)}+\text{\&c.}=\frac{n-1}{1\,.\,n}$$

$$+\frac{n-1}{2\,(n+1)}+\frac{n-1}{3\times(n+2)}\cdots\frac{n-1}{(m-1)\,.\,(n+m-2)};\ \text{\& sic deinceps.}$$

Cor. 1. Sit data ſeries $t+t'+t^2+t^3+t^4+$ &c. ubi per t, t', t^2, t^3, &c. deſignentur ſucceſſivi ejus termini, & erit terminus generalis ſeriei ex datâ ſerie per hanc methodum deductæ $=at^m+bt^n+ct^r+dt^s+$ &c.; ubi per t^m, t^n, t^r, t^s, &c. deſignantur termini ad diſtantias m, n, r, s, &c. reſpective a primo vel quocunque alio datæ ſeriei termino, & $a+b+c+d+$ &c. $=0$; ſit m minor quam n, n quam r, r quam s, & m minimus & l maximus index; tum erit ſeriei, cujus generalis terminus eſt $(at^m+bt^n+ct^r+dt^s+$ &c.$)$, ſumma $=a(t^m+t^{m+1}+t^{m+2}\ldots+t^{l-1})+b(t^n+t^{n+1}+t^{n+2}\ldots+t^{l-1})+c(t^r+t^{r+1}+t^{r+2}\ldots+t^{l-1})+d(t^s+t^{s+1}+t^{s+2}\ldots+t^{l-1})+$ &c.: e. g. ſit data ſeries $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+$ &c., tum ſumma ſeriei, cujus terminus eſt $a(t^m-t^n)=\frac{a}{z+m}-\frac{a}{z+n}=\frac{an-am}{z+m\,.\,z+n}$, erit $=a$

$$\left(\frac{1}{z+m}+\frac{1}{z+m+1}+\frac{1}{z+m+2}\cdots\frac{1}{z+n-1}\right).$$

Cor. 2. Hinc facile conſtat ſemper eandem evadere ſeriem, utrum primum ſubtrahatur e datâ ſerie eadem ab m terminis multata, & deinde ex ſerie reſultante B ſubtrahatur eadem B ab n terminis multata; vel primum ſubtrahatur e datâ ſerie eadem ab n terminis multata, & deinde ex ſerie reſultante ſubtrahatur eadem ab m terminis multata. Et conſimilia etiam affirmari poſſunt de pluribus conſimilibus repetitis operationibus.

2. Altera vero methodus erit, ut aſſumptâ quâlibet ſerie infinitâ, cujus termini tum ad nihil perpetuo convergant, tum procedant per

poteſtates

poteſtates indeterminatæ x, ſi multiplicetur ſeries aſſumpta per binomium vel multinomium utcunque conflatum ex quantitatibus datis & indeterminatâ x, deinde ponatur binomium vel multinomium illud nihilo æquale, &c. hinc elicietur valor indeterminatæ x, ad quem ſi quantitas iſta reſtringatur, tunc ſeries ex hâc multiplicatione genita evadet etiam nihilo æqualis, quapropter ſi transferantur termini ejus primi ad alteram æquationis partem, ſeries reſidua æqualis erit terminis tranſlatis.

Ex. 1. Sumatur ſeries $1 + \frac{1}{2}x + \frac{1}{3}x^2 + \frac{1}{4}x^3 + \frac{1}{5}x^4 +$ &c. quæ ſi multiplicetur per binomium $1 - ax^m = 0$ erit ſeries genita $1 + \frac{1}{2}x + \frac{1}{3}x^2$ $\ldots \frac{1}{m}x^{m-1} - \frac{(m+1)\times a - 1}{m+1} x^m - \frac{(m+2)a - 2}{(m+2).2} x^{m+1} - \frac{(m+3)\times a - 3}{(m+3).3}$ $\times x^{m+2} -$ &c. unde $1 + \frac{1}{2}x + \frac{1}{3}x^2 \ldots \frac{1}{m}x^{m-1} = P = \frac{(m+1).a - 1}{m+1}$ $x^m + \frac{(m+2)a - 2}{(m+2).2} x^{m+1} + \frac{(m+3)a - 3}{(m+3).3} x^{m+2} +$ &c. multiplicetur hæc æquatio reſultans in $1 - bx^n$ & reſultat $(1 - bx^n) \times P =$ $\frac{(m+1)\times a - 1}{m+1} x^m + \frac{(m+2).a - 2}{(m+2).2} x^{m+1} + \frac{(m+3).a - 3}{(m+3).3} x^{m+2} \ldots +$ $\frac{(m+n)a - n}{(m+n)\times n} x^{m+n-1} - \frac{((n+1)\times(m+1)\times(m+n+1)a - (m+n+1)\times(n+1))}{(m+1)\times(n+1)\times}$ $\frac{b - (m+1)\times((m+n+1)a - (n+1))}{(m+n+1)} x^{m+n} + \frac{2(m+n+2)\times(m+2)a - 2}{2.m+2.}$ $\frac{(m+2)\times(n+2) - ((m+2)\times a - 2)\times(m+n+2)\times(n+2)b}{n+2.(m+n+2)} x^{m+n+1} +$ $\frac{3(m+n+3).(m+3)a - 3(m+3).(n+3) - ((m+3)\times a - 3)\times}{3.m+3.n+3.}$ $\frac{(m+n+3).(n+3)b}{m+n+3} \times x^{m+n+2} +$ &c. unde $(1 - bx^n) \times P -$ $\frac{(m+1)\times a - 1}{m+1} x^m - \frac{(m+2)a - 2}{(m+2).2} x^{m+1} - \frac{(m+3)a - 3}{(m+3).3} x^{m+2} \ldots -$ $(m+n)$

$$\frac{(m+n)a-n}{(m+n).n}x^{m+n-1}=\frac{(m+1).(m+n+1)a-(m+1).(n+1)}{(m+n+1)\times(n+1)}$$

$$\frac{-((m+1)a-1)\times(m+n+1)\times(n+1)b}{\times(n+1)\times(m+1)}+\frac{2(m+2).(m+n+2)}{m+n+2}$$

$$\frac{a-2(m+2).(n+2)-((m+2)a-2)\times(m+n+2)\times(n+2)b}{.n+2.m+2..2}$$

$+$ &c.

Et ſic iteratâ operatione novæ exurgent ſeries.

Hæ methodi fallunt, cum ſeries inventa ſit infinita quantitas.

Cor. 1. Secunda methodus eadem fit ac prior, ſi modo pro x ſcribatur unitas, & vice verſâ prima methodus ſemper continetur in ſecundâ.

Cor. 2. Sit terminus $\frac{A}{az+b.cz+d.ez+f.gz+h.\&c.}=\frac{B}{az+b}+\frac{B'}{cz+d}+\frac{B''}{ez+f}+\frac{B'''}{gz+h}+\&c.=\frac{C}{az+b.cz+d}+\frac{C'}{ez+f}+\frac{C''}{gz+h}+\&c.=\frac{c}{az+b.cz+d}+\frac{c'}{ez+f.gz+h}+\&c.=\frac{D}{az+b.cz+d.ez+f}+\frac{D'}{gz+h}+\&c.=\frac{d}{az+b.cz+d}+\frac{d'}{gz+h.ez+f}+\&c.=\&c.$, ubi in denominatoribus harum fractionum contineantur omnes diviſores denominatoris datæ fractionis & nulli alii.

Cor 3. Datis ſeriebus, ex his methodis ſæpe deduci poſſunt earum ſummæ.

Aſſumatur ſeries, ita ut factores in denominatore contenti contineantur in duobus vel pluribus diverſis terminis aſſumptæ ſeriei, quorum diſtantiæ a ſe invicem ſint m, n, &c. & deinde per præcedentes methodos inveniatur ſeries, quæ fiat datæ ſeriei æqualis, & fit coroll.

Ex. 1. Sit ſeries $S = \frac{4n+5}{2n+1.2n+2.2n+3.2n+4}x + \frac{4n+9}{2n+3.2n+4.2n+5.2n+6}x^2 + \frac{4n+13}{2n+5.2n+6.2n+7.2n+8}x^3 +$ &c. quâpropter aſſumptâ ſerie $\frac{1}{2n+1.2n+2} + \frac{1}{2n+3.2n+4}x + \frac{1}{2n+5.2n+6}x^2 +$ &c. $= S$, quoniam ſi $2n+1$ & $2n+2$ ſint factores in denominatore unius termini, tum $2n+3.2n+4$ erit denominator ſubſequentis termini: multiplicetur ſeries aſſumpta per $x-1$, & reſultat $(x-1)S = \left(\frac{-1}{2n+1.2n+2}\right) + \frac{8n+10}{2n+1.2n+2.2n+3.2n+4}x +$ &c. $= -\frac{1}{2n+1.2n+2} + 2S$, pone $x-1=0$ & evadet datæ ſeriei ſumma $S = \frac{1}{2n+1.2n+2}$.

Cor. . Pone in prædictâ ſummâ S pro n duos diverſos valores l & $l+m$, ubi m ſit integer numerus, & erit duarum quantitatum reſultantium differentia ſumma m terminorum, e. g. in hoc exemplo ſcribe in ſummâ $\frac{1}{2n+1.2n+2}$ valores n & $n+m$ pro n, & evadet differentia $\frac{1}{2n+1.2n+2} - \frac{1}{2n+m+1.2n+m+2}$ ſumma m primorum terminorum.

Ex. 2. Sit ſeries $B = \frac{1}{1.2.3.4.5} + \frac{4}{4.5.6.7.8} + \frac{9}{7.8.9.10.11} + \frac{16}{10.11.12.13.14} +$ &c.

Sit n numerus deſignans locum termini, tum terminus ipſe erit $\frac{\frac{1}{3}n}{3n-2.3n-1.3n+1.3n+2}$; & ſeries poſt n terminos erit

$$\frac{\frac{1}{3}\times(n+1)}{(3n+1)\cdot(3n+2)\cdot(3n+4)\cdot(3n+5)}+\frac{\frac{1}{3}(n+2)}{(3n+4)\cdot(3n+5)\cdot(3n+7)\cdot(3n+8)}+\frac{\frac{1}{3}(n+3)}{(3n+7)\times(3n+8)\times(3n+10)\times(3n+11)}$$

+ &c. quoniam denominator quilibet hujus ſeriei e factoribus quatuor conſtat, quorum bini quique in termino proxime ſequente repetuntur, ſumatur idcirco ſeries $\frac{1}{3n+1\cdot 3n+2}+\frac{x}{3n+4\cdot 3n+5}+\frac{x^2}{3n+7\cdot 3n+8}+$ &c. $= S$, cujus bini factores in ſingulis denominatoribus ſemel tantummodo occurrunt; deinde, quoniam in ſerie datâ denominatoris cujuſlibet pars ea quæ iterum occurrit in termino proxime ſequente conſpicitur: multiplicetur ſeries data per $x-1$ (ſin vero ita accideret ut terminus unus vel duo vel plures terminis repetitis interjaceant, fiat congruens multiplicatio ſeriei datæ per x^2-1 vel x^3-1, &c.) prodibit ſeries $-\frac{1}{(3n+1)\times(3n+2)}+$

$$\frac{18n+18}{3n+1\cdot 3n+2\cdot 3n+4\cdot 3n+5}x+\frac{18n+36}{3n+4\cdot 3n+5\cdot 3n+7\cdot 3n+8}$$

$$\frac{18n+54}{3n+7\cdot 3n+8\cdot 3n+10\cdot 3n+11}x^3$$ &c. $=(x-1)S$: ponatur $x=1$, deinde fiat tranſlatio primi termini ad alteram æquationis partem; tum dividantur omnia per 54, hinc exurget ſumma deſiderata

$$\frac{\frac{1}{3}\times(n+1)}{3n+1\cdot 3n+2\cdot 3n+4\cdot 3n+5}+\frac{\frac{1}{3}(n+2)}{3n+4\cdot 3n+5\cdot 3n+7\cdot 3n+8}$$

$+$ &c. $=\frac{1}{54\cdot 3n+1\cdot 3n+2}$.

Ex. 3. Proponatur ſeries $\frac{19}{1\cdot 2\cdot 3}\times\frac{1}{4}+\frac{28}{2\cdot 3\cdot 4}\times\frac{1}{8}+\frac{39}{3\cdot 4\cdot 5}\times\frac{1}{16}+\frac{52}{4\cdot 5\cdot 6}\times\frac{1}{32}+$ &c. cujus ſumma requiritur.

Aſſumatur ſeries $1 + \frac{1}{2}x + \frac{1}{3}x^2 + \frac{1}{4}x^3 + \frac{1}{5}x^4 +$ &c. $= S$, in quâ inveniuntur omnes factores e quibus conflantur denominatores ſeriei propoſitæ: fingantur multiplicatores bini $2x - 1$ & $ax - b$; prior aſſumatur, quoniam ſeries data procedit per poteſtates fractionis $\frac{1}{2}$; præterea cum ex horum multiplicatorum mutuo ductu generetur quantitas $2ax^2 - 2bx - ax + b$ multiplicetur ſeries aſſumpta per productum iſtud, quo facto emerget ſeries $b - \frac{3b+2a}{1.2}x + \frac{9a-4b}{1.2.3}x^2 + \frac{16a-10b}{2.3.4}x^3 + \frac{25a-18b}{3.4.5}x^4 +$ &c. $= (2x-1) \times (ax-b) \times S$, tranſlatione factâ erit $\frac{9a-4b}{1.2.3}x^2 + \frac{16a-10b}{2.3.4}x^3 + \frac{25a-18b}{3.4.5}x^4$ &c. $= (2x-1) \times (ax-b) \times S - b + \frac{3b+2a}{1.2}x$. Comparetur nunc primus terminus hujus æquationis cum termino primo ſerie datæ, itemque ſecundus cum ſecundo; hinc erit $9a - 4b = 19$, & $16a - 10b = 28$, unde invenietur $a = 3$, $b = 2$, & ſumma ſeriei propoſitæ erit $= 1$.

Eâdem operâ invenietur (ſi ponatur $ax - b$ in $3x - 2$, erit jam $x = \frac{2}{3}$) $\frac{19}{1.2.3} \times \frac{4}{9} + \frac{28}{2.3.4} \times \frac{8}{27} + \frac{39}{3.4.5} \times \frac{16}{81} + \frac{52}{4.5.6} \times \frac{32}{243}$ &c. $= 2$.

Ex. 4. Sit ſeries $1 + \frac{1}{2}x + \frac{1}{3}x^2 + \frac{1}{4}x^3 + \frac{1}{5}x^4 +$ &c. $= S$, multiplicetur ſeries aſſumpta per trinomium $a + bx + cx^2$, unde erit ſeries genita $\frac{2a+3b+6c}{1.2.3}x^2 + \frac{6a+8b+12c}{2.3.4}x^3 + \frac{12a+15b+20c}{3.4.5}x^4$ &c. $= (a + bx + cx^2) \times S - a - \frac{1}{2}ax - bx$; fiat comparatio terminorum ſeriei genitæ cum terminis ſeriei propoſitæ, e. g. ſit ſeries $\frac{19x^2}{1.2.3} + \frac{28x^3}{2.3.4} + \frac{39x^4}{3.4.5} +$ &c. hinc orientur tres æquationes $2a + 3b + 6c = 19$, $6a + 8b + 12c = 28$, $12a + 15b + 20c = 39$,

$= 39$, quarum ope invenientur $a = 2$, $b = -7$, $c = 6$; erit igitur ſumma deſiderata $= (2 - 7x + 6x^2) \times S - a + 6x$: jam ſi requiratur, ut irrationalitas tollatur, pone $2 - 7x + 6x^2 = o$, hinc elicietur valor duplex quantitatis x nempe $\frac{1}{2}$ & $\frac{2}{3}$, quapropter ſi x reſtinguatur ad valorem $\frac{1}{2}$ erit ſumma ſeriei $= 1$; ſin vero x reſtinguatur ad valorem $\frac{2}{3}$, erit ſumma ſeriei $= 2$.

3. Exempla hujus multiplicationis etiam ex præcedente additione ulterius promotâ facile erui poſſunt. E. g. Sit ſeries $p + qx + rx^2 + sx^3 +$ &c. $= S$, multiplicetur hæc ſeries in a, in bx & in cx^2; & reſultant $ap + aqx + arx^2 +$ &c., $bpx + bqx^2 + brx^3 +$ &c., & $cpx^2 + cqx^3 +$ &c.; quibus ad unam ſummam adjunctis, ea reſultat $ap + (aq + bp)x + (ar + bq + cp)x^2 +$ &c. $= (a + bx + cx^2) \times S$: ſi $a + bx + cx^2 = o$, & termini ad infinitam diſtantiam nihilo evadant æquales; tum ſeries reſultans ſemper converget.

Sint enim termini ad infinitam diſtantiam $\frac{1}{z^o}$, ubi o eſt quantitas quam minima, at finita; tum differentia inter duos ſucceſſivos terminos $\frac{1}{z^o}$ & $\frac{1}{(z+1)^o}$, ubi z eſt infinita quantitas $= \frac{(z+1)^o - z^o}{z^o \times (z+1)^o} = \frac{-o}{z^{o+1}}$ prope; unde termini ad infinitam diſtantiam decreſcunt in reciprocâ ratione $(o + 1)$ diſtantiæ a primo ſeriei termino, quæ eſt major quam ſimplex (1); ergo ſumma ſeriei erit finita.

THEOR. XIII.

Sit ſeries $a + bx + cx^2 +$ &c. ſecundum dimenſiones quantitatis x progrediens, cujus coefficientes terminorum proxime ſubſequentium ad infinitam diſtantiam hanc habent inter ſe rationem, viz. ſingulus præcedens ad ejus ſubſequentem $:: r : 1$; ducatur hæc ſeries in functionem ipſius x nihilo æqualem, cum $x = \alpha$; tum, ſi α major ſit quam r, ſeries reſultans ſemper diverget; ſin minor vero converget.

Facile conſtat hujuſce theorematis demonſtratio.

THEOR. XIV.

Sit ſeries $a+b+c+d+e+$ &c. $=S$, cujus termini generaliter exprimantur per datam functionem (ϕ) quantitatis z diſtantiæ a primo ſeriei termino: addantur ſimul quique duo ſucceſſivi termini, c. g. $(a+b)+(c+d)+(e+f)+$ &c. & reſultat ſeries $(A)=S$ a primo termino incipiens; vel ſeries $(B)=(b+c)+(d+e)+(f+g)+$ &c. $=S-a$ a ſecundo incipiens; in functione ϕ pro z ſcribatur $z+1$ & reſultet functio ϕ'; deinde in quantitate $\phi+\phi'$ ſi termini datæ ſeriei ſint omnes affirmativi; vel in $\phi-\phi'$ ſi termini ſint alternatim affirmativi vel negativi, pro z ſcribatur $2z'$, ubi z' eſt diſtantia a primo ſeriei termino in ſerie A, & reſultat generalis terminus ſeriei A: & ſimiliter in functione ϕ pro z ſcribantur $z+1$ & $z+2$, & reſultent functiones ϕ' & ϕ'', tum in quantitate $\phi'+\phi''$ vel $\phi'-\phi''$ pro z ſcribatur $2z'$, & reſultat generalis terminus ſeriei B, ubi z' denotat diſtantiam a primo ſeriei termino.

Ex. 1. Sit ſeries $1+\frac{1}{2}+\frac{1}{3}+$ &c., cujus generalis terminus eſt $\frac{1}{z+1}=\phi$; tum, ſi pro z ſcribatur $z+1$, reſultabit $\phi'=\frac{1}{z+2}$; in functione $\phi+\phi'=\frac{1}{z+1}+\frac{1}{z+2}=\frac{2z+3}{z+1\cdot z+2}$ pro z ſcribatur $2z'$, & reſultat $\frac{4z'+3}{2z'+1\cdot 2z'+2}$ generalis terminus ſeriei $(1+\frac{1}{2})+(\frac{1}{3}+\frac{1}{4})+(\frac{1}{5}+\frac{1}{6})+$ &c. $=\frac{3}{2}+\frac{7}{12}+\frac{11}{30}+$ &c., ubi z' eſt diſtantia a primo ſeriei termino.

Ex. 2. Sit ſeries $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+$ &c., cujus generalis terminus eſt $\frac{1}{z+1}$; pro z in functione $\frac{1}{z+1}=\phi$ ſcribatur $z+1$, & reſultat $\phi'=\frac{1}{z+2}$; & quoniam termini ſunt alternatim negativi & affirmativi in functione $\phi-\phi'=\frac{1}{z+1}-\frac{1}{z+2}=\frac{1}{z+1\cdot z+2}$ pro z

ſcribatur $2z'$ & reſultat $\frac{1}{2z'+1\,.\,2z'+2}$ generalis terminus ſeriei $(1-\frac{1}{2})+(\frac{1}{3}-\frac{1}{4})+(\frac{1}{5}-\frac{1}{6})+$ &c. $=\frac{1}{1\,.\,2}+\frac{1}{3\,.\,4}+\frac{1}{5\,.\,6}+$ &c.; literâ z' eandem quantitatem ac prius denotante.

Ex. 3. Sit data ſeries $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+$ &c., & requiratur generalis terminus ſeriei $(\frac{1}{2}+\frac{1}{3})+(\frac{1}{4}+\frac{1}{5})+(\frac{1}{6}+\frac{1}{7})+$ &c. $=\frac{5}{2\,.\,3}+\frac{9}{4\,.\,5}+\frac{13}{6\,.\,7}+$ &c.: in generali termino $\frac{1}{z+1}$ datæ ſeriei pro z ſcribatur $z+1$ & $z+2$; & reſultant $\frac{1}{z+2}=\phi'$ & $\frac{1}{z+3}=\phi''$: in termino $\phi'+\phi''=\frac{2z+5}{z+2\,.\,z+3}$ pro z ſcribatur $2z'$ & reſultat $\frac{2z'+5}{2z'+2\,.\,2z'+3}$ generalis terminus quæſitus.

2. Si modo addantur ſimul quique (n) ſucceſſivi termini datæ ſeriei, cujus generalis terminus eſt ϕ, a primo, ſecundo, & denique $(m+1)$ termino incipiens; & reſultet ſeries A, cujus generalis terminus quæritur: in termino ϕ pro z ſcribantur (n) quantitates $z+m$, $z+m+1$, $z+m+2, \ldots z+m+n-2$, $z+m+n-1$, & reſultent quantitates ϕ', ϕ'', ϕ''', &c.; in ſummâ $\phi+\phi'+\phi''+\phi'''+$ &c. pro z ſcribatur nz, & reſultabit generalis terminus ſeriei A quæſitus.

Ex. Sit data ſeries $1+\frac{1}{h+1}+\frac{1}{2h+1}+$ &c., cujus generalis terminus eſt $\frac{1}{zh+1}$: invenire generalem terminum ſeriei, cujus termini ſint ſumma quorumcunque (3) ſucceſſivorum terminorum a ſecundo incipientium. Pro z in generali termino ſcribatur $z+1$, quoniam terminus a ſecundo incipit & exorietur $\frac{1}{(z+1)h+1}$; in hoc termino pro z ſcribantur z, $z+1$ & $z+2$, & reſultant $\frac{1}{(z+1)}$

$h+1$

$\frac{}{h+1}$, $\frac{1}{(z+2)h+1}$ & $\frac{1}{(z+3)h+1}$, cujus ſumma eſt $\frac{h^2(3z^2+12z+11)+h(6z+12)+3}{(z+1)h+1\,.\,(z+2)h+1\,.\,(z+3)h+1}$; in hâc ſummâ pro z ſcribatur $3z'$ & reſultabit $\frac{h^2(27z'^2+36z'+11)+h(18z'+12)+3}{(3z'+1)h+1\,.\,(3z'+2)h+1\,.\,(3z'+3)h+1}$ generalis terminus ſeriei $\left(\frac{1}{h+1}+\frac{1}{2h+2}+\frac{1}{3h+1}\right)+\left(\frac{1}{4h+1}+\frac{1}{5h+1}+\frac{1}{6h+1}\right)+$ &c., ubi z' denotat diſtantiam a primo ſeriei termino.

3. Si datæ ſeriei termini ſint alternatim negativi & affirmativi, & n numerus terminorum ſimul adjunctorum ſit impar; tum erit ſeriei reſultantis termini alternatim negativi & affirmativi; ſin n ſit par numerus, tum omnes termini idem habebunt ſignum.

Cor. Ex ſeriebus per hanc methodum transformatis & principiis in prob. præced. traditis facile acquiri poſſunt multæ ſeries, quarum ſummæ innoteſcunt.

Ex. Sit data ſeries $1+\frac{1}{h+1}+\frac{1}{2h+1}+\frac{1}{3h+1}+$ &c., cujus generalis terminus eſt $\frac{1}{hz+1}$; ſimul addantur tres ſucceſſivi termini $\frac{1}{hz+1}+\frac{1}{hz+h+1}+\frac{1}{hz+2h+1}=\frac{3h^2z^2+6h^2z+6hz+2h^2+6h+3}{hz+1\,.\,hz+h+1\,.\,hz+2h+1}=A$; etiamque tres ſucceſſivi termini $\frac{1}{hz+h+1}+\frac{1}{hz+2h+1}+\frac{1}{hz+3h+1}=\frac{3h^2z^2+12h^2z+6hz+11h^2+12h+3}{hz+h+1\,.\,hz+2h+1\,.\,hz+3h+1}=B$: in quantitatibus A & B per theor. pro z ſcribatur $3z$, & reſultant quantitates $\frac{27h^2z^2+18h^2z+18hz+2h^2+6h+3}{3hz+1\,.\,3hz+h+1\,.\,3hz+3h+1}=\beta$

$=\beta$ & $\frac{27h^2z^2+(36h^2+18h)z+11h^2+12h+3}{3hz+h+1 \cdot 3hz+2h+1 \cdot 3hz+3h+1}=\gamma$: inveniatur differentia $\beta-\gamma$ inter has duas quantitates, quæ erit $\frac{27h^3z^2+(27h^3+18h^2)z+6h^3+9h^2+3h}{3hz+1 \cdot 3hz+h+1 \cdot 3hz+2h+1 \cdot 3hz+3h+1}=\tau$; & conſequenter ſumma ſeriei, cujus termini τ progrediuntur a valore o quantitatis z uſque ad valorem infinitum ejuſdem quantitatis, erit 1; i.e. ſumma ſeriei a valore o ad valorem infinitum quantitatis z, cujus generalis terminus eſt $\frac{9h^2z^2+(9h^2+6h)z+2h^2+3h}{3hz+1 \cdot 3hz+h+1 \cdot 3hz+2h+1 \cdot 3hz+3h+1}$, erit $\frac{1}{3h}$.

Ex. Sit eadem ſeries $1+\frac{1}{h+1}+\frac{1}{2h+1}+$ &c.; addantur tres ſucceſſivi termini $\frac{1}{hz+2h+1}+\frac{1}{hz+3h+1}+\frac{1}{hz+4h+1}$ $=\frac{3h^2z^2+(18h^2+6h)z+26h^2+18h+3}{hz+h+1 \cdot hz+2h+1 \cdot hz+3h+1}=C$: in quantitate C pro z ſcribatur $3z$, & reſultat $\frac{27h^2z^2+(54h^2+18h)z+26h^2+18h+3}{3hz+2h+1 \cdot 3hz+3h+1 \cdot 3hz+4h+1}=\delta$; inveniatur differentia inter quantitates γ & δ & reſultat $\frac{27h^3z^2+(45h^3+18h^2)z+18h^3+15h^2+3h}{3hz+h+1 \cdot 3hz+2h+1 \cdot 3hz+3h+1 \cdot 3hz+4h+1}=\sigma$, & conſequenter ſumma ſeriei, cujus termini σ progrediuntur a valore o quantitatis z ad valorem infinitum ejuſdem quantitatis erit $\frac{1}{h+1}$; unde ſumma ſeriei, cujus generalis terminus eſt $\frac{9h^2z^2+(15h^2+6h)z+6h^2+5h+1}{3hz+h+1 \cdot 3hz+2h+1 \cdot 3hz+3h+1 \cdot 3hz+4h+1}$, a valore ($o$) quantitatis z ad infinitum erit $\frac{1}{3h \cdot h+1}$.

Eadem etiam applicari poſſunt ad alias ſeries, quarum termini ſunt alternatim affirmativi & negativi: in hoc caſu, cum numerus terminorum, qui ſimul adjunguntur, ſit impar; tum termini ſeriei reſultantis etiam erunt alternatim affirmativi & negativi, ſin aliter non.

Conſimilia etiam prædicari poſſunt de caſibus in quibus quicunque n termini ſint affirmativi & m negativi, vel quocunque alio modo termini ſint affirmativi & negativi.

4. Sint T^m, T^n, T^r, T^s, &c. termini ſeriei A, quorum ſit T^m quicunque terminus datæ ſeriei; & T^n, T^r, T^s, &c. termini prædictæ ſeriei, quorum diſtantiæ a termino T^m ſint reſpective $n - m$, $r - m$, $s - m$, &c.; i. e. in generali termino ſeriei A, pro z diſtantiâ a primo ſeriei termino ſcribantur reſpective $z + m$, $z + n$, $z + r$, $z + s$, &c.; & reſultabunt termini T^m, T^n, T^r, T^s, &c.

Sint $T^{m'}$, $T^{n'}$, $T^{r'}$, $T^{s'}$, &c. termini ſeriei B, cujus generalis terminus, qui eſt functio quantitatis z diſtantiæ a primo ſeriei termino, ſit ϕ; & ſi in ϕ pro z ſcribantur $z + m'$, $z + n'$, $z + r'$, $z + s'$, &c., reſultabunt termini $T^{m'}$, $T^{n'}$, $T^{r'}$, $T^{s'}$, &c.

Sint $T^{m''}$, $T^{n''}$, $T^{r''}$, $T^{s''}$, &c. termini ſeriei C, cujus generalis terminus ſit ϕ' functio quantitatis z diſtantiæ a primo ſeriei termino, & ſi in termino ϕ' pro z ſcribantur reſpective $z + m''$, $z + n''$, $z + r''$, $z + s''$, &c., reſultabunt termini $T^{m''}$, $T^{n''}$, $T^{r''}$, $T^{s''}$, &c.

Aſſumatur quantitas $aT^m + bT^n + cT^r + dT^s +$ &c. $+ a'T^{m'} + b'T^{n'} + c'T^{r'} + d'T^{s'} +$ &c. $+ a''T^{m''} + b''T^{n''} + c''T^{r''} +$ &c. $+$ &c., pro termino ſeriei quæſitæ; tum ex ſummis (S, S', S'', &c.) datarum ſerierum A, B, C, &c. acquiri poteſt ſumma ſeriei reſultantis quæſitæ, erit enim $(a + b + c + d +$ &c.$) \times S + (a' + b' + c' + d' +$ &c.$) \times S' + (a'' + b'' + c'' + d'' +$ &c.$) \times S'' +$ &c. $- (a + b + c + d +$ &c.$) (T + T' + T'' + \ldots T^{m-1}) - (b + c + d +$ &c.$) (T^m + T^{m+1} + T^{m+2} \ldots T^{n-1}) - (c + d +$ &c.$) (T^n + T^{n+1} +$ &c. $\ldots + T^{r-1}) -$ &c. $- (a' + b' + c' + d' +$ &c.$) (T^{1'} + T^{2'} + T^{3'} \ldots T^{m'-1}) - (b' + c' + d' +$ &c.$) (T^{m'} + T^{m'+1} \ldots T^{n'-1}) -$ &c. $-$ &c.

Cor. Sit data ſeries $\frac{1}{a} + \frac{1}{a+1}x + \frac{1}{a+2}x^2 + \frac{1}{a+3}x^3 +$ &c. $= S$;

$= S$; ex ſummâ S datâ ſemper acquiri poteſt ſumma cujuſcunque ſeriei, cujus generalis terminus eſt $\frac{b z^m + c z^{m-1} + \&c.}{a+z+\alpha . a+z+\beta . a+z+\gamma .\&c.} x^z$; ubi $\alpha, \beta, \gamma, \delta$, &c. ſunt integri numeri, & m eſt numerus minor quam numerus factorum in denominatore: aſſumatur $\frac{A}{a+z+\alpha} + \frac{B}{a+z+\beta} + \frac{C}{a+z+\gamma} + \&c. = \frac{b z^m + c z^{m-1} + \&c.}{a+z+\alpha . a+z+\beta . a+z+\gamma .\&c.}$ & exinde inveniri poſſunt coefficientes A, B, C, &c.; deinde ſumantur $A = d' x^\alpha$, $B = b' x^\beta$, $C = c' x^\gamma$, &c.; unde conſtabunt quantitates d', b', c', &c. & exinde erui poteſt ſumma ſeriei quæſita $= (d' + b' + c' + \&c.) \times S - d' \left(\frac{1}{a} + \frac{1}{a+1} x + \frac{1}{a+2} x^2 \ldots + \frac{1}{\alpha+\alpha-1} x^{\alpha-1}\right) - b' \left(\frac{1}{a} + \frac{1}{a+1} x \ldots \frac{1}{a+\beta-1} x^{\beta-1}\right) - c' \left(\frac{1}{a} + \frac{1}{a+1} x \ldots + \frac{1}{a+\gamma-1} x^{\gamma-1}\right) + \&c. = (d' + b' + c' + \&c.) \left(S - \left(\frac{1}{a} + \frac{1}{a+1} \ldots + \frac{1}{a+\alpha-1}\right) - (b' + c' + \&c.) \left(\frac{1}{a+\alpha} + \frac{1}{a+\alpha+1} \ldots \frac{1}{a+\beta-1}\right) - (c' + \&c.) \left(\frac{1}{a+\beta} + \frac{1}{a+\beta+1} + \frac{1}{a+\gamma-1}\right) - \&c.$

2. Sint duæ datæ ſeries $\frac{1}{a} + \frac{1}{a+1} x + \frac{1}{a+2} x^2 + \&c. = S$; & $\frac{1}{b} + \frac{1}{b+1} x + \frac{1}{b+2} x^2 + \frac{1}{b+3} x^3 + \&c. = S'$; & $\frac{1}{c} + \frac{1}{c+1} x + \frac{1}{c+2} x^2 + \&c. = S''$; &c.; tum facile ex his ſummis datis acquiri poteſt ſumma cujuſcunque ſeriei $\frac{A z^m +}{a+z+\alpha . a+z+\beta . a+z+\gamma . \&c \times}$

$$\frac{B z^{m-1} + C z^{m-2} + \&c.}{b+z+\alpha' . b+z+\beta' . b+z+\gamma' . \&c. \times c+z+\alpha'' . c+z+\beta'' . \&c. . \&c.} x^{z+b},$$

x^{z+b}, ubi m, α, β, γ, &c.; α', β', γ', &c.; α'', β'', &c. ſunt integri numeri.

Cor. 2. Sit data ſeries $\frac{1}{a} - \frac{1}{a+1}x + \frac{1}{a+2}x^2 - \frac{1}{a+3}x^3 + \&c.$; tum ex eâ acquiri poteſt ſumma cujuſcunque ſeriei, cujus generalis terminus eſt $\frac{a'z^m + bz^{m-1} + cz^{m-2} + \&c.}{2z+a \,.\, 2z+a+1 \,.\, 2z+a+2 \,..\, 2z+a+l+1}x^z$, ubi z denotat diſtantiam a primo ſeriei termino & m & l ſunt integri numeri.

Cor. 3. Ex ſummis ſerierum $\frac{1}{\alpha} + \frac{1}{\alpha+1}x + \frac{1}{\alpha+2}x^2 + \&c. = S$, $\frac{1}{\beta} + \frac{1}{\beta+1}x + \frac{1}{\beta+2}x^2 + \&c. = S'$, $\frac{1}{\gamma} + \frac{1}{\gamma+1}x + \frac{1}{\gamma+2}x^2 + \&c. = S''$, &c., etiamque ſummis ſerierum $\frac{1}{\pi} - \frac{1}{\pi+1}x + \frac{1}{\pi+2}x^2 - \&c. = \Sigma$, $\frac{1}{\rho} - \frac{1}{\rho+1}x + \frac{1}{\rho+2}x^2 - \&c. = \Sigma'$, $\frac{1}{\sigma} - \frac{1}{\sigma+1}x + \frac{1}{\sigma+2}x^2 - \&c. = \Sigma''$, &c. deduci poteſt ſumma cujuſcunque ſeriei, quæ habet generalem terminum, cujus numerator eſt $az^m + bz^{m-1} + cz^{m-2} + \&c.$ in x^{bz+a}, ubi m eſt integer numerus & z diſtantia a primo ſeriei termino, & cujus denominator eſt $z+\alpha \,.\, z+\alpha+1 \,.\, z+a+2 \,.\, \&c. \times z+\beta \,.\, z+\beta+1 \,.\, z+\beta+2 \,.\, \&c. \times z+\gamma \,.\, z+\gamma+1 \,.\, z+\gamma+2 \,.\, \&c. \times \&c. \times 2z+\pi \,.\, 2z+\pi+1 \,.\, 2z+\pi+2 \,.\, 2z+\pi+3 \,.\, \&c. \times 2z+\rho \,.\, 2z+\rho+1 \,.\, 2z+\rho+2 \,.\, 2z+\rho+3 \,.\, \&c. \times 2z+\sigma \,.\, 2z+\sigma+1 \,.\, 2z+\sigma+2 \,.\, 2z+\sigma+3 \,.\, 2z+\sigma+4 \,.\, \&c. \times \&c.$

THEOR. XV.

Quam plurimæ ſeries deduci poſſunt ex dividendo alteram quantitatem per alteram & mutando terminos denominatoris continuo.

Ex. 1. Sit fractio $\frac{1}{1+1}$; aſſumantur diverſi denominatores $1+1 = (1+\frac{1}{2})+\frac{1}{2} = (1+\frac{3}{4})+\frac{1}{4} = (1+\frac{7}{8})+\frac{1}{8} = (1+\frac{15}{16})+\frac{1}{16} =$ &c.; & operatio erit

$$\begin{array}{r|l|l}
1+1 & 1 & 1 \\
 & 1+1 & \\
(1+\frac{1}{2})+\frac{1}{2} & -1 & -\frac{2}{3} \\
 & -1-\frac{1}{3} & \\
(1+\frac{3}{4})+\frac{1}{4} & +\frac{1}{3} & +\frac{4}{3\cdot7} \\
 & \frac{1}{3}+\frac{1}{3\cdot7} & \\
(1+\frac{7}{8})+\frac{1}{8} & -\frac{1}{3\cdot7} & \frac{-8}{3\cdot7\cdot15}
\end{array}$$

&c.; unde reſultat $\frac{1}{1+1} = \frac{1}{2} =$

$$1-\frac{2}{3}+\frac{4}{3\cdot7}-\frac{8}{3\cdot7\cdot15}+\frac{16}{3\cdot7\cdot15\cdot31}-\frac{32}{3\cdot7\cdot15\cdot31.63}+\frac{64}{3\cdot7\cdot15\cdot31.63.127}-\text{&c.}$$

Ex. 2. Sit fractio $\frac{m}{m+n}$, tum

$$\begin{array}{r|l|l}
m+n & m & 1 \\
 & m+n & \\
(m+\frac{r}{s}n)+\frac{s-r}{s}n & -n & \frac{-sn}{sm+rn} \\
 & -n-\frac{(s-r)\times n^2}{sm+rn} & \\
(m+(\frac{r}{s}+\frac{r\times(s-r)}{s^2})n)+\frac{(s-r)^2}{s^2}n & +\frac{(s-r)\times n^2}{sm+rn} & \frac{+(s-r)\times s^2n^2}{(smr+rn)(s^2m+2rs-r^2)n^2} \\
 & \text{&c.} & \text{&c.}
\end{array}$$

unde erit $\frac{m}{m+n} = 1 - \frac{sn}{sm+rn} + \frac{(s-r)\times snA}{(s^2m+2rs-r^2)n} - \frac{(s-r)snB}{(s^3m-(s-r)^3+s^3)n} + \frac{(s-r)snC}{(s^4m-(s-r)^4+s^4)n} \ldots \frac{s\times(s-r)nL}{(s^bm-(s-r)^b+s^b)n}$, ubi $A, B, C, \ldots L$ denotant præcedentes terminos & h deſignat diſtantiam a primo ſeriei termino.

Hic continuo adjicitur ad primum terminum denominatoris pars $\frac{r}{s}$ reſidui: in primo exemplo $\frac{r}{s} = \frac{1}{2}$.

Hæc methodus generaliter reddi poteſt ex aſſumendo diverſos valores pro quantitatibus $\frac{r}{s}$, vel diverſas quantitates ad primum terminum adjungendas ad ſingulas diviſiones: etiamque ex aſſumendo plures terminos continuo & in numeratore & denominatore contentos.

Eadem principia etiam ad extractiones radicum applicari poſſunt, & ex hâc complexâ methodo dividendi & radices extrahendi quamplurimæ infinitæ ſeries deduci poſſunt, quarum ſummæ innoteſcunt.

PROB. X.

Ex datis quibuſdam ſeriebus alias invenire, quarum ſummæ innoteſcunt.

1. E datis ſeriebus inveniatur earum quæcunque functio, quæ ſit convergens ſeries; & invenitur ſeries, cujus ſumma e datis ſeriebus & datâ functione facile deduci poteſt.

2. E datis ſeriebus, quæ haud ſunt ſummabiles ſæpe deduci poſſunt ſeries, quæ ſunt ſummabiles: inveniatur enim talis functio e ſingulis ſeriebus, ut ſint ſummabiles, i. e. evaneſcant e ſummis ſerierum reſultantium ſummæ ſerierum, quæ haud ſunt ſummabiles.

Ex. 1. Sit $\dot{v} = \frac{\dot{x}}{1 - x}$, & conſequenter $v = x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 +$ &c. ducatur hæc æquatio in $x^n \dot{x}$, & reſultat æquatio $v x^n \dot{x} = x^{n+1}\dot{x} + \frac{1}{2}x^{n+2}\dot{x} + \frac{1}{3}x^{n+3}\dot{x} +$ &c. inveniatur fluens ex utrâque æquationis parte, & reſultat $\frac{v x^{n+1}}{n+1} + \frac{x^{n+1}}{(n+1).n+1} + \frac{x^n}{n+1.n} \ldots + \frac{x}{n+1} -$

$n+1$

$\frac{v}{n+1} = \frac{x^{n+2}}{n+2} + \frac{x^{n+3}}{2\times(n+3)} + \frac{x^{n+4}}{3\times(n+4)} + \&c.$ Fiant termini $\frac{v x^{n+1}}{n+1} - \frac{v}{n+1} = 0$, unde $x = 1$, & reſultant ſeries ſummabiles $\frac{1}{n+1} \times \left(\frac{1}{n+1} + \frac{1}{n} + \frac{1}{n-1} + \frac{1}{n-2} + \&c.\right)$ (quæ ad tot terminos continuantur quot ſunt unitates in $n+1$) $= \frac{1}{n+2} + \frac{1}{2.(n+3)} + \frac{1}{3.(n+4)} + \&c.$

Sit n impar numerus, tunc ex æquatione $\frac{v x^{n+1}}{n+1} - \frac{v}{n+1} = 0$ poterit deduci $x = -1$, adeoque ſeries infinita $-\frac{1}{n+2} + \frac{1}{2.(n+3)} - \frac{1}{3\times(n+4)} + \frac{1}{4\times(n+5)} - \&c. = \frac{1}{n+1} \times \left(\frac{1}{n+1} - \frac{1}{n} + \&c.\right)$ ad tot terminos continuatæ quot ſunt unitates in $n+1$; quapropter ſi n ſit numerus impar e ſummâ præcedentium ſerierum emerget nova ſeries infinita $\frac{1}{2\times(n+3)} + \frac{1}{4\times(n+5)} + \frac{1}{6\times(n+7)} + \frac{1}{8\times(n+9)} + \&c.$ æqualis ſeriei finitæ $\frac{1}{n+1} \times \left(\frac{1}{n+1} + \frac{1}{n-1} + \frac{1}{n-3} + \frac{1}{n-5} + \&c.\right)$ ad tot terminos continuatæ quot ſunt unitates in $\frac{n+1}{2}$: ex earundem ſerierum differentiâ emerget altera ſeries infinita $\frac{1}{n+2} + \frac{1}{3\times(n+4)} + \frac{1}{5\times(n+6)} + \frac{1}{7\times(n+8)} + \frac{1}{9\times(n+10)} + \&c.$ æqualis ſeriei finitæ $\frac{1}{n+1} \times \left(\frac{1}{n} + \frac{1}{n-2} + \frac{1}{n-4} + \frac{1}{n-6}\ \&c.\right)$ ad tot terminos continuatæ quot ſunt unitates in $\frac{n+1}{2}$.

2. Ducatur prædicta æquatio $\frac{v x^{n+1}}{n+1} + \frac{x^{n+1}}{n+1 \cdot n+1} + \frac{x^{n}}{n+1 \cdot n} \ldots - \frac{v}{n+1} = \frac{x^{n+2}}{n+2} + \frac{x^{n+3}}{2 \times (n+3)} + \frac{x^{n+4}}{3 \times (n+4)} + \text{\&c.}$ in $x^{m}\dot{x}$ & reſultat $\frac{v x^{m+n+1}\dot{x}}{n+1} + \frac{x^{n+m+1}\dot{x}}{n+1 \cdot n+1} + \frac{x^{n+m}\dot{x}}{n+1 \cdot n} \ldots - \frac{v x^{m}\dot{x}}{n+1} = \frac{x^{n+m+2}\dot{x}}{n+2} + \frac{x^{n+m+3}\dot{x}}{2 \cdot n+3} + \frac{x^{n+m+4}\dot{x}}{3 \cdot n+4} + \text{\&c.}$ inveniantur fluéntes ex utrâque æquationis parte, & reſultat $\frac{v x^{n+m+2}}{n+1 \cdot n+m+2} + \frac{x^{n+m+2}}{n+1 \cdot n+m+2 \cdot n+m+2} + \frac{x^{n+m+1}}{n+1 \cdot n+m+2 \cdot n+m+1} + \frac{x^{n+m}}{n+1 \cdot n+m+2 \cdot n+m} \ldots - \frac{v}{n+1 \cdot n+m+2} + \frac{x^{n+m+2}}{(n+1) \cdot n+1 \cdot n+m+2} + \frac{x^{n+m+1}}{n+1 \cdot n \cdot n+m+1} + \frac{x^{n+m}}{n+1 \cdot n-1 \cdot n+m} \ldots + \frac{x^{m+2}}{n+1 \cdot m+2} - \frac{v x^{m+1}}{n+1 \cdot m+1} - \frac{x^{m+1}}{n+1 \cdot m+1 \cdot m+1} - \frac{x^{m}}{n+1 \cdot m+1 \cdot m} - \frac{x^{m-1}}{n+1 \cdot m+1 \cdot m-1} \ldots + \frac{v}{n+1 \cdot m+1} = \frac{x^{n+m+3}}{n+2 \cdot n+m+3} + \frac{x^{n+m+4}}{2 \cdot n+3 \cdot n+m+4} + \frac{x^{n+m+5}}{3 \cdot n+4 \cdot n+m+5} + \text{\&c.}$ Ut hæc ſeries fiat finita neceſſe eſt nihilo evadere æquales tres terminos $\frac{v x^{n+m+2}}{n+1 \cdot n+m+2} - \frac{v x^{m+1}}{n+1 \cdot m+1} + \frac{v}{m+1 \cdot n+m+2} \left(\frac{-v}{n+1 \cdot n+m+2} + \frac{v}{n+1 \cdot m+1} \right)$ in quibus invenitur quantitas logarithmica v.

Et ſic ducatur hæc æquatio reſultans in $x^{r}\dot{x}$, tum inveniatur fluens ex utrâque æquationis parte; & deinde ducatur æquatio reſultans in $x^{s}\dot{x}$, & inveniatur fluens ex utrâque æquationis parte; & ſic deinceps; & novæ continuo oriuntur ſeries, quarum ſummæ inveniuntur.

3. Sit $\dot{z} = \frac{\dot{x}}{1+x^2}$, & exinde $z = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \frac{1}{9}x^9 -$ &c. ducatur hæc æquatio in $x^n\dot{x}$, & inveniatur fluens ex utrâque æquationis parte: 1mo. ſit $\frac{n+1}{2}$ integer numerus, & erit ſumma ſeriei finitæ $\frac{1}{n+1} \times (x^{n+1}z - \frac{x^n}{n} + \frac{x^{n-2}}{n-2} - \frac{x^{n-4}}{n-4} + \frac{x^{n-6}}{n-6}$ &c. $\pm z)$ æqualis ſummæ infinitæ ſeriei $\frac{x^{n+2}}{n+2} - \frac{x^{n+4}}{3 \cdot n+4} + \frac{x^{n+6}}{5 \cdot n+6} -$ &c. Jam ſi fuerit $\frac{n+1}{2}$ numerus par, ultimus terminus z afficietur ſigno negativo; ſin fuerit $\frac{n+1}{2}$ numerus impar, ultimus terminus ſigno affirmativo afficietur.

Cor. . Sit $\frac{n+1}{2}$ numerus par, & ſi modo fiat $x^{n+1}z - z = 0$, i. e. termini, qui haud inveniri poſſunt, nihilo æquales; tum erit $x = \pm 1$ & exinde deduci poſſunt ſeries, quæ exprimuntur finitis terminis.

3. 2do. Sit n par numerus, & aſſumantur fluentes, erit $\frac{x^{n+2}}{n+2} - \frac{x^{n+4}}{3 \times (n+4)} + \frac{x^{n+6}}{5 \times (n+6)} - \frac{x^{n+8}}{7 \times (n+8)} +$ &c. in infinitum æqualis ſeriei finitæ $\frac{1}{n+1} \times (x^{n+1}z - \frac{x^n}{n} + \frac{x^{n-2}}{n-2} - \frac{x^{n-4}}{n-4}$ &c. $\pm \log. \surd(1+x^2))$, ubi primus & ultimus terminus ſemper ſervatur, & tot intermedii quot unitates in $\frac{1}{2}n$.

Eodem proceſſu repetito plurium detegentur ſerierum ſummæ, &c. Et ſic deinceps.

4. Sit ſeries aſſumpta $x + \frac{1}{3}x^3 + \frac{1}{5}x^5 + \frac{1}{7}x^7 +$ &c. $= v$, ubi $\dot{v} = \frac{\dot{x}}{1-x^2}$ ducatur utraque æquationis pars in $x^n\dot{x}$, & æquationis reſul-

tantis inveniatur fluens, & exinde reſultabit finitæ ſeriei ſumma $\frac{v x^{n+1}}{n+1} + \frac{x^n}{n+1\cdot n} + \frac{x^{n-2}}{n+1\cdot n-2} + \frac{x^{n-4}}{n+1\cdot n-4} \ldots + \frac{x^2}{n+1\cdot 2}$ $+ \frac{1}{2}\log. \times (1 - x^2)$ æqualis ſummæ infinitæ ſeriei $\frac{x^{n+2}}{n+2} + \frac{x^{n+4}}{3\cdot n+4}$ $+ \frac{x^{n+6}}{5\cdot n+6} + \frac{x^{n+8}}{7\cdot n+8} +$ &c. ſi n ſit par numerus.

Si vero n ſit impar numerus, tum erit ſumma finitæ ſeriei $\frac{v x^{n+1}}{n+1}$ $+ \frac{x^n}{n+1\cdot n} + \frac{x^{n-2}}{n+1\cdot n-2} + \frac{x^{n-4}}{n+1\cdot n-4} \ldots + \frac{x}{n+1\cdot 1} -$ $\frac{v}{n+1} = \frac{x^{n+2}}{n+2} + \frac{x^{n+4}}{3\cdot n+4} +$ &c.

Eodem proceſſu repetito plures detegentur ſeries.

5. In genere ſit ſeries aſſumpta $v = x \mp \frac{x^{n+1}}{n+1} + \frac{x^{2n+1}}{2n+1} \mp \frac{x^{3n+1}}{3n+1} + \frac{x^{4n+1}}{4n+1} \mp$ &c. ubi $\dot{v} = \frac{\dot{x}}{1 \mp x^n}$, ducatur utraque æquationis pars in $x^m \dot{x}$, & quoniam $\frac{\dot{x}}{1 \mp x^n}$ dividi poteſt per notas regulas in quantitates hujuſce generis $\frac{\dot{x} \times (\alpha + \pi x)}{x^2 - 2ax + 1} + \frac{\dot{x} \times (\beta + \rho x)}{x^2 - 2bx + 1}$ $+ \frac{\dot{x} \times (\gamma + \sigma x)}{x^2 - 2cx + 1} +$ &c. igitur e præcedentibus caſibus facile inveniri poteſt fluens fluxionis $v x^m \dot{x}$ per finitos terminos, &c. ergo invenitur ſumma infinitæ ſeriei $\frac{x^{m+2}}{m+2} \mp \frac{x^{n+m+2}}{n+1\cdot n+m+2} + \frac{x^{2n+m+2}}{2n+1 \times 2n+m+2}$ $\mp \frac{x^{3n+m+2}}{(3n+1) \times (3n+m+2)} +$ &c. per finitos terminos, &c.; & ſic continuo repetitâ hâc operatione plures ſequuntur ſeries ſummabiles.

6. Sit fractio finita $\frac{1}{x^n - p x^{n-1} + q x^{n-2} - \text{\&c.}} = x^r + P x^{r-1} +$ $Q x^{r-2} +$

$\mathcal{Q}x^{r-2} + Rx^{r-3} +$ &c. in infinitum; ſint $\alpha, \beta, \gamma, \delta,$ &c. radices æquationis $x^n - px^{n-1} + qx^{n-2} -$ &c. $= 0$; & ita transformetur finita fractio, ut fiat $\frac{1}{x^n - px^{n-1} + qx^{n-2} - \&c.} = \frac{a}{x-\alpha} + \frac{b}{x-\beta} + \frac{c}{x-\gamma} + \frac{d}{x-\delta} +$ &c. (ſi vero duæ fractiones $\frac{a}{x-\alpha} + \frac{b}{x-\beta}$ involvant correſpondentes impoſſibiles radices, tum reducendæ ſunt ad unam fractionem $\frac{(b+a)x - (\beta a + b\alpha)}{(x-\alpha).(x-\beta)}$, & evaneſcent impoſſibiles quantitates); ducatur æquatio $\frac{a}{x-\alpha} + \frac{b}{x-\beta} + \frac{c}{x-\gamma} +$ &c. $= x^r + Px^{r-1} +$ &c. in datam finitam quantitatem $(Ax^m + Bx^{m-1} + Cx^{m-2} + \&c.) \times \dot{x}$, e duobus præcedentibus caſibus conſtat fluentem ſeriei finitæ $(Ax^m + Bx^{m-1} + Cx^{m-2} + \&c.) \times \left(\frac{a}{x-\alpha} + \frac{b}{x-\beta} + \frac{c}{x-\gamma} + \&c.\right) \times \dot{x}$ inveniri poſſe in finitis terminis, circularibus arcubus & logarithmis; in his duobus caſibus fiant partes, quæ involvunt circulares arcus & logarithmos, nihilo reſpective æquales; & exinde erui poſſunt caſus, in quibus ſeries, ſi modo unquam, evadet finita quantitas; ergo ſumma inventa finitæ ſeriei æqualis erit ſummæ infinitæ ſeriei, i. e. fluenti fluxionis $\dot{x} \times (Ax^m + Bx^{m-1} + Cx^{m-2} + \&c.) \times (x^r + Px^{r-1} + \mathcal{Q}x^{r-2} + \&c.)$; & ſic deinceps.

PROB. XI.

Datâ quantitate $\frac{\mathrm{A}z^m + \mathrm{B}z^{m-1} + \mathrm{C}z^{m-2} + \&c.}{z+\alpha . z+\beta . z+\gamma . z+\delta . \&c.} \times x^{hz+m}$, *quæ exprimit ſingulos datæ ſeriei terminos, ubi* z *denotat diſtantiam termini a primo, literæ vero* A, B, C, D, &c., $\alpha, \beta, \gamma, \delta,$ &c. h, *datas quantitates, &* m *integrum numerum, reſpective denotant; invenire ſummam datæ ſeriei.*

1. Sit $v = \frac{1}{1 \mp x^h} = 1 \pm x^h + x^{2h} \pm x^{3h} + x^{4h} \pm$ &c. ducatur data

æquatio in $x^{h\alpha-1}\dot{x}$ & fit $\frac{x^{h\alpha-1}\dot{x}}{1\mp x^{h}} = x^{h\alpha-1}\dot{x} \pm x^{h\alpha+h-1}\dot{x} + x^{h\alpha+2h-1}\dot{x} \pm x^{h\alpha+3h-1}\dot{x}$ + &c. Inveniantur fluentes fluxionum ex utrâque æquationis parte, viz. $\int.\frac{x^{h\alpha-1}\dot{x}}{1\mp x^{h}} = W = \frac{x^{h\alpha}}{h\alpha} \pm \frac{x^{h\alpha+h}}{h\times(\alpha+1)} + \frac{x^{h\alpha+2h}}{h\times(\alpha+2)}$ ± &c. ducatur hæc æquatio in $\frac{x^{h\beta-1}\dot{x}}{x^{h\alpha}}$, & resultat æquatio $Wx^{h\beta-h\alpha-1}\dot{x} = \frac{x^{h\beta-1}\dot{x}}{h\alpha} \pm \frac{x^{h\beta+h-1}\dot{x}}{h(\alpha+1)} + \frac{x^{h\beta+2h-1}\dot{x}}{h\times(\alpha+2)}$ ± &c. inveniantur fluentes fluxionum ex utrâque æquationis parte X, & evadunt $\int.Wx^{h\beta-h\alpha-1}\dot{x} = X = \frac{x^{h\beta}}{h^{2}\times\alpha\beta} \pm \frac{x^{h\beta+h}}{h^{2}\times(\alpha+1)\times(\beta+1)} + \frac{x^{h\beta+2h}}{h^{2}\times(\alpha+2)\times(\beta+2)}$ ± &c. ducatur hæc æquatio in $\frac{x^{h\gamma-1}}{x^{h\beta}}\dot{x}$, & resultat $Xx^{h\gamma-h\beta-1}\dot{x} = \frac{x^{h\gamma-1}\dot{x}}{h^{2}\alpha\beta} \pm \frac{x^{h\gamma+h-1}}{h^{2}\times(\alpha+1)\times(\beta+1)}\dot{x} + \frac{x^{h\gamma+2h-1}}{h^{2}\times(\alpha+2)\times(\beta+2)}$ ± &c. inveniantur fluentes fluxionum ex utrâque æquationis parte, & erunt $\int.Xx^{h\gamma-h\beta-1} = Y = \frac{x^{h\gamma}}{h^{3}\times\alpha\beta\gamma} \pm \frac{x^{h\gamma+h}}{h^{3}\times(\alpha+1)\times(\beta+1)\times(\gamma+1)} + \frac{x^{h\gamma+2h}}{h^{3}\times(\alpha+2)\times(\beta+2)\times(\gamma+2)}$ ± &c. ducatur hæc æquatio in $\frac{x^{h\delta-1}}{x^{h\gamma}}\dot{x}$, & resultat æquatio $Yx^{h\delta-h\gamma-1}\dot{x} = \frac{x^{h\delta-1}\dot{x}}{h^{3}\times\alpha\beta\gamma} \pm \frac{x^{h\delta+h-1}\dot{x}}{h^{3}\times(\alpha+1)\times(\beta+1)\times(\gamma+1)} + \frac{x^{h\delta+2h-1}\dot{x}}{h^{3}\times(\alpha+2)\times(\beta+2)\times(\gamma+2)}$ ± &c. cujus fluens est $\int.Yx^{h\delta-h\gamma-1}\dot{x} = Z = \frac{x^{h\delta}}{h^{4}\alpha\beta\gamma\delta} \pm \frac{x^{h\delta+h}}{h^{4}\times(\alpha+1)\times(\beta+1)\times(\gamma+1)\times(\delta+1)} + \frac{x^{h\delta+2h}}{h^{4}\times(\alpha+2)\times(\beta+2)\times(\gamma+2)\times(\delta+2)}$ ± &c. & sic continuo repetitâ operatione, usque donec includuntur in denominatore omnes divisores $(z+\alpha).(z+\beta).(z+\gamma).(z+\delta)$ &c. quorum ultimo inventus sit $z+\pi$ & numerus n, & fluens V;

ergo ſumma ſeriei cujus termini ſemper ſint $A \times \frac{1}{z+\alpha . z+\beta . z+\gamma . z+\delta \, \&c. \, z+\pi} x^{bz+m}$ erit $b^n A \times \frac{x^m}{x^{h\pi}} \times V$.

Per ex. prob. erit $X = \int . W x^{h\beta - h\alpha - 1} \dot{x} = \frac{1}{b(\beta - \alpha)} x^{h(\beta-\alpha)} \int . \frac{x^{h\alpha-1}\dot{x}}{1 \pm x^b} + \frac{1}{b(\alpha-\beta)} \int . \frac{x^{h\beta-1}\dot{x}}{1 \pm x^b}$, & $Y = \int . X x^{h\gamma - h\beta - 1} \dot{x} = \frac{1}{b^2(\gamma-\alpha)(\beta-\alpha)} x^{h(\gamma-\alpha)} \int . \frac{x^{h\alpha-1}\dot{x}}{1 \pm x^b} + \frac{1}{b(\gamma-\beta)(\alpha-\beta)} \times x^{h(\gamma-\beta)} \int . \frac{x^{h\beta-1}\dot{x}}{1 \pm x^b} + \frac{1}{(\gamma-\alpha)(\gamma-\beta)} \int . \frac{x^{h\gamma-1}\dot{x}}{1 \pm x^b}$; & ſic deinceps.

Cor. Si omnes differentiæ $\alpha-\beta$, $\alpha-\gamma$, $\beta-\gamma$; $\alpha-\delta$, $\beta-\delta$, $\gamma-\delta$, &c. ſint integri numeri, tum fluentes omnium fluxionum $\frac{x^{h\alpha-1}\dot{x}}{1 \pm x^b}$, $\frac{x^{h\beta-1}\dot{x}}{1 \pm x^b}$, $\frac{x^{h\gamma-1}\dot{x}}{1 \pm x^b}$, &c. a ſe invicem pendent & deduci poſſunt; & conſequenter ſummæ omnium ſerierum, quarum generalis terminus eſt $\frac{(Az^m + Bz^{m-1} + \&c.) \times x^{bz+m}}{(z+\alpha)(z+\beta)(z+\gamma)(z+\delta)\&c.}$, a fluente fluxionis $\frac{x^{h\alpha-1}\dot{x}}{1 \pm x^b}$ per diviſionem, &c. deduci poſſunt.

Cor. 2. Si vero omnes reliqui indices γ, δ, &c. a duobus α & β diſtent per integros numeros, tum ſummæ ſerierum inveniri poſſunt a fluentibus fluxionum $\frac{x^{h\alpha-1}\dot{x}}{1 \pm x^b}$ & $\frac{x^{h\beta-1}\dot{x}}{1 \pm x^b}$; & ſic deinceps.

Exhinc facile deduci poſſunt caſus particulares, in quibus prædictæ ſeries æquant finitas quantitates.

Cor. Hinc ex finitis terminis, circularibus arcubus & logarithmis deduci poſſunt ſummæ omnium ſerierum, quarum termini ſint prædicti, ni duo α & β vel tres vel plures valores incognitarum quantitatum α, β, γ, &c. ſint inter ſe æquales, nam in iſto caſu $\alpha - \beta = 0$, &c., & conſequenter ejus reciproca evadit infinita: minime refert, annon poſſibiles vel impoſſibiles ſint prædictæ radices α, β, γ, δ, &c.

2. Nunc data ſeries vel dividi poteſt in plures, quorum termini ſint reſpective $\frac{Az^{m}x^{bz+m'}}{z+\alpha\,.\,z+\beta\,.\,z+\gamma\ \&c.}$, $\frac{Bz^{m-1}x^{bz+m'}}{z+\alpha\,.\,z+\beta\,.\,z+\gamma\ \&c.}$, $\frac{Cz^{m-2}x^{bz+m'}}{z+\alpha\,.\,z+\beta\,.\,z+\gamma\ \&c.}$, vel quocunque alio modo ita ut ſumma e ſingularum ſerierum terminis æqualis ſit datæ quantitati; deinde inveniatur per ſubſequentem methodum ſumma e ſingulis hiſce ſeriebus, quibus ad unam ſummam adjunctis, ea erit ſumma ſeriei datæ quæſita.

2. Sit $V=$ ſumma ſeriei, cujus termini ſint $\frac{1}{z+\alpha\,.\,z+\beta\,.\,z+\gamma\ \&c.}x^{bz}$, i. e. $\frac{1}{\alpha\,.\,\beta\,.\,\gamma\,.\,\delta} \pm \frac{1}{\alpha+1\,.\,\beta+1\,.\,\gamma+1\,.\,\delta+1\,.\ \&c.}x^{b} + \frac{1}{\alpha+2\,.\,\beta+2\,.\,\gamma+2\,.\,\delta+2\,.\ \&c.}x^{2b} + \&c.$ Inveniatur fluxio utriusque æquationis partis, & reſultat $\dot{V} = \frac{\pm b}{\alpha+1\,.\,\beta+1\,.\ \&c.}x^{b-1}\dot{x} + \frac{2b}{\alpha+2\,.\,\beta+2\,.\ \&c.}x^{2b-1}\dot{x} \pm \frac{3b}{\alpha+3\,.\,\beta+3\,.\ \&c.}x^{3b-1}\dot{x} + \&c.$ ducatur hæc æquatio in $\frac{x}{\dot{x}}$, & reſultat $\frac{\dot{V}x}{\dot{x}} = \frac{\pm b}{\alpha+1\,.\,\beta+1\,.\,\gamma+1\,.\ \&c.}x^{b} + \frac{2b}{\alpha+2\,.\,\beta+2\,.\,\gamma+2\,.\ \&c.}x^{2b} \pm \frac{3b}{\alpha+3\,.\,\beta+3\,.\,\gamma+3\,.\ \&c.}x^{3b} + \&c.$ cujus flux. ſit flux. quan. $\frac{\dot{V}x}{\dot{x}} = \dot{W} = \frac{\pm b^{2}}{\alpha+1\,.\,\beta+1\,.\,\gamma+1\,.\ \&c.}x^{b-1}\dot{x} + \frac{2\,.\,2b^{2}}{\alpha+2\,.\,\beta+2\,.\ \&c.}x^{2b-1}\dot{x} \pm \frac{3\cdot 3b^{2}}{\alpha+3\,.\,\beta+3\,.\ \&c.}x^{3b-1}\dot{x} + \&c.$ ducatur hæc æquatio in $\frac{x}{\dot{x}}$, & reſultat $\frac{x\dot{W}}{\dot{x}} = L = \frac{\pm b^{2}}{\alpha+1\,.\,\beta+1\,.\ \&c.}x^{b} + \frac{2\,.\,2b^{2}}{\alpha+2\,.\,\beta+2\,.\ \&c.}x^{2b} \pm \frac{3\cdot 3b^{2}}{\alpha+3\,.\,\beta+3\,.\ \&c.}x^{3b} + \&c.$ inveniatur fluxio hujus æquationis, & reſultat $\dot{L} = \frac{\pm b^{3}}{\alpha+1\,.\,\beta+1\,.\ \&c.}$

$x^{b-1}\dot{x} + \frac{2.2.2h^3}{\alpha+2.\beta+2.\&c.}x^{2b}\dot{x} + \frac{3.3.3h^3}{\alpha+3.\beta+3.\&c.}x^{3b}\dot{x} + \&c.$ & ſic continuo repetitâ operatone tandem invenietur ſumma ſeriei; cujus terminus eſt $\frac{Az^m}{z+\alpha.z+\beta.\&c.}$; & ſic de reliquis.

Aliter: ſit $A.(z+\pi).(z+\rho).(z+\sigma).(z+\tau).\&c. = Az^m + Bz^{m-1} + Cz^{m-2} + \&c.$ & ſit ſumma ſeriei, cujus termini in genere ſint $\frac{x^{bz}}{z+\alpha.z+\beta.z+\gamma.\&c.}$, V, i. e. $V = \frac{1}{\alpha.\beta.\gamma.\delta\,\&c.} + \frac{1}{\alpha+1.\beta+1.\&c.}x^b + \frac{1}{\alpha+2.\beta+2.\&c.}x^{2b} + \&c.$ ducatur hæc æquatio in $x^{\pi h}$, & reſultat $V \times x^{\pi h} = \frac{x^{\pi h}}{\alpha\beta\gamma\delta\&c.} + \frac{x^{\pi h+h}}{\alpha+1.\beta+1.\gamma+1.\&c.} + \frac{x}{\alpha+2.\beta+2.\gamma+2.\&c.} + \&c.$ inveniatur fluxio hujus æquationis, & erit flux. quan. $(V \times x^{\pi h}) = \dot{W} = \frac{h\pi x^{\pi h-1}\dot{x}}{\alpha\beta\gamma\delta\,\&c.} + \frac{h\times(1+\pi)}{\alpha+1.\beta+1.}$
$\frac{x^{h\pi+h-1}\dot{x}}{\gamma+1.\&c.} + \frac{h\times(2+\pi)x^{h\pi+2h-1}\dot{x}}{\alpha+2.\beta+2.\gamma+2.\&c.}$; ducatur hæc æquatio in $\frac{x^{\rho h}}{x^{\pi h-1}\dot{x}}$, & reſultat $\frac{\dot{W}x^{\rho h}}{x^{\pi h-1}\dot{x}} = U = \frac{h\pi x^{\rho h}}{\alpha\beta\gamma\delta.\&c.} + \frac{h\times(1+\pi)x^{\rho h+h}}{\alpha+1.\beta+1.\gamma+1.\&c.} + \frac{h\times(2+\pi)x^{\rho h+2h}}{\alpha+2.\beta+2.\gamma+2.\&c.}$, inveniatur fluxio utriuſque partis æquationis, & reſultat flux. quan. $\frac{\dot{W}x^{\rho h}}{x^{\pi h-1}\dot{x}} = \dot{U} = \frac{h^2\pi\rho x^{\rho h-1}\dot{x}}{\alpha\beta\gamma\delta\,\&c.} + \frac{h^2\times(1+}{\alpha+1.\beta}$
$\frac{\pi)\times(1+\rho)x^{\rho h+h-1}\dot{x}}{+1.\gamma+1.\&c.} + \frac{h^2.(2+\pi).(2+\rho).x^{\rho h+2h-1}\dot{x}}{\alpha+2.\beta+2.\gamma+2.\&c.} + \&c.$ ducatur hæc æquatio in $\frac{x^{\sigma h}}{x^{\rho h-1}\dot{x}}$ & reſultat $\frac{\dot{U}\times x^{\sigma h}}{x^{\rho h-1}\dot{x}} = M = \frac{h^2\pi\rho x^{\sigma h}}{\alpha\beta\gamma\delta\,\&c.} + \frac{h^2\times(\pi+1).(\rho+1)x^{\sigma h+h}}{\alpha+1.\beta+1.\gamma+1.\&c.} + \frac{h^2.(\pi+2).(\rho+2).x^{\sigma h+2h}}{\alpha+2.\beta+2.\gamma+2.\delta+2.\&c.}$;

hujus æquationis inveniatur fluxio, & reſultat $\dot{M} = \frac{b^3 \pi \rho \sigma x^{\sigma h - 1} \dot{x}}{\alpha \beta \gamma \delta \,\&c.} + \frac{b^3 . (\pi + 1) . (\rho + 1) . (\sigma + 1) x^{\sigma h + h - 1} \dot{x}}{\alpha + 1 . \beta + 1 . \gamma + 1 . \delta + 1 . \&c.} + \frac{b^3 . (\pi + 2) . (\rho + 2) . (\sigma + 2) x^{\sigma h + 2h - 1} \dot{x}}{\alpha + 2 . \beta + 2 . \gamma + 2 . \delta + 2 . \&c.}$ unde facile deduci poteſt ſumma ſeriei, cujus termini ſunt prædictæ datæ quantitates.

Si $\alpha + z$ vel $\beta + z$ vel $\gamma + z$, &c. unquam evadant nihilo æquales, tum in detegendis fluentibus terminorum prædictorum neceſſe erit invenire fluentes fluxionum hujuſce formulæ $x^{\pi} \dot{x}\, \text{ſ}. \left(\frac{\dot{x}}{x}\right)^{n}$, ubi n denotat integrum numerum; hæ fluentes in libri primi capite ſecundo dantur.

Cor. 1. Hinc facile iterum conſtat, ſi modo $\alpha, \beta, \gamma, \delta$, &c. ſint inæquales quantitates, ſummam cujuſcunque ſeriei, cujus termini reſpective denotantur per quantitatem $\frac{A z^m + B z^{m-1} \,\&c.}{H . (z + \alpha) . (z + \beta) . (z + \gamma) . \&c.}$, ubi m eſt integer numerus; inveniri poſſe ope finitorum terminorum circularium arcuum & logarithmorum.

Cor. 2. Facile etiam e præcedentibus principiis deduci poteſt ſumma m terminorum inter quoſcunque datos terminos contenta, & e principiis in medit. algeb. traditis conſequetur ſumma terminorum, quorum diſtantiæ a primo ſeriei termino ſint reſpective n, $n + m$, $n + 2m$, $n + 3m$, $n + 4m$, &c.

Cor. 3. Sit data ſeries $A x^e + B x^{e+n} + C x^{e+2n} + D x^{e+3n} +$ &c., cujus ſumma ſit functio quantitatis x, cujus fluxio inveniri poteſt; ſit z diſtantia a primo ſeriei termino & aſſumatur functio $a z^m + b z^{m-1} + c z^{m-2} + d z^{m-3} + \&c. = P$, ubi m eſt integer numerus; in functione P pro z ſcribantur 0, 1, 2, 3, 4, &c. & reſultent quantitates $\alpha, \beta, \gamma, \delta$, &c.: tum ex datâ ſerie ſemper acquiri poteſt ſumma ſeriei $\alpha A z^e + \beta B z^{e+n} + \gamma C z^{e+2n} +$ &c.

Cor. 4. Ducantur termini prædictæ ſeriei $A x^e + B x^{e+n} + C x^{e+2n} +$ &

&c. reſpective in terminos $\frac{\beta}{\alpha}$, $\frac{\beta\times(\beta+n)}{\alpha.(\alpha+n)}$, $\frac{\beta\times(\beta+n)(\beta+2n)}{\alpha(\alpha+n)(\alpha+2n)}$, &c.; & reſultat ſeries $\frac{\beta}{\alpha}Ax^{e}+\frac{\beta\times(\beta+n)}{\alpha(\alpha+n)}Bx^{e+n}+\frac{\beta(\beta+n)(\beta+2n)}{\alpha(\alpha+n)(\alpha+2n)}Cx^{e+2n}+$ &c., cujus ſumma e ſummâ datæ ſeriei ſemper deduci poteſt, ſi $\beta=\alpha+n\times r$, & r ſit integer numerus; nam in hoc caſu prædicti termini $\frac{\beta}{\alpha}$, $\frac{\beta}{\alpha}\times\frac{\beta+n}{\alpha+n}$, &c. evadunt functio

$$\frac{(\alpha+rn+nz)(\alpha+rn+nz-n)(\alpha+rn+nz-2n).(\alpha+rn+nz-3n)..(\alpha+nz+n)}{\alpha\times(\alpha+n)\times(\alpha+2n)(\alpha+3n).(\alpha+4n)\ldots(\alpha+(r-1)n)},$$

ubi z eſt diſtantia a primo ſeriei termino.

Cor. 5. Si fluens fluxionis $\dot{x}\times\phi:x$ detegi poſſit, tum ſumma ſeriei, quæ eſt fluens fluxionis $\dot{x}\phi:(x)\times(az^{m}+bz^{m+1}+\&c.)$ etiam exprimi poteſt.

PROB. XII.

Datâ æquatione relationem inter T, T′, T″, T$^{\prime n-1}$ *ſucceſſivos ſeriei terminos, & z diſtantiam a primo ſeriei termino, ſeriem ejus invenire. Aſſumantur pro* n *primis ſeriei terminis reſpective* A, B, C, D, *&c. & e datâ æquatione continuo inveniantur ſucceſſivi termini quæſitæ ſeriei, & exinde invenietur ſeries quæſita.*

PROB. XIII.

Datâ æquatione relationem inter ſucceſſivos ſeriei terminos exprimente; invenire æquationem, ſive ea algebraica ſive fluxionalis ſit, (ſi modo hujuſce generis detur æquatio), cujus radix erit ſeries quæcunque data, quæ definitur æquatione, in quâ termini ſeriei ſunt unius tantum dimenſionis.

1. Invenire æquationem, cujus radix eſt ſeries $y=A+Bx^{m}+Cx^{2m}+$ &c. in quâ relatio coefficientium eſt conſtans, ſcilicet $tD+sC+rB+qA+$ &c. $=0$, ubi A, B, C, D, &c. ſucceſſivos terminos, & q, r, s, &c. invariabiles quantitates reſpective denotant.

Sit

$$\begin{array}{llllll} \text{Sit } y = & A + & Bx^{m} + & Cx^{2m} + & Dx^{3m} + & Ex^{4m} + \&c. \text{ tum erit} \\ ty = & tA + & tBx^{m} + & tCx^{2m} + & tDx^{3m} + & tEx^{4m} + \&c. \ \& \\ sx^{m}y = & & sAx^{m} + & sBx^{2m} + & sCx^{3m} + & sDx^{4m} + \&c. \\ rx^{2m}y = & & & rAx^{2m} + & rBx^{3m} + & rCx^{4m} + \&c. \\ qx^{3m}y = & & & & qAx^{3m} + & qBx^{4m} + \&c. \\ \&c. & & & & \&c. & \end{array}$$

ſed ex hypotheſi $tD + sC + rB + qA = 0$, $tE + sD + rC + qB = 0$, & ſic in infinitum; unde evaneſcent omnes ſubſequentes termini, adeoque reſultabit $(t + sx^{m} + rx^{2m} + qx^{3m})y = tA + (tB + sA)x^{m} + (tC + sB + rA)x^{2m}$.

2. Invenire fluxionalem æquationem exprimentem ſeriem (y), cujus ſeriei æquatio relationem inter ſucceſſivos $(T, T', T''$, &c.) datæ æquationis terminos deſignans ſit $(az^{\alpha} + bz^{\alpha-1} + cz^{\alpha-2} + \&c.)Tx^{m} + (ez^{\beta} + fz^{\beta-1} + gz^{\beta-2} + \&c.)T'x^{p\pm 1m} + (hz^{\gamma} + lz^{\gamma-1} + \&c.)T''x^{p\pm 2m} + \&c. = 0$, ubi z denotat diſtantiam a primo ſeriei termino, & a, b, c, &c. e, f, g, &c. h, l, &c. conſtantes quantitates; etiamque α, β, γ, &c. integros numeros reſpective denotant.

Fingatur $y = Ax^{n} + Bx^{n+m} + Cx^{n+2m} \ldots Tx^{n+rm} + T'x^{n+(r+1)m} + T''x^{n+(r+2)m} + \&c.$ nunc primo per methodum in prob. præced. traditam ex aſſumptâ æquatione $y = Ax^{n} + Bx^{n+m} + \&c.$ inveniatur æquatio fluxionalis $\overset{a}{\dot{y}} + \&c. =$ ſeriei, cujus generalis terminus ſit $(az^{\alpha} + bz^{\alpha-1} + \&c.)T$; & ſic deducantur fluxionales æquationes, quarum generales termini ſerierum ſint $(ez^{\beta} + fz^{\beta-1} + \&c.)T'$, &c. & ſic deinceps; tum ita reducantur hæ fluxionales æquationes, i. e. ducantur in tales dimenſiones quantitatis x, ut eædem ejus inveniantur dimenſiones in correſpondentibus terminis $(T, T'$, &c.); & exinde conſequentur fluxionales æquationes quæſitæ.

Ex. . Requiratur fluxionalis æquatio, cujus radix ſit $1 - \frac{1}{4}x^{2} - \frac{3}{64}x^{4} - \frac{3}{256}x^{6} - \&c.$ i. e. cujus relatio inter ſucceſſivos terminos ſit $(2z - 1) \times (2z + 1)Tx^{2} - (2z + 2)^{2}T' = 0$; aſſumatur æquatio $y = A + Bx^{2} + Cx^{4} + Dx^{6} + \&c.$ ſed quoniam factor T ductus fuit in x^2, conſequitur $m = 2$; & conſequenter omnes factores in quantitates T, T', T'', &c. ducti, qui ſint $lz - \alpha$, multiplicentur in $\frac{2}{l}$, ita ut evadant factores hujuſce generis $2z - \beta$; factores $(2z - 1, 2z + 1, 2z + 2)$ vero in datâ æquatione contenti hanc habent formulam ſine ullâ reductione.

Nunc inveniatur talis fluxionalis functio quantitatum y & x & earum fluxionum, ut æqualis ſit ſeriei, cujus generalis terminus ſit $(2z - 1) \times (2z + 1) \times T$, ubi T ſit generalis terminus ſeriei aſſumptæ: per prob. prædic. ducatur æquatio aſſumpta in x, & reſultat $yx = Ax + Bx^3 + Cx^5 +$ &c. cujus fluxio erit $y\dot{x} + x\dot{y} = A\dot{x} + 3Bx^2\dot{x} + 5Cx^4\dot{x} +$ &c. $(2z + 1)Tx^{2z}\dot{x}$, dividatur hæc æquatio per x & evadet $\frac{y\dot{x}}{x} + \dot{y} = \frac{A\dot{x}}{x} + 3Bx\dot{x} + 5Cx^3\dot{x} +$ &c. cujus fluxio erit $\frac{\dot{y}\dot{x}}{x} - \frac{y\dot{x}^2}{x^2} + \ddot{y} = -\frac{A\dot{x}^2}{x^2} + 3B\dot{x}^2 + 3 \cdot 5Cx^2\dot{x}^2 \ldots (2z + 1) \times (2z - 1) T x^{2z-2}\dot{x}^2$; ducatur hæc æquatio in x^2 & evadet (P) $x\dot{x}\dot{y} - y\dot{x}^2 + x^2\ddot{y} = -A\dot{x}^2 + 3Bx^2\dot{x}^2 + 3 \cdot 5Cx^4\dot{x}^2 +$ &c. & per eandem methodum inveniatur (Q) $x\ddot{y} + \dot{x}\dot{y} = 4Bx\dot{x}^2 + 16Cx^3\dot{x}^2 \ldots 2z \times 2zTx^{2z-1}\dot{x}^2$; ducatur æquatio (P) in (x) talem dimenſionem quantitatis x, ut dimenſiones quantitatis x in terminum T æquationis P ductæ inveniantur eædem ac dimenſiones quantitatis x in terminum ſucceſſivum T' æquationis (Q) ductæ; unde reſultat $-(x^2\dot{x}\dot{y} - xy\dot{x}^2 + x^3\ddot{y} - (x\ddot{y} + \dot{x}\dot{y})) = (A + 4B)x\dot{x}^2 + (16C - 3B)x^3\dot{x}^2 \ldots ((2z - 1) \times (2z + 1)T - (2z + 2)^2 T')x^{2z+1} = 0$, unde fluxionalis æquatio quæſita erit $(x^3 - x)\ddot{y} - (1 - x^2)\dot{y}\dot{x} - xy\dot{x}^2 = 0$.

Cor. . Ordo fluxionalis æquationis quæſitæ ſemper æquat indicem vel α vel β vel γ, &c. ubi α, β, γ, &c. integri ſint numeri, qui invenitur maximus, & data ſeries erit ejus particularis reſolutio.

Et ex datâ particulari reſolutione deprimi poteſt quæſita æquatio ad inferiorem.

3. Si vero numerus factorum functionis quantitatis z in datâ æquatione relationem inter ſucceſſivos terminos T, T', &c. exprimente

continuo augeatur, tum omnino ex iiſdem principiis petenda eſt ſolutio.

4. Si vero diverſæ poteſtates ſubſequentium terminorum in datâ æquatione contineantur, tum aſſumendo ſeriem pro ſummis & ex eâ deducendo ſingulos datæ æquationis terminos; & correſpondentes reſultantis æquationis terminos inter ſe æquando deduci poteſt ſeries quæſita.

PROB. XIV.

1. *Datis duabus vel pluribus convergentibus ſeriebus* S, S', S'', *&c. ex iis deducere ſeries, quarum conſequentur ſummæ.*

Sint α, β, γ, δ, &c. quicumque ſucceſſivi termini ex unâ ſerie; π, ρ, σ, τ, &c. ex alterâ, &c. tum erit $a\alpha + b\beta + c\gamma + d\delta +$ &c. $k\pi + l\rho + m\sigma + n\tau +$ &c. terminus generalis ſeriei, cujus ſumma facile erui poteſt e ſummis datarum convergentium ſerierum, ubi a, b, c, d, &c. $k. l, m$, &c. ſunt quæcumque finitæ quantitates.

2. Si vero quædam e prædictis ſeriebus S, S', &c. haud ſint convergentes, ſed earum termini continuo ad nihil vergant, & ultimo minores ſint quam quævis aſſignandæ quantitates; S'', S''', &c. vero ſint convergentes ſeries; tum erui poteſt ſumma ſeriei, cujus generalis terminus eſt $a\alpha + b\beta + c\gamma +$ &c. $+ k\pi + l\rho + m\sigma +$ &c. $+$ &c. $+ Ap + Bq +$ &c. $+ Dr + Es +$ &c. ubi α, β, γ, &c. π, ρ, σ, &c. &c. ſint termini ſucceſſivi ſerierum datarum haud convergentium; ſed p, q, &c. r, s, &c. termini ſucceſſivi ſerierum convergentium; etiamque ſint a, b, c, &c. k, l, m, &c. &c. A, B, &c. D, E, &c. quæcunque finitæ quantitates, ita vero ut in coefficientibus ſerierum, quæ haud convergunt, fingantur $a + b + c +$ &c. $= 0$, $k + l + m$ $+$ &c. $= 0$.

Si modo ſeries progrediatur ſecundum dimenſiones quantitatis x, tum a, b, c, &c. k, l, m, &c. A, B, C, &c. D, E, &c. ſint functiones quæcunque ipſius x.

Hæc etiam principia applicari poſſunt ad ſummas ſerierum, quæ ſunt tales functiones datarum convergentium ſerierum, quæ nullas in ſe reciprocas poteſtates involvunt; vel convergentes ſeries præbent. Inveniantur enim functiones datarum ſerierum & quantitatum, quibus hæ ſeries æquales ſunt, & erunt ſummæ ſerierum reſultantium æquales ſummæ quantitatum reſultantium e prædictis quantitatibus.

Cor. 1. Ex primis terminis ſerierum, quarum termini ad nihil vergunt; & ex datis primis terminis, & ſummis convergentium ſerierum, quæ haud in quantitates ſimul ſumptas nihilo æquales ducuntur, facile innoteſcet ſumma ſeriei quæſitæ.

Cor. 2. Datis ſummis quarumcunque ſerierum, facile plerumque detegi poteſt, utrum ſumma cujuſcunque ſeriei ex iis per methodos prius traditas deduci poteſt, necne: ex obſervatis factoribus in denominatore prædictæ ſeriei & ſerierum, quarum dantur ſummæ, conſequitur ſubſtitutio, e quâ erui poteſt in ſummis datarum ſerierum ſumma ſeriei quæſitæ.

Cor. 3. Ex datâ ſerie $y = x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 +$ &c. erui poteſt ſumma cujuſcunque ſeriei, cujus generalis terminus eſt $= \frac{Az^b + Bz^{b-1} + \&c.}{z \times (z+m) \times (z+n) \times (z+p) \times \&c.} x^z$; ubi b, m, n, p, &c. ſunt integri numeri: erunt enim $\frac{1}{z}$, $\frac{1}{z+m}$, $\frac{1}{z+n}$, &c. diverſi termini datæ ſeriei; & conſequenter $\frac{ax^z}{z} + \frac{bx^{m+z}}{z+m} + \frac{cx^{n+z}}{z+n} + \frac{dx^{p+z}}{z+p} = a\alpha + b\beta + c\gamma + d\delta +$ &c. ubi α, β, γ, δ, &c. ſint termini ad diſtantias a primo $m-1$, $n-1$, $p-1$, &c. fiat $\frac{ax^z}{z} + \frac{bx^{z+m}}{z+m} + \frac{cx^{z+n}}{z+n} +$ &c. $= \frac{Az^b + Bz^{b-1} + \&c.}{z \,.\, z+m \,.\, z+n \,.\, z+p \,.\, \&c.} x^z$, & inveniantur coefficientes a, b, c, &c. & id, quod requiritur, exinde facile erui poteſt.

THEOR. XVI.

1. Ex fluentibus fluxionum, quæ finitis terminis exprimi poſſunt, acquiri poſſunt ſeries quantitatum, quarum ſummæ innoteſcunt.

Reducantur datæ fluxiones ad infinitas ſeries ſecundum terminos, quorum fluentes finitis terminis exprimi poſſunt, progredientes; inveniantur fluentes terminorum reſultantium, & reſultant ſeries, quarum ſummæ innoteſcunt.

Ex. Sit fluxio $(a+bx^n)^{\frac{h}{m}}x^{rn-1}\dot{x}$, cujus fluens (exceptis excipiendis) in finitis terminis exprimi poteſt, cum r ſit integer numerus; reducatur data fluxio $(a+bx^n)^{\frac{h}{m}}x^{rn-1}\dot{x}$ ad ſeriem $a^{\frac{h}{m}}(x^{rn-1}\dot{x}(1+\frac{hb}{ma}x^n+\frac{h}{m}\times\frac{h-m}{2m}\times\frac{b^2}{a^2}x^{2n}+\&c.)$ ſecundum dimenſiones quantitatis x^n progredientem, cujus fluens erit $a^{\frac{h}{m}}x^{rn}\left(\frac{1}{rn}+\frac{1}{(r+1)n}\times\frac{hb}{ma}x^n+\frac{1}{(r+2)n}\times\frac{h}{m}\times\frac{h-m}{2m}\times\frac{b^2}{a^2}x^{2n}+\frac{1}{(r+3)n}\times\frac{h}{m}\times\frac{h-m}{2m}\times\frac{h-2m}{3m}\times\frac{b^3}{a^3}x^{3n}+\&c.\right)$; hæc fluens autem innoteſcit, ergo ſumma ſeriei $\frac{1}{rn}+\frac{1}{(r+1)n}\times\frac{h}{m}\times\frac{b}{a}x^n+\frac{1}{(r+2)n}\times\frac{h}{m}\times\frac{h-m}{2m}\times\frac{b^2}{a^2}x^{2n}+\frac{1}{(r+3)n}\times\frac{h}{m}\times\frac{h-m}{2m}\times\frac{h-2m}{3m}\times\frac{b^3}{a^3}x^{3n}+\&c.$ ſemper finitis terminis exprimi poteſt.

1. 2. Si nova ſeries incipiat ab $(l+1)$ termino datæ ſeriei, tum evadet $\frac{b^l}{a^l}x^{ln}\times\frac{h}{m}\times\frac{h-m}{2m}\cdot\frac{h-2m}{3m}\ldots\frac{h-(l-1)m}{lm}\left(\frac{1}{(r+l)n}+\frac{1}{(r+l+1)n}\times\frac{h-lm}{(l+1)m}\times\frac{b}{a}x^n+\frac{1}{(r+l+2)n}\times\frac{h-lm}{(l+1)m}\cdot\frac{h-(l+1)m}{(l+2)m}\times\frac{b^2}{a^2}x^{2n}+\&c.\right)$, ergo ſumma ſeriei $\frac{1}{(r+l)n}+\frac{1}{(r+l+1)n}\times\frac{h-lm}{(l+1)m}\times\frac{b}{a}x^n+\frac{1}{(r+l+2)n}\times\frac{h-lm}{(l+1)m}\times\frac{h-(l+1)m}{(l+2)m}\times\frac{b^2}{a^2}x^{2n}+\&c.$ cum r & l ſint integri numeri, in finitis terminis exprimi poteſt.

1. 3. Per cap. 3. medit. algebr. erui poteſt in finitis ſumma e termini

prædictæ ſeriei ad k diſtantiam a ſe poſitis, e. g. $\frac{1}{(r+l)n} + \frac{1}{(r+l+k)n} \cdot$

$$\times \frac{b-lm}{(l+1)m} \times \frac{b-(l+1)m}{(l+2)m} \cdot \frac{b-(l+2)m}{(l+3)m} \ldots \frac{b-(l+k-1)m}{(l+k)m} \times$$

$$\frac{b^k}{a^k} x^{kn} + \frac{1}{(r+l+2k)n} \times \frac{b-lm}{(l+1)m} \cdot \frac{b-(l+1)m}{(l+2)m} \ldots \frac{b-(l+2k-1)m}{(l+2k)m}$$

$$\times \frac{b^{2k}}{a^{2k}} x^{2kn} + \&c.$$

1. 4. Si requiratur ut ſeries progrediatur ſecundum dimenſiones quantitatis x^{-n}, tum evadet $b^{\frac{b}{m}} x^{(r+\frac{b}{m})n} \left(\frac{1}{(r+\frac{b}{m})n} + \frac{1}{(r+\frac{b}{m}-1)n} \times \frac{b}{m} \times \right.$

$$\frac{a}{b x^n} + \frac{1}{(r+\frac{b}{m}-2)n} \times \frac{b}{m} \cdot \frac{b-m}{2m} \times \frac{a^2}{b^2 x^{2n}} + \&c.) = \int . x^{rn-1} \dot{x} \, (b x^n + a)^{\frac{b}{m}} :$$

& ſic detegi poteſt ſumma ſeriei, quæ ab $(l+1)$ termino hujuſce ſeriei incipiat; vel quæ erit ſumma ſingulorum ad k diſtantiam a ſe invicem poſitorum terminorum ejuſdem ſeriei.

In his ſeriebus pro b ſcribatur $-b$, & evadunt termini ſeriei alternatim affirmativi & negativi, & $b-m$, $b-2m$, &c. fiunt $-b-m$, $-b-2m$, &c.

1. 5. Si $-\frac{b}{m}-r$ ſit integer affirmativus numerus, tum ob $(a + b x^n)^{\frac{b}{m}} x^{rn-1} \dot{x} = (a x^{-n} + b)^{\frac{b}{m}} x^{\frac{b}{m}n+rn-1} \dot{x}$ pro r in ſeriebus prius traditis ſcribatur $+\frac{b}{m}+r$, & pro a ſcribatur b & pro b, a; & pro x^n, x^{-n}; & reſultant ſeries ejuſdem formulæ, quarum ſummæ innoteſcunt.

Ex. 2. Fluens fluxionis $(a + b x^n)^{\frac{b}{m}} x^{rn-1} \dot{x}$ per infinitam ſeriem ſic exprimi poteſt $\frac{1}{rn} x^{rn} \times (a + b x^n)^{\frac{b}{m}} \times \left(1 - \frac{b}{m} \times \frac{1}{(r+1)} \times \frac{b x^n}{a + b x^n} + \right.$

$$\frac{b}{m} \cdot \frac{b-m}{m} \times \frac{1}{r+1} \cdot \frac{1}{r+2} \cdot \frac{b^2 x^{2n}}{(a+b x^n)^2} - \frac{b}{m} \cdot \frac{b-m}{m} \cdot \frac{b-2m}{m} \times \frac{1}{r+1}$$

$\cdot\frac{1}{r+2}\cdot\frac{1}{r+3}\times\frac{b^3x^{3n}}{(a+bx^n)^3}+$ &c.); unde ſi r ſit integer numerus (prædictis exceptis) ſumma hujuſce ſeriei in finitis terminis exprimi poteſt.

Et per transformationes prius traditas plures ſeries erui poſſunt.

2. Ex fluentibus fluxionum, quæ exprimi poſſunt ex finitis terminis, circularibus, hyperbolicis & ellipticis arcubus, logarithmis & fluentibus datarum fluxionum, inveniri poſſunt ſeries, quarum ſummæ per prædictas fluentes exprimi poſſunt.

Series inveniri poſſunt ex reducendo datas fluxiones ad infinitas ſeries & earum fluentes inveniendo, ut in priori caſu docetur.

Ex. Sit fluxio $(a+bx^n)^{\pm\frac{1}{2}}\times x^{rn+\frac{1}{2}n-1}\dot{x}$, ubi r eſt integer numerus vel affirmativus vel negativus, cujus fluens per ſeriem ſic exprimi poteſt $x^{rn+\frac{1}{2}n}\Big(\frac{2}{(2r+1)n}a^{\pm\frac{r}{2}}\pm\frac{1}{2}\times\frac{2}{(2r+3)n}a^{\pm\frac{1}{2}-1}bx^n\pm\frac{1}{2}\times$ $\frac{\pm 1-2}{2\,.\,2}\times\frac{2}{(2n+5)n}\times a^{\pm\frac{1}{2}-2}b^2x^{2n}\pm\frac{1}{2}\times\frac{\pm 1-2}{2\,.\,2}\times\frac{\pm 1-4}{2\,.\,3}\times$ $\frac{2}{(2r+7)n}\times a^{\pm\frac{1}{2}-3}b^3x^{3n}\pm\ldots\pm\frac{1}{2}\times\frac{\pm 1-2}{2\times 2}\times\frac{\pm 1-4}{2\,.\,3}\times\frac{\pm 1-6}{2\,.\,4}$ $\ldots\frac{\pm 1-2z}{2\,.\,z+1}\times\frac{2}{(2r+2z+3)n}\times a^{\pm\frac{1}{2}-z-1}b^{z+1}x^{(z+1)n}+$ &c.)

Et ſimiliter reducenda eſt fluens ad ſeriem ſecundum reciprocas dimenſiones quantitatis x^n progredientem.

Ex. Sit fluxio $(a+bx^n)^{\pm\frac{1}{3}\text{ vel }\frac{2}{3}\text{ vel }\frac{1}{4}\text{ vel }\frac{3}{4}}\times x^{rn-\frac{1}{3}n\text{ vel }\frac{1}{4}n\text{ vel }\frac{1}{2}n\text{ \&c.}}\dot{x}$, reducatur hæc quantitas ad terminos ſecundum dimenſiones quantitatis x^n vel ejus reciprocas progredientes; & reſultabit ſeries, cujus ſumma per ellipticos, hyperbolicos, arcus, &c. innoteſcit.

Eadem principia etiam ad fluentes omnium fluxionum applicari poſſunt.

Ex. Aſſumatur fluxio $(a+bx^2)^{\pm\frac{1}{2}+m}\times x^r\dot{x}$, ubi literæ m & r integros denotant numeros vel affirmativos vel negativos, cujus fluens per algebraicam functionem circularium arcuum, logarithmorum &

finitorum terminorum exprimi poteſt; reducatur fluxio ad infinitam ſeriem, & inveniatur ejus fluens, quæ erit $a^{m\pm\frac{1}{2}}x^{r+1}\Big(\frac{1}{r+1}+(m\pm\frac{1}{2})\times\frac{r+1}{r+3}\times A\times\frac{b}{a}x^{2}+\frac{m\pm\frac{1}{2}-1}{2}\times\frac{r+3}{r+5}\times B\times\frac{b^{2}}{a^{2}}x^{4}+\frac{m\pm\frac{1}{2}-2}{3}\times\frac{r+5}{r+7}\times C\times\frac{b^{3}}{a^{3}}x^{6}+\frac{m\pm\frac{1}{2}-3}{4}\times\frac{r+7}{r+9}\times D\times\frac{b^{4}}{a^{4}}x^{8}+\ldots+P\times\frac{b^{l}}{a^{l}}\times x^{2l}\Big(\frac{m\pm\frac{1}{2}-l+1}{l}\times\frac{r+2l-1}{r+2l+1}+\frac{m\pm\frac{1}{2}-l}{l+1}\times\frac{r+2l+1}{r+2l+3}\times \mathcal{Q}\,\frac{b}{a}x^{2}+\frac{m\pm\frac{1}{2}-l-1}{l+2}\times\frac{r+2l+3}{r+2l+5}\times R\times\frac{b^{2}}{a^{2}}x^{4}+\text{\&c.}\Big)\Big)$, ubi literæ A, B, C, &c.; P, $\mathcal{Q}$, R, &c. denotant coefficientes præcedentium terminorum & l eſt integer affirmativus numerus.

Cor. Hinc innoteſcit ſumma ſeriei $\frac{m\pm\frac{1}{2}-l+1}{l}\times\frac{r+2l-1}{r+2l+1}+\frac{m\pm\frac{1}{2}-l}{l+1}\times\frac{r+2l+1}{r+2l+3}\times\mathcal{Q}\times\frac{b}{a}x^{2}+\frac{m\pm\frac{1}{2}-l-1}{l+2}\times\frac{r+2l+3}{r+2l+5}\times R\times\frac{b^{2}}{a^{2}}x^{4}+\text{\&c.}$, ubi $\mathcal{Q}$, R, &c. prædecentes coefficientes denotant.

Ducantur termini hujuſce ſeriei in functionem $e+fz+gz^{2}+bz^{3}+\text{\&c.}$ quantitatis z diſtantiæ a primo ſeriei termino, i. e. ducantur primus terminus in e, ſecundus in $e+f+g+b+\text{\&c.}$, tertius in $e+2f+4g+8b+\text{\&c.}$, quartus in $e+3f+9g+27b+\text{\&c.}$; & ſic deinceps; & reſultat ſeries, cujus ſumma innoteſcit: vel ducantur termini ſucceſſivi datæ ſeriei in terminos $\frac{\beta}{\alpha}$, $\frac{\beta}{\alpha}\times\frac{\beta+n}{\alpha+n}$, $\frac{\beta}{\alpha}\times\frac{\beta+n}{\alpha+n}\times\frac{\beta+2n}{\alpha+2n}$, &c., & reſultat ſeries $\frac{m\pm\frac{1}{2}-l+1}{l}\times\frac{r+2l-1}{r+2l+1}\times\frac{\beta}{\alpha}+\frac{m\pm\frac{1}{2}-l}{l+1}\times\frac{r+2l+1}{r+2l+3}\times\frac{\beta}{\alpha}\times\frac{\beta+n}{\alpha+n}\times\mathcal{Q}\times\frac{b}{a}x^{2}+\text{\&c.}$, cujus ſumma innoteſcit; ſi $\beta-\alpha=m'n$, ubi m' eſt integer affirmativus numerus: nam ſeries $\frac{\beta}{\alpha}$, $\frac{\beta}{\alpha}\cdot\frac{\beta+n}{\alpha+n}$, $\ldots$ $\frac{\beta\,.\,\overline{\beta+n}\,.\,\overline{\beta+2n}\ldots\overline{\beta+zn}}{\alpha\,.\,\overline{\alpha+n}\,.\,\overline{\alpha+2n}\ldots\overline{\alpha+zn}}=$

$\beta+zn$

$\dfrac{\beta + zn \,.\, \beta + (z-1)n \,.\, \beta + (z-2)n \,.\,.\,.\, \beta + (z-m+1)n}{\alpha \,.\, \alpha + n \,.\, \alpha + 2n \,.\,.\,.\, \alpha + (m-1)n}$ prædicta functio quantitatis z.

Ad eundem modum ex reducendo fluxionem $(bx^2 \pm a)^{m \pm \frac{1}{2}} \times x^r \dot{x}$ ad seriem $b^{m \pm \frac{1}{2}} x^{2m \pm 1 + r + 1} \left(\dfrac{1}{2m \pm 1 + r + 1} + (m \pm \tfrac{1}{2}) \times \dfrac{2m \pm 1 + r + 1}{2m \pm 1 + r - 1} \times A \times \dfrac{a}{bx^2} + \text{\&c.} \right)$, ubi A, &c. præcedentes coefficientes denotat, & principiis prius traditis erui poſſunt aliæ ſeries, quarum ſummæ innoteſcunt.

Cor. Sit ſeries $a + bx^n + cx^{2n} + dx^{3n} + \text{\&c.} = S$, cujus fluxio deduci poteſt; tum ex eâ erui poteſt ſumma cujuſcunque ſeriei $a \times \dfrac{\alpha}{\beta} \times \dfrac{\alpha'}{\beta'} \times \dfrac{\alpha''}{\beta''} \times \dfrac{\alpha'''}{\beta'''} \times \text{\&c.} + b \times \dfrac{\alpha + n}{\beta + n} \times \dfrac{\alpha' + n}{\beta' + n} \times \dfrac{\alpha'' + n}{\beta'' + n} \times \dfrac{\alpha''' + n}{\beta''' + n} \times \text{\&c.} \times Ax^n + c \times \dfrac{\alpha + 2n}{\beta + 2n} \times \dfrac{\alpha' + 2n}{\beta' + 2n} \times \dfrac{\alpha'' + 2n}{\beta'' + 2n} \times \dfrac{\alpha''' + 2n}{\beta''' + 2n} \times \text{\&c.} \times Bx^{2n} + d \times \dfrac{\alpha + 3n}{\beta + 3n} \times \dfrac{\alpha' + 3n}{\beta' + 3n} \times \dfrac{\alpha'' + 3n}{\beta'' + 3n} \times \dfrac{\alpha''' + 3n}{\beta'' + 3n} \times \text{\&c.} \times Cx^{3n} + \text{\&c.}$, ubi $\beta - \alpha$, $\beta' - \alpha'$, $\beta'' - \alpha''$, $\beta''' - \alpha'''$, &c. per n diviſæ integri ſunt affirmativi numeri & literæ A, B, C, D, &c. præcedentes coefficientes reſpective denotant.

PROB. XV.

Ex datâ convergente ſerie, vel ſerie cujus termini ultimo nihilo fiunt æquales, & cujus termini ſecundum dimenſiones quantitatis x *progrediuntur, viz.* $(\mathrm{a}\mathrm{x}^{\mathrm{s}} + \mathrm{b}\mathrm{x}^{\mathrm{r}+\mathrm{s}} + \mathrm{c}\mathrm{x}^{2\mathrm{r}+\mathrm{s}} + \mathrm{d}\mathrm{x}^{3\mathrm{r}+\mathrm{s}} + \text{\&c.})$ *ita ut ad infinitam diſtantiam dimenſiones evadant infinite magnæ vel negativæ vel affirmativæ, & cujus ſumma* (S) *datur, ſi modo ſeries ſit convergens; infinitas alias conſimiles detegere.*

Ducatur data ſeries in quotcunque quantitates hujuſce generis $x^{\alpha}\dot{x}$; fluxionum reſultantium inveniantur fluentes, viz. $\int . Sx^{\alpha}\dot{x}$; & ſic ducantur ſeries reſultantes in quantitates hujuſce formulæ $x^{\beta}\dot{x}$, & in-

& inveniantur fluxionum reſultantium $x^{\beta} \dot{x} \int . S x^{\alpha} \dot{x}$ fluentes, & ſi ſeries data ſit convergens, tum a fortiori ſeries reſultans etiam erit convergens; & ſic deinceps: deinde ducatur data ſeries in quantitates hujuſce formulæ x^{π}, & inveniantur fluxiones quantitatum reſultantium, dividantur hæ fluxiones per $\dot{x}$, & reſultant novæ ſeries, quarum ſummæ erunt $\left(\frac{\pi x^{\pi-1} S \dot{x} + x^{\pi} \dot{S}}{\dot{x}}\right)$, ſi modo hæ reſultantes ſeries convergant. Hæ reſultantes ſeries in pleriſque caſibus convergent, ſi modo data ſeries convergat. Et ſic de fluxionibus novarum ſerierum, &c. uſque donec hæ ſeries evadant tales, ut termini ad infinitam diſtantiam haud nihilo fiant æquales, tum ex omnibus his ſeriebus per prob. præced. inveniri poſſunt ſeries, quarum ſummæ innoteſcunt.

Et ſic e datis quibuſcunque ſeriebus prædicti generis, facile deduci poſſunt infinitæ aliæ per methodos in prob. præc. traditas, quarum ſummæ innoteſcunt; vel ex multiplicando datas ſeries in quaſcunque quantitates, ita ut reſultent ſeries quarum fluxiones vel fluentes vel integrales vel incrementa producant convergentes ſeries.

Et vice verſâ ex principiis prius traditis inveſtigari poteſt, utrum ſumma datæ ſeriei e ſummis quarumque ſerierum datis inveſtigari poſſit, nence.

Ex. 1. Sit æquatic data $S = 1 + \frac{1}{1 . 2} x + \frac{1}{1 . 2 . 3} x^2 + \frac{1}{1 . 2 . 3 . 4} x^3 +$ &c. ducatur hæc ſeries in $1 - x$, & reſultat ſeries $(1 - x) \times S = 1 - \frac{1}{1 . 2} x - \frac{1}{1 . 3} x^2 - \frac{1}{1 . 2 . 4} x^3 - \frac{1}{1 . 2 . 3 . 5} x^4 -$ &c. in infinitum; ſit $x = 1$, & evadet $1 = \frac{1}{2} + \frac{1}{1 \times 3} + \frac{1}{1 . 2 \times 4} + \frac{1}{1 . 2 . 3 \times 5} + \frac{1}{1 . 2 . 3 . 4 \times 6} +$ &c. & ſic de conſimilibus e præcedentibus principiis deducendis.

Ducatur data æquatio in $a + bx + cx^2 + dx^3 +$ &c. $= 0$, & reſultat ſeries $a + (\frac{1}{2} a + b) x + \left(\frac{1}{1 . 2 . 3} a + \frac{1}{1 . 2} b + c\right) x^2 +$ &c. $= 0$.

Ex. 2. Sit ſeries $(1-x^2)^{-m} = 1 + mx^2 + m.\frac{m+1}{2}x^4 + m.\frac{m+1}{2}.\frac{m+2}{3}x^6 + \&c.$ ducatur æquatio in $\dot{x}$, & inveniatur ejus fluens, & reſultat $x + \frac{mx^3}{3} + \frac{m.m+1\times x^5}{2.5} + \frac{m.m+1.m+2.x^7}{2.3.7} + \&c. = \int.\frac{\dot{x}}{(1-x^2)^m}$; ſit $m = \frac{1}{2}$, & erit $\int.\frac{\dot{x}}{\surd(1-x^2)} = x + \frac{x^3}{2.3} + \frac{1.3.3.x^5}{1.2.3.4.5} + \frac{1.3.3.5.5x^7}{1.2.3.4.5.6.7} + \frac{1.3.3.5.5.7.7x^9}{1.2.3.4.5.6.7.8.9} + \&c.$ Ducantur ſucceſſivi termini hujuſce ſeriei reſpective in quamcunque algebraicam & integralem functionem quantitatis z diſtantiæ a primo ſeriei termino. E. g. Ducantur termini reſpective in terminos $\frac{\beta}{\alpha}$, $\frac{\beta.(\beta+n)}{\alpha.(\alpha+n)}$, $\frac{\beta.\beta+n.\beta+2n}{\alpha.\alpha+n.\alpha+2n}$, &c., ſi $\frac{\beta-\alpha}{n}$ ſit integer numerus; tum reſultabit ſeries, cujus ſumma e prædictâ fluente deduci poteſt: ſi ex termino ad diſtantiam (l) a primo incipiat ſeries, cujus termini facile exprimi poſſunt per functionem vel algebraicam vel fluxionalem vel incrementialem diſtantiæ a primo ſeriei termino; e medit. algebr. deduci poteſt ſumma e ſingulis alternis, & terminis ſeriei ad (k) diſtantiam a ſe poſitis. Ex his facile deduci poſſunt infinitæ aliæ, &c.

PROB. XVI.

Datis ſeriebus formularum, quæ haud per algebraicas vel fluxionales vel incrementiales æquationes exprimi poſſunt; invenire, annon earum ſummæ finitis terminis exprimi poſſunt.

Quæratur, annon poteſt eſſe ſeries continuo ad nihil vergens & ultimo ad id accedens, quæ datæ ſeriei fiat æqualis; ſi modo ſeries inventa ablatis quibuſdam ejus terminis auferatur a ſe, & ſic deinceps; vel quod magis generale erit, quæ datæ ſeriei per quaſdam quantitates auctæ vel diminutæ æqualis erit, ſi modo ſeries prædicti generis ducatur in quantitatem inveſtigandam nihilo æqualem.

Ex. 1. Sit ſeries, $\frac{2\cdot3\ldots(n)-1}{1\cdot2\cdot3\ldots n}+\frac{3\cdot4\ldots(n+1)-1}{1\cdot2\cdot3\ldots(n+1)}+\frac{4\cdot5\ldots(n+2)-1}{1\cdot2\cdot3\ldots(n+2)}+$ &c. cujus generalis terminus erit $\frac{(z+2)\cdot(z+3)\ldots(z+n)-1}{1:2\cdot3\cdot\cdot(z+n)}$.

Facile conſtat e denominatoribus, &c. datæ ſeriei ut ſeries quæſita ſit $1+\frac{1}{1\cdot2}+\frac{1}{1\cdot2\cdot3}+\frac{1}{1\cdot2\cdot3\cdot4}+$ &c. & conſequenter ſumma datæ ſeriei quæſita erit $1+\frac{1}{1\cdot2}+\frac{1}{1\cdot2\cdot3}\ldots+\frac{1}{1\cdot2\cdot3\cdot\cdot(n-1)}$.

Ex. 2. Sit ſeries $\frac{e^{2n+n^2}-1}{e^{(1+n)^2}}+\frac{e^{4n+n^2}-1}{e^{(2+n)^2}}+\frac{e^{6n+n^2}-1}{e^{(3+n)^2}}+$ &c. & ſeries quæſita erit $\frac{1}{e}+\frac{1}{e^4}+\frac{1}{e^9}+\frac{1}{e^{16}}+$ &c. & conſequenter ſumma ſeriei datæ quæſita erit $\frac{1}{e}+\frac{1}{e^4}+\frac{1}{e^9}+\frac{e}{e^{16}}\ldots\frac{1}{e^{n^2}}$.

Ex obſervatâ lege quam habent termini datæ ſeriei plerumque facile erui poteſt ſeries quæſita.

PROB. XVII.

1. *Invenire, annon datæ ſeriei ſumma e pluribus aliis datis & finitis ſeriebus inveſtigari poteſt.*

Inveſtigetur, annon data ſeries ſit quæcunque directa functio ſerierum datarum & finitarum quantitatum, & perficitur problema.

2. Datâ particulari fluente, infinitæ ſunt diverſæ fluxionales æquationes, quarum ea eſt reſolutio, & quarum diverſæ ſunt generales reſolutiones; etiamque hoc verum erit in ſeriebus, quæ particulares

fluentes

fluentes reſpective denotant, quamvis forſan vulgaris methodus unam ſolummodo deducat.

Ex. 1. Sit $y = z + 1 . 2 z^2 + 1 . 2 . 3 z^3 +$ &c. ſeries, quæ ſemper divergit; tum erit $y z = z^2 + 1 . 2 z^3 +$ &c. & exinde $\frac{(y\dot{z} + z\dot{y})}{\dot{z}} \times z = 1 \times 2 z^2 + 1 \times 2 . 3 z^3 +$ &c. $= y - z$, unde $y\dot{z} + z\dot{y} = \frac{y - z}{z} \dot{z}$ una æquatio, cujus particularis reſolutio eſt prædicta ſeries; hujuſce fluxionalis æquationis innoteſcit generalis reſolutio: inveniatur igitur generalis reſolutio, & ita corrigatur ut fiat prædicta particularis reſolutio ($P = o$), deinde quantitati P addatur quæcunque functio Q quantitatum y & z, quæ nihilo evadat æqualis, cum $P = o$, tum erit $P + Q = o$ æquatio, cujus particularis reſolutio erit data ſeries; &c.

Aliter: aſſumatur data æquatio $y = z + 1 . 2 z^2 +$ &c. aucta vel diminuta per quaſcunque functiones quantitatum z & y ductas in directas poteſtates novæ invariabilis quantitatis A ad libitum aſſumendæ; deinde inveniatur per methodum prius traditam fluxionalis æquatio, cujus æquatio aſſumpta ſit generalis fluens, & erit fluxionalis æquatio quæſita.

PROB. XVIII.

Invenire infinitas ſeries, quæ ſint generales reſolutiones quæſitarum fluxionalium æquationum.

Aſſumantur quantitates tot invariabiles quantitates ad libitum aſſumendas involventes, quot ſit ordo fluxionalis æquationis quæſitæ, quæ facile reduci poſſunt ad infinitas ſeries; harum quantitatum inveniantur fluxionales æquationes, quarum generales ſint fluentes, & perficitur prob.

PROB.

PROB. XIX.

Invenire, quando termini datæ ſeriei fiant maximi. Inveniantur, quando proximi termini datæ ſeriei fiant æquales, & exinde conſtabit, quando fiant maximi; unde fiant maximi, cum ſeries nec convergat nec divergat; vel quod idem eſt, cum fluxio termini nihilo ſit æqualis.

2. Invenire punctum inflexûs in datâ ſerie, cujus termini vel relationes inter terminos dantur.

Aſſumantur tres termini in genere ſucceſſivi Q, R & S, ſupponantur $Q - R = R - S$, & exinde detegentur quantitates Q, R & S, & invenitur punctum inflexûs quam proxime; & idem deduci poteſt e principiis fluxionum prius traditis.

Et ſic de inveniendis punctis vel terminis, in quibus conſiſtat quæcunque data relatio: aſſumantur enim in genere termini datam habentes inter ſe relationem, & ex æquationibus reſultantibus facile conſtabit reſolutio.

PROB. XX.

Invenire ſummabiles ſeries.

Aſſumatur quæcunque quantitas pro ſummâ, & reducatur ea ad ſeriem, cujus termini ſecundum quamcunque legem progrediuntur, ita autem ut ſeries reſultans converget, & perficitur prob. vel quod ad idem redit: ſit quæcunque finita quantitas vel rationalis vel irrationalis, cujus inveniatur proximus valor; deinde differentiæ inter hunc valorem & datam quantitatem inveniatur valor prope, & ſic deinceps in infinitum, & reſultabit convergens ſeries.

Ex. 1. Erit $\frac{1}{a-b} = \frac{1}{a} + \frac{b}{a \times (a-b)} = \frac{1}{a} + \frac{b}{a \times (a+c)} + \frac{b \times (b+c)}{a \times (a+c) \times (a-b)} = \frac{1}{a} + \frac{b}{a.(a+c)} + \frac{b.(b+c)}{a.(a+c).(a+d)} + \frac{b.(b+c).(b+d)}{a.(a+c).(a+d).(a-b)} = \frac{1}{a} + \frac{b}{a.(a+c)} + \frac{b.(b+c)}{a.(a+c).(a+d)} +$

$+ \frac{b.(b+c).(b+d)}{a.(a+c).(a+d).(a+e)} + \frac{b.(b+c).(b+d).(b+e)}{a.(a+c).(a+d).(a+e)\times(a-b)}$ $=$ &c.

2. Erit etiam $\frac{1}{a+b} = \frac{1}{a} - \frac{b}{a\times(a+b)} = \frac{1}{a} - \frac{b}{a\times(a-c)} + \frac{b\times(b+c)}{a.(a-c)\times(a+b)} = \frac{1}{a} - \frac{b}{a.(a-c)} + \frac{b.(b+c)}{a.(a-c).(a-d)} - \frac{b.(b+c).(b+d)}{a.(a-c).(a-d)\times(a+b)} =$ &c.

Facile demonſtrari poſſint hæc theoremata e reductione fractionum ad communes denominatores.

Cor. . Ex aſſumptis diverſis ſeriebus numerorum pro quantitatibus a, b, c, d, e, f, &c. facile conſequentur ſeries, quarum innoteſcunt ſummæ.

Ex. 2. Sit ſeries $1 + b + bc + bcd + bcde + bcdef +$ &c. cujus termini continuo decreſcunt, & ultimo nihilo æquales fiunt, & erit $1 = 1 - b + b\times(1-c) + bc\times(1-d) + bcd\times(1-e) + bcde\times(1-f) +$ &c.

Hæc ſeries non convergit celerius quam data.

Cor. . Sint $b = \frac{m}{a}$, $c = \frac{m+p}{a+p}$, $d = \frac{m+q}{a+q}$, $e = \frac{m+r}{a+r}$, &c. & erit $1 = 1 - b + b\times(1-c) +$ &c. $= 1 - \frac{m}{a} + \frac{m}{a} . \frac{a-m}{a+p} + \frac{m}{a} \times \frac{m+p}{a+p} \times \frac{a-m}{a+q} + \frac{m}{a} \times \frac{m+p}{a+p} . \frac{m+q}{a+q} . \frac{a-m}{a+r} +$ &c. nunc ſtatuantur $q = 2p$, $r = 3p$, $s = 4p$, &c. & evadet hæc ſeries $\frac{a-m}{a} + \frac{m}{a} . \frac{a-m}{a+p} + \frac{m}{a} . \frac{m+p}{a+p} . \frac{a-m}{a+q} +$ &c. $= \frac{\beta - p - m}{\beta - p} (1 + \frac{m}{\beta} + \frac{m}{\beta} . \frac{m+p}{\beta+p} + \frac{m . m+p . m+2p}{\beta . \beta+p . \beta+2p} +$ &c. ubi $\beta = a + p$.

Ex. 3. Sint $b - \pi = \beta$, $c - \rho = \gamma$, $d - \sigma = \delta$, $e - \tau = \varepsilon$, &c. tum erit $o = \frac{\alpha}{a} - \frac{\alpha}{a} \times \frac{\beta}{b} - \frac{\alpha}{a} \times \frac{\pi}{b} \times \frac{\gamma}{c} - \frac{\alpha}{a} \times \frac{\pi}{b} \times \frac{\rho}{c} \times \frac{\delta}{d}$ — &c. ſi modo fractiones novæ ultimo ſint minores quam unitas, i. e. $o = 1 - \frac{\beta - \beta'}{\beta} - \frac{\beta'}{\beta} \times \frac{\gamma - \gamma'}{\gamma} - \frac{\beta'}{\beta} \times \frac{\gamma'}{\gamma} \times \frac{\delta - \delta'}{\delta} - \frac{\beta'}{\beta} \times \frac{\gamma'}{\gamma} \times \frac{\delta'}{\delta} \times \frac{\varepsilon - \varepsilon'}{\varepsilon}$ — &c., ubi β', γ', δ', ε', &c. ſunt quantitates minores quam β, γ, δ, ε, &c.

Si modo addantur quantitates, quæ continuo decreſcant in uno termino, & detrahantur in ſucceſſivis, vel prope; tum reſultabit ſeries, cujus ſumma innoteſcit; hæc principia continent methodum ſeries deducendi, quarum ſummæ innoteſcunt.

Hæc principia etiam applicari poſſunt ad inveniendas ſeries convergentes, quarum ſummæ ſint quæcunque irrationales quantitates: ſed perraro uſui inſerviunt conſimiles transformationes, 1$^{\text{mo}}$. enim neceſſe eſt, ut detur methodus inveniendi quantitates a, b, c, d, &c; α, β, γ, &c. e datis ſeriei terminis, aliter non datur ſeriei transformatio; i. e. non ex datâ ſerie acquiritur ſeries convergens.

2. Datâ ſerie convergente, inveniatur ejus functio vel algebraica vel fluxionalis vel fluentialis vel incrementialis vel integralis e principiis prius traditis, quæ evadit convergens, & perficitur prob.

PROB. XXI.

Invenire ſummas ſerierum, quarum ſinguli termini dantur ex infinitis ſeriebus.

1. Si vero dentur termini ex infinitis ſeriebus: per infinitas ſeries inveniantur vel ſummæ harum infinitarum ſerierum, vel approximationes ſatis propinquæ ad illas ſummas; quibus in terminis ſeriei, cujus ſumma requiritur, pro ſuis valoribus ſubſtitutis, reſultat ſeries, cujus ſumma inveniatur prope, & erit ſumma quæſita prope.

Tum continuo per methodos prius traditas corrigendæ ſunt approximationes inventæ, & exinde corrigetur ſumma ſeriei quæſita; & ſic deinceps.

2. Si vero termini e datis æquationibus pendeant, tum e datis æquationibus vel deducantur valores terminorum datæ ſeriei vel approximationes ad terminos datæ ſeriei ſatis appropinquantes, & tum per methodos prædictas continuo corrigatur ſumma inventa, & continuo reſultabunt propiores approximationes ad ſummam quæſitam.

Ex. Literâ $z - 1$ denotante diſtantiam termini cujuſcunque e primo, terminus prædictus exprimatur per reciprocam ſeriei $1 + \frac{1}{1.2} z^{-1} + \frac{1}{1.2.3} z^{-2} + \frac{1}{1.2.3.4} z^{-3} + \&c.$ ductam in $\frac{1}{2^{z}}$, i. e. ſeries ſit

$$\frac{\frac{1}{2}}{1 + \frac{1}{1.2} + \frac{1}{2.3} + \&c.} + \frac{\frac{1}{4}}{1 + \frac{2}{1.2} + \frac{2^2}{2.3} + \&c.} + \frac{\frac{1}{8}}{1 + \frac{3}{1.2} + \frac{3^2}{1.2.3} + \&c.} + \&c.$$

quæ ſeries vere convergit; & facile erit dijudicare de numero terminorum in fractione $1 + \frac{z}{1.2} + \frac{z^2}{2.3} + \&c.$ ad ſingulas operationes aſſumendorum, &c.: ſingulorum enim terminorum præſtat, ut inveniantur approximationes, quæ a veris ſummis per quantitates prope æquales diſtant, & ſi requiratur ſumma ſeriei vera ad quendam limitem, tum ita inveniantur approximationes, ut earum aggregatum non produceret errorem limitem prædictum exſuperantem.

In his æque ac in omnibus aliis ſeriebus ex relatione, quam habent termini ſucceſſivi ad infinitam diſtantiam dijudicari poteſt convergentia ſeriei.

PROB. XXII.

Conceſsâ methodo detegendi generaliter, annon una data quantitas ſit algebraica functio aliarum datarum quantitatum, & datâ ſerie ſecundum dimenſiones incognitæ quantitatis x *progrediente, cujus ſumma ſit* y; *invenire* *annon aſſignari poteſt æquatio, ſive algebraica ſive fluxionalis ſit, quæ deſignat relationem inter quantitates* x *&* y, *& earum fluxiones.*

1. Sit data ſeries $ax^n + bx^{n+m} + cx^{n+2m} + \&c. = y$, & e conceſsâ methodo ſequitur, utrum data ſeries y ſit algebraica functio literæ x, i. e. utrum detur algebraica æquatio, quæ exprimit relationem inter quantitates x & y, necne.

2. Inveniantur primæ, ſecundæ, &c. fluxiones datæ ſeriei, quæ erunt reſpective $\dot{y} = nax^{n-1}\dot{x} + (n+m)bx^{n+m-1}\dot{x} + (n+2m)cx^{n+2m-1}\dot{x} + \&c.$ &c. jam vero e conceſsâ methodo detegatur utrum data ſeries $ax^n + bx^{n+m} + \&c. = y$ ſit algebraica functio datæ quantitatis x, & datarum ſerierum $nax^{n-1} + (n+m)bx^{n+m-1} + \&c. = \frac{\dot{y}}{\dot{x}}$, $n.(n-1)ax^{n-2} + (n+m) \times (n+m-1)bx^{n+m-2} + \&c. = \frac{\ddot{y}}{\dot{x}^2}$, &c. necne; & conſit problema.

Ex. Sit data ſeries $1 + \frac{x^n}{1.2.3..n} + \frac{x^{2n}}{1.2.3..2n} + \frac{x^{3n}}{1.2.3...3n} + \&c. = y$, cujus fluxio n ordinis erit $\overset{n}{\dot{y}} = (1 + \frac{x^n}{1.2.3...n} + \frac{x^{2n}}{1.2.3..2n} + \&c.)\dot{x}^n$; unde $\overset{n}{\dot{y}} = y\dot{x}^n$.

Cor. Generalis fluens æquationis $\overset{n}{\dot{y}} = y\dot{x}^n$ erit $y = a(1 + \frac{x^n}{1.2.3..n} + \frac{x^{2n}}{1.2.3..2n} + \frac{x^{3n}}{1.2.3..3n} + \frac{x^{4n}}{1.2.3..4n} + \&c.$ in infinitum$) + bx(1 + \frac{x^n}{2.3..n+1} + \frac{x^{2n}}{2.3..2n+1} + \frac{x^{3n}}{2.3..3n+1} + \&c.) + \frac{cx^2}{1.2}(1 + \frac{x^n}{3.4..n+2} + \frac{x^{2n}}{3.4..2n+2} + \frac{x^{3n}}{3.4..3n+2} + \&c.) + \frac{dx^3}{1.2.3}(1 + \frac{x^n}{4.5..n+3} + \frac{x^{2n}}{4.5..2n+3} + \frac{x^{3n}}{4.5..3n+3} + \&c.)$

$+$ &c.) $+ \frac{e x^4}{1 \cdot 2 \cdot 3 \cdot 4} (1 + \frac{x^n}{5 \cdot 6 \, . \, . \, n+4} +$ &c.) $+$ &c. . . . $+$ $\frac{b x^{n-1}}{1 \cdot 2 \cdot 3 \, . \, . \, n-1} \times (1 + \frac{x^n}{n \cdot n+1 \, . \, . \, 2n-1} + \frac{x^{2n}}{n(n+1)(n+2) \, . \, . \, (3n-1)}$ $+$ &c. in infinitum); ubi literæ a, b, c, d, e, &c. & b invariabiles coefficientes ad libitum aſſumendas reſpective denotant.

3. Si termini ſeriei $y = P$ generaliter exprimantur per fractionem hujuſce generis $\frac{a z^m + b z^{m-1} + c z^{m-2} + \&c.}{A z^n + B z^{n-1} + C z^{n-2} + \&c.} \times x^{rz+rs}$, ubi literæ r, s, a, b, c, &c. A, B, C, &c. invariabiles denotant quantitates, & m & n ſunt integri numeri, & z diſtantia a primo ſeriei termino: tum e prob. præced. deduci poteſt fluxionalis æquatio, cujus radix particularis invenitur $y = P$.

Cor. 1. Ex hinc facile colligi poteſt fluentem generalis fluxionalis æquationis hujuſce formulæ $\overset{n}{\dot{y}} + \frac{a}{x} \overset{n-1}{\dot{y}} \dot{x} + \frac{b}{x^2} \overset{n-2}{\dot{y}} \dot{x}^2 + \frac{c}{x^3} \overset{n-3}{\dot{y}} \dot{x}^3 +$ &c. $= \frac{d' - b'^{z+1} x^{z+1}}{x^n - b' x^{n+1}}$ eſſe $y = A x^{-\alpha} + B x^{-\beta} + C x^{-\gamma} + D x^{-\delta} +$ &c. $+$ $\frac{d'}{\alpha \cdot \beta \cdot \gamma \cdot \delta \cdot \&c.} + \frac{d' b' x}{\alpha+1 \cdot \beta+1 \cdot \gamma+1 \cdot \delta+1 \cdot \&c.} + \ldots$ $\frac{d' b'^z x^z}{\alpha+z \cdot \beta+z \cdot \gamma+z \cdot \delta+z \cdot \&c.}$, ubi a, b, c, &c. ſunt invariabiles coefficientes facile deducendæ ex (n) invariabilibus quantitatibus α, β, γ, δ, &c. datis, & (n) quantitates A, B, C, D, &c. ſunt quæcunque invariabiles quantitates ad libitum aſſumendæ, & a', b', &c. ſunt datæ invariabiles quantitates; & z eſt integer numerus.

4. Sit æquatio $(a + a'x + a''x^2 + \&c.) \dot{x}^n + (b + b'x + \&c.) y \dot{x}^n$ $+ (c + \&c.) \dot{y} \dot{x}^{n-1} + (d + \&c.) \ddot{y} \dot{x}^{n-2} \ldots + (g + \&c.) \overset{n-1}{\dot{y}} \dot{x} + b \overset{n}{\dot{y}}$ $= 0$, tum ſi modo ejus generalis fluens ſit $y = A + Bx + Cx^2 \ldots$ $G x^{n-1} + H x^n + I x^{n+1} + K x^{n+2} +$ &c., ubi (n) literæ A, B, C, &c. ſunt invariabiles quantitates ad libitum aſſumendæ, cæteræ vero H, I, &c. invariabiles, erit $a + bA + cB + 2dC + \ldots + 1 \cdot 2 \cdot 3$ $. \, . \, (n-1)$

$\ldots(n-1)gG = -1.2.3\ldots n\times bH$, $bB + 1.2cC + 2.3dD + \ldots 2.3.4\ldots(n+1)Ib = 0$, &c.

PROB. XXIII.

1. *Invenire quantitatem, quæ in ſeriem inveniendam magis celeriter convergentem ducta præbet datam ſeriem.*

Hujuſce problematis reſolutio ſolummodo deduci poteſt e terminis ad infinitam diſtantiam poſitis; inveniatur enim quantitas ſi modo talis deduci poſſit, quæ vel in unum terminum cujuſcunque formulæ, vel in plures terminos proxime convergentes ducta, præbet datam ſeriem, & perficitur problema.

Ex. 1. Sit ſeries infinita $(A)\ 1 + b + b^2 + b^3 + \text{\&c.}$ & erit $(1+b)\times(1 + b^2 + b^4 + b^6 + \text{\&c.} = B)$, & $(1+b)(1+b^2)(1 + b^4 + b^8 + \text{\&c.} = C)$, & ſimiliter $(1+b)(1+b^2)(1+b^4)(1 + b^8 + b^{16} + \text{\&c.} = D)$, &c. ubi b ſit vel negativa vel affirmativa quantitas, & erit A minus convergens quam B, B quam C, C quam D, &c.

Ex. 2. Sit ſeries $(P)\ x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \text{\&c.}$ aſſumatur $(1+x)^{-1}\times(x + Ax^2 + Bx^3 + Cx^4 + \text{\&c.}) = P$, & invenientur $A = \frac{1}{1.2}$, $B = \frac{-1}{2.3}$, $C = \frac{1}{3.4}$, &c. i. e. ſeries erit $(\mathcal{Q})\ x + \frac{1}{2}x^2 - \frac{1}{2.3}x^3 \ldots \frac{1}{z.z+1}x^{z+1}$; aſſumatur $2.(1+x)^{-1}\times(\frac{1}{2}x + ax^2 + bx^3 + \text{\&c.}) = \mathcal{Q}$; & invenientur $a = +\frac{3}{4}$, $b = +\frac{1}{1.2.3}$, $c = -\frac{1}{2.3.4}$, $d = +\frac{1}{3.4.5}$, &c. i. e. ſeries erit $(R)\ \frac{1}{2}x + \frac{3}{4}x^2 + \frac{1}{1.2.3}x^3 - \frac{1}{2.3.4}x^4 \ldots \frac{1}{z.z+1.z+2}x^{z+2}$, & ſic aſſumatur $3(1+x)^{-1}(\frac{1}{6}x + \alpha x^2 + \beta x^3 + \gamma x^4 + \text{\&c.}) = R$, & invenietur ſeries $(S)\ \frac{1}{6}x + \frac{5}{12}x^2 + \frac{11}{36}x^3 + \frac{1}{1.2.3.4}x^4 - \frac{1}{2.3.4.5}x^5 \ldots \frac{1}{z.z+1.z+2.z+3}x^{z+3}$,

x^{z+3}, & ſic deinceps: unde $P = \frac{\mathcal{Q}}{1+x} = \frac{R}{2.(1+x).(1+x)} = \frac{S}{2.3.(1+x).(1+x)^2} = \frac{T}{2.3.4.(1+x).(1+x)^3} =$ &c.

Cor. Sit x minor quam 1, & omnes ſeries P, $\mathcal{Q}$, R, S, &c. evadent convergentes; ſit x eadem quantitas in omnibus prædictis ſeriebus, & erunt termini ſerierum (P, $\mathcal{Q}$, R, S, &c.) ad infinitam diſtantiam reſpective inter ſe ut $\frac{1}{z}$, $\frac{1}{z.z-1}$, $\frac{1}{z.z-1.z-2}$, $\frac{1}{z.z-1.z-2.z-3}$, & ſic deinceps; ubi z denotat diſtantiam a primo ſeriei termino; & conſequenter ad infinitam diſtantiam minor erit convergentia ſeriei P quam $\mathcal{Q}$, $\mathcal{Q}$ quam R, R quam S, &c. termini enim ad prædictam diſtantiam ſeriei P infinite majores ſunt quam termini ſeriei $\mathcal{Q}$, & ſic deinceps: in ſeriebus enim per functiones algebraicas quantitatis z deſignatis, convergentia pendet ex differentiâ inter dimenſiones quantitatis z in numeratore & denominatore contentas.

2. Ducatur ſeries quæcunque (P) in quantitatem $\varkappa = 0$, (e.g. ſit $1 - x = 0$, & conſequenter $\varkappa = 1 - x$); & ſi ſeries (P) ſit convergens, tum ſeries reſultans erit $= 0$; ſi ſeries (P) ſit divergens, tum ſeries reſultans poteſt eſſe convergens vel divergens.

3. Datâ functione quantitatis z, quæ denotat quemcunque terminum ſeriei (P), cujus diſtantia a primo ſit z; reducatur data functio ad ſeriem deſcendentem ſecundum dimenſiones quantitatis z; deinde inveniatur quantitas A, ita ut, ſi modo reducatur finita quantitas A ad infinitam ſeriem, termini ad infinitam diſtantiam quam proxime iidem evadant ac termini datæ ſeriei; inveniatur ſeries $\mathcal{Q}$, quæ in quantitatem A ducta, præbet datam ſeriem (P), & fere ſeries $\mathcal{Q}$ magis celeriter converget quam ſeries P.

4. Si vero detur relatio inter ſucceſſivos terminos ſeriei (P) ad infinitam diſtantiam, inveniatur quantitas A, quæ reducta ad ſeriem infinitam, cujus termini ſecundum dimenſiones quantitatis z deſcendentes, eandem prope habent relationem inter ſucceſſivos terminos ac data relatio; tum deducatur ſeries $\mathcal{Q}$, ita ut $A \times \mathcal{Q} = P$, & fere $\mathcal{Q}$ erit ſeries magis celeriter convergens.

Facile inveſtigari poſſint infinitæ quantitates *A*, quarum termini ſucceſſivi ad infinitam diſtantiam vel quam proxime ſint iidem, vel habeant inter ſe relationem, quam habent datæ ſeriei termini.

E convergentiâ totius ſeriei, i. e. terminorum ad infinitam diſtantiam minime dici poteſt convergentia omnis partis; ſed fere facile inveniri poſſunt termini, cum tranſeat ſeries e divergentiâ in convergentiam, vel cum fiat maxime convergens, vel convergentiæ incrementum evadat maximum, &c.

THEOR. XVII.

Sit $y = a - br^z - cr^{2z} - dr^{3z} - er^{4z} -$ &c. ubi r, a, b, c, d, e, &c. ſunt quantitates invariabiles; & ſint *A*, *B*, *C*, *D*, *E*, &c. diverſi dati valores quantitatis *y*, reſpondentes diverſis valoribus quantitatis *z*, qui ſunt 0, 1, 2, 3, 4, &c. unde $A = a - b - c - d - e -$ &c. $B = a + br - cr^2 - dr^3 -$ &c. $C = a - br^2 - cr^4 - dr^6 - er^8 -$ &c. $D = a - br^3 - cr^6 - dr^9 - er^{12} -$ &c. &c., ubi tot aſſumantur æquationes, quot ſunt incognitæ quantitates a, b, c, d, e, &c. & pro *r* aſſumi poteſt quicunque numerus exceptis 1 & -1; ex his ſimplicibus æquationibus inveniri poſſunt coefficientes a, b, c, d, &c.; & conſequenter æquatio $y = a - br^z - cr^{2z} -$ &c.

Cor. 1. Sit *z* infinita quantitas & *r* minor quam 1, tum erit $y = a$.

Cor. 2. Fiat *z* negativa quantitas, & erit $y = a - \frac{b}{r^z} - \frac{c}{r^{2z}} -$ &c. & ſit *z* infinita, & *r* major quam 1; tum erit $y = a$.

Cor. 3. Si $r = 1$ vel -1, tum aſſumenda eſt ſeries $y = a + \frac{b}{z} + \frac{c}{z^2} + \frac{d}{z^3} +$ &c. vel $a + bz + cz^2 +$ &c.

Differentiæ quarumcunque quantitatum ad perparvas diſtantias (*o*) a ſe invicem poſitarum erunt plerumque in ratione diſtantiarum ipſarum; & conſequenter quantitas exprimi poteſt per quantitatem hujuſce formulæ $A + Bo$, & magis generaliter per quantitatem $A + Bo + Co^2 + Do^3 + \&c.$

Hoc verum eſt haud ſolummodo in algebraicis ſed etiam in exponentialibus quantitatibus.

Hinc prope corrigi poſſunt errores, qui irrepunt in pleraſque operationes ad perparva intervalla inſtitutas.

THEOR. XVIII.

Sint A, B, C, D, E, &c. quantitates quæcunque invariabiles, z vero variabilis, tum in expreſſione $A + Bz + Cz \,.\, \frac{z-n}{2} + Dz \,.\, \frac{z-n}{2} \,.\, \frac{z-2n}{3}$. &c. ſcribe numeros quoſvis æquidiſtantes o, n, $2n$, $3n$, &c. pro z, & quantitates provenientes dicantur reſpective a, b, c, d, e, &c. ſubtrahatur quiſque terminus ſeriei $a + b + c + d + e + \&c.$ de proxime præcedente, vocenturque termini reſiduorum ($a - b$, $b - c$, $c - d$, &c.) primæ differentiæ; deinde ſubtrahatur differentia quæque prima a proxime præcedente, & reſidua reſultantia vocentur ſecundæ differentiæ; ſubtrahatur etiam quæque ſecunda a proxime præcedente, & reſidua reſultantia dicantur tertiæ differentiæ; & ſic deinceps: ſit a primus ſeriei terminus, d' prima primarum differentiarum, d'' prima ſecundarum, & ſic deinceps; & erit expreſſio $a - d'\frac{z}{n} + \frac{z}{n} \,.\, \frac{z-n}{2n} d'' - \frac{z}{n} \,.\, \frac{z-n}{2n} \,.\, \frac{z-2n}{3n} d''' + \&c. =$ termino, cujus diſtantia a primo ſit z.

Cor. 1. Hinc $d' = a - b$; $d'' = (a - b) - (b - c) = a - 2b + c$; & ſic $d''' = a - 3b + 3c - d$; & denique $\overset{n}{d'} = a - nb + n \,.\, \frac{n-1}{2} c - n \,.\, \frac{n-1}{2} \,.\, \frac{n-2}{3} d + \&c.$

Cor. 2. 1. Sint a, b, c, d, e, &c. ordinatæ ad curvam, quarum communis diſtantia a ſe invicem ſit n, i. e. quarum abſciſſæ ſunt o, n, $2n$,

&c.

&c. & æquatio relationem inter abſciſſam z & ejus correſpondentes ordinatas y deſignans ſit $y = A - \frac{Bz}{n} + \frac{Cz}{n} . \frac{z-n}{2n} - D\frac{z}{n} . \frac{z-n}{2n} . \frac{z-2n}{3n} +$ &c. tum erunt $A = a$, $B = d' = a - b$, $C = d'' = a - 2b + c$, $D = d''' = a - 3b + 3c - d$, &c.

2. Sint $\underset{\cdot}{A}n = d'$, $\underset{\cdot\cdot}{A}n^2 = d''$, $\underset{\because}{A}n^3 = d'''$, &c. tum erit $y = A - \underset{\cdot}{A}z + \underset{\cdot\cdot}{A}z . \frac{z-n}{2} - \underset{\because}{A}z . \frac{z-n}{2} . \frac{z-2n}{3} +$ &c. fiat $n = o$ & evadent $\underset{\cdot}{A}$, $\underset{\cdot\cdot}{A}$, $\underset{\because}{A}$, &c. primæ, ſecundæ, tertiæ, &c. fluxiones primæ ordinaiæ A, & exinde ordinata $y = A - \frac{\dot{A}}{\dot{z}}z + \frac{1}{2}\frac{\ddot{A}}{\dot{z}^2}z^2 - \frac{1}{2.3}\frac{\dddot{A}}{\dot{z}^3}z^3 + \frac{1}{2.3.4}\frac{\ddddot{A}}{\dot{z}^4}z^4 -$ &c. & ejus area $= Az - \frac{1}{2}\frac{\dot{A}}{\dot{z}}z^2 + \frac{1}{6}\frac{\ddot{A}}{\dot{z}^2}z^3 - \frac{1}{24}\frac{\dddot{A}}{\dot{z}^3}z^4 +$ &c. pro A, $\dot{A}$, $\ddot{A}$, &c. ſcribantur y, $\dot{y}$, $\ddot{y}$, &c. & fit area $= yz - \frac{1}{2}\frac{\dot{y}}{\dot{z}}z^2 + \frac{1}{6}\frac{\ddot{y}}{\dot{z}^2}z^3 - \frac{1}{24}\frac{\dddot{y}}{\dot{z}^3}z^4 +$ &c. ſint $\dot{y}$, $\dddot{y}$, &c. negativæ quantitates, & fit area $= yz + \frac{1}{2}\frac{\dot{y}}{\dot{z}}z^2 + \frac{1}{6}\frac{\ddot{y}}{\dot{z}^2}z^3 +$ &c.

In hoc caſu abſciſſa ſemper ſupponitur incipere a primâ vel ultimâ ordinatâ.

Cor. . Hinc literæ a, b, c, &c. inveniuntur e differentiis a, d', d'', d''', &c. fere per eandem legem ac differentiæ d', d'', d''', &c. e literis a, b, c, &c. i. e. $b = a - d'$, $c = a - 2d' + d''$, $d = a - 3d' + 3d'' - d'''$, $e = a - 4d' + 6d'' - 4d''' + d''''$, &c.

Ex. 1. Sint termini ſucceſſivi $\frac{1}{z}$, $\frac{1}{z+1}$, $\frac{1}{z+2}$, $\frac{1}{z+3}$, &c. & invenientur prædictæ differentiæ $\frac{1}{z} - \frac{1}{z+1} = \frac{1}{z\,.\,z+1} = d'$, $\frac{1}{z} - \frac{2}{z+1} + \frac{1}{z+2} = \frac{1 \times 2}{z\,.\,z+1\,.\,z+2} = d''$; & ſic $\frac{1 \times 2 \times 3}{z\,.\,z+1\,.\,z+2\,.\,z+3} = d'''$,

$= d'''$, & ſic deinceps: & vice verſâ ſint termini reſpectivi $\frac{1}{z}$, $\frac{1}{z.z+1}$, $\frac{1.2}{z.z+1.z+2}$, $\frac{1.2.3}{z.z+1.z+2.z+3}$, & erunt differentiæ prædictæ d', d'', d''', &c. reſpective $\frac{1}{z+1}$, $\frac{1}{z+2}$, $\frac{1}{z+3}$, $\frac{1}{z+4}$, &c.

Ex. 2. Sint termini a, ab, ab^2, ab^3, &c. in geometricâ progreſſione, & erunt differentiæ prædictæ d', d'', d''', &c. reſpective $a = d$, $a - ab = a \times (1 - b) = d'$, $a - 2ab + ab^2 = a \times (1 - b)^2 = d''$, &c. i. e. differentiæ ſucceſſivæ a, $a(1 - b)$, $a \times (1 - b)^2$; $a \times (1 - b)^3$, &c. quæ ſint a, d', d'', &c. erunt quantitates in geometricâ progreſſione, & vice verſâ ſint prædictæ differentiæ in geometricâ progreſſione, & erunt quantitates ipſæ in geometricâ progreſſione.

Ex. 3. Sit ſeries terminorum a, b, $a + b$, $a + 2b$, $2a + 3b$, $3a + 5b$, $5a + 8b$, &c., ubi quique ſucceſſivus terminus eſt ſumma duorum præcedentium, tum termini differentiarum n ordinis erunt iidem ac termini datæ ſeriei (n primis exceptis); primus terminus differentiarum n ordinis erit $ma - hb$, ſi $mb + ha$ ſit terminus in datâ ſerie, cujus diſtantia a primo ſit $n + 1$.

Si termini ſint ſummæ (m) præcedentium terminorum in quaſcunque datas coefficientes ductorum, vel eorum quæcunque functiones; tum exinde erui poſſunt differentiæ.

PROB. XXIV.

Datâ ſerie $a + b + c + d + e +$ &c. *in infinitum pergente & e terminis ſucceſſivis* a, b, c, d, &c. *conſtante, & inventis differentiis* $d' = a - b$, $d'' = a - 2b + c$, &c. *ſit* z *diſtantia a primo ſeriei termino, tum terminus, cujus diſtantia a primo ſit* z, *per theor. præc. erit* $a - d'z + z . \frac{z-1}{2} d'' - z \times \frac{z-1}{2} \times \frac{z-2}{3} d''' +$ &c. *quæ ſeries in infinitum excurret, ni hæ differentiæ fiant ultimo nihilo æquales. Invenire igitur, utrum hæc ſeries in infinitum progrediens, converget; necne.*

1^{mo}. Sit data ſeries talis, ut terminus quiſque ultimo ſit ad terminum ſucceſſivum in permagnâ ratione fere ut infinita quantitas ad finitam, e. g. ſit ſeries $1 + \frac{1}{1.2} + \frac{1}{1.2.3} + \frac{1}{1.2.3.4}$, & denique $\frac{1}{1.2.3..z} + \frac{1}{1.2.3..z+1}$; ſit z infinita quantitas & $\frac{1}{1.2.3..z}$ erit ad $\frac{1}{1.2.3..z.(z+1)}$ ut infinita $z+1$ ad 1. In hoc caſu ſeries data maxime convergit, ſed ſeries reſultans $1 - d'(1 - \frac{1}{2})z + d''\left(1 - 2 \times \frac{1}{2} + \frac{1}{1.2.3}\right)z \times \frac{z-1}{2} - d''' \times z \times \frac{z-1}{2}.\frac{z-2}{3} + \&c.$ minime converget: hoc conſtare poteſt e ſubſequente ratiocinatione; ſit primus terminus ſeriei a, omnes vero reliqui vel $= o$, vel ſinguli ſucceſſivi infinite ſint minores præcedentibus; & erit ſeries prope $a + o' + o'' + o''' + \&c.$ & omnes primæ differentiæ primarum, ſecundarum, &c. differentiarum erunt reſpective a; unde ſeries erit $a - az + az \times \frac{z-1}{2} - az \times \frac{z-1}{2} \times \frac{z-2}{3} + \&c.$ quæ nunquam celeriter converget, ni z ſit integer numerus.

2^{do}. Si ultimus datæ æquationis terminus habeat ad ejus ſucceſſivum rationem majorem quam æqualitatis, i. e. ultimi termini datæ ſeriei ſint in geometricâ progreſſione, cujus communis ratio eſt $1 : r$; tum, ſi omnes termini ſeriei ſint in geometricâ progreſſione, erunt differentiæ d', d'', d''', &c. in geometricâ progreſſione, cujus communis ratio eſt $1, 1-r$: unde conſtat, ſi 1 major ſit quam $1-r$, tum converget hæc ſeries, ſin aliter vero non. Eadem etiam affirmari poſſunt de ſerie, cujus termini ultimo ſunt in geometricâ progreſſione.

3^{tio}. Si termini ad infinitam diſtantiam propiores accedant ad rationem æqualitatis quam pro quâvis finitâ differentiâ; tum, utrum ſeries deducta ſit convergens, necne; pendet e ratione, quam habent ultimo ſucceſſivæ differentiæ d', d'', d''', &c. i. e. ex æquatione $a - zd' + z \times \frac{z-1}{2} d'' - \&c.$ prius traditâ, i. e. ex ratione, quam habent inter ſe ultimo ſucceſſivi ſeriei termini, conſtabit, utrum ſeries reſultans ſit convergens, necne.

Hinc conſtat, ut hæc ſeries plerumque maxime, cum data ſeries minime convergat; ſemper autem minime cum maxime convergat data ſeries: pendet enim e rationibus, quas habent inter ſe ſucceſſivæ differentiæ d, d', d'', &c.

THEOR. XIX.

1. Sit convergens ſeries $a + b + c + d + e + f +$ &c. quorum terminorum primæ differentiæ ſint reſpective a', b', c', d', e', &c. ſecundæ vero a'', b'', c'', d'', &c. & ſic deinceps: differentiæ vero n ordinis ſint reſpective α, β, γ, δ, ε, &c. tum erit $a = \alpha + n\beta + n \, . \, \frac{n+1}{2} \gamma + n \, . \, \frac{n+1}{2} \, . \, \frac{n+2}{3} \delta + n \, . \, \frac{n+1}{2} \, . \, \frac{n+2}{3} \, . \, \frac{n+3}{4} \varepsilon +$ &c. in infinitum.

Cor. . Si modo ſeries data ſit convergens; tum omnes ſeries, quorum termini ſunt differentiæ cujuſcunque ordinis erunt etiam convergentes.

Facile conſtat, ſi modo ſeries ſit affirmativa; ſin termini ejuſdem ſeriei, (i. e. cujus affirmativi & negativi termini eandem prorſus obſervant legem) ſint alternatim affirmativi & negativi, tum erunt prædictæ differentiæ alternatim affirmativæ & negativæ, & ſeries erit convergens.

2. Summa ſeriei $a + a' + a'' + a''' +$ &c. $\overset{n}{a'} = (n+1)a - n \, . \, \frac{n+1}{2} b + (n-1) \, . \, \frac{n}{2} \, . \, \frac{n+1}{3} c - (n-2) \, . \, \frac{n-1}{2} \, . \, \frac{n}{3} \, . \, \frac{n+1}{4} d +$ &c.

THEOR. XX.

1. Sint a', a'', a''', a'''', &c. ut antea ſucceſſivæ primæ differentiæ terminorum ſeriei $a + b + c + d + e +$ &c. i. e. cujus termini ſint ſucceſſive a, b, c, d, e, &c. Sit etiam ſeries, cujus termini ſunt ſucceſſive

ceſſive $E+a+b+c+d+e+$ &c., $E+b+c+d+e+$ &c., $E+c+d+e+$ &c., $E+d+e+$ &c., $E+e+$ &c. & ſint primæ differentiæ ſucceſſivæ reſpective D', D'', D''', &c. tum erit $D'=a$, $D''=d'$, $D'''=d''$, &c. & ſeries, quæ eſt ſumma prædictorum terminorum per theor. præc. $A-D'(z+1)+D''(z+1)\times\frac{z}{2}-D'''\times(z+1)\times\frac{z}{2}\times\frac{z-1}{3}+$ &c. ubi A ſit quantitas ad libitum aſſumenda.

2. Sint ſucceſſivæ prædictæ ſummæ reſpective S, S', S'', S''', &c. termini ſucceſſivi novæ ſeriei; $\delta', \delta'', \delta'''$, &c. prædictæ differentiæ; tum erit ſumma novæ ſeriei $B(z+2)-\delta'\times(z+2)\times\frac{z+1}{2}+\delta''\times(z+2)\times\frac{z+1}{2}.\frac{z}{3}+$ &c. ubi quantitas B ad libitum aſſumenda eſt; & ſic deinceps.

Cor. 1. Hinc tot quantitates in hâc ſerie aſſumi ad libitum poſſunt, quot ſucceſſivæ ſummæ requiruntur.

3. Sit ſeries $a+b+c+d+e+$ &c. addatur unus terminus A, & erit $A+a+b+c+d+e+$ &c. ſint ſucceſſivæ differentiæ prioris ſeriei d', d'', d''', &c. reſpective, & erunt ſucceſſivæ differentiæ poſterioris ſeriei $A-a=D'$, $d'=D'-D''$, $d''=D''-D'''$, $d'''=D'''-D''''$ &c. ſit ſumma prioris ſeriei $a-zd'+z\times\frac{z-1}{2}d''-$ &c. & e præced. conſtat etiam ſummam poſterioris eſſe $A-(z+1)D'+(z+1)\times\frac{z}{2}D''+$ &c. ſi vero $A=0$ tum ſumma hujus ſeriei eadem erit ac præcedentis ſumma.

Cor. 2. Et ſic ſi modo ab alio puncto incipiamus: lex enim fere eadem erit, ut conſtat e præced. Et tot aſſumi poſſunt ad libitum incognitæ quantitates, quot ſunt illæ differentiæ, quæ ortum ſuum ducunt ante primam differentiam ſeriei prædictæ $a-b$.

3. Erit $a-zd'+z.\frac{z-1}{2}d''-z.\frac{z-1}{2}.\frac{z-3}{3}d'''+$ &c. $=a$

$$(1 - z + z \cdot \frac{z-1}{2} - z \cdot \frac{z-1}{2} \cdot \frac{z-2}{3} + z \cdot \frac{z-1}{2} \cdot \frac{z-2}{3} \cdot \frac{z-3}{4} - \&c.) + bz(1 - (z-1) + (z-1) \cdot \frac{z-2}{2} - (z-1) \cdot \frac{z-2}{2} \cdot \frac{z-3}{3} + \&c.) + c \cdot z \cdot \frac{z-1}{2}(1 - (z-2) + (z-2) \cdot \frac{z-3}{2} - (z-2) \cdot \frac{z-3}{2} \cdot \frac{z-4}{3} + \&c.) + d \times z \cdot \frac{z-1}{2} \cdot \frac{z-2}{3}(1 - (z-3) + (z-3) \cdot \frac{z-4}{2} - (z-3) \cdot \frac{z-4}{2} \cdot \frac{z-5}{3} + \&c.) - e, \&c.$$

ſi modo data ſeries ſit $a + b + c + d +$ &c.

THEOR. XXI.

1. Sint ſeries quantitatum a^m, $(a + b)^m$, $(a + 2b)^m$, $(a + 3b)^m$, &c. quarum radices ſunt in arithmeticâ progreſſione, ubi a & b ſunt affirmativæ quantitates; tum præcedentes differentiæ ſingulorum ordinum, qui ſunt minores quam m, erunt minores quam ejuſdem ordinis ſubſequentes; e contra erunt majores quam ſubſequentes in ordinibus qui proxime majores ſunt quam m; & præcedentes differentiæ primi ordinis majores erunt quam ſubſequentes, ſi m negativa ſit quantitas, vel minor quam unitas.

Sit m integer affirmativus numerus, tum erunt differentiæ ordinis m inter ſe æquales.

Hæc principia etiam applicari poſſunt ad plures compoſitas quantitates.

2. Invenire valorem quantitatis n, cum præcedentes primæ differentiæ evadant minores quam ſubſequentes, i. e. $a^m - n(a + b)^m + n \cdot \frac{n-1}{2}(a + 2b)^m - n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3}(a + 3b)^m +$ &c. fiat major vel minor quam $a^m - (n + 1)(a + b)^m + (n + 1) \cdot \frac{n}{2}(a + 2b)^m -$ &c.; & conſimiliter invenire valorem prædictum, cum primæ differentiæ evadant

majores vel minores quam ſecundæ, &c. i. e. $a^m - n(a+b)^m + n.\frac{n-1}{2}(a+2b)^m -$ &c. evadat major vel minor quam $(a+b)^m - n(a+2b)^m + (n-1).\frac{n}{2}(a+3b)^m -$ &c. inveniatur prope valor quantitatis n, cum prædictæ quantitates evadant æquales; & id, quod requiritur, conficitur: & ſimiliter reſolvi poteſt hoc problema, cum dentur quantitates n & m; vel n & ratio, quam habent quantitates a & b inter ſe, & requiritur prædicta ratio, vel quantitas m.

Cor. 1. Erit quantitas (A) $a^m - l.(a+b)^m + l.\frac{l-1}{2}(a+2b)^m - l.\frac{l-1}{2}.\frac{l-2}{3}(a+3b)^m +$ &c. ad finem ſeriei nihilo æqualis, ſi m minor ſit quam l, & m & l ſint integri affirmativi numeri; ſi vero $m = l$, erit prædicta quantitas $= \pm 1.2.3.4.5\ldots l \times b^m$: ſignum affixum erit + ſi m ſit par, ſin aliter −.

Hæc enim quantitas erit prima differentia (l) ordinis prædictarum quantitatum in arithmeticâ progreſſione, quæ ſemper erit nihilo æqualis, cum l major ſit quam m, & l & m ſint integri numeri.

Conſimiles propoſitiones affirmari poſſunt de caſibus, in quibus vel a vel b vel utræque ſunt negativæ quantitates, in utriſque prioribus caſibus quantitates creſcant uſque ad infinitum, & deinde decreſcant.

PROB. XXV.

Detur ſeries ordinatarum intervallis quibuſcunque ab ſe invicem diſtantium, pergens vero ex unâ tantum parte in infinitum; invenire lineam parabolicam, quæ tranſit per extremitates omnium.

Sint A, $A1$, $A2$, $A3$, $A4$, &c. ordinatæ inſiſtentes abſciſſæ in angulis rectis, ſitque R punctum quodlibet in abſciſsâ: & ponatur $a = RA$, $b = RA1$, $c = RA2$, $d = RA3$, &c. Attamen deſignet T ordinatam quamvis in genere, cujus diſtantia a puncto R ſit z; ponatur etiam

$B=\frac{A1-A}{b-a}$, $C=\frac{B1-B}{c-a}$, $D=\frac{C1-C}{d-a}$, $E=\frac{D1-D}{e-a}$, &c.

$B1=\frac{A2-A1}{c-b}$, $C1=\frac{B2-B1}{d-b}$, $D1=\frac{C2-C1}{e-b}$, &c.

$B2=\frac{A3-A2}{d-c}$, $C2=\frac{B3-B2}{e-c}$, $D2=\frac{C3-C2}{f-c}$, &c.

&c. &c. &c.

atque erit ordinata $T=A+B\times(z-a)+C\times(z-a)\times(z-b)+D\times(z-a)\times(z-b)\times(z-c)+E\times(z-a)\times(z-b)\times(z-c)\times(z-d)+$ &c. facile conſtabit hoc prob. e ſcribendo in datâ æquatione pro z in ordinata $z=A+B(z-a)+$ &c. ejus ſucceſſivos valores a, b, c, &c.

Cor. . Hinc, ſi modo ſcribantur fluxiones radicum pro differentiis ordinatarum; & pro earum intervallis fluxiones abſciſſæ, inveſtigari poteſt radix fluxionalis æquationis.

PROB. XXVI.

Detur ſeries ordinatarum utrinque excurrens in infinitum, invenire lineam parabolicam, quæ tranſibit per extremitates omnium.

1. Deſignet a ordinatam in medio omnium, ſintque $a2$, $a4$, $a6$, $a8$, &c. eæ ex unâ parte; & $2a$, $4a$, $6a$, $8a$, &c. ex alterâ, pergente progreſſione utrinque in infinitum. Collige earum differentias primas $7B$, $5B$, $3B$, $1B$, $B1$, $B3$, $B5$, $B7$; ſecundas $6b$, $4b$, $2b$, b, $b2$, $b4$, $b6$; tertias $5C$, $3C$, $1C$, $C1$, $C3$, $C5$; quartas $4c$, $2c$, c, $c2$, $c4$; & ſic in reliquis, auferendo ſemper antecedentes de conſequentibus ut in prob. præc. Sint jam a, b, c, d, e, &c. ordinata media & mediæ differentiæ in ordinibus alternis reſpective; ſintque $1B$ & $B1$, $1C$ & $C1$, $1D$ & $D1$, &c. duæ mediæ differentiæ in reliquis ordinibus, & ponantur $B=1B+B1$, $C=1C+C1$, $D=1D+D1$, $E=1E+E1$; ſitque intervallum inter quamvis ordinatam T & mediam a ad intervallum commune æquidiſtantium ordinatarum ut z ad unitatem; eritque ordinata $T=a+\frac{Bz+bz^2}{1.2}+\frac{2Cz+cz^2}{1.2}$

×

$\times \frac{z^2 - 1}{3 \cdot 4} + \frac{3Dz + dz^2}{1 \cdot 2} \times \frac{z^2 - 1}{3 \cdot 4} \times \frac{z^2 - 4}{5 \cdot 6} + \frac{4Ez + ez^2}{1 \cdot 2} \times \frac{z^2 - 1}{3 \cdot 4}$ $\times \frac{z^2 - 4}{5 \cdot 6} \times \frac{z^2 - 9}{7 \cdot 8} +$ &c. ubi notandum eſt abſciſſam z eſſe negativam, quando ordinata quæſita jacet ad contrarias partes ordinatæ mediæ.

2. Sint jam $1A$ & $A1$ ordinatæ duæ mediæ; $A3$, $A5$, $A7$, $A9$, &c. eæ ex unâ parte; & $3A$, $5A$, $7A$, $9A$, &c. ex alterâ. Collige earum differentias primas $8a$, $6a$, $4a$, $2a$, a, $a2$, $a4$, $a6$, $a8$, &c. ſecundas $7B$, $3B$, $1B$, $B1$, $B3$, $B5$, $B7$; tertias $6b$, $4b$, $2b$, b, $b2$, $b4$, $b6$, &c. & ſic deinceps; ſubducendo ubique priores de poſterioribus: excerpe nunc differentias medias a, b, c, d, e, &c. ut & duas medias in aliis ordinibus, ſcilicet $1A$ & $A1$, $1B$ & $B1$, $1C$ & $C1$, &c. & ponantur $A = 1A + A1$, $B = 1B + B1$, $C = 1C + C1$, &c. & cujuſvis ordinatæ diſtantia T a medio puncto ſit ad intervallum commune æquidiſtantium ut z ad binarium; eritque $T = \frac{A + az}{2} +$ $\frac{3B + bz}{2} \times \frac{z^2 - 1}{4 \cdot 6} + \frac{5C + cz}{2} \times \frac{z^2 - 1}{4 \cdot 6} \times \frac{z^2 - 9}{8 \cdot 10} + \frac{7D + dz}{2} \times$ $\frac{z^2 - 1}{4 \cdot 6} \times \frac{z^2 - 9}{8 \cdot 10} \times \frac{z^2 - 25}{12 \cdot 14} +$ &c.

Uterque caſus facile demonſtrari poteſt, ſcribendo in priori caſu in ordinatâ $T = a +$ &c. pro z, ejus ſucceſſivos valores 0, 1 & -1, 2 & -2, 3 & -3, &c. & pro a, b, c, d, &c. earum reſpectivos valores a, $2a - 2 \times a + a2$, $4a - 4 \times 2a + 6 \times a - 4 \times a2 + a4$, $6a - 6 \times 4a + 15 \times 2a - 20 \times a + 15 \times a2 - 6 \times a3 + a6$, &c. & pro B, C, D, &c. reſpective $2a - a2$, $4a - 2 \times 2a + 2 \times a2 - a4$, $6a - 4 \times 4a + 5 \times 2a - 5 \times a2 + 4 \times a4 - a6$, &c. quæ erunt reſpective ſummæ quantitatum $2a - a$ & $a - a2$; $4a - 3 \times 2a + 3 \times a - a2$ & $2a - 3 \times a + 3 \times a2 - a4$; $6a - 5 \times 4a + 10 \times 2a - 10 \times a + 5 \times a2 - a4$ & $4a - 5 \times 2a + 10 \times a - 10 \times a2 + 5 \times a4 - a6$, &c.: in poſteriori vero caſu pro a, b, c, d, e, &c. ſcribendo reſpective $1A - A1$, $3A - 3 \times 1A + 3 \times A1 - A3$, $5A - 5 \times 3A + 10 \times 1A - 10 \times A1 + 5 \times A3 - A5$, &c.; & pro A, B, C, D, E, &c.; $1A + A1$, $3A - 1A - A1 + A3$, &c.; quæ erunt reſpective ſummæ quantitatum $1A$ & $A1$; $3A - 2 \times 1A + A1$ & $1A - 2 \times A1 + A3$; $5A - 4 \times 3A + 6 \times 1A - 4 \times A1 + A3$ & $3A - 4 \times 1A + 6 \times A1 - 4 \times A3 + A5$; $7A - 6 \times 5A + 15 \times 3A - 20 \times 1A + 15 \times A1 - 6 \times A3 + A5$ & $5A - 6 \times 3A + 15 \times 1A - 20 \times A1 + 15 \times A3 - 6 \times A5 + A7$; &c.

Eodem plane modo deduci poteſt inveſtigatio, cum incipiat abſciſſa z a quocunque alio puncto, quod inter quaſcunque alias ordinatas jacet. Hi quidem omnes caſus e prob. præc. facile erui poſſunt.

Hic vero animadvertendum eſt ſeries ex his methodis deductas haud convergere, ni ordinata quæſita quam proxime exprimi poſſit per ſeriem in terminis abſciſſæ x hujuſce formulæ $A + Bx + Cx^2 + Dx^3 +$ &c. deſignatam.

Ex. Sit formula ſeriei $Q \times (a + bx + cx^2 + dx^3 + ex^4 + \&c.) = y$, ubi Q ſit quæcunque data functio quantitatis x, & n ſint incognitæ coefficientes a, b, c, d, e, &c. tum facile e datis n correſpondentibus valoribus quantitatum x & y per regulas prius traditas, ſi modo ſcribatur $\frac{y}{Q} = v$, acquiri poſſunt (n) coefficientes a, b, c, d, &c. Et ſic de infinitis aliis.

In caſibus prius traditis præſtat, ſi terminus vel ordinata quæſita longe diſtet a terminis vel ordinatis datis, ita transformare datas ordinatas, ut inveniantur aliæ, quæ haud longe diſtant a quæſitâ ordinatâ.

Ex. 2. Sit æquatio $ax^n + bx^{n-1} + cx^{n-2} + \&c. \ldots fx + g + \frac{h}{x} + \frac{k}{x^2} + \frac{l}{x^3} \ldots + \frac{p}{x^m} = y$; ducatur hæc æquatio in x^m, & reſultat $ax^{n+m} + bx^{n+m-1} + cx^{n+m-2} + \&c. \ldots + p = yx^m$, cujus coefficientes a, b, c, d, &c. inveniri poſſunt e datis correſpondentibus valoribus quantitatum x & y, eâdem methodo ac coefficientes æquationis $ax^{n+m} + bx^{n+m-1} + cx^{n+m-2} \ldots + p = v$; ſcribatur enim in priori æquatione pro yx^m ejus valor v, & reſultat ſecunda.

Ex. 3. Sit æquatio $ax^n + bx^{n-1} + cx^{n-2} \ldots + fx + g + \frac{bx^s}{x + \alpha}$

+

$+\frac{kx^{s+1}}{(x+\alpha)^2}+$ &c. $+\frac{lx^t}{x+\beta}+$ &c. $=y$, ducatur hæc æquatio in $(x+\alpha)^2\times(x+\beta)\times$ &c. & reſultat æquatio, cujus reſolutio (ſi modo dentur α, β, γ, &c.) haud differt a præcedente.

Cor. 3. Hinc per n puncta duci poteſt parabolico-hyperbolica curva eâdem facilitate ac parabolica curva per eundem punctorum numerum.

Ex. 4. Sit $y=\frac{1}{ax^n+bx^{n-1}+cx^{n-2}+\&c.}$, ſi modo pro y ſcribatur $\frac{1}{v}$, reſultabit præcedens æquatio $v=ax^n+bx^{n-1}+cx^{n-2}+$ &c. ad parabolicam curvam.

Ex. 5. Sit æquatio $\frac{ax^n+bx^{n-1}+cx^{n-2}+\&c.}{Ax^m+Bx^{m-1}+Cx^{m-2}+\&c.}=y$, & ejus reſolutio aliquantulum differt a præcedente.

PROB. XXVII.

Invenire ſolidum algebraicum, quod per m+n+1 *data puncta tranſeat.*

Aſſumatur æquatio $a+bx+cx^2+dx^3\ldots+fx^n+Bz+Cz^2\ldots Fz^m=y$, ubi x & z ſunt duæ abſciſſæ & y earum correſpondens ordinata; ex datis $m+n+1$ correſpondentibus valoribus duarum abſciſſarum & ordinatæ erui poſſunt coefficientes a, b, c, &c. B, C, &c. Hæc eſt maxime ſimplex formula, quam forſan recipere poteſt problema.

PROB. XXVIII.

1. *Sit* P *algebraica functio quantitatum* x, y, z, &c. *dati generis, quæ* m *incognitas quantitates invariabiles habet, & dentur* m *correſpondentes functionis* P *& quantitatum* x, y, z, &c. *valores; invenire* m *incognitas quantitates* a, b, c, d, &c.

Scribantur in datâ functione P pro literis x, y, z, &c. earum correſpondentes valores reſpective & pro quantitatibus reſultantibus (P)

earum reſpectivi valores, & exorientur m æquationes totidem incognitas quantitates a, b, c, d, &c. habentes, e quibus inveniri poſſunt valores prædictarum quantitatum a, b, c, d, &c.

2. Si vero dentur duæ vel plures quantitates P, Q, &c. quæ ſunt functiones quantitatum x, y, z, &c. & correſpondentes valores quantitatum P, Q, &c. x, y, z, &c. pro ſuis quantitatibus reſpective ſubſtituantur, & ſi exoriantur tot æquationes quot incognitæ quantitates; tum ex iis deduci poſſunt incognitæ quantitates quæſitæ.

Cor. 2. Si æquationes exortæ ſint ſimplices, tum e ſimplicibus æquationibus facile conſtabunt incognitæ quantitates quæſitæ.

PROB. XXIX.

1. *Invenire ſummam fractionum, quarum numeratores creſcunt juxta progreſſionem ſeriei numerorum differentiam conſtantem habentium, vel quarum ultimæ differentiæ ejuſdem ordinis fere ſunt inter ſe in ratione æqualitatis, quarumque denominatores conſtituant progreſſionem quamlibet geometricam.*

Sint $1 : 1 - x$ communis denominatorum ratio, etiamque d', d'', d''', &c. differentiæ primæ, ſecundæ, tertiæ, &c. numeratorum datæ ſeriei $A + Bx + Cx^2 + Dx^3 + \&c.$ & erit per theor. præc. quique terminus, cujus diſtantia e primo ſit $(p) = (A - p\,d' + p \,.\, \frac{p-1}{2} d'' - p \,.\, \frac{p-1}{1} \,.\, \frac{p-2}{2} d''' + \&c.)\, x^p$; ſed erit $\frac{A}{1-x} = A(1 + x + x^2 \ldots x^p + \&c.)$, $\frac{d'}{(1-x)^2} = d'(1 + 2x + 3x^2 \ldots p\,x^{p-1} + \&c.)$, $\frac{d''}{(1-x)^3} = d''(1 + 3x + 6x^2 \ldots p \,.\, \frac{p+1}{2} x^{p-1} + \&c.)$, $\frac{d'''}{(1-x)^4} = d'''(1 + 4x + 10x^2 \ldots + p \,.\, \frac{p+1}{2} \,.\, \frac{p+2}{3} x^{p-1} + \&c.)$, &c. unde ſumma ſeriei erit $\frac{A}{1-x} - \frac{d'x}{(1-x)^2} + \frac{d''x^2}{(1-x)^3} - \frac{d'''x^3}{(1-x)^4} + \&c.$

In hâc ſummâ inventâ ponatur $x = \frac{1}{z}$, & evadet ſeries data $A + \frac{B}{z} + \frac{C}{z^2} + \frac{D}{z^3} +$ &c. & ejus ſumma $\frac{Az}{z-1} - \frac{d'z}{(z-1)^2} + \frac{d''z}{(z-1)^3} - \frac{d'''z}{(z-1)^4} +$ &c.

Hæc facile conſtant ex hâc propoſitione, nempe $(1-x)^{-p} = 1 + px + p \cdot \frac{p+1}{2} x^2 + p \cdot \frac{p+1}{2} \cdot \frac{p+2}{3} x^3 +$ &c.

2. Si vero requiratur ſumma n primorum terminorum ſeriei $A + Bx + Cx^2 + Dx^3 +$ &c. inveniatur ſumma ſeriei in infinitum progredientis, viz. $\frac{A}{1-x} - \frac{d'x}{(1-x)^2} + \frac{d''x^2}{(1-x)^3} -$ &c. e quâ ſubtrahatur ſumma ſeriei, cujus diſtantia a primo ſit n, in infinitum progrediens, i. e. ſit $\mathcal{Q}x^n$ terminus ſeriei oriundus poſt n numerum terminorum, ſit præterea D' prima primarum differentiarum exurgentium poſt terminum $\mathcal{Q}x^n$, D'' prima ſecundarum, D''' prima tertiarum, &c. tum erit ſeries incipiens a termino $\mathcal{Q}x^n$ atque in infinitum pergens $= \frac{\mathcal{Q}x^n}{1-x} - \frac{D'x^{n+1}}{(1-x)^2} + \frac{D''x^{n+2}}{(1-x)^3} -$ &c. ſed $\mathcal{Q} = A - nd' + n \cdot \frac{n-1}{2} d'' - n \cdot \frac{n-1}{2} \cdot \frac{n-2}{4} d''' +$ &c. $D' = d' - nd'' + n \cdot \frac{n-1}{2} d''' -$ &c. $D'' = d'' - nd''' +$ &c. unde $\frac{\mathcal{Q}x^n}{1-x} - \frac{D'x^{n+1}}{(1-x)^2} + \frac{D''x^{n+2}}{(1-x)^3} -$ &c. $=$

$$x^n \left(\begin{array}{l} \frac{A}{1-x} - \frac{nd'}{1-x} + \frac{n \cdot \frac{n-1}{2} d''}{1-x} - \text{\&c.} \\ \qquad - \frac{xd'}{(1-x)^2} + \frac{nxd''}{(1-x)^2} - \text{\&c.} \\ \qquad\qquad + \frac{x^2 d''}{(1-x)^3} - \text{\&c.} \end{array} \right)$$

, ſcribatur $x = \frac{1}{z}$ & reſultat

$$\frac{1}{z^{n-1}}\left(\begin{array}{l} \frac{A}{z-1}-\frac{nd'}{z-1}+\frac{n.\frac{n-1}{2}d''}{z-1}-\frac{n.\frac{n-1}{2}.\frac{n-2}{3}d'''}{z-1}+\&c. \\ \qquad -\frac{d'}{(z-1)^2}+\frac{nd''}{(z-1)^2}-\frac{n.\frac{n-1}{2}d'''}{(z-1)^2}+\&c. \\ \qquad\qquad +\frac{d''}{(z-1)^3}-\frac{nd'''}{(z-1)^3}+\&c. \\ \qquad\qquad\qquad -\frac{d''''}{(z-1)^4}+\&c. \end{array}\right)$$

THEOR. XXII.

Sit convergens ſeries $a+bx+cx^2+dx^3+ex^4+$ &c. in infinitum, cujus termini lente convergant; ſint primæ primarum, ſecundarum, tertiarum, &c. differentiæ reſpective D, D', D'', D''', &c. i. e. $a-b=D$, $a-2b+c=D'$, $a-3b+3c-d=D''$, &c. & erit $a+bx+cx^2+dx^3+ex^4+fx^5+$ &c. $=\frac{a}{1-x}-\frac{Dx}{(1-x)^2}+\frac{D'x^2}{(1-x)^3}-\frac{D''}{(1-x)^4}+$ &c. $=\frac{a}{1-x}-\frac{(a-b)x}{(1-x)^2}+\frac{(a-2b+c)x^2}{(1-x)^3}\frac{(a-3b+3c-d)x^3}{(1-x)^4}+$ &c. $=\frac{a}{1+x}+\frac{(a+b)x}{(1+x)^2}+\frac{(a+2b+c)x^2}{(1+x)^2}+\frac{(a+3b+3c+3d)x^3}{(1+x)^4}+$ &c. ubi a, $a+b$, $a+2b+c$, $a+3b+3c+d$, &c. ſunt primæ ſucceſſivæ ſummæ reſpective.

2. Sit convergens ſeries $a-bx+cx^2-dx^3+$ &c. tum erit $a-bx+cx^2-dx^3+$ &c. $=\frac{a}{1+x}+\frac{(a-b)x}{(1+x)^2}+\frac{(a-2b+c)x^2}{(1+x)^3}+\frac{(a-3b+3c-d)x^3}{(1+x)^4}+$ &c. $=\frac{a}{1-x}-\frac{(a+b)x}{(1-x)^2}+\frac{(a+2b+c)x^2}{(1-x)^3}-\frac{(a+3b+3c+d)x^3}{(1-x)^4}+$, &c.

In quibuſdam caſibus ſeries ſic transformatæ magis celeriter convergant, in aliis vero minime. E. g. 1. Si data ſeries conſtet e terminis in geometricâ progreſſione, viz. ſit ſeries $a + a^2x + a^3x^2 + a^4x^3 + \&c.$ tum erit ſeries hoc modo derivata $= \frac{a}{1-x} - \frac{a-a^2}{(1-x)^2}x + \frac{a-2a^2+a^3}{(1-x)^3}x^2 - \&c. = \frac{a}{1-x} - \frac{1-a}{1-x}Px + \frac{1-a}{1-x}Qx - \frac{1-a}{1-x}Rx + \&c.$ ubi P, Q, R, &c. præcedentes terminos reſpective denotant, unde conſtat ſeriem derivatam etiam eſſe geometricam ſeriem, cujus communis ratio eſt $1 : \frac{1-a}{1-x}x$. Si hæc ratio major ſit quam $1 : ax$, tum magis celeriter converget ſeries reſultans quam data; ſin aliter vero non. 2. Si termini ad infinitam diſtantiam conſtituti magis celeriter convergant quam geometrica ſeries, tum ſeries derivata nunquam converget in majore ratione quam ſeries e terminis in geometricâ progreſſione conſtans; forſan vero in minore ratione, & forſan etiam diverget. E. g. Si $1-x$ major ſit quam x, tum ſemper ſeries prædicta $\frac{a}{1-x} - \frac{Dx}{(1-x)^2}$, &c. evadet ultimo in geometrica progreſſione; ſi minor ſit, ſemper diverget; ſi vero $1-x = x$, tum vel diverget vel converget in ratione minori quam quæcunque ſeries conſtans e terminis in geometricâ progreſſione. 3. Si termini ad infinitam diſtantiam conſtituti evadant prope in ratione æqualitatis; tum, ſi $1-x$ major ſit quam x, ſeries vel converget in ratione ſeriei, cujus termini ſint in geometricâ progreſſione, vel in majore; ſin aliter ſeries reſultantes vel poſſunt eſſe convergentes in ratione prædictâ geometricâ, vel in majore, vel in minore, vel divergentes; ſed plerumque ſi modo conſtent ex algebraicis functionibus diſtantiæ a primo ſeriei termino, & data ſeries ſit convergens, reſultans etiam erit convergens.

3. Erit $1 + bx + b^2x + b^3x^3 + b^4x^4 + \&c. = \frac{1}{1-bx} = \frac{1}{1-ax} - \frac{(a-b)x}{(1-ax)^2} + \frac{(a^2-2ba+b^2)x^2}{(1-ax)^3} - \frac{(a^3-3ba+3ba^2-b^3)x^3}{(1-ax)^4} + \&c.$

4. Infinitæ ſeries haud nunquam reduci poſſunt ad diverſas geometricas ſeries, ita ut termini ſucceſſivi magis celeriter convergant.

E terminis ſeriei datæ, qui maximi ſint ad infinitam diſtantiam facile erui poteſt prima geometrica ſeries; ſubtrahatur hæc ſeries de datâ infinitâ ſerie, & e terminis reſultantis ſeriei, qui maximi ſint ad infinitam diſtantiam, inveniatur ſecunda geometrica ſeries; ſubtrahatur hæc ſeries de ſerie prius reſultante, & per methodum præcedentem continuo repetitam progrediendum eſt.

Si vero ſeries per hanc methodum inventæ haud celeriter convergant, transformetur ea in ſeriem quantitatum hujuſce formulæ $\frac{ax^{r}}{x-\alpha} + \frac{bx^{r'}}{x^2-\beta x-\gamma} + \frac{cx^{r''}}{x^3-\delta x^2+\varepsilon x-\eta} +$ &c. vel hujuſce $\frac{ax^{q}+bx^{r}}{cx^{s}+dx^{t}}$, vel infinitarum aliarum formularum; vel in ſeriem e diverſis ſeriebus, vel quantitatibus, vel rationalibus, vel irrationalibus utcunque in ſeſe ductis, &c. ita vero ut ſeries reſultans maxime celeriter convergat. E. g.

Sit data ſeries $A + Bx + Cx^2 + Dx^3 +$ &c. aſſumatur $\frac{a}{1-px} + \frac{bx}{(1-px)^2} + \frac{cx^2}{(1-px)^3} +$ &c. vel $\frac{a}{1-px} + \frac{bx}{1-qx} + \frac{cx^2}{1-rx} +$ &c. vel quæcunque alia ſeries conſtans e terminis vel rationalibus vel irrationalibus; ſi modo hæ ſeries fiant æquales datæ ſeriei, quicunque ſit valor quantitatis x, & convergant.

Sit formula $Ax + Bx^3 + Cx^5 +$ &c. & aſſumi poteſt ſeries $\frac{Ax}{1+x^2} + \frac{ax^3}{(1+x^2)^2} + \frac{bx^5}{(1+x^2)^3} +$ &c.

Eadem etiam principia applicari poſſunt ad datas ſeries exprimendas cujuſcunque aliæ formulæ vel rationalis vel irrationalis vel aſcendentis vel deſcendentis. E. g. Sit ſeries data $A + \frac{B}{x} + \frac{C}{x^2} + \frac{D}{x^3} +$ &

&c. aſſumatur ſeries $\frac{ax}{x-p}+\frac{b}{(x-p)^2}+\frac{c}{(x-p)^3}+\frac{d}{(x-p)^4}+$ &c. reducatur hæc ſeries ad alteram ſecundum dimenſiones reciprocas quantitatis x progredientem, & evadet $a+\frac{pa}{x}+\frac{(ap^2+b)}{x^2}+\frac{(ap^3+2bp+c)}{x^3}+\frac{(ap^4+3bp^2+3cp+d)}{x^4}+$ &c. & ex æquatis correſpondentibus hujus & datæ æquationis terminis reſultant $a=A$, $p=\frac{B}{A}$, $b=C-ap^2$, $c=D-ap^3-2bp$, & ſic deinceps.

REGULA.

1. In ſummatione ſerierum ſimul addantur plures primi termini, & ex ultimâ ratione terminorum ad infinitam diſtantiam inveniatur prima approximatio (A) ad ſummam e reliquis terminis; deinde inveniatur terminus (t), cujus ſumma eſt A, etiamque differentia (D) inter hunc terminum & ejus correſpondentem datæ ſeriei terminum; ex ultimâ ratione differentiæ D inveniatur prima approximatio (B) ad ſummam, cujus terminus eſt D, & erit $A \pm B$ propior approximatio ad ſummam datæ ſeriei quæſitam; & ſic ex operatione continuo repetitâ reſultabunt novæ approximationes ad ſummam datæ ſeriei quæſitam magis magiſque appropinquantes.

1. 2. Inveniatur e quâcunque methodo quantitas a ſummâ haud longe diſtans; deinde inveniatur differentia inter terminos hujuſce & datæ ſeriei; & ſeriei, cujus terminus eſt differentia prædicta, inveniatur proxime ſumma; hujuſce quantitatis terminorum & differentiæ prædictæ inveniatur differentia; deinde inveniatur quantitas haud longe diſtans e ſummâ ſeriei, cujus terminus eſt ea differentia; & ſic deinceps; & ultimo ad ſummam ſeriei accedere liceat.

Ex. 1. Sit terminus generalis $\frac{az^m+bz^{m-1}+cz^{m-2}+\text{\&c.}}{z^n+pz^{n-1}+qz^{n-2}+\text{\&c.}}$, ubi z ſit diſtantia a primo ſeriei termino, m minor ſit quam n per quantitatem majorem quam 1; i. e. ſeries ſit convergens: addantur plurimi primi termini, ita quidem ut z multo major fiat quam quæcunque radix vel poſſibilis vel impoſſibilis æquationum $z^n + pz^{n-1} + qz^{n-2} +$ &c. $= 0$, & $az^m + bz^{m-1} + cz^{m-2} +$ &c. $= 0$; deinde ultima ratio generalis termini ad infinitam diſtantiam erit $\frac{az^m}{z^n} = \frac{a}{z^{n-m}}$, & conſequenter prima approximatio A ad ſummam datæ ſeriei erit $\int. \frac{a\dot{z}}{z^{n-m}} = \frac{-a}{(n-m-1)z^{n-m-1}}$; terminus vero t (cujus ſumma eſt A) erit $\frac{-a}{n-m-1} \times \left(\frac{1}{z^{n-m-1}} - \frac{1}{(z+1)^{n-m-1}}\right) = \frac{-a}{n-m-1} \times \frac{(n-m-1)z^{n-m-2} + (n-m-1)\left(\frac{n-m-2}{2}\right)z^{n-m-3} + \text{\&c.}}{z^{2n-2m-2} + (n-m-1)z^{2n-2m-3} + \text{\&c.}}$; differentia vero (D) inter hunc terminum & terminum $\frac{az^m + bz^{m-1} + \text{\&c.}}{z^n + pz^{n-1} + \text{\&c.}}$ erit $\frac{-((n-m-2)\left(\frac{n-m-1}{2}\right)a - (n-m-1)(b-ap))z^{n-m-3} + \text{\&c.}}{(n-m-1)z^{2n-2m-2} + \text{\&c.}}$, & conſequenter erit $B = (-(n-m-2) \times \frac{1}{2}a + (b-ap))\ (\alpha) \int. \frac{\dot{z}}{z^{n-m+1}} = (\alpha) \times \frac{-1}{(n-m)z^{n-m}}$, & exinde invenitur propior approximatio $A \pm B$; & ſic deinceps.

Aliter: pro ſummâ ſeriei, cujus generalis terminus eſt $\frac{a}{z^{n-m}}$, aſſumatur approximatio $\frac{a'}{z^{\frac{n-m-1}{r}} \times (z+1)^{\frac{n-m-1}{r}} \times (z+2)^{\frac{n-m-1}{r}} \, .. \, (z+r-1)^{\frac{n-m-1}{r}}}$, ubi r eſt quicunque integer numerus, & per methodum prius datam erui poſſunt ſucceſſivæ approximationes.

2. Et ſic detegi poſſunt approximationes ad ſummam datæ ſeriei, cujus lex exprimitur per quamcunque irrationalem functionem quan-

titatis z; etiamque conſtat numerus terminorum, qui primo ſimul addendi ſunt: e. g. ſit generalis terminus $\frac{\pi}{\rho}$; tum ſumma totidem primorum terminorum inveniatur, ut z evadat major quam ulla radix æquationis $\pi = o$ vel $\rho = o$; quo magis quantitas (z) exſuperat omnes prædictas radices, eo magis converget ſeries per hanc methodum reſultans, quamvis non in eâdem ratione: vel hæc ſumma e notis regulis inveſtigari poteſt, irrationalis functio reducatur ad ſeriem terminorum progredientium ſecundum legem, cujus deduci poſſunt ad reſpectivos terminos continuæ approximationes; & ſic deinceps.

E. g. Datis ſummis ſerierum $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \&c. = A$, $1 + \frac{1}{8} + \frac{1}{27} + \frac{1}{64} + \&c. = B$, $1 + \frac{1}{16} + \frac{1}{81} + \frac{1}{256} + \&c. = C$, &c. in infinitum, etiamque quantitate, quæ eſt talis functio quantitatis z diſtantiæ a primo datæ ſeriei termino, ut reducatur ad ſeriem ſubſequentis formulæ $az^{-2} + bz^{-3} + cz^{-4} + \&c.$ tum ſumma ſeriei erit $aA + bB + cC + \&c.$

Ex principiis in hoc problemata traditis erui poteſt lex, quam obſervat convergentia datæ ſeriei ex hâc methodo derivatæ.

Ex. 2. Sit ſeries, cujus termini ultimo ſint prope in geometricâ progreſſione, tum inveniatur ſumma iſtius geometricæ ſeriei; deinde eruatur terminus geometricæ ſeriei, ſubtrahatur hic terminus ab ejus correſpondente datæ ſeriei termino, & deinde per eandem methodum inveniatur approximatio ad ſummam ſeriei, cujus terminus eſt reſultans differentia; & ſic continuo repetitâ operatione tandem exorietur ſumma quæſita.

E. g. Sit infinita ſeries $a + b + c + d + e + f + g + b + k + \&c.$ ſint $r^2a - c = a'$, $r^4a - r'^2a' + e = a''$, $r^6a - r'^4a' + r''^2a'' - g = a'''$, &c. etiamque $r^2b - d = b'$, $r^4b - r'^2b' + f = b''$, $r^6b - r'^4b' + r''^2b'' - b = b'''$, & ſic deinceps; ubi $r = \frac{b}{a}$, $r' = \frac{b'}{a'}$, $r'' = \frac{b''}{a''}$, &c. tum erit ſumma ſeriei $\frac{a^2}{a-b} - \frac{a'^2}{a'-b'} + \frac{a''^2}{a''-b'} - \&c. = a + b + c + d + \&c.$

Subſtituantur pro d', b'; a'', b''; &c. earum reſpective valores, & facile conſtat exemplum.

Ex aſſumptis diverſis denominatoribus facile diverſis modis dividi poteſt data infinita ſeries in infinitas ſeries geometricas.

2. Sit ſeries $a - bx + cx^2 - dx^3 + \&c.$ & erit ſumma quæſita

$$\frac{a}{1+x} + \frac{d'x}{(1+x)^2} + \frac{d''x^2}{(1+x)^3} + \&c. = (1-x)\,(a + (a-b)x + (a-b+c)x^2 + (a-b+c-d)x^3 + \&c.) = (1-x)\left(\frac{a}{1-x} - \frac{bx}{(1-x)^2} + \frac{b+c}{(1-x)^3}x^2 - \frac{b+2c+d}{(1-x)^4}x^3 + \frac{b+3c+3d+c}{(1-x)^5}x^4 - \&c.\right),$$

ubi d', d'', d''', &c. reſpective ſint $a-b$, $a-2b+c$, $a-3b+3c-d$, &c. i. e. prima primarum, ſecundarum, tertiarum, &c. differentiarum.

2. 2. Sit ſeries $a + bx + cx^2 + dx^3 + \&c.$ & erit ſumma

$$\frac{a}{1-x} - \frac{d'x}{(1-x)^2} + \frac{d''x^2}{(1-x)^3} - \&c. = (1+x)\,(a - (a-b)x + (a-b+c)x^2 - (a-b+c-d)x^3 + \&c.) = (1+x)\left(\frac{a}{1+x} + \frac{bx}{(1+x)^2} + \frac{(c+b)}{(1+x)^3}x^2 + \frac{b+2c+d}{(1+x)^4}x^3 + \frac{b+3c+3d+e}{(1+x)^5}x^4 + \frac{b+4c+6d+4e+f}{(1+x)^6}x^5 + \&c.\right)$$

cujus ſeriei lex cuicunque eam inſpicienti facile conſtabit.

Sit $x = -1$ in priori ſerie $a - bx + cx^2 - \&c.$ vel $x = 1$ in poſteriori $a + bx + cx^2 + \&c.$ & ex his methodis nihil plerumque deduci poteſt, quoniam in his caſibus ſeries evadant infinite magnæ.

Et ſic multiplicetur vel dividatur data ſeries vel per $1-x$ vel $1+x$ vel per quamcunque aliam quantitatem; & exorientur novæ ſeries, quarum ſummæ vel per hanc, vel per alias methodos traditas, forſan innoteſcent.

Ex. 4. Sit generalis terminus ſeriei, cujus ſumma requiritur, $=$

$1\,r^x$;

$\frac{1}{z^2}r^z$; aſſumatur $\frac{1}{z^2} \times \frac{r^z}{1-r}$ pro approximatione ad ſummam ſeriei quæſitam; ſi vero hæc ſit ſumma ſeriei, tum ejus terminus erit $\frac{1}{z^2} \times \frac{r^z}{1-r} - \frac{1}{(z+1)^2} \times \frac{r^{z+1}}{1-r} = \frac{(1-r)z^2+2z+1}{z^2(z+1)^2} \times \frac{r^z}{1-r}$, ſubtrahatur hic terminus de termino $\frac{1}{z^2}r^z$, & reſultat $\frac{((z+1)^2(1-r)-}{z^2(z+1)^2}$ $\frac{(1-r)z^2-2z-1)r^z}{(1-r)} = \frac{-(2z+1)r^{z+1}}{z^2(z+1)^2(1-r)}$; deinde pro ſubſequente approximatione aſſumatur $\frac{-2}{z^3} \times \frac{r^{z+1}}{(1-r)^2}$, unde propior approximatio ad ſummam ſeriei erit $\frac{1}{z^2} \times \frac{r^z}{1-r} - \frac{2}{z^3} \times \frac{r^{z+1}}{(1-r)^2}$ &c.; & ſic deinceps.

Cor. . Hinc deduci poteſt ſumma cujuſcunque ſeriei hujuſce generis, i. e. cujus terminus generalis ſit functio algebraica quantitatis z in r^z; reduci enim poteſt algebraica functio (ſi haud per alias notas methodos detegi poſſit ſumma quæſita) ad terminos ſecundum reciprocas dimenſiones quantitatis z progredientes, & deinde per methodum hic traditam deduci poteſt ſumma ſeriei quæſita.

Cor. . Approximatio hujuſcemodi ſeriei ſic inventa pendet e ratione prius prolatâ, etiamque e ratione quam habet $1-r:r$.

Et ſic inveniri poteſt aggregatum e pluribus hujuſcemodi ſeriebus, i. e. pluribus ſeriebus, quarum termini ultimo ſint in geometricâ progreſſione.

Hæc principia etiam applicari poſſunt ad ſeries, quarum termini involvunt functionem quantitatis z continuo creſcentem; ſed hæ ſeries, cum z ſit permagna quantitas, ſine ullâ transformatione celerrime convergunt.

3. Summa ſeriei, cujus generalis terminus datur, etiam detegi poteſt ex reducendo datum terminum ad ſeriem ſecundum dimenſiones quantitatis z diſtantiæ a primo termino progredientem; deinde ex

aſſumendo ſeriem formulæ prius traditæ pro ejus integrali, & ejus incrementum deducendo, & ſeriei generalem terminum exprimenti æquale reddendo.

Ex. 1. Sit generalis terminus z^n; aſſumatur pro ejus integrali $az^{n+1} + bz^n + cz^{n-1} + dz^{n-2} + ez^{n-3} +$ &c., cujus inveniatur incrementum $a((z+1)^{n+1} - z^{n+1}) + b((z+1)^n - z^n) + c((z+1)^{n-1} - z^{n-1}) +$ &c. $= ((n+1)a)z^n + (nb + (n+1) \cdot \frac{n}{2}a)z^{n-1} + ((n-1)c + n \cdot \frac{n-1}{2}b + (n+1) \cdot \frac{n}{2} \cdot \frac{n-1}{3}a)z^{n-1} + ((n-2)d + (n-1) \cdot \frac{n-2}{2}c + n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3}b + (n+1) \cdot \frac{n}{2} \cdot \frac{n-1}{3} \cdot \frac{n-2}{4}a)z^{n-3} +$ &c. $= z^n$; & exinde $a = \frac{1}{n+1}$, $b = -\frac{1}{2}$, $c = \frac{1}{12}n$, $d = 0 \times n \cdot \frac{n-1}{2}$, $e = -\frac{1}{120} \times n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3}$, &c.

Si n ſit negativus numerus, tum pro n ſcribatur $-n$ in præcedente ſerie, & reſultat ſeries quæſita.

Ex. 2. Invenire ſummam logarithmorum, quorum numeri ſunt a, $a+n$, $a+2n$, ... $z-n$; hujuſce ſummæ decrementum ſit log. $z-n$, quod inveniatur log. $z - \frac{n}{z} - \frac{n^2}{2z^2} - \frac{n^3}{3z^3} - \frac{n^4}{4z^4} -$ &c.: pro integrali hujuſce decrementi aſſumatur Az log. $z + bz + c -$ integ. incr. $\frac{n}{2z} + \frac{d}{z} + \frac{e}{z^2} + \frac{f}{z^3} +$ &c., cujus decrementum erit $A(z$ log. $z - (z-n)$ log. $(z-n)) + bn - \frac{n}{2z} + d\left(\frac{1}{z} - \frac{1}{z-n}\right) + e\left(\frac{1}{z^2} - \frac{1}{(z-n)^2}\right) + f\left(\frac{1}{z^3} - \frac{1}{(z-n)^3}\right) +$ &c. $= A(n$ log. $z + n - \frac{n^2}{2z} - \frac{n^3}{2 \cdot 3z^2} - \frac{n^4}{3 \cdot 4z^3} - \frac{n^5}{4 \cdot 5z^5} -$ &c.$) + bn - \frac{n}{2z} - \frac{dn}{z \cdot z-n} - \frac{2nz-n^2}{z^2(z-n)^2} \times e - \frac{3z^2n - 3n^2z + n^3}{z^3(z-n)^3} \times f$
— &c.

$-\&c. = An \log. z + An + bn - \frac{n}{2z} - \frac{n^2}{2z}A - \left(\frac{1}{2.3}An^3 + dn\right)z^{-1} - \left(\frac{1}{3\cdot4}An^4 + dn^2 + 2ne\right)z^{-3} - \left(\frac{1}{4\cdot5}An^5 + dn^3 + 3n^2e + 3nf\right) z^{-4} - \&c.$; fiant correſpondentes hujuſce & dati incrementi termini inter ſe æquales, & reſultant $An = 1$, unde $A = \frac{1}{n}$; $An = -bn$; & $\frac{1}{2\cdot3}An^3 + dn = \left(\frac{1}{2\cdot3}n^2 + dn\right) = \frac{n^2}{2}$, unde $d = \frac{n}{3}$; & $\frac{1}{3\cdot4}An^4 + dn^2 + 2ne = \frac{n^3}{3}$ & exinde $\frac{1}{3\cdot4}n^3 + \frac{n^3}{3} + 2ne = \frac{n^3}{3}$, & $e = \frac{-n^2}{2.12}$, &c.; & ſeries quæſita $\frac{1}{n}z \log. z - \frac{1}{n}z + c -$ integ. incr. $\frac{n}{2z} + \frac{n}{3z} - \frac{n^2}{24z^2} - \&c.$

Cum duo valores α & β quantitatis (z), inter quos requiritur integralis, ſint permagni; & inveniatur integralis incrementi $\frac{1}{z}$ inter prædictos valores α & β poſita, converget data ſeries: ſi vero α & β vel α vel β habeat perparvam rationem ad quantitatem n; tum vel inveniatur ſeries ſecundum dimenſiones quantitatis z aſcendens, vel interpolentur plures ſeries inter valores α & β quantitatis z prædictos.

Cor. Interpolare quantitates inter ſucceſſivos terminos contenti $1 \cdot 2 \cdot 3 \cdot 4 \ldots z$; inveniatur ejus log. qui erit $l.1 + l.2 + l.3 \ldots + l.z$; hujuſce ſeriei inveniatur ſumma pro dato valore quantitatis z, & invenitur log. quantitatis quæſitæ; e dato logarithmo inveniatur numerus ei correſpondens, & perficitur problema.

Hæc principia etiam applicari poſſunt ad quam plurima conſimilia contenta.

4. Sit æquatio data relationem inter ſucceſſivos ſeriei datæ terminos (t & t') exprimens, & facile per methodos infinitarum ſerierum prius datas deduci poteſt ſeries, quæ exprimit ſummam datæ ſeriei, i. e. rejiciantur ex datâ vel datis æquationibus omnes termini, qui perparvi ſint reſpectu habito ad reliquos, & ex æquatione vel æquatio-

nibus reſultantibus inveniatur proximus valor ſummæ quæſitæ, & ſic deinceps: vel aſſumi poſſunt ſeries, quæ continent integrales quæſitas in terminis quantitatum z & quorundam præcedentium terminorum, quibus in datis æquationibus pro ſuis valoribus ſubſtitutis, & correſpondentibus terminis reſultantium æquationum inter ſe æquatis exinde deduci poſſunt ſeries quæſitæ. E. g. Sit ſeries $(z^{\theta} + az^{\theta-1} + bz^{\theta-2} + \&c.)\, t = (z^{\theta} + ez^{\theta-1} + fz^{\theta-2} + \&c.)\, t'$, & exinde $t' = \frac{z^{\theta} + az^{\theta-1} + \&c.}{z^{\theta} + ez^{\theta-1} + \&c.}\, t = \left(1 + \frac{a - e}{z} + \frac{b - ae + e^2 - f}{z^2} + \&c.\right) t$; aſſumatur quantitas bz^{α} pro ſummâ quæſitâ, & conſequenter duo ſucceſſivi termini t & t' erunt reſpective $b(z^{\alpha} - (z+1)^{\alpha}) = -b\left(\alpha z^{\alpha-1} + \alpha \cdot \frac{\alpha-1}{2} z^{\alpha-2} + \&c.\right)$, & $b((z+1)^{\alpha} - (z+2)^{\alpha}) = -b\left(\alpha z^{\alpha-1} + 3\alpha \cdot \frac{\alpha-1}{2} z^{\alpha-2} + \&c.\right)$, unde $t' = \frac{-b\left(\alpha z^{\alpha-1} + 3\alpha \cdot \frac{\alpha-1}{2} z^{\alpha-2} + \&c.\right)}{-b\left(\alpha z^{\alpha-1} + \alpha \cdot \frac{\alpha-1}{2} z^{\alpha-2} + \&c.\right)}\, t = \left(1 + \frac{\alpha \cdot (\alpha-1)}{z} + \&c.\right) t$, & $\alpha \cdot (\alpha - 1) = a - e$, & conſequenter $\alpha = \frac{1}{2} \pm \sqrt{(a - e + \frac{1}{4})}$; & ſic ex continuo repetitâ operatione, i. e. ſcribendo $bz^{\alpha} + kz^{\beta} + \&c.$ pro ſummâ quæſitâ facile deduci poteſt approximatio propior ad veram ſummam; & ſic deinceps.

Et ſic de quâcunque datâ relatione hujuſmodi inter ſucceſſivos terminos t, t', t'', &c.

Ex. 2. Sit $t' = \left(\frac{rz^{\theta} + az^{\theta-1} + bz^{\theta-2} + \&c.}{z^{\theta} + ez^{\theta-1} + fz^{\theta-2} + \&c.}\right) t = \left(r + \frac{a - re}{z} + \frac{b - rf - ae + re^2}{z^2} + \&c.\right) t$; pro ſummâ quæſitâ ſcribatur $\frac{z^{\alpha} r^{z-\pi}}{1-r}$; & termini ſucceſſivi t & t' erunt $\left(\frac{z^{\alpha}}{1-r} - \frac{(z+1)^{\alpha} r}{1-r}\right) r^{z-\pi} = \left(z^{\alpha} - \frac{\alpha z^{\alpha-1} r}{1-r} + \&c.\right) r^{z-\pi}$, & $\left(\frac{(z+1)^{\alpha} r - (z+2)^{\alpha} r^2}{1-r}\right) r^{z-\pi} =$

$r^{z-\pi} = \left(r z^{\alpha} - \frac{(2r^2 - r)}{1-r} \alpha z^{\alpha-1} + \&c.\right) r^{z-\pi}$; unde $t' = \left(\frac{r z^{\alpha} - \frac{2r^2 - r}{1-r} \alpha z^{\alpha-1} + \&c.}{z^{\alpha} - \frac{\alpha z^{\alpha-1} r}{1-r} + \&c.}\right) t = \left(r + r \cdot \frac{\alpha}{z} + \&c.\right) t$; deinde fiat $r + \frac{a - re}{z} = r + r \cdot \frac{\alpha}{z}$, & conſequenter $r\alpha = (a - re)$; & ſic deinceps.

Hinc etiam e principiis hic traditis deduci poſſunt propiores approximationes.

Si vero indices datæ æquationis ſint reſpective θ, $\theta - \sigma$, $\theta - 2\sigma$, &c. vel data æquatio ſit quæcunque algebraica vel etiam fluxionalis vel incrementialis functio ſucceſſivorum ſeriei quæſitæ terminorum, & quantitatis z diſtantiæ a primo ſeriei termino; ex principiis hic traditis deduci poteſt ejus ſumma.

Convergentia ſeriei e principiis prius traditis dijudicari poteſt.

Sit æquatio $(z^{\theta} + a z^{\theta-1} + \&c.) t = (r z^{\theta-\pi} - e z^{\theta-\pi-\sigma} + \&c.) t' + (s z^{\theta-\pi'} + \&c.) t'' + \&c.$, ubi π, π', σ, &c. ſunt affirmativæ quantitates; & ultimo erit ſeries maxime convergens.

Hæ ſeries transformari poſſunt per methodos prius traditas, ita ut haud raro magis celeriter convergant.

Ex. 3. Sit æquatio $(z^{\theta} + a z^{\theta-1} + b z^{\theta-2} + \&c.) t = (r z^{\theta} + s z^{\theta-1} + t z^{\theta-2} + \&c.) t'$; reducatur data æquatio ad æquationem $t = \left(r + \frac{a'}{z} + \frac{b'}{z^2} + \&c.\right) t'$; aſſumatur ſumma ſeriei $S = \alpha \times \frac{z+m}{z+n} t$ prope; tum erit $S' = \alpha \times \frac{z+m+1}{z+n+1} t'$ prope; & exinde $S - S' = \alpha \times \left(\frac{z+m}{z+n} t - \frac{z+m+1}{z+n+1} t'\right) = t$, unde $t = \frac{-\alpha (z+n) \cdot (z+m+1)}{(1-\alpha) z + n - \alpha m \cdot z + n + 1} t'$ prope; reducatur hæc æquatio ad æquationem $t_{,} = \left(\frac{-\alpha}{1-\alpha} + \frac{\beta}{z} + \right.$

$\gamma_{,} +$

$\frac{\gamma}{z^2} + \&c.)\,t'$; & ex æquatis correſpondentibus hujuſce & datæ æquationis terminis reſultant tres æquationes $\frac{-\alpha}{1-\alpha} = r$, $\beta = a'$ & $\gamma = b'$; e quibus erui poſſunt $\alpha = \frac{r}{r-1}$, m & n: deinde aſſumatur ſumma feriei $S = \alpha \times \frac{z+m}{z+n}\,t + \pi \times \frac{z+m'}{z+n'}\,t'$ prope; & exinde per methodum prius traditam, i. e. inveniendo differentiam $S - S' = \alpha \times \left(\frac{z+m}{z+n}\,t - \frac{z+m+1}{z+n+1}\,t'\right) + \pi\left(\frac{z+m'}{z+n'} - \frac{z+m'+1}{z+n'+1}\right)t'' = t$; & in hâc æquatione ſcribendo pro t'' ejus valorem $(r + \frac{a'}{z+1} + \frac{b'}{(z+1)^2} + \&c. = r + \frac{a'}{z} + \frac{b'-a'}{z^2} + \&c.)\,t'$, & æquando correſpondentes reſultantis & datæ æquationis terminos deduci poſſunt coefficientes π, m' & n'; & ſic deinceps: vel aſſumatur ſumma $S = \alpha \times \frac{z^b + mz^{b-1} + nz^{b-2} + \&c.}{z^s + m'z^{s-1} + n'z^{s-2} + \&c.} \times t$ vel $S = (\alpha z^b + mz^{b-1} + nz^{b-2} + \&c.)\,t$; pro z in his æquationibus ſcribatur $z+1$, & reſultabunt $S' = \alpha \times \frac{(z+1)^b + m \times (z+1)^{b-1} + \&c.}{(z+1)^s + m' \times (z+1)^{s-1} + \&c.} \times t'$; &c.; & ex æquatione $S - S' = t$ & æquatione exinde reſultante $t = (az^k + bz^{k-1} + cz^{k-2} + \&c.)\,t'$, acquiri poſſunt coefficientes α, m, n, &c., m', n', &c.

5. Datâ æquatione relationem inter diverſos terminos t, t', t'', ... t^n exprimente; invenire ſummam feriei, quæ prædictam habeat relationem: aſſumatur $t = a + \frac{b}{z} + \frac{c}{z^2} + \frac{d}{z^3} + \&c.$, & exinde $t' = a + \frac{b}{z+1} + \frac{c}{(z+1)^2} + \&c. = a + \frac{b}{z} + \frac{c-b}{z^2} + \&c.$, $t'' = a + \frac{b}{z+2} + \frac{c}{(z+2)^2} + \&c. = a + \frac{b}{z} + \frac{c-2b}{z^2} + \&c.$, &c.;

ſcribantur hæ quantitates pro ſuis valoribus in datâ æquatione; & ex æquatis inter ſe vel nihilo correſpondentibus æquationum reſultantium terminis erui poſſunt coefficientes a, b, c, &c.; & exinde ſumma $S = az +$ integ. increm. $\left(\frac{b}{z}\right) + \frac{c}{z} + \frac{d-c}{2z\,.\,z+1} +$ &c.: vel aſſumatur $t = a + \frac{b}{z} + \frac{c}{z\,.\,z+1} + \frac{d}{z\,.\,z+1\,.\,z+2} +$ &c., & exinde $t' = a + \frac{b}{z+1} + \frac{c}{z+1\,.\,z+2} +$ &c. $= a + \frac{b}{z} + \frac{c-b}{z\,.\,z+1} +$ &c., $t'' = a + \frac{b}{z+2} + \frac{c}{z+2\,.\,z+3} +$ &c. $= a + \frac{b}{z} + \frac{c-2b}{z\,.\,z+1} +$ &c., &c. ſcribantur hæ quantitates pro ſuis valoribus in datâ æquatione, & ex æquatis correſpondentibus reſultantis æquationis terminis erui poſſunt coefficientes a, b, c, &c.; unde ſumma quæſita erit $az +$ integ. increm. $\left(\frac{b}{z}\right) + \frac{c}{z} + \frac{d}{2z\,.\,z+1} +$ &c.

Si vero termini haud exprimi poſſint per præcedentes ſeries, tum aſſumatur ſeries $\frac{((z+1)^s - z^s)\,a}{z^s\,.\,(z+1)^s} + \frac{((z+1)^{s+1} - z^{s+1})\,b}{z^{s+1}\,.\,(z+1)^{s+1}} + \frac{((z+1)^{s+2} - z^{s+2})\,c}{z^{s+2}(z+1)^{s+2}} +$ &c., vel $\frac{((z+1)^s - z^s)\,a'}{z^s(z+1)^s} + \frac{((z+1)^{s+1} - z^{s+1})\,b'}{z^{s+1}(z+1)^{s+1}} + \frac{((z+1)^{s+2} - z^{s+2})\,c'}{z^{s+2}(z+1)^{s+2}} +$ &c. $+ \frac{((z+1)^{s+r} - z^{s+r})\,a''}{z^{s+r}(z+1)^{s+r}} + \frac{((z+1)^{s+r+1} - z^{s+r+1})\,b''}{z^{s+r+1}(z+1)^{s+r+1}} +$ &c. $+ \frac{((z+1)^{s+2r} - z^{s+2r})\,a'''}{z^{s+2r}(z+1)^{s+2r}} + \frac{((z+1)^{s+2r+1} - z^{s+2r+1})\,b'''}{z^{s+2r+1}(z+1)^{s+2r+1}} +$ &c. vel $\frac{((z+1)^s - z^s)\,a}{z^s(z+1)^s} + \frac{((z+2)^s - z^s)\,b}{z^s(z+1)^s(z+2)^s} + \frac{((z+3)^s - z^s)\,c}{z^s(z+1)^s\ldots(z+3)^s}$ vel $\frac{((z+1)^s - z^s)\,a}{z^s(z+1)^s} + \frac{((z+1)^{s+r} - z^{s+r})\,b}{z^{s+r}(z+1)^{s+r}} + \frac{((z+1)^{s+2r} - z^{s+2r})\,c}{z^{s+2r} \times (z+1)^{s+2r}} +$ &c., &c., vel aſſumatur ſeries $a\frac{((z+1)^r - z^r e^s)}{z^r(z+1)^r}e^{sz} + b\frac{((z+1)^t - z^t e^l)}{z^t(z+1)^t} \times e^{lz} +$ &c., &c.; harum ſerierum terminorum ſummæ innoteſcunt; vel infinitæ ſeries haud multum diſſimiles aſſumi poſſunt; reducantur hæ quantitates ad ſeries ſecundum dimenſiones quantitatis z progredientes, & ex æquatis correſpondentibus reſultantis & datæ æquationis terminis erui poſſunt coefficientes quæſitæ: vel magis generaliter aſſumatur quæcunque ſeries ſecundum reciprocas dimenſiones quantitatis z pro ſummâ S, & deinde inveniatur ejus ſucceſſiva ſumma S', & exinde earum differentia $S - S'$, cujus termini fiant æquales eorum correſpondentibus terminis in datâ ſerie; & exinde erui poſſunt coefficientes quæſitæ.

6. Eodem modo pro termino quæſito aſſumi poteſt quæcunque ſeries ſecundum reciprocas dimenſiones quantitatis z progrediens, &c.: vel aſſumi poteſt pro ſummâ $S = At + Bt' + Ct'' +$ &c., ubi literæ t, t', t'', &c. ſucceſſivos terminos; & A, B, C, &c. functiones quantitatis z reſpective denotant: vel quæcunque aliæ functiones terminorum t, t', t'', &c., & quantitatis z: vel pro S in quibuſdam caſibus aſſumantur exponentiales quantitates $Ae^{ax^n + bx^{n+m} + cx^{n+2m} + \&c.} +$ &c.

In datis æquationibus relationem inter ſummas S, S', &c. & terminos t, t', &c. exprimentibus rejiciantur omnes termini, qui ex datâ hypotheſi evadant multo minores quam reliqui; & ex æquationibus reſultantibus inveniantur approximationes (π, &c.) ad ſummas vel terminos quæſitos, &c.; pro ſummis vel terminis prædictis ſubſtituantur π, &c. per incognitas quantitates auctæ, & ex æquationibus reſultantibus per methodum hìc traditam inveniantur propiores approximationes; & ſic deinceps.

Hæ reſolutiones ſæpe ſolummodo præbent particularem valorem ſeriei, quod quidem evenit, quoniam pro primâ approximatione ſumitur particularis valor; i. e. haud incipitur a primo ſeriei termino: invenire autem generalem valorem perraro uſui inſerviunt; primo enim præſtat valores ad valorem quæſitum maxime appropinquantes aſſumere, quod reſtinguit reſolutionem ad particularem, quæ nullâ difficultate urgetur.

7. Si generalis valor per ſeriem infinitam quæratur; tum in ſerie ex incrementiali deductâ tot erunt invariabiles quantitates ad libitum

aſſumendæ, quot continentur in generali integrali datæ incrementialis æquationis.

Methodus has generales ſeries inveniendi ex incrementialibus æquationibus vel æquationibus relationes inter ſucceſſivas ſummas & terminos exprimentibus iiſdem difficultatibus urgetur ac methodus inveniendi ſeries ex fluxionalibus æquationibus datis. Series incrementiales in fluxionales infinitas ſeries transformari poſſunt.

Ex. 1. Sit æquatio $t' = \frac{z^2}{z^2+r} t$; aſſumatur $t = a + \frac{b}{z} + \frac{c}{z.\overline{z+1}} + \frac{d}{z.\overline{z+1}.\overline{z+2}} + \&c.$ unde $t' = a + \frac{b}{z+1} + \frac{c}{\overline{z+1}.\overline{z+2}} + \&c. = a + \frac{b}{z} + \frac{c-b}{z.\overline{z+1}} + \&c.$ ſcribantur hi valores pro t & t' in datâ æquatione, & reſultat $t = a + \frac{rA}{z} + \frac{r+1}{2.\overline{z+1}} B + \frac{r+4}{3.\overline{z+2}} C + \frac{r+9}{4.\overline{z+3}} D + \&c.$

Si vero requiratur generalis valor radicis t; aſſumatur $t = az^n + bz^{n-1} + \&c.$ unde $t' = a(z+1)^n + b(z+1)^{n-1} + \&c.$ ſcribantur hæ quantitates pro ſuis valoribus in datâ æquatione, & ex æquatis inter ſe correſpondentibus reſultantis æquationis terminis erui poſſunt coefficientes a, b, &c. quæſitæ.

Hæc propoſitio haud nunquam magis facilem recipiet ſolutionem, ſi modo aſſumatur $s = yt$, vel $t = yt'$, vel $s = Pyt$, &c. ubi P ſit functio quantitatis z, & s ſit ſumma ſeriei quæſita, t vero & t' duo ſucceſſivi termini, &c.; & deinde ita reducantur æquationes ut reſultet æquatio relationem inter y & z exprimens, e quâ inveniatur y in terminis quantitatis z, qui progrediuntur ſecundum formulas in præcedenti libro traditas.

PROB. XXX.

Transformare æquationem inter ſucceſſivas ſummas & terminos deſignantem in infinitam fluxionalem æquationem relationem inter ſummam S vel *terminum* t, *ejus fluxiones, & z diſtantiam a primo ſeriei termino &* $\dot{z}$ *exprimentem.*

In datâ æquatione (cum ſolummodo contineantur ſucceſſivi termini $t, t', t'', t''',$ &c., z & $\underset{.}{z}$) pro $t', t'', t''', t'''',$ &c. $\overset{m}{t'}$, ſcribantur reſpective $t' = t + \frac{\dot{t}}{\dot{z}} \times \underset{.}{z} + \frac{\ddot{t}}{2\dot{z}^2} \underset{.}{z}^2 + \frac{\dddot{t}}{2.3\dot{z}^3} \times \underset{.}{z}^3 + \frac{\ddddot{t}}{2.3.4\dot{z}^4} \times \underset{.}{z}^4 +$ &c.,

$t'' = t + \frac{2\dot{t}}{\dot{z}} \times \underset{.}{z} + \frac{4\ddot{t}}{2\dot{z}^2} \times \underset{.}{z}^2 + \frac{8\dddot{t}}{2.3\dot{z}^3} \times \underset{.}{z}^3 + \frac{\ddddot{t}}{2.3.4\dot{z}^4} \times \underset{.}{z}^4$ vel denique $\overset{m}{t'} = t + \frac{m\dot{t}}{\dot{z}} \times \underset{.}{z} + \frac{m^2\ddot{t}}{2\dot{z}^2} \times \underset{.}{z}^2 + \frac{m^3\dddot{t}}{2.3\dot{z}^3} \times \underset{.}{z}^3 + \frac{m^4\ddddot{t}}{2.3.4\dot{z}^4} \times \underset{.}{z}^4$ + &c. & reſultat æquatio relationem exprimens inter t & ejus fluxiones, z & $\dot{z}$; ubi $\underset{.}{z}$ & $\dot{z}$ ſunt invariabiles quantitates.

Si vero $s, s', s'',$ &c., $\overset{m}{s'}$; $t, t', t'',$ &c., $\overset{n}{t'}$ in datâ æquatione contineantur; pro $\overset{m}{s'}$ ſcribatur $s + \frac{m\dot{s}}{\dot{z}} \times \underset{.}{z} + \frac{m^2\ddot{s}}{2\dot{z}^2} \times \underset{.}{z}^2 + \frac{m^3\dddot{s}}{2.3\dot{z}^3} \times \underset{.}{z}^3 + \frac{m^4\ddddot{s}}{2.3.4\dot{z}^4} \times \underset{.}{z}^4$ + &c.; & pro $t' = \overset{r}{S'} - \overset{r+1}{S'} = -\frac{\dot{s}}{\dot{z}} \times \underset{.}{z} - \frac{((r+1)^2 - r^2)\ddot{s}}{2\dot{z}^2} \times \underset{.}{z}^2 - \frac{((r+1)^3 - r^3)\dddot{s}}{2.3\dot{z}^3} \times \underset{.}{z}^3 - \frac{((r+1)^4 - r^4)\ddddot{s}}{2.3.4\dot{z}^4} \times \underset{.}{z}^4 -$ &c.

Si $y = t \pm \frac{\dot{t}}{\dot{z}} \times \underset{.}{z} + \frac{\ddot{t}}{2\dot{z}^2} \times \underset{.}{z}^2 \pm \frac{\dddot{t}}{2.3\dot{z}^3} \times \underset{.}{z}^3 + \frac{\ddddot{t}}{2.3.4\dot{z}^4} \times \underset{.}{z}^4 \pm$ &c.; cum $y = t$ prope, tum erit $t = y \mp \frac{\dot{y}}{\dot{z}} \times \underset{.}{z} + \frac{\ddot{y}}{2\dot{z}^2} \times \underset{.}{z}^2 \mp \frac{\dddot{y}}{2.3\dot{z}^3} \times \underset{.}{z}^3$ + &c.; & ſimiliter ſi $y = t \pm \frac{m\dot{t}}{\dot{z}} \times \underset{.}{z} + \frac{m^2\ddot{t}}{2\dot{z}^2} \underset{.}{z}^2 \pm \frac{m^3\dddot{t}}{2.3\dot{z}^3} \times \underset{.}{z}^3$ + &c.; cum $y = t$ prope, tum erit $t = y \mp \frac{m\dot{y}}{\dot{z}} \times \underset{.}{z} + \frac{m^2\ddot{y}}{1.2\dot{z}^2} \times \underset{.}{z}^2 \mp$ &c.

Hæc conſtant ex reverſione ſerierum.

Erit $\underset{m}{\underset{.}{s}} = \underset{m-1}{\underset{.}{t}} = \frac{\overset{m}{\dot{s}}}{\dot{z}^m} \times \underset{.}{z}^m + \alpha \frac{\overset{m+1}{\dot{s}}}{\dot{z}^{m+1}} \times \underset{.}{z}^{m+1} + \beta \frac{\overset{m+2}{\dot{s}}}{\dot{z}^{m+2}} \times \underset{.}{z}^{m+2} + \gamma \frac{\overset{m+3}{\dot{s}}}{\dot{z}^{m+3}} \times \underset{.}{z}^{m+3}$

$+\delta$ &c.; ubi erit coefficiens primi termini $= 1 = \frac{\mp 1}{1 . 2 . 3 . 4 . . m} \times (m \times 1^m - m . \frac{m-1}{2} \times 2^m + m . \frac{m-1}{2} . \frac{m-2}{3} \times 3^m - m . \frac{m-1}{2} . \frac{m-2}{3} . \frac{m-3}{4} \times 4^m + m . \frac{m-1}{2} . \frac{m-2}{3} . \frac{m-3}{4} . \frac{m-4}{5} \times 5^m -$ &c.), unde $m \times 1^m - m . \frac{m-1}{2} \times 2^m + m . \frac{m-1}{2} . \frac{m-2}{3} \times 3^m -$ &c. $= \pm 1 . 2 . 3 . . . m$; ſignum affixum erit $+$, ſi m ſit impar numerus; ſin aliter $-$: ſi m ſit negativus numerus vel fractio, tum prædicta infinita ſeries erit æqualis termino, cujus diſtantia a primo ſit $m+1$, ſeriei $1 - 1 . 2 + 1 . 2 . 3 - 1 . 2 . 3 . 4 +$ &c.

Erunt $\pm 1 . 2 . 3 . 4 . . (m+1) \alpha = m \times 1^{m+1} - m . \frac{m-1}{2} \times 2^{m+1} + m . \frac{m-1}{2} . \frac{m-2}{3} \times 3^{m+1} - m . \frac{m-1}{2} . \frac{m-2}{3} . \frac{m-3}{4} \times 4^{m+1} + m . \frac{m-1}{2} . \frac{m-2}{3} . \frac{m-3}{4} . \frac{m-4}{5} \times 5^{m+1} +$ &c.; $1 . 2 . 3 . . (m+2) \beta = m \times 1^{m+2} - m . \frac{m-1}{2} \times 2^{m+2} + m . \frac{m-1}{2} . \frac{m-2}{3} \times 3^{m+2} - m . \frac{m-1}{2} . \frac{m-2}{3} . \frac{m-3}{4} \times 4^{m+1} +$ &c.; $1 . 2 . 3 . . (m+3) \gamma = m \times 1^{m+3} - m . \frac{m-1}{2} \times 2^{m+3} + m . \frac{m-1}{2} . \frac{m-2}{3} \times 3^{m+3} - m . \frac{m-1}{2} . \frac{m-2}{3} . \frac{m-3}{4} \times 4^{m+3} +$ &c.; & ſic deinceps.

Erit $m \times 1^r - m . \frac{m-1}{2} 2^r + m . \frac{m-1}{2} . \frac{m-2}{3} 3^r - m . \frac{m-1}{2} . \frac{m-2}{3} 4^r +$ &c. $= 0$, ſi modo ſit r numerus minor quam m.

Substituantur hæ quantitates pro ſuis valoribus in datâ æquatione, & reſultat æquatio quæſita.

Ex. Sit $t = az^2 t'$; in hâc æquatione pro t & t' ſcribantur reſpective

$s - s' = \frac{\dot{s}}{\dot{z}} \times z - \frac{\ddot{s}}{2\dot{z}^2} \times z^2 +$ &c., & $s' - s'' = \frac{\dot{s}}{\dot{z}} \times z - \frac{3\ddot{s}}{2\dot{z}^2} \times z^2 +$ &c., quibus ſubſtitutis reſultat fluxionalis æquatio $2\dot{s}\dot{z} \times z - \ddot{s}z^2$ &c. $= az^2(2\dot{s}\dot{z}z - 3\ddot{s}z^2$ &c.$)$, ex quâ inveniri poteſt prima approximatio ad ſummam s; & ſic progredi liceat ad inveniendas reliquas approximationes.

THEOR. XXIII.

1. Datis n ſummis quantitatis (P) $\frac{a\overset{m}{\dot{y}}}{\overset{r}{\dot{y}}\dot{x}^{n}} + \frac{b\overset{m-1}{\dot{y}}}{\overset{r}{\dot{y}}\dot{x}^{n-1}} + \frac{c\overset{m-2}{\dot{y}}}{\overset{r}{\dot{y}}\dot{x}^{n-2}} +$ &c. (ubi a, b, c, &c. ſunt datæ coefficientes & $m = n + r$) ad n diverſos valores abſciſſæ x, & ordinatæ y; pro relatione inter abſciſſam x & ejus correſpondentes ordinatas y aſſumatur æquatio $A + Bx + Cx^2 + Dx^3 \ldots x^{n-1} = y$, ex hâc æquatione inveniatur valor quantitatis P, in quantitate reſultante pro x & y ſcribantur earum n correſpondentes valores, & reſultabunt n ſimplices æquationes totidem incognitas quantitates A, B, C, D, &c. habentes, e quibus conſtabunt coefficientes A, B, C, D, &c. quæſitæ.

2. Sint duæ vel tres vel denique n diverſæ quantitates (P) ejuſdem generis, i. e. ſint datæ coefficientes a, b, c, d, &c. in diverſis ſummis diverſæ; tum ex n datis vel ejuſdem vel diverſi prædicti generis quantitatibus ad n diverſos datos valores abſciſſæ x & ordinatæ y, per methodum hìc traditam erui poteſt æquatio $A + Bx + Cx^2 \ldots x^{n-1} = y$, cui correſpondent n datæ ſummæ.

Cor. . Ex inventis æquationibus $A + Bx + Cx^2 +$ &c. $= y$ erui poſſunt quæcunque quantitates, quæ ad variabiles x & y habent relationem facile inveſtigandam.

Si vero aſſumatur æquatio $ax^r + bx^{r+s} + cx^{r+2s} + dx^{r+3s} +$ &c. $= y$, ſcribatur $z = x^s$, & reſultat æquatio prædictæ formulæ $a + bz + cz^2 + dz^3 +$ &c. $= \frac{y}{x^r} = v$, cujus reſolutio prius datur.

Omnia hæc etiam ad æquationes curvas deſignantes, quæ habent aſymptotos, facile applicari poſſunt.

THEOR.

THEOR. XXIV.

1. Sit ſeries $y = a + bx + cx^2 + dx^3 +$ &c. ſupponantur $E, \dot{E}, \ddot{E}, \dddot{E}$, &c. reſpectivi valores quantitatum $y, \dot{y}, \ddot{y}, \dddot{y}$, &c. cum $x = 0$; tum erit integralis $y = E + \frac{\dot{E}x}{\dot{x}} + \frac{\ddot{E}x^2}{1 . 2\,\dot{x}^2} + \frac{\dddot{E}x^3}{1 . 2 . 3\,\dot{x}^3} + \frac{\ddddot{E}x^4}{1 . 2 . 3 . 4\,\dot{x}^4}$ + &c. erunt enim $E = a$, $\frac{\dot{E}}{\dot{x}} = b$, $\frac{\ddot{E}}{1 . 2\,\dot{x}^2} = c$, $\frac{\dddot{E}}{1 . 2 . 3\,\dot{x}^3} = d$, & ſic deinceps.

Fluens vero fluxionis $\int . y\dot{x} = ax + \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{dx^4}{4} +$ &c. $=$ $Ex + \frac{\dot{E}x^2}{2\dot{x}} + \frac{\ddot{E}x^3}{1 . 2 . 3\,\dot{x}^2} + \frac{\dddot{E}x^4}{1 . 2 . 3 . 4\,\dot{x}^4}$ + &c.

FIG. 2. Sit baſis $AP = z$ & $\dot{z} = 1$, cujus ordinata $PM = y$; ſit prima ordinata $AF = a$, $AB = 1$ & area $ABEF = A$; ſint A, B, C, D, E, &c. ſucceſſivæ areæ per ordinatas $y, \dot{y}, \ddot{y}, \dddot{y}$, &c. reſpective generatæ, i. e. fluentes fluxionum $y\dot{z}, \dot{y}\dot{z}, \ddot{y}\dot{z}, \dddot{y}\dot{z}$, &c. tum erit $AF = a$. $= A - \frac{B}{2} + \frac{C}{12} - \frac{E}{720} + \frac{F}{30240}$ — &c. nam $A = a + \frac{\dot{a}}{2} + \frac{\ddot{a}}{2 . 3} + \frac{\dddot{a}}{2 . 3 . 4}$ + &c. & ſic $B = \dot{a} + \frac{\ddot{a}}{2} + \frac{\dddot{a}}{6}$ + &c. $C = \ddot{a} + \frac{\dddot{a}}{2}$ + &c. $D = \dddot{a} + \frac{\ddddot{a}}{2}$ + &c. &c. reducantur hæ æquationes ita ut exterminentur $\dot{a}, \ddot{a}, \dddot{a}$, &c. & reſultat $a = A - \frac{B}{2} + \frac{C}{12}$ — &c. i. e. ex integralibus fluentis & ejus ſucceſſivarum fluxionum invenitur primus terminus: lex coefficientium ita dici poteſt; ſint $k = \frac{1}{2}$, $l = \frac{1}{6}$, $m = \frac{1}{24}$, $n = \frac{1}{120}$, &c. etiamque ſint $K = k = \frac{1}{2}$, $L = Kk - l = \frac{1}{12}$, $M = kL - lK + m = 0$, $N = kM - lL + mK - n = -\frac{1}{720}$, &c. & erit $a = A - KB + LC - MD +$ &c.

Cor. 1. Quoniam areæ A, B, C, D, &c. generantur, dum tranſit ordinata PM ab AF ad BE, ob $\dot{z} = 1$, & exinde area $B =$ differ. inter ordinatas BE & AF, $C =$ differen. inter fluxiones prædictarum ordinatarum, $D =$ differen. inter earum tertias fluxiones, &c. ſcribantur α, β, δ, ϵ, &c. igitur pro $BE - AF$, $\dot{B}E - \dot{A}F$, $\ddot{B}E - \ddot{A}F$, $\ddddot{B}E - \ddddot{A}F$, &c. & evadet $a = A - \frac{\alpha}{2} + \frac{\beta}{12} - \frac{\delta}{720} +$ &c.

Dividatur baſis Aa in æquales ſucceſſivas partes AB, BC, CD, &c. $= \dot{z}$, pro ſummâ æquidiſtantium ordinatarum $AF + BE + CK +$ &c. (excipiatur ultima ordinata af) ſubſtituatur S, & pro areâ $AFfa$, & differentiis $af - AF$, $\dot{a}f - \dot{A}F$, $\ddot{a}f - A\ddot{F}$, &c. ſcribantur reſpective A, α, β, δ, ζ, &c. tum erit $S = \frac{A}{\dot{z}} - \frac{\alpha}{2} + \frac{z\beta}{12\dot{z}} - \frac{z^3\delta}{720\dot{z}^3} + \frac{z^5\zeta}{30240\dot{z}^5} - \frac{z^7\theta}{1209600\dot{z}^7} +$ &c.

2. Poſitis ordinatâ $AF = a$, & lineis AR & Ar ad oppoſita latera puncti A inter ſe æqualibus; lineis RV & rv ordinatis ad R & r, quæ terminant aream $RVvr$; & quâcunque ordinatâ $PM = y$; fluat uniformiter baſis, & denotent literæ A, C, E, &c. areas ſuper baſim Rr a reſpectivis ordinatis y, $\ddot{y}$, $\ddddot{y}$, &c. generatas: ſupponatur $AR = z$, & erit media ordinata $AF = a = \frac{A}{2z} - \frac{zC}{12\dot{z}^2} + \frac{7z^3E}{720\dot{z}^4} - \frac{31z^5G}{30240\dot{z}^6} +$ &c. per theor. enim $\frac{RrvV}{2z} = \frac{A}{2z} = a + \frac{z^2\ddot{a}}{2 \cdot 3\dot{z}^2} + \frac{z^4\ddddot{a}}{2 \cdot 3 \cdot 4 \cdot 5\dot{z}^4} +$ &c. vel $a = \frac{A}{2z} - \frac{z^2\ddot{a}}{6\dot{z}^2} - \frac{z^4\ddddot{a}}{120\dot{z}^4} -$ &c. & ſimiliter $\ddot{a} = \frac{C}{2z} - \frac{z^2\ddddot{a}}{6\dot{z}^2} -$ &c. $\ddddot{a} = \frac{E}{2z} -$ &c. ita reducantur hæ æquationes, ut exterminentur $\ddot{a}$, $\ddddot{a}$, &c. & conſtabit. prop.

Lex hujuſce ſeriei ſic enuntiari poteſt; ſint $k = \frac{z^2}{2 \cdot 3\dot{z}^2}$, $l = \frac{z^2k}{4 \cdot 5\dot{z}^2}$, $m = \frac{\dot{z}^2l}{6 \cdot 7\dot{z}^2}$, $n = \frac{z^2m}{8 \cdot 9\dot{z}^2}$, &c. etiamque $K = \frac{k}{2z} = \frac{z}{12\dot{z}^2}$ $L = kK$ —

$-\frac{l}{2z}$, $M = kL - lK + \frac{m}{2z}$, $N = kM - lL + mK - \frac{n}{2z}$, &c. tum erit $a = \frac{A}{2z} - KC + LE - MG + NI -$ &c. & quoniam areæ C, E, &c. ſunt reſpective fluentes fluxionum $\ddot{y}\dot{z}$, $\ddddot{y}\dot{z}$, &c. ſi reſpectivæ differentiæ fluxionum primi, tertii, quinti, &c. ordinis ordinatarum rv & RV exprimantur per β, δ, γ, θ, &c. tum erit $a = \frac{A}{2z} - \frac{z\beta}{12\dot{z}} + \frac{7z^3\delta}{720\dot{z}^3} - \frac{31z^5\gamma}{30240\dot{z}^5} + \frac{127z^7\theta}{1209600\dot{z}^7} -$ &c.

Cor. 1. Sint AF, BE, CK, &c. ſeries ordinatarum æquidiſtantium ſuper baſim Aa, quarum AF ſit prima & af ultima; AB eorum communis differentia ſit $2z$, & linea $ARabA$ retro ſumpta $= z = \frac{1}{2}AB$ & ar ſumpta in contrariâ directione $\frac{1}{2}ab = z$, & ſi area $RVrv$ ab ordinatis RV & rv terminata dicatur A, & differentiæ inter primas, tertias, quintas, &c. fluxiones ordinatarum rv & RV exprimantur per β, δ, γ, θ, &c. & ſumma ordinatarum $AF + BE + CK \ldots + af = S$; erit $S = \frac{A}{2z} - \frac{z\beta}{12\dot{z}} + \frac{7z^3\delta}{720\dot{z}^3} - \frac{31z^5\gamma}{30240\dot{z}^5} + \frac{127z^7\theta}{1209600\dot{z}^7} - \frac{511z^9\kappa}{47900160\dot{z}^9} +$ &c. unde $A = 2zS + \frac{z^2\beta}{6\dot{z}} -$ &c.

Cor. 1. Hinc inveniri poteſt $m^r + (m+e)^r + (m+2e)^r \ldots n^r = \frac{1}{(r+1).e}(n^r - m^r) + \frac{n^r - m^r}{2} + \frac{re}{12}(n^{r-1} - m^{r-1}) - \frac{r.r-1.r-2}{720} e^3 \times (n^{r-3} - m^{r-3}) +$ &c.

Et ſic $(m+e)^r + (m+3e)^r + (m+5e)^r + \ldots + (n-e)^r = \frac{1}{(r+1).2e} \times (n^{r+1} - m^{r+1}) - \frac{re}{12} \times (n^{r-1} - m^{r-1}) + \frac{7r.r-1.r-2.e^3}{720}(n^{r-3} - m^{r-3}) -$ &c.

Si $r = -s$ & ſumma $\frac{1}{m^s} + \frac{1}{(m+e)^s} + \frac{1}{(m+2e)^s} + \frac{1}{(m+3e)^s} \ldots$, $\frac{1}{n^s}(o) = \frac{1}{(s-1)em^{s-1}} + \frac{1}{2m^s} + \frac{se}{12m^{s+1}} - \frac{s.s+1.s+2.e^3}{720m^{s+3}} +$ &c.

$s.s+1$

$\frac{s\,.\,s+1\,.\,s+2\,.\,s+3\,.\,s+4\,.\,e^5}{30240\,m^{s+5}}$ + &c. & ſic colligi poteſt $\frac{1}{(m+e)^s} + \frac{1}{(m+3e)^s} + \frac{1}{(m+5e)^s} + \&c. = \frac{1}{(s-1)\,2e\,m^{s-1}} - \frac{se}{12\,m^{s+1}} + \frac{7s\,.\,s+1\,.\,s+2\,.\,e^3}{720\,m^{s+3}} - \&c.$

Cor. . Si termini ſeriei ſint alternatim affirmativi & negativi, in priori caſu per theor. erit ſumma affirmativorum $= \frac{A}{2e} - \frac{\alpha}{2} + \frac{2e\beta}{112} - \frac{8e^3\delta}{720} + \&c.$ ſumma vero negativorum per cor. 1. $\frac{A}{2e} - \frac{e\beta}{12} + \cdot\frac{7e^3\delta}{720} - \&c.$ ubi $2e$ ſit communis diſtantia inter ordinatas, & $\alpha = af - AF$; & β, δ, &c. ut antea; ergo differentia inter has duas invenietur $\frac{AF-af}{2} + \frac{e\beta}{4} - \frac{e^3\delta}{48} + \frac{e^5\gamma}{480} - \frac{17e^7\theta}{80640} - \&c.$

Ex. 2. Sint $EA = 2$, $Ea = n$, $AF =$ log. 2, $af =$ log. n; & β, δ, γ, &c. denotent fluxiones diverſas ipſius AF, tum logar. ultimi valoris contenti $\frac{2}{3} \times \frac{4}{5} \times \frac{6}{7} \times \ldots \times \frac{n-2}{n-1} \times 2\surd(n) = \frac{l,2 - l,n}{2} - \frac{\beta}{4} + \frac{\delta}{48} - \frac{\gamma}{480} + \&c. + l\,.\,2 + \frac{l,n}{2} = \frac{3}{2} \times l\,.\,2 - \frac{\beta}{4} + \frac{\delta}{48} - \frac{\gamma}{480} + \&c. =$ (ob $\frac{3}{2} \times l\,.\,2 = 1 + \frac{\beta}{12} - \frac{7\delta}{720} + \frac{31\gamma}{30240} - \&c.$) $1 - \frac{2\beta}{12} + \frac{8\delta}{720} - \&c. =$ (ob $\beta = \frac{1}{2}$, $\delta = \frac{2}{8}$, $\gamma = \frac{24}{32}$, &c.) $1 - \frac{1}{12} + \frac{1}{360} - \frac{1}{1260} + \frac{1}{1680} - \&c.$ ſed quoniam circumferentia circuli (c) cujus radius eſt unitas, ſit $c = 8\,.\,\frac{8}{9}\,.\,\frac{24}{25}\,.\,\frac{48}{49}\,.\,\frac{80}{81}\,.\,\&c.$ quæ ſeries ita ſcribi poteſt $c = 4 \times \frac{4}{9}\,.\,\frac{16}{25}\,\frac{36}{49}\,.\,\frac{64}{81} \cdots \frac{(n-2)^2}{(n-1)^2} \times n$, unde $\surd(c) = \frac{2}{3} \times \frac{4}{5} \times \frac{6}{7} \times \frac{8}{9} \times \&c.$ in infinitum $\times\, 2\surd(n)$, & ejus log. $= \frac{l,c}{2} = 1 - \frac{1}{12} + \frac{1}{360} - \frac{1}{1260} + \frac{1}{1680} - \&c.$

Hæc applicari poſſunt ad inveniendas ſummas logarithmorum quantitatum in arithmeticâ progreſſione.

1. Sit $3 \times \frac{21}{25} \times \frac{77}{81} \times \frac{165}{169} \times \frac{285}{289} \times$ &c. cujus ultimus valor ſit p, & erit $\sqrt{(p)}$ ultimus valor contenti $\frac{3}{5} \times \frac{7}{9} \times \frac{11}{13} \times \frac{15}{17} \ldots \frac{n-4}{n-2}\sqrt{(n)}$; ſit r quicunque numerus in progreſſione 3, 7, 11, 15, &c. & N numerus, cujus logarithmus eſt $\frac{1}{2r} - \frac{1}{3r^3} + \frac{8}{5r^5} -$ &c. tum erit $\sqrt{(p)} = \frac{\sqrt{r}}{N} \times \frac{3}{5} \times \frac{7}{9} \times \frac{11}{13} \times \frac{15}{17} \ldots \frac{r-4}{n-2}$.

2. Et ex his principiis deduci poteſt unicam medii termini ad ſummam unicarum binomii eſſe ut :: $1 : \frac{\sqrt{(c \times (n+1))}}{2N}$ prope, ubi N ſit numerus, cujus log. eſt $\frac{1}{4 \times (r+1)} - \frac{1}{24 \times (r+1)^2} + \frac{1}{20 \times (r+1)^5}$ — &c. & r ſit poteſtas binomii cum ſit par; vel $r+1$ ſit poteſtas, cum ſit impar.

3. Supponatur baſis Aa dividi in æquales partes, & ſit area $AFfa = \mathcal{Q}$, ſumma extremarum ordinatarum $AF + af = A$, & ſumma omnium intermediarum $BE + CK +$ &c. $= B$, baſis $Aa = R$, & eædem quantitates ut antea per β, δ, γ, &c. denotentur; tum erit area $AFfa = \mathcal{Q} = \left(\frac{A}{2n+2} + \frac{nB}{n^2-1}\right)R - \frac{R^4\delta}{720n^2} + \frac{R^6\gamma}{30240} \times \frac{n^2+1}{n^4} -$ &c. nam ſupponâtur $e = \frac{R}{n}$, $S + \frac{af - AF}{2} = B + \frac{A}{2} = \frac{n\mathcal{Q}}{R} + \frac{R\beta}{12n} - \frac{R^3\delta}{720n^3} +$ &c. deinde $\frac{AF + af}{2} = \frac{A}{2} = \frac{\mathcal{Q}}{R} + \frac{R\beta}{12} - \frac{R^3\delta}{720} +$ &c. exterminetur β ex his duabus æquationibus, & reſultat area prædicta.

Rejiciantur δ, γ, θ; &c. & erit area $AFfa = \frac{AR}{2n+2} + \frac{nBR}{n^2-1}$; ſupponatur $n = 2$, unde tres ſolummodo erunt ordinatæ & B media ordinata, & erit area $AFfa = \frac{A+4B}{6}R - \frac{R^4\delta}{4\cdot720} + \frac{5R^6\zeta}{16\cdot30240}$ — &c. ſit $n = 3$ & quatuor ſolummodo ſunt ordinatæ, & erit $B =$ ſummæ primæ & ſecundæ ordinatarum; & area $AFfa = \frac{A+3B}{8} \times R - \frac{R^4\delta}{9\cdot720} + \frac{R^6\zeta}{81\cdot3024}$ — &c. ſi vero rejiciantur δ, ζ, &c. tum erit area $AFfa = \frac{A+3B}{8} \times R$.

Sint quinque ordinatæ, A ſumma primæ & ultimæ, B ſumma ſecundæ & quartæ, C vero media ordinata; erit area $AFfa = \frac{7A+32B+12C}{90} \times R - \frac{31R^6\zeta}{6\times16\times16\times30240}$ + &c. nam per regulam pro tribus ordinatis erit $\frac{\mathcal{Q}}{R} = \frac{A+4C}{6} \times R - \frac{R^3\delta}{4\times720} + \frac{5R^5\zeta}{16\cdot30240}$ — &c. dividatur ſingula baſis in duas æquales partes, & inveniatur per eandem regulam area e ſingulâ parte, addantur hæ duæ partes in unam & reſultat $\frac{2\mathcal{Q}}{R} = \frac{A+4B+2C}{6} - \frac{R^3\delta}{32\cdot720} + \frac{5R^5\zeta}{2\times16\times16\times30240}$ — &c. ex his duabus æquationibus exterminetur δ, & conſtabit area quæſita.

3. Ex his principiis etiam interpolari poſſunt ſeries. Sint FNz figura, cujus ſucceſſivæ ordinatæ ad puncta A, B, C, D, &c. ſemper ſint æquales ſucceſſivis ſummis ordinatarum figuræ FMf, i. e. $AF = AF$, $B\varepsilon = AF + BE$, $C\kappa = B\varepsilon + CK$, $D\lambda = C\kappa + DL$, &c. & requiratur quæcunque intermedia ordinata PN figuræ FNz; occurrat ordinata PN curvæ FMf in M; ſint $AF = a$, $PM = y$, & communis diſtantia ordinatarum $AB = e$, & area $AFMP = \mathcal{Q}$, $\dot{y} - \ddot{a} = \beta$, $\ddot{y} - \dddot{a} = \delta$, &c. & $PN = \frac{\mathcal{Q}}{e} + \frac{a+y}{2} + \frac{e\beta}{12} - \frac{e^3\delta}{720} + \frac{e^5\zeta}{30240}$ — &c. quod demonſtrari poteſt ſupponendo PN ſuc-

ceſſive in locos ordinatarum figuræ FNz ad puncta A, B, C, D, &c. movere, & exinde ejus ſucceſſivos valores per theor. inveniri: ſi vero exterminanda eſt area $\mathcal{Q}$, ſupponatur $AP = m$, & quoniam $\frac{a+y}{2} = \frac{\mathcal{Q}}{m} + \frac{m\beta}{12} - \frac{m^3\delta}{720} + \frac{m^5\daleth}{30240} -$ &c. ſequitur $PN = \frac{a+y}{2} \times \frac{e+m}{e} + \frac{e^2-m^2}{12e} \times \beta - \frac{e^4-m^4}{720e} \times \delta +$ &c.

Sumantur AR & Pr in oppoſitis directionibus reſpective $= \frac{1}{2}AB$, RV & rv occurrant FMf in V & v; ſit $\mathcal{Q}$ area $RVvr$; & $\beta, \delta, \daleth$, &c. differentiæ; quibus prima, tertia, quinta, &c. fluxiones ordinatæ rv ſuperant reſpectivas fluxiones quantitatis RV, & $AB = e$ ut prius; tum erit ordinata $PN = \frac{\mathcal{Q}}{e} - \frac{e\beta}{2\,.\,12} + \frac{7e^3\delta}{8\,.\,720} - \frac{31e^5\daleth}{32\,.\,30240} -$ &c.

Ordinatæ ad puncta A, B, C, D, &c. dicuntur primariæ ordinatæ figuræ FNz vel FMf. Si $Pp = AB$, & pn occurrat FNz in n & FMf in m, tum $pn = PN + pm$ vel $PN - pm$, prout Pp in hâc vel illâ directione ab P ſumatur; & hinc ex quâcunque (PN) intermediâ ordinatâ datâ omnes aliæ ordinatæ figuræ FNz, quarum diſtantiæ ab eâ (PN) ſit $n \times AB$, ubi n eſt integer numerus, facile deduci poſſunt vel addendo vel ſubtrahendo intermedias ordinatas figuræ FMf.

Cor. 1. Sint $\mathcal{T}X$ & $\mathcal{T}'X'$ primariæ ordinatæ figuræ FMf adjacentes ad ordinatam intermediam PN; biſeca $\mathcal{T}\mathcal{T}'$ in x, & occurrat ordinata xy curvæ FMf in y; area $xyvr = q$, ordinata $\mathcal{T}\tau$ ad $\mathcal{T}$ curvæ FNz ſit s, $xy = y$, $rv = u$; tum $PN = s + \frac{q}{e} - \frac{e}{2\,.\,12} \times (\dot{u} - \dot{y}) + \frac{7e^3}{8\,.\,720}(\ddot{u} - \ddot{y}) -$ &c. nam ſi $Rv = a$ & area $RVvr = \mathcal{Q}$, tum $RVyx = \mathcal{Q} - q$ & $PN = \frac{\mathcal{Q}}{e} - \frac{e}{2\,.\,12} \times (\dot{u} - \dot{a}) + \frac{7e^3}{8\,.\,720} \times (\ddot{u} - \dddot{a}) -$ &c. per præced. & $\mathcal{T}\tau = s = \frac{\mathcal{Q} - q}{e} - \frac{e}{2\,.\,12} \times (\dot{y} - \dot{a}) + \frac{7e^3}{8\,.\,720} \times (\ddot{y} - \dddot{a}) -$ &c. unde $PN - s = \frac{q}{e} - \frac{e}{24}(\dot{u} - \dot{y}) + \frac{7e^3}{8\,.\,720} \times (\ddot{u} - \ddot{y})$, & $PN = s + \frac{q}{e} - \frac{e}{24} \times (\dot{u} - \dot{y}) +$ &c.

Cor. 2. Iiſdem figuris Fnz & FMf manentibus, ſit baſis Ff aſymptos poſterioris figuræ; & ſit $A\pi$ præcedens primum terminum AF, minor quam AB, & ſumantur $A\pi = bB = cC = Dd =$ &c. PN ultimo in hoc caſu erit $= T\tau$; i.e. ſupponendo $(PT' = \pi A)$, $\pi n + be + ck + dl +$ &c. ultimo $= AF + BE + CK +$ &c. unde $\pi n = AF - be + BE - ck + CK - dl +$ &c.

Ex. 1. Sint $AF = 1$, $BE = \frac{1}{2}$, $CK = \frac{1}{3}$, $DL = \frac{1}{4}$, &c. tum erunt primariæ ordinatæ figuræ FNz reſpective $1, \frac{3}{2}, \frac{11}{6}, \frac{25}{12}, \frac{147}{60}$, &c. ſit $A\pi = \frac{1}{2}AB$, & quoniam intermediæ differentiæ $be = \frac{2}{3}$, $ck = \frac{2}{5}$, $dl = \frac{2}{7}$, &c. erit $\pi n = 1 - \frac{2}{3} + \frac{1}{2} - \frac{2}{5} + \frac{1}{3} - \frac{2}{7} + \frac{1}{5} - \frac{2}{9}$ &c. $= 2 \times (1 - \log. 2)$.

Ex. 2. Sit $AF = 1$, $BE = \frac{1}{4}$, $CK = \frac{1}{9}$, $DL = \frac{1}{16}$, &c. & ſit $A\pi = -\frac{1}{2}AB$, tum erunt intermediæ differentiæ be, ck, dl, &c. reſpective $\frac{4}{9}$, $\frac{4}{25}$, $\frac{4}{49}$, &c. & ordinatæ AF, $B\varepsilon$, $C\kappa$, $D\lambda$, &c. erunt 1, $1 + \frac{1}{4}$, $1 + \frac{1}{4} + \frac{1}{9}$, &c. & $\pi n = 1 - \frac{4}{9} + \frac{1}{4} - \frac{4}{25} + \frac{1}{9} - \frac{4}{49} +$ &c.

Ex. 3. Sit $AF = 1$, $BE = \frac{1}{5}$, $CK = \frac{1}{9}$, $DL = \frac{1}{13}$, &c. & $A\pi = \frac{1}{2}AB$, tum erit $\pi n = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} -$ &c. $=$ octuplæ parti circumferentiæ circuli, cujus radius eſt 1,

Cor. . Cum termini progrediantur ſine fine, & eorum ſecundæ differentiæ decreſcant, ita ut ultimo evaneſcant; ſit K ultimus valor primarum differentiarum terminorum, & erit $\pi n = \frac{AB - A\pi}{AB} \times K + AF + BE + CK +$ &c. $- be - ck - dl -$ &c. quoniam in hoc caſu fluxiones quantitatum rv & xy ultimo evaneſcunt, PN erit ultimo $= s + \frac{q}{e}$, $q = K \times rx = K \times (AB - A\pi)$; & conſequenter $\pi n = \frac{AB - A\pi}{AB} \times K + AF - be + BE - ck +$ &c.

Conſimile etiam deduci poteſt theorema, cum ſecundæ differentiæ terminorum ad limitem appropinquent.

Sit ſeries 1, 1×1, 1×2, $1 \times 2 \times 3$, $1 \times 2 \times 3 \times 4$, &c. propoſita, invenire terminum, qui intermedius ſit inter duos primos terminos: logarithmus termini quæſiti erit $\frac{K}{2} + \log. 1 - \log. \frac{3}{2} + \log. 2 - \log. \frac{5}{2} + \log. 3 - \log. \frac{7}{2} + \log. 4 -$ &c. $=$ logar. ultimi valoris contenti $\frac{2}{3} \times \frac{4}{5} \times \frac{6}{7} \times \frac{8}{9} \times \ldots \frac{n}{n-1} \surd(n+1) = \log. \frac{\surd(c)}{2}$, & conſequenter terminus ipſe æqualis erit dimidio radicis, quæ poteſt quadratum ſemicircumferentiæ circuli, cujus radius eſt 1.

Hic ultimo pauca adjicienda ſunt de convergentiâ ſerierum in hâc methodo æque ac præcedentibus deductarum.

1. Si ordinata curvæ, cujus fluens requiritur, in infinitum progrediatur, forſan ejus area ſit finita; i. e. fluens ſit finita, quamvis ejus fluxio ſit infinita; tum nunquam ſeries pro areâ, in quâ continetur prædicta infinita ordinata, converget; ni area, i. e. fluens terminet ad ordinatam vel fluxionem, quæ eſt infinita.

1. Si ordinata haud ſit infinita, ſed ejus fluxio cujuſcunque ordinis ſit infinita; tum hæc ſeries haud ultimo converget, ni ab ordinatâ, cujus fluxio eſt infinita, terminetur.

3. Si fluxio areæ vel fluentis cujuſcunque ordinis evadat nihilo æqualis; tum hæc ſeries pro areâ, in quâ continetur prædicta fluxio nihilo æqualis, nunquam converget; ni area terminetur ad ordinatam, in quâ invenitur prædicta fluxio nihilo æqualis. Conſtant e præceden.

4. Eo magis ceteris paribus converget ſeries reſultans, quo magis diſtant termini datæ ſeriei a prædictis punctis; & quo plures proprie interponantur termini, eo magis ſaltem in ratione numeri terminorum interpolandorum converget ſeries.

5. Si valores quantitatis x longe diſtent ab omni radice æquationum reſultantium ex ſupponendo generales terminos, &c. $= o$; tum ſeries y proprie per ſeriem hujuſce formulæ $a + bx + cx^2 +$ &c. exhiberi poteſt, ſin aliter non: ſunt caſus, cum x ſit perparva, in quibus y per ſeriem $a + bx^r + cx^{2r} +$ &c. vel $ax^s + bx^{s+r} + cx^{s+2r} +$ &c., &c., deſignari poteſt; cum autem x ſit permagna quantitas, ſunt caſus, in quibus y per ſeriem $a + bx^{-1} + cx^{-2} + dx^{-3} +$ &c., vel per $a + bx^{-r} + cx^{-2r} +$ &c., vel per $ax^{-s} + bx^{-s-r} + cx^{-s-2r} +$ &c.; &c.; deſignari poteſt; & ad hos caſus conſimilia iis, quæ in hoc problemate ad ſeriem $a + bx + cx^2 +$ &c. dantur, etiam applicari poſſunt.

THEOR.

THEOR. XXIV.

Sit $t = v$, & erit $t' = t + \underset{\cdot}{t} = v + \dot{v} + \frac{1}{2}\ddot{v} + \frac{1}{6}\dddot{v} + \frac{1}{24}\ddddot{v}$ + &c. & exinde $\underset{\cdot}{t} = \dot{v} + \frac{1}{2}\ddot{v} + \frac{1}{6}\dddot{v} + \frac{1}{24}\ddddot{v}$ + &c. unde $\underset{\cdot}{t} + \underset{\cdot\cdot}{t} = \dot{v} + (\frac{1}{2} + 1)\ddot{v} + (\frac{1}{6} + \frac{1}{2} + \frac{1}{2})\dddot{v}$ + &c. & conſequenter $\underset{\cdot\cdot}{t} = \ddot{v} + \dddot{v} + \frac{7}{12}\ddddot{v} + \frac{1}{4}\overset{\cdot\cdot\cdot\cdot\cdot}{v}$ + &c. & ex iiſdem operationibus repetitis erui poſſunt $\underset{\because}{t} = \dddot{v} + 1\frac{1}{2}\ddddot{v} + \frac{5}{4}\overset{\cdot\cdot\cdot\cdot\cdot}{v}$ + &c. $\underset{::}{t} = \ddddot{v} + 2\overset{\cdot\cdot\cdot\cdot\cdot}{v}$ + &c. &c. lex, quam obſervat hæc ſeries, ſic enuntiari poteſt, ſit $\underset{\substack{\cdot \\ n-1}}{t} = \overset{n-1}{\dot{v}} + \alpha\overset{n}{\dot{v}} + \beta\overset{n+1}{\dot{v}} + \gamma\overset{n+2}{\dot{v}}$ + &c. & erit $\underset{\substack{\cdot \\ n}}{t} = \overset{n}{\dot{v}} + \left(\frac{1}{1\,.\,2} + \alpha\right)\overset{n+1}{\dot{v}} + \left(\frac{1}{1\,.\,2\,.\,3} + \frac{1}{1\,.\,2}\alpha + \beta\right)\overset{n+2}{\dot{v}} + \left(\frac{1}{1\,.\,2\,.\,3\,.\,4} + \frac{1}{1\,.\,2\,.\,3}\alpha + \frac{1}{1\,.\,2}\beta + \gamma\right)\overset{n+3}{\dot{v}}$ + &c. unde facile conſtabit lex ſeriei quæſita.

Cor. . Hinc ex datâ incrementiali æquatione relationem inter t, $\underset{\cdot}{t}$, $\underset{\cdot\cdot}{t}$, &c. z & $\dot{z}$ exprimente deduci poteſt infinita fluxionalis æquatio: ſcribantur enim pro t, $\underset{\cdot}{t}$, $\underset{\cdot\cdot}{t}$, &c. in datâ æquatione earum valores, & reſultat æquatio quæſita.

THEOR. XXVI.

Ex reverſione prædictarum ſerierum inveniri poſſunt $\dot{v} = \underset{\cdot}{t} - \frac{1}{2}\underset{\cdot\cdot}{t} + \frac{1}{3}\underset{\because}{t} - \frac{1}{4}\underset{::}{t} + \frac{1}{5}\underset{\cdot\cdot\cdot\cdot\cdot}{t}$ — &c. $\ddot{v} = \underset{\cdot\cdot}{t} - \underset{\because}{t} + \frac{11}{12}\underset{::}{t} - \frac{10}{12}\underset{\cdot\cdot\cdot\cdot\cdot}{t}$ + &c. $\dddot{v} = \underset{\because}{t} - 1\frac{1}{2}\underset{::}{t} + \frac{7}{4}\underset{\cdot\cdot\cdot\cdot\cdot}{t}$ — &c. $\ddddot{v} = \underset{::}{t} - 2\underset{\cdot\cdot\cdot\cdot\cdot}{t}$ + &c. &c.

Cor. . Scribantur hæ quantitates in datâ fluxionali æquatione pro ſuis valoribus v, $\ddot{v}$, $\dddot{v}$, &c. & reſultat infinita incrementialis æquatio.

Series diſtingui poſſunt in diverſos ordines; prout quantitates, quæ exprimunt earum terminos, continent unam, duas, tres vel plures independentes & variabiles quantitates x, y, z, v, &c. E. g. Ea ſeries, cujus termini exprimi poſſunt per quantitatem, in quâ ſolummodo continetur una variabilis quantitas, dici poteſt ſeries primi ordinis; ea, quæ continet duas variabiles quantitates, dici poteſt ſeries ſecundi ordinis; & ſic deinceps.

PROB.

PROB. XXXI.

1. *Conceſsâ omnium ſerierum primi ordinis ſummas generaliter inveniendi methodo; invenire ſummam cujuſcunque ſeriei* (A) *ſecundi, tertii, &c. ordinis.*

Aſſumantur omnes variabiles quantitates (y, z, v, &c.) præter unam x in datâ quantitate, quæ exprimit ſeriei terminos tanquam invariabiles; & exinde e conceſsâ methodo inveniatur ſeriei reſultantis ſumma; deinde in ſummâ reſultante aſſumantur quantitas x & omnes reliquæ præter y tanquam invariabiles, & e prædictâ conceſsâ methodo inveniatur ſeriei reſultantis ſumma; & ſic progrediendum eſt uſque donec omnes invariabiles & independentes quantitates (x, y, z, &c.) ſejunctim tanquam variabiles aſſumptæ fuerint, i. e. toties inveniantur ſummæ reſultantium ſerierum, quot variabiles & independentes quantitates (x, y, z, v, &c.) in datâ ſerie A contineantur; ultima ſumma reſultans vere correcta erit ſumma quæſita.

Ex. 1. Sint z & v quicunque integri numeri 1, 2, 3, 4, 5, &c. & ſit data quantitas A, in cujus formulâ continentur omnes termini ſeriei, cujus ſumma requiritur, $\frac{1}{2z+v} \times \frac{1}{2z+v+1} \times \frac{1}{2z+v+2}$, i. e. ſit ſeries data

$$+\frac{1}{3\cdot4\cdot5}+\frac{1}{4\cdot5\cdot6}+\frac{1}{5\cdot6\cdot7}+\text{\&c. ubi } z=1 \text{ \& } v=1,2,3,\text{\&c.}$$

$$+\frac{1}{5\cdot6\cdot7}+\frac{1}{6\cdot7\cdot8}+\frac{1}{8\cdot9\cdot10}+\text{\&c. ubi } z=2 \text{ \& } v=1,2,3,\text{\&c.}$$

$$+\frac{1}{7\cdot8\cdot9}+\frac{1}{8\cdot9\cdot10}+\frac{1}{9\cdot10\cdot11}+\text{\&c. ubi } z=3 \text{ \& } v=1,2,3,\text{\&c.}$$

&c. &c. &c.

ubi z & v creſcunt per unitatem.

Aſſumo z tanquam invariabilem quantitatem, pro quâ ſcribatur α; & ſeriei, cujus termini exprimuntur per quantitatem reſultantem $\frac{1}{2\alpha+v} \times \frac{1}{2\alpha+v+1} \times \frac{1}{2\alpha+v+2}$ inveniatur ſumma, quæ erit

(B)

(*B*) $\frac{1}{2\alpha+v}\times\frac{1}{2(2\alpha+v+1)}$; in hâc ſummâ pro α ſcribatur ejus prior valor z, & reſultat ſumma $\frac{1}{2z+v}\times\frac{1}{2(2z+v+1)}$: nunc aſſumo v invariabilem, pro quâ ſcribatur β; z vero variabilem, & evadet ſumma (*B*) $\frac{1}{2z+\beta}\times\frac{1}{2(2z+\beta+1)}$; inveniatur ſumma ſeriei, cujus termini exprimuntur per quantitatem $\frac{1}{2(2z+\beta)(2z+\beta+1)}$, quæ erit (*C*) $\frac{1}{2(2z+\beta)}$, pro β in hâc ſummâ (*C*) ſcribatur iterum v, & reſultat ſumma quæſita $\frac{1}{2(2z+v)}$.

2. Ex hinc facile conſtant ſummæ quæſitæ inter quoſcunque valores, vel negativos vel affirmativos quantitatum x, y, z, v, &c. contentæ.

Aſſumantur omnes quantitates (y, v, z, &c.) præter unam (x) in prædictâ quantitate (*P*) tanquam invariabiles; & exinde e conceſsâ methodo inveniatur ſeriei reſultantis ſumma, quæ ſit *B*; in hâc ſummâ pro x ſcribantur α & a; ubi a & α ſint quantitates, inter quas continentur valores quantitatis x; & ſint quantitates reſultantes A & A'; & ſumma ſeriei inter valores a & α quantitatis x erit $A'-A=\mathcal{Q}$: deinde in quantitate $\mathcal{Q}$ ita aſſumantur omnes quantitates a, α, v, z, &c. præter unam y tanquam invariabiles, & exinde e conceſsâ methodo inveniatur ſumma ſeriei, cujus termini ſunt $\mathcal{Q}$, in quâ valores quantitatis y inter b & β ponuntur; & ſic de reliquis; ultima ſumma ſic inventa erit quæſita.

Ex. Invenire ſummam ſeriei, cujus terminus eſt $\frac{1}{2z+v}\times\frac{1}{2z+v+1}\times\frac{1}{2z+v+2}$; in quo incrementum quantitatis v eſt 1 & quantitatis z eſt $\frac{1}{2}$; & valores termini (v) inter 1 & 4 ponuntur; valores autem termini (z) inter 1 & 3 ponuntur.

Primo ſuppono z eſſe invariabilem quantitatem, & invenio ſummam ſeriei exinde reſultantis, quæ erit $\frac{1}{2z+v}\times\frac{1}{2(2z+v+1)}$ (B); at quoniam valores quantitatis (z) ſupponuntur inter 1 & 4 conſiſtere; per prob.   in hâc quantitate (B) pro v ſcribantur reſpective 1 & $4+1=5$, unde reſultat ſumma $\frac{1}{2z+1}\times\frac{1}{2(2z+2)}$ & $\frac{1}{(2z+5)}\times\frac{1}{2(2z+6)}$, & ſumma inter valores prædictos contenta erit $\frac{1}{2z+1}\times\frac{1}{2(2z+2)}-\frac{1}{2z+5}\times\frac{1}{2\times(2z+6)}$: 2^{do}. ſumma ſeriei, cujus terminus eſt $\frac{1}{2z+1}\times\frac{1}{2(2z+2)}-\frac{1}{2z+5}\times\frac{1}{2(2z+6)}$, erit $\frac{\frac{1}{2}}{2z+1}-\frac{\frac{1}{2}}{2z+5}$; at, quoniam valores quantitatis z inter 1 & 3 continentur, in hâc ſummâ pro z ſcribantur reſpective 1 & $3+z,=3+\frac{1}{2}=\frac{7}{2}$, & reſultant reſpective $\frac{\frac{1}{2}}{2+1}-\frac{\frac{1}{2}}{2+5}=\frac{2}{21}$ & $\frac{\frac{1}{2}}{7+1}-\frac{\frac{1}{2}}{7+5}=\frac{1}{48}$; unde ſumma quæſita erit $\frac{2}{21}-\frac{1}{48}=\frac{25}{336}$.

Cor. In his caſibus haud refert, an prius aſſumatur (z) tanquam invariabilis quantitas, an v; & ſic de reliquis variabilibus quantitatibus; nam ex ſingulis hiſce diverſis modis eadem reſultabit ſumma quæſita.

PROB. XXXII.

Invenire generales terminos ſerierum hujuſce generis, quarum ſummæ innoteſcunt.

Aſſumatur quæcunque functio quantitatum $(x, y, z, v, \&c.)$ pro ſummâ quæſitâ; deinde (datis incrementis quantitatum x, y, z, v, &c.) inveniatur generalis terminus ſeriei, cujus ſumma eſt data functio, ex hypotheſi quod (x) ſolummodo eſt variabilis; & ſit quantitas reſultans P: deinde aſſumatur (P) pro ſummâ, & ex hypotheſi quod y ſolummodo ſit variabilis inveniatur generalis terminus ſeriei, cujus ſumma eſt P; & ſic deinceps, de ſingulis reliquis incognitis quantitatibus: terminus ultimo reſultans erit quæſitus.

Ex. Sit $\frac{1}{2z+v}$ functio quantitatum z & v pro ſummâ aſſumptâ: $z_{,}$ & $v_{,}$ incrementa quantitatum z & v: aſſumatur (v) tanquam invariabilis quantitas, & incrementum datæ functionis erit $P = \frac{1}{2z+v} - \frac{1}{2z+v+2z_{,}} = \frac{2z_{,}}{(2z+v)(2z+v+2z_{,})}$; deinde in hoc primo incremento aſſumantur z & $z_{,}$ tanquam invariabiles quantitates, & inveniatur incrementum quantitatis P, quod erit $\frac{2z_{,}}{(2z+v)(2z+v+2z_{,})} - \frac{2z_{,}}{(2z+v+v_{,})(2z+2z_{,}+v+v_{,})} = \mathfrak{Q}$; ergo $\mathfrak{Q}$ erit quantitas, vel terminus, cujus ſumma poteſt eſſe data functio.

Cor. Quamvis unica ſolummodo datur formula termini, quæ correſpondet datæ ſummæ; attamen dantur infinitæ diverſæ ſummæ, quæ correſpondent eidem termino: e. g. ſit π functio (n) quantitatum $(x, y, z, v, w, \&c.)$, cujus terminus ſit R; tum erit generaliter $\pi + \phi(y, z, v, w, \&c.) + \phi'(x, z, v, w, \&c.) + \phi''(x, y, v, w, \&c.) + \phi'''(x, y, z, w, \&c.) + \&c.$; ubi $\phi(y, z, v, w, \&c.)$ denotat functionem quamcunque omnium quantitatum y, z, v, w, &c. præter unam x; & $\phi'(x, z, v, w, \&c.)$ denotat quamcunque functionem quantitatum $(x, z, v, w, \&c.)$ præter unam y; & ſic deinceps; ſumma, cujus terminus eſt R.

Cor. 2. Sit π functio quantitatum $(x, y, z, v, w, \&c.)$, & ſint $\alpha, \beta, \gamma, \delta, \varepsilon$, &c. reſpective incrementa quantitatum x, y, z, v, w, &c.: in functione π pro x, y, z, v, w, &c. ſcribantur reſpective $x+\alpha$, $y+\beta$, $z+\gamma$, $v+\delta$, $w+\varepsilon$, &c., & ſint quantitates reſultantes reſpective A, B, C, D, E, &c.; ſimiliter in prædicta functione pro x & y; x & z, y & z, z & v, &c. ſcribantur reſpective $x+\alpha$ & $y+\beta$, $x+\alpha$ & $z+\gamma$, $y+\beta$ & $z+\gamma$, $x+\alpha$ & $v+\delta$, &c. & ſint quantitates reſultantes reſpective AB, AC, BC, AD, &c.; deinde in π pro x & y & z, x & y & v,

x & z & v, y & z & v, x & y & w, &c. ſcribantur reſpective $x + \alpha$ & $y + \beta$ & $z + \gamma$, $x + \alpha$ & $y + \beta$ & $v + \delta$, $x + \alpha$ & $z + \gamma$ & $v + \delta$, $y + \beta$ & $z + \gamma$ & $v + \delta$, &c. & ſint quantitates reſultantes ABC, ABD, ACD, BCD, &c.: & ſic pro x & y & z & v, &c. ſcribantur in functione π reſpective $x + \alpha$ & $y + \beta$ & $z + \gamma$ & $v + \delta$, &c., & reſultet quantitas $ABCD$, &c.; & ſic deinceps; tum erit terminus (cujus ſumma eſt π) $= \pi - (A + B + C + D +$ &c.$) + (AB + AC + BC + AD + BD + CD +$ &c.$) - (ABC + ABD + ACD + ABE +$ &c.$) + (ABCD + ABCE + ABDE + ACDE + BCDE +$ &c.$) - (ABCDE +$ &c.$) + (ABCDEF +$ &c.$) -$ &c.

PROB. XXXIII.

Iiſdem conceſſis; invenire ſummam ſeriei, cujus termini exprimuntur per datam quantitatem, in quâ continentur variabiles quantitates (x, y, z, v, *&c.*) *ubi literæ* x, y, z, *&c. diverſos denotent numeros vel quantitates, i. e. in dato termino ſit* x *nec idem uumerus ac* y *vel* z, *&c. nec idem numerus ſit* y *ac* z, *&c. &c.*

Dicatur ſumma per prædictam methodum inventa A.

Pro duabus literis x & y, quæ neceſſario diverſæ ſupponuntur, ſcribatur eadem litera α, & inveniatur per prædictam methodum ſumma (a) ſeriei, cujus termini exprimuntur per quantitatem reſultantem; ſcribatur etiam in datâ quantitate pro x & z (quæ ſupponuntur ſemper diverſæ) eadem litera β; & ſic pro y & z ſcribatur eadem litera γ, &c. & per methodum præcedentem inveniantur ſummæ (b, c; &c.) ſerierum, quarum termini exprimuntur per reſultantes quantitates: aggregatum ſummarum ($a + b + c +$ &c.) e ſingulis hujuſmodi ſeriebus dicatur B.

Pro tribus literis x, y & z quæ ſupponuntur inter ſe diverſæ, in datâ quantitate ſcribatur eadem litera ε, & per præcedentem methodum inveniatur ſumma (e) ſeriei, cujus termini exprimuntur per ſeriem reſultantem; & ſi pro ſingulis tribus literis diverſos numeros eſſe ſuppoſitis ſcribantur in datâ quantitate eædem literæ reſpective, & per prædictam methodum inveniantur ſummæ (f, g, &c.) ſerierum, quarum termini exprimuntur per reſultantes quantitates reſpective; & aggregatum ($e+f+g+$ &c.) e ſingulis ſummis dicatur C.

Pro quatuor literis (x, y, z, v), quæ ſupponuntur eſſe diverſæ, ſcribatur in datâ quantitate eadem litera, & inveniatur ſumma (l) ſeriei, cujus termini exprimuntur per quantitatem reſultantem; & ſic ſcribantur pro ſingulis quatuor literis diverſos numeros eſſe ſuppoſitis eædem literæ, & inveniantur ſummæ ſerierum, quarum termini exprimuntur per literas reſultantes; & aggregatum ex his ſummis dicatur D.

Pro literis x & y, quæ diverſæ ſupponuntur, ſcribatur in datâ quantitate eadem litera; etiamque pro z & v, quæ diverſæ ſunt, ſcribatur eadem litera, ſed e priori diverſa; & inveniatur ſumma ſeriei, cujus termini exprimuntur per quantitatem reſultantem; & ſic inveniantur ſummæ ſerierum, quarum termini exprimuntur per quantitates ex conſimilibus ſubſtitutionibus deductas: aggregatum ex his ſummis dicatur BB; &c.

Summa quæſita erit $A - 1 . B + 1 . 2 C - 1 . 2 . 3 D +$ &c.
$+ 1 . 1 BB -$ &c.

Hujus ſeriei lex e prob. 3. medit. algebraic. colligi poteſt.

Ex. 1. Sit quantitas, quæ denotat reſpectivos terminos datæ ſeriei, cujus ſumma requiritur, $\frac{1}{x^4+y^3}$, ubi x & y integros quoſcunque numeros 1, 2, 3, &c. a ſe invicem diverſos reſpective denotant.

Aſſumatur x tanquam invariabilis quantitas (a), & inveniatur ſumma ſeriei, cujus termini erunt $\frac{1}{a^4+y^3}$; deinde in ſummâ inventâ pro a ſcribatur x, & aſſumatur y tanquam invariabilis quantitas, & inveniatur ſumma ſeriei reſultantis, quæ dicatur A.

Deinde pro x & y in datâ quantitate $\frac{1}{x^4+y^3}$ ſcribatur eadem litera x, & inveniatur ſumma ſeriei, cujus termini exprimuntur per quantitatem reſultantem $\frac{1}{x^4+x^3}$, quæ dicatur B; tum ſumma quæſita erit $A-B$.

Si vero in datâ quantitate contineantur quæcunque irrationales quantitates, eadem erit methodus problema reſolvendi ac præcedens.

Cor. 1. Si vero inter datas quantitates contineantur valores quantitatum variabilium (*x*, *y*, *z*, &c.); tum ſumma acquiſita corrigi poteſt e ſubſtitutione prædictarum quantitatum pro ſuis valoribus; &c.

Cor. 2. Haud neceſſario ſemper denotant integros numeros literæ (*x*, *y*, *z*, *v*, &c.) ſed quantitates cujuſcunque generis, & ex iiſdem principiis conſtabit problematis reſolutio.

Cor. 3. Si quantitas *A* inveniri poſſit, tum ſemper inveniri poſſunt quantitates *B*, *C*, *D*, *BB*, *E*, *BC*, &c. i. e. datâ quantitate, quæ exprimit ſingulos ſeriei terminos duas vel plures variabiles quantitates (*x*, *y*, *z*, &c.) involventis; ſi modo ſumma hujuſce ſeriei inveniri poſſit; tum ſumma omnis ſeriei, cujus termini exprimuntur per quantitatem reſultantem e ſubſtitutione invariabilium quantitatum pro quibuſlibet literis *x*, *y*, *z*, &c. vel *x* pro *y*, &c. ſemper inveniri poteſt.

Cor. 4. Aſſumatur quæcunque quantitas, quæ duas vel plures variabiles quantitates (*x*, *y*, *z*, *v*, &c.) involvat pro ſummâ ſeriei; & e differentiâ inter hanc quantitatem aſſumptam & ejus ſucceſſivam ſummam, & ſic deinceps; deduci poteſt quantitas, quæ exprimit ſingulos prædictæ ſeriei terminos.

Cor. 5. Si vero maximæ dimenſiones variabilium quantitatum (*x*, *y*, *z*, &c.) in denominatore haud ſuperent maximas dimenſiones prædictarum quantitatum in numeratore contentas per quantitatem majorem quam numerum diverſarum variabilium quantitatum (*x*, *y*, *z*, *v*, &c.) in datâ quantitate contentarum; tum ſeries in infinitum progrediens neceſſario evadet infinita.

Summa vero hujuſmodi ſeriei, ſi haud in finitis terminis, ſemper per infinitas ſeries exprimi poteſt; per hoc problema enim reduci poteſt ad reſolutionem ſerierum, in quibus una ſolummodo continetur variabilis quantitas; vel ex aſſumptâ ſerie formulæ e formulâ dati termini facile acquirendæ, cujus terminus deducatur, & ex æquatis correſpondentibus datæ & reſultantis ſeriei terminis erui poteſt prædicta reſolutio.

Convergentiæ harum ſerierum ex iiſdem principiis ac convergentiæ ſerierum unam variabilem quantitatem ſolummodo habentium dijudicari poſſunt.

THEOR. XXVI.

Series (n) ordinis dividi poſſunt in infinitas ſeries ($n-1$) ordinis; ita quidem ſæpe ut ſummæ ex ſingulis prædictis infinitis ſeriebus innoteſcant; & hæ ſummæ præbeant convergentem ſeriem.

Ex. Sit ſeries ſecundi ordinis, cum terminus generalis eſt $\frac{1}{2z+1\cdot 2v+1}$, ubi literæ z & v quoſcunque integros numeros denotant; ſit vero terminus negativus, cum $\frac{v-z}{4}$ ſit integer numerus, ſin aliter affirmativus: dividatur hæc ſeries ſecundi generis in infinitam ſeriem ſerierum primi generis, quarum ſummæ detegi poſſunt, & exorientur novæ infinitæ ſeries: ex. g. dividatur data ſeries in alias $\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+$ &c. (ubi differentia inter v & z ſemper eſt 2), $\frac{1}{1\cdot 5}+\frac{1}{3\cdot 7}+\frac{1}{5\cdot 9}+$ &c. (ubi prædicta differentia eſt 4), $\frac{1}{1\cdot 7}+\frac{1}{3\cdot 9}+\frac{1}{5\cdot 11}+$ &c. &c. tum innoteſcunt ſummæ harum ſerierum reſultantium, viz. $\frac{1}{2}\times 1$, $\frac{1}{4}(1+\frac{1}{3})=\frac{1}{3}$, $\frac{1}{6}(1+\frac{1}{3}+\frac{1}{5})=\frac{23}{90}$, &c. tum erit ſumma datæ ſeriei quæſita $=\frac{1}{1\cdot 2}-\frac{1}{1\cdot 3}+\frac{23}{2\cdot 3\times 1\cdot 3\cdot 5}-\frac{176}{2\cdot 4\cdot 1\cdot 3\cdot 5\cdot 7}+\frac{1689}{2\cdot 5\cdot 1\cdot 3\cdot 5\cdot 7\cdot 9}-$ &c.

Et ſic de pluribus hujuſcemodi ſeriebus ad novam ſeriem reducendis.

Si vero contineantur plures (n) variabiles quantitates in dato termino, tum reducenda eſt data ſeries ad infinitas alias, quæ continent $n-1$ variabiles quantitates, & quarum ſummæ innoteſcunt; & exinde ſequuntur novæ infinitæ ſeries, &c.

PROB.

PROB. XXXIV.

Datâ ſerie ex irrationalibus terminis conſtante, invenire ejus ſummam.

Primo dentur ſolummodo n irrationales quantitates; &, ſi modo poſſibile ſit, ita dividatur ſeries in plures, ut unica, &c. ſolummodo contineatur irrationalis quantitas in ſingulis; & per methodos præcedentes inveniatur ſumma e ſingulis hiſce ſeriebus.

2^do^. Si modo infinitæ dentur irrationales quantitates in datâ ſerie, nonnunquam ita transformari poſſunt irrationales quantitates e numeratore in denominatorem; & vice verſâ, e denominatore in numeratorem per medit. algebraic. &c. ut evaneſcant e ſerie omnes irrationales quantitates.

Ex. 1. Sit data ſeries $\frac{1}{2\cdot3\cdot(\surd(\frac{1}{2})+\surd(\frac{1}{3}))}+\frac{1}{3\cdot4\cdot(\surd(\frac{1}{3})+\surd(\frac{1}{4}))}+\frac{1}{4\cdot5\cdot(\surd(\frac{1}{4})+\surd(\frac{1}{5}))}+\frac{1}{5\cdot6\cdot(\surd(\frac{1}{5})+\surd(\frac{1}{6}))}+$ &c. transformentur irrationales quantitates e denominatore in numeratorem; & quoniam $(\surd(\frac{1}{2})+\surd(\frac{1}{3}))\times(\surd(\frac{1}{2})-\surd(\frac{1}{3}))=\frac{1}{2}-\frac{1}{3}=\frac{1}{6}$; erit $\frac{1}{2\cdot3\cdot(\surd(\frac{1}{2})+\surd(\frac{1}{3}))}=\surd(\frac{1}{2})-\surd(\frac{1}{3})$, & ſic $\frac{1}{3\cdot4\cdot(\surd(\frac{1}{3})+\surd(\frac{1}{4}))}=\surd(\frac{1}{3})-\surd(\frac{1}{4})$, & ſic $\frac{1}{4\cdot5\cdot(\surd(\frac{1}{4})+\surd(\frac{1}{5}))}=\surd(\frac{1}{4})-\surd(\frac{1}{5})$; unde ſeries erit in genere $\surd(\frac{1}{2})-\surd(\frac{1}{3})+\surd(\frac{1}{3})-\surd(\frac{1}{4})+\surd(\frac{1}{4})-$ &c. in infinitum; & ſumma erit $\surd(\frac{1}{2})$.

Cor. 1. Series conſtantes ex irrationalibus terminis, quarum ſummæ dantur, facile deduci poſſunt ex principiis prius traditis, quæ deducunt ſeries ſummabiles, quæ haud involvunt irrationales quantitates.

Ex. 1. Aſſumatur ſeries, haud refert utrum ſummabilis ſit, necne; modo ejus termini ad nihil continuo vergant, & denique propiores ad id accedant, quam ulla quævis differentia; & ſubtrahatur ipſamet a ſeipsâ demptis r primis terminis; deinde transformentur termini reſultantes e numeratore in denominatorem per medit. algeb. vel in alios diverſæ formulæ mutentur, & invenitur ſeries quæſita. E. g. Sit ſeries aſſumpta $1+\left(\frac{1}{2}\right)^{\frac{1}{n}}+\left(\frac{1}{3}\right)^{\frac{1}{n}}+\left(\frac{1}{4}\right)^{\frac{1}{n}}+\left(\frac{1}{5}\right)^{\frac{1}{n}}+$ &c. ſubtrahatur ipſamet a ſe ipſâ demptis r primis terminis, i. e. $\left(\frac{1}{r}\right)^{\frac{1}{n}}+\left(\frac{1}{r+1}\right)^{\frac{1}{n}}+$ &c. & reſiduum erit $1-\left(\frac{1}{r}\right)^{\frac{1}{n}}+\left(\frac{1}{2}\right)^{\frac{1}{n}}-\left(\frac{1}{r+1}\right)^{\frac{1}{n}}+\left(\frac{1}{3}\right)^{\frac{1}{n}}-\left(\frac{1}{r+2}\right)^{\frac{1}{n}}+\&c.=1+\left(\frac{1}{2}\right)^{\frac{1}{n}}+\left(\frac{1}{3}\right)^{\frac{1}{n}}\ldots\left(\frac{1}{r-1}\right)^{\frac{1}{n}}$: transformentur hæ irrationales quantitates é numeratore in denominatorem, & reſultabunt $1-\left(\frac{1}{r}\right)^{\frac{1}{n}}=\frac{r-1}{r\times\left(1+\left(\frac{1}{r}\right)^{\frac{1}{n}}+\left(\frac{1}{r}\right)^{\frac{2}{n}}\ldots\left(\frac{1}{r}\right)^{\frac{n-1}{n}}\right)}$

$$\&\left(\frac{1}{2}\right)^{\frac{1}{n}}-\left(\frac{1}{r+1}\right)^{\frac{1}{n}}=\frac{r-1}{2\times(r+1)\times\left(\frac{1}{2}^{\frac{n-1}{n}}+\frac{1}{2}^{\frac{n-2}{n}}\left(\frac{1}{r+1}\right)^{\frac{1}{n}}+\frac{1}{2}^{\frac{n-3}{n}}\left(\frac{1}{r+1}\right)^{\frac{2}{n}}+\ldots\left(\frac{1}{r+1}\right)^{\frac{n-1}{n}}\right)}$$

& ſic deinceps; & erit ſumma $1+\left(\frac{1}{2}\right)^{\frac{1}{n}}+\left(\frac{1}{3}\right)^{\frac{1}{n}}+\left(\frac{1}{4}\right)^{\frac{1}{n}}\ldots\left(\frac{1}{r-1}\right)^{\frac{1}{n}}=\frac{r-1}{1\times r\times\left(1+\left(\frac{1}{r}\right)^{\frac{1}{n}}+\left(\frac{1}{r}\right)^{\frac{2}{n}}+\&c.\left(\frac{1}{r}\right)^{\frac{n-1}{n}}\right)}$

$$+\frac{r-1}{2\times(r+1)\times\left(\frac{1}{2}^{\frac{n-1}{n}}+\&c.\right)}+\&c.$$

Ex. 2. Sit ſeries aſſumpta $\surd\left(\frac{1}{1\cdot 2}\right)-\surd\left(\frac{1}{2\cdot 3}\right)x+\surd\left(\frac{1}{3\cdot 4}\right)x^2-$ &c. ducatur hæc ſeries in quamcunque quantitatem. E. g. $1+x$ & reſultat quantitas $\surd\left(\frac{1}{1\cdot 2}\right)+\left(\surd\left(\frac{1}{1\cdot 2}\right)-\surd\left(\frac{1}{2\cdot 3}\right)\right.$ $=\left(\frac{2}{1\cdot 2\cdot 3\cdot\left(\surd\left(\frac{1}{1\cdot 2}\right)+\surd\left(\frac{1}{2\cdot 3}\right)\right)}\right)x-\left(\surd\left(\frac{1}{2\cdot 3}\right)-\right.$

$\surd(1)$

$$\sqrt{\left(\frac{1}{3\cdot4}\right)} = \frac{2}{2\cdot3\cdot4\cdot\left(\sqrt{\left(\frac{1}{2\cdot3}\right)}+\sqrt{\left(\frac{1}{3\cdot4}\right)}\right)}x^2 + \&c.$$

3. Ducantur ſeries hujuſmodi in quaſcunque alias quantitates, & facile inveniri poſſunt ſeries irrationales ſummabiles omnino per eundem modum ac ſeries rationales: & ſic de reliquis methodis inveniendi rationales ſummabiles ſeries applicandis ad irrationales ſeries ſummabiles inveniendas; etiamque irrationales ſeries ope aliarum ſerierum deducendas.

Ex. 3. Sit ſeries aſſumpta $\frac{1}{\sqrt{(2)}}+\frac{1}{\sqrt{(3)}}x+\frac{1}{\sqrt{(4)}}x^2+$ &c. ducatur hæc ſeries in quantitatem $1-2x$, & reſultat $\frac{1}{\sqrt{(2)}}+\frac{1}{\sqrt{(3)}}x$ $+\frac{1}{\sqrt{(4)}}x^2+\&c. - \frac{2}{\sqrt{(2)}}x - \frac{2}{\sqrt{(3)}}x^2 - \frac{2}{\sqrt{(4)}}x^3 - \&c. = \frac{1}{\sqrt{(2)}}$ $-\frac{2\sqrt{(3)}-\sqrt{(2)}}{\sqrt{(2\cdot3)}}x - \frac{2\sqrt{(4)}-\sqrt{(3)}}{\sqrt{(3\cdot4)}}x^2 - \frac{2\sqrt{(5)}-\sqrt{(4)}}{\sqrt{(4\cdot5)}}x^3 -$ &c. Fingatur $2x-1=0$, & reſultat $x=\frac{1}{2}$ & ſumma ſeriei reſultantis $\frac{2\sqrt{(3)}-\sqrt{(2)}}{\sqrt{(6)}}\times\frac{1}{2}+\frac{2\sqrt{(4)}-\sqrt{(3)}}{\sqrt{(12)}}\times\frac{1}{4}+\&c. =$ $\frac{1}{\sqrt{(2)}}$.

Omnia, quæ prius de rationalibus ſeriebus quoad earum convergentiam, &c. tradita ſunt, etiam ad has irrationales applicari poſſunt.

THEOR. XXVII.

In ſeriebus interpolandis, juxta ac in iis ſummandis, neceſſe eſt ut inveniantur limites, inter quos conſiſtant termini quæſiti, aliter de iis nihil affirmandum eſt: e quibuſcunque finitis datis ordinatis ad quaſcunque datas diſtantias a ſe invicem poſitis minime ullas alias interponere poſſumus, ni detur lex, quam obſervant termini in genere, haud enim ſunt data, e quibus deduci poſſunt quæſita: ſi vero detur lex, tum exinde deduci poteſt ſubſtitutio, ex quâ interpolari poſſunt.

Facile conſtat, quod haud interpolari poſſunt ſeries per methodum differentiarum, ni ultimæ differentiæ ejuſdem ordinis ſint prope in ratione æqualitatis.

THEOR. XXVIII.

1. Sit curva parabolica $Q\alpha b\beta c\gamma$, &c. invenire ordinatam $PM(y)$ ad abſciſſam AP; aſſumatur alia parabolica curva $R\pi b\rho c\sigma$ &c. quæ habet ordinatas (v) præcedenti curvæ communes bl, cm, dn, eo, &c. quarum una abſciſſa An ſit z; pro radicibus $A\alpha$, $A\beta$, $A\gamma$, $A\delta$, &c. prioris curvæ ſcribantur α, β, γ, δ, &c. reſpective; & pro radicibus poſterioris curvæ, quæ dicatur approximans curva, ſubſtituantur π, ρ, σ, τ, &c. & pro AP, x; tum erit approximatio inventa ex hâc methodo interpolandi, i. e. correſpondens ordinata $LP(v)$ in curvâ aſſumptâ ad ordinatam $PM(y)$ quæſitam :: $\frac{(x-\pi)\times(x-\rho)\times(x-\sigma)\times \&c.}{(z-\pi)\times(z-\rho)\times(z-\sigma)\times \&c.}$: $\frac{(x-\alpha)\times(x-\beta)\times(x-\gamma)\times \&c.}{(z-\alpha)\times(z-\beta)\times(z-\gamma)\times \&c.}$ prope, ob $y=H\times(x-\alpha).(x-\beta).(x-\gamma).\&c.$, & $v=K\times(x-\pi).(x-\rho).(x-\sigma).\&c.$

2. Si vero curva data habeat crura ad finitas abſciſſas $(a, b, c, \&c.)$ in infinitum pergentia; tum radices $x-a$, $x-b$, $x-c$, &c. $z-a$, $z-b$, $z-c$, &c. quæ per prædictas abſciſſas denotantur, in denominatore continentur; & ſic de abſciſſis ad aſymptotos in curvâ aſſumptâ; & exinde ſequitur ratio, quam habet ordinata in curvâ aſſumptâ ad ordinatam quæſitam.

3. Si vero impoſſibiles evadant quædam radices prædictæ, tum ex quibuſque duabus correſpondentibus impoſſibilibus radicibus conſequetur quadratica quantitas poſſibilis, quæ & in curvâ datâ & aſſumptâ pro multiplicatione duarum radicum in ſeſe aſſumenda eſt.

Cor. . Hinc approximatio plerumque maxime pendet e radicibus quæ finitimæ ſunt ordinatæ quæſitæ; & in hâc æque ac in omnibus aliis curvis pendet e rationibus, quas habent differentiæ inter abſciſſam AP & eas, quæ habent communes ordinatas; ad diſtantias earum a punctis, in quibus ordinatæ vel poſſibiles vel impoſſibiles evadant nihil vel infinite magnæ.

Eadem principia etiam applicari poſſunt ad inveniendam convergentiam ſerierum hujuſce generis $ax^{s} + bx^{s+t} + cx^{s+2t} +$ &c. ubi per s & t quæcunque fractiones denotentur, vel irrationales cujuſcunque generis quantitates.

THEOR. XXIX.

Omnis ſeries eſt interpolabilis, cujus termini ſunt interpolabiles. Interpolentur enim diverſi termini & interpolatur data ſeries.

THEOR. XXX.

Erit $\dfrac{r.r+1.r+2.r+3\ldots q}{p.p+1.p+2.p+3\ldots p+q-r} = \dfrac{r.r+1.r+2\ldots p-1}{q+1.q+2\ldots p+q-r}$ reducantur enim hæ fractiones ad communem denominatorem, & conſtat theor.

Ex. 1. Sit ſeries $\dfrac{2\times 4\times 6\times 8\ldots \&c.}{1\times 3\times 5\times 7\ldots \&c.}$; ſed quoniam factorum incrementum eſt binarius, divide numeratores & denominatores per binarium, & evadet ſeries $\dfrac{1\times 2\times 3\times 4\times \&c.}{\frac{1}{2}\times 1\frac{1}{2}\times 2\frac{1}{2}\times \&c.} = \dfrac{r\times(r+1)\times(r+2)\times \&c.}{p\times(p+1)\times(p+2)\times \&c.}$ & conſequenter erunt $r = 1$ & $p = \frac{1}{2}$; nunc ſit m diſtantia inter primum ſeriei terminum & quemvis alium q, tum erit $q = m + 1$, & ſeries $\dfrac{r.r+1.r+2\,..\,p-1}{q+1\ldots p+q-r}$ evadet $= \dfrac{1.2.3\,..\,p-1}{m+2.m+3\,..\,p+m}$; ergo terminus ſeriei $\left(\dfrac{1\times 2\times 3\,..\,m+1}{\frac{1}{2}\times 1\frac{1}{2}\times 2\frac{1}{2}\ldots m+\frac{1}{2}}\right)$, cujus diſtantia a primo ſit m, æquat terminum ſeriei $\left(\dfrac{1\times 2\times 3\ldots p-1}{m+2.m+3\ldots m+\frac{1}{2}}\right)$, cujus diſtantia a primo ſit $p - 2 = \frac{1}{2} - 2 = -1\frac{1}{2}$.

Cor. . Hinc ſeries hujuſce generis ſemper interpolari poteſt, viz. $\dfrac{r.r+1.r+2\,..\,p-1}{q+1.q+2\,..\,p+q-r}$, ubi q ſuperat r per integrum numerum; requiratur enim terminus, cujus diſtantia a primo ſit m: aſſumatur

$p = r + m + 1$, & erit terminus quæſitus $\frac{r \,.\, r+1 \,.\, r+2 \,..\, q}{p \,.\, p+1 \,.\, p+2 \,..\, p+q-r}$.

THEOR. XXXI.

1. In interpolandis quantitatibus, quæ ſunt functiones numeri z, quarum continuo augetur factorum numerus, ubi z denotat ſucceſſive numeros 1, 2, 3, 4, &c. ſæpe ex interpolatione indicum conſtabit formula ſeriei, quæ terminum quæſitum exprimet, ex quâ per ſubſtitutionem acquiri poteſt ſeries ipſa, i. e. terminus quæſitus.

Ex. 1. Sint quantitates hujuſce generis 1, $1 \,.\, 2$, $1 \,.\, 2 \,.\, 3$, $1 \,.\, 2 \,.\, 3 \,.\, 4$, & in genere $1 \,.\, 2 \,.\, 3 \ldots z$, vel quod idem eſt $z \,.\, z-1 \,.\, z-2 \ldots 1$, $z+1 \,.\, z \,.\, z-1 \,.\, z-2 \ldots 1$, invenire terminum intermedium inter $(T)\; z \,.\, z-1 \,.\, z-2 \,..\, 1$ & $(T')\; z+1 \,.\, z \,.\, z-1 \,.\, z-2 \,..\, 1$: dimenſiones quantitatis z in his duobus terminis datis contentæ per unitatem differunt, ergo termini intermedii dimenſiones different a dimenſionibus quantitatis z in duobus prædictis terminis per $\frac{1}{2}$: aſſumatur igitur ſeries $T(az^{\frac{1}{2}} + bz^{-\frac{1}{2}} + cz^{-1\frac{1}{2}} +$ &c.) pro termino intermedio quæſito, in ſerie $(S)\; az^{\frac{1}{2}} + bz^{-\frac{1}{2}} + cz^{-1\frac{1}{2}} +$ &c. pro z ſcribatur $z + \frac{1}{2}$, & reſultat $(S')\; az^{\frac{1}{2}} + (b + \frac{1}{4}a)\, z^{-\frac{1}{2}} + (c - \frac{1}{4}b - \frac{1}{8} \times \frac{1}{4}a)\, z^{-1\frac{1}{2}} +$ &c. ducantur hæ duæ ſeries S & S' in ſeſe; & productum fiat æquale factori $z + 1$; & ex æquatis correſpondentibus terminis ſequitur ſeries $(S)\; z^{\frac{1}{2}} + \frac{3}{8}z^{-\frac{1}{2}} - \frac{7}{128}z^{-\frac{3}{2}} + \frac{9}{1024}z^{-\frac{5}{2}} -$ &c. & conſequenter terminus quæſitus erit $T \times S$.

2. Ex iiſdem principiis ulterius promotis inveniri poteſt terminus ad quamcunque diſtantiam a dato termino: ſed major erit calculi labor.

3. Sint quantitates x, $x \,.\, x+n$, $x \,.\, x+n \,.\, x+n^2$, $x \,.\, x+n \,.\, x+n^2 \,.\, x+n^3$, &c. ubi n multo minor eſt quam x & unitas, tum ſeries e multiplicatione deducta pro generali termino converget. E. g. Series erit $x^{z+1} + \frac{n - n^{z+1}}{1-n}x^z + \frac{1}{2}\left(\left(\frac{n - n^{z+1}}{1-n}\right)^2 - \frac{n^2 - n^{2z+2}}{1-n^2}\right)x^{z-1}$ + &c.

In hâc ſerie ſcribatur pro z ejus valor aſſignatus, & conſtat terminus, cujus diſtantia e primo ſit z.

Lex hujuſce ſeriei conſtat e noſtris medit. algebr.

4. Datâ quâcunque ſerie, & æquatione relationem inter terminos datæ & novæ ſeriei exprimente, ita ut e datæ ſeriei terminis conſtent novæ æquationis correſpondentes termini, & ſi termini datæ ſeriei, tum etiam termini novæ ſeriei, interpolari poſſunt.

Indices interpolari poſſunt per eandem methodum ac aliæ algebraicæ quantitates.

THEOR. XXXII.

1. Si ſucceſſivi termini continuo exoriantur e majori numero factorum, qui ſunt functiones quantitatis z, in ſeſe ductorum; tum inveniantur logarithmi terminorum ſucceſſivorum, & deduci poteſt e differentiis logarithmorum, &c. ſeries exprimens logarithmum ſummæ, quæ forſan progreditur ſecundum vulgares leges. Si vero numerus factorum in ſerie deductâ deſignetur etiam per functionem quantitatis z conſtantem continuo e majori numero factorum, tum inveniantur logarithmi prædictorum logarithmorum, & ſic deinceps; & tandem reſultabit ſeries exprimens logarithmum logarithmi, &c. cujus termini prope etiam per vulgares leges exprimi poſſunt, ni numerus factorum continuo exprimatur per functionem quantitatis z; quæ continuo conſtat e majori numero factorum. E. g. Sit ſeries 1, $1 \times 2 \times 3$, $1 \times 2 \times 3 \times 4 \times 5 \times 6$, &c. ubi n numerus factorum continuo fit $= z \, . \, \frac{z+1}{2}$; inveniantur logarithmi e ſingulis terminis, & erunt eorum ſucceſſivæ differentiæ $l2 + l3$, $l4 + l5 + l6$, $l7 + l8 + l9 + l10$, &c. & deinde inveniantur ſecundæ differentiæ, &c.

Hæ vero differentiæ exprimi poſſunt per notas methodos, ergo ſeries ipſa.

2. Sit ſeries $a + b + c + d + e +$ &c. $= S$, tum continuo inveniantur la; $l(a+b) - la$; $l(a+b+c) - l(a+b)$, $l(a+b+c+d)$

$c + d) - l(a + b + c)$, &c. & tandem invenietur ſeries, cujus ſumma eſt lS.

3. Hæ transformationes ſæpe uſui inſervire poſſunt in interpolatione ſerierum hujuſmodi, quæ aliter interpolationem vix aut ne vix recipiant: facile enim eſt interpolare ſeriem reſultantem, eâ vero interpolatâ, inveſtigari poteſt interpolatio ſeriei, ex quâ deducta fuit.

4. Si vero ſeries exprimatur per terminos, quorum indices per prædictas functiones progrediuntur; tum e præcedente methodo inveniantur logarithmi ſeriei, & ſæpe deduci poteſt logarithmus ſeriei ſummæ in terminis ſecundum vulgares leges progredientibus.

Cor. . Fluxiones, vel incrementa, &c. quantitatum $1 . 2 . 3 \ldots z$ vel $1 . 2 . 3 \,.\,. z^z$, vel infinitarum aliarum, facile deduci poſſunt e principiis prius traditis pro interpolationibus conſimilium quantitatum.

Numerus factorum vel dimenſiones quantitatum, quæ ex hâc methodo interpolantur, deduci poſſunt.

PROB. XXXV.

Invenire fluentes & exinde ſeries per datam fluentem interpolabiles.

Aſſumantur quæcunque quantitates in datâ fluente tanquam conſtantes; vel eædem in datâ & quæſitâ quantitate variabiles, quæcunque vero aliæ variabiles, i. e. diverſæ relationem aſſignabilem inter ſe habentes; reducantur quantitates reſultantes ad infinitas ſeries, & reſultant ſeries, quæ per datam fluentem interpolabiles erunt.

Ex. 1. Sit data fluens $\int . (a^2 - x^2)^{\frac{1}{2}}\dot{x}$, aſſumatur index $\frac{1}{2}$ variabilis, i. e. pro indice $\frac{1}{2}$ ſcribantur reſpective 0, 1, 2, 3, 4, 5, &c. denique l; quæ ſeries per $\frac{1}{2}$ interpolabilis eſt, & reſultant $\int . (a^2 - x^2)^0\dot{x}$, $\int . (a^2 - x^2)^1\dot{x}$, $\int . (a^2 - x^2)^2\dot{x}$, $\int . (a^2 - x^2)^3\dot{x}$, &c. & conſequenter ſeries intermedia inter has fluentes inter eoſdem valores quantitatis x contentas, erit $\int . (a^2 - x^2)^{\frac{1}{2}}\dot{x}$, quæ per aream circuli exprimi poteſt, unde intermedius terminus inter primum & ſecundum ſeriei $\frac{1}{1}$, $\frac{1}{1} - \frac{1}{3}$, $\frac{1}{1} - \frac{2}{3} + \frac{1}{5}$, $\frac{1}{1} - \frac{3}{3} + \frac{3}{5} - \frac{1}{7}$, $\frac{1}{1} - \frac{4}{3} + \frac{6}{5} - \frac{4}{7} + \frac{1}{9}$, &c. erit $\frac{1}{8}$ peripheriæ circuli, cujus radius eſt 1.

2. Aſſumatur index 2 variabilis, & erit prædicta fluens intermedia inter fluentes fluxionum $(a^2 - x^0)^{\frac{1}{2}}\dot{x}$, $(a^2 - x^4)^{\frac{1}{2}}\dot{x}$, $(a^2 - x^8)^{\frac{1}{2}}\dot{x}$, &c.

3. Et ſic aſſumi poſſunt indices 2 & $\frac{1}{2}$ invariabiles, & quantitas a variabilis, ita vero ut ſimul ſint interpolabiles omnes prædictæ variabiles per datam fluentem. Inveniantur per infinitas ſeries fluentes prædictarum fluxionum, & reſultant ſeries, quæ per datam fluentem interpolabiles erunt.

Ex. 2. Sit $\int . \dot{x} \surd (x^2 + 1)$ data fluens, & erunt fluentes fluxionum $\dot{x} \times (x^2 + 1)^0$, $\dot{x}(x^2 + 1)^1$, $\dot{x}(x^2 + 2)^2$, &c. per eam interpolabiles.

Cor. Fluentes fluxionum $\dot{x}(1 \pm x^2)^{\frac{1}{3}}$, $\dot{x} \times (1 \pm x^2)^{\frac{1}{4}}$, $\dot{x}(1 \pm x^2)^{\frac{2}{3}}$, $\dot{x} \times (1 \pm x^2)^{\frac{3}{4}}$ (& conſequenter arcus elliptici vel hyperbolici), & generaliter $\dot{x}(1 \pm x^2)^{\frac{n}{m}}$ & $\dot{x}(1 \pm x^2)^{-\frac{n}{m}}$ deduci poſſunt ex interpolatione ſerierum, quæ oriuntur e fluentibus fluxionum $(1 \pm x^2)^0 \dot{x}$, $(1 \pm x^2)^1 \dot{x}$, $(1 \pm x^2)^2 \dot{x}$, &c.

Ex. 3. Sit $\int . \frac{\dot{x}}{x^2} \surd (1 + x^4)$ data fluens, quæ exprimit hyperbolicum arcum, ergo hyperbolicus arcus erit intermedius terminus inter primum & ſecundum ſeriei expreſſæ per fluentes fluxionum $\frac{\dot{x}}{x^2}(1 + x^2)^0$, $\frac{\dot{x}}{x^2} \times (1 + x^4)^1$, $\frac{\dot{x}}{x^2} \times (1 + x^4)^2$, &c. terminum.

Cor. Arcus hyperbolicus exprimi poteſt per ſeriem $-\frac{1}{x} + \frac{1}{2 \times 3} x^3 - \frac{1}{2^2 . 2 \times 7} x^7 + \frac{1 . 3}{2^3 \times 2 . 3 \times 11} x^{11} - \frac{1 . 3 . 5}{2^4 . 2 . 3 . 4 \times 15} x^{15} + \frac{1 . 3 . 5 . 7}{2^5 . 2 . 3 . 4 . 5 \times 19} x^{19} -$ &c. ubi x denotat abſciſſam ad aſymptoton.

Si vero requiratur deſcendens ſeries, tum erit $x - \frac{1}{2 \times 3} x^{-3} + \frac{1}{2^2 . 2 \times 7} x^{-7} - \frac{1 . 3}{2^3 . 2 . 3 . 11} x^{-11} +$ &c. quæ quoad coefficientes attinet, prorſus eandem obſervat legem ac præcedens.

Ex. 4. Sint fluentes fluxionum $\frac{\dot{x}}{(1+x)^0}$, $\frac{\dot{x}}{(1+x)^2}$, $\frac{\dot{x}}{(1+x)^4}$, $\frac{\dot{x}}{(1+x)^6}$, &c. . . . $\frac{\dot{x}}{(1+x)^{2n}}$, quæ erunt x, $\frac{-1}{1+x}$, $\frac{-1}{3(1+x)^3}$, $\frac{-1}{5(1+x)^5}$, &c.

hæ vero fluentes inter valores 0 & 1 quantitatis x contentæ erunt reſpective 1, $\frac{1}{2}$, $\frac{7}{24}$, $\frac{31}{5\times 32}$, ... $\frac{2^{2n-1}-1}{(2n-1)\cdot 2^{2n-1}}$; & medius terminus inter duos primos erit hyp. log. 2.

Omnia hæc etiam ad incrementiales & integrales quantitates applicari poſſunt. Facile conſimili methodo inveniri poſſunt infinitæ interpolationes incrementialium quantitatum.

THEOR. XXXIII.

Si numerus factorum in ſucceſſivis terminis contentorum per arithmeticam ſeriem deſignetur, tum forſan relatio inter ſucceſſivos terminos ſit conſtans, i. e. numerus factorum in eâ contentus idem ſemper maneat. E. g. Sint ſucceſſivi termini 1, 1×2, $1\times 2\times 3$, $1\times 2\times 3\times 4$, &c. quorum numerus factorum ſemper augetur per unitatem, i. e. ſunt 1, 2, 3, 4, &c. ſint t & t' duo ſucceſſivi ſeriei termini, z vero diſtantia termini t a primo ſeriei termino, tum erit $(z+2)\times t = t'$ conſtans relatio. Cum vero relatio detur conſtans inter ſucceſſivos terminos, tum ex quibuſdam datis primis terminis erui poſſunt reliqui; & conſequenter ſi interpoletur quantitas (π) inter primum & ſecundum terminum, cujus diſtantia ex primo termino ſit $\frac{n}{m}$, e termino vero ſecundo ſit $\frac{m-n}{m}$; tum ex prædictâ conſtante relatione erui poteſt quicunque terminus, cujus diſtantia a termino $\overset{r}{t}$ ſit $\frac{n}{m}$, ubi r eſt integer numerus; a termino vero proxime ſubſequente ſit $\frac{m-n}{m}$; nam erit $\pi\times\left(\frac{n}{m}+2\right)\left(\frac{n}{m}+3\right)\left(\frac{n}{m}+4\right)\ldots\left(\frac{n}{m}+r+1\right)$. E. 2. Sit $\varphi:(z)\times t = t'$, & $\varphi:(z+1)\times t' = t''$, tum erit $\varphi:(z+r-1)\overset{r-1}{t'} = t^{r}$, $\varphi:(z+r)\overset{r}{t'} = t^{r+1}$, &c.: & ſic ratiocinari liceat, cum detur quæcunque relatio inter ſucceſſivos terminos.

Conſimilia etiam applicari poſſunt ad ſeries, quarum primæ, ſecundæ, tertiæ, &c. differentiæ numerorum terminorum erunt in arithmeticâ progreſſione; hæ ſeries, ut prius oſtenditur, interpolari poſſunt e ſucceſſivis logarithmis ejuſdem ordinis ac differentiæ numerorum, quæ conſtantes evadunt.

Si quantitates interpolandæ vel conſtent ex interpolabilibus factoribus; vel ſint quæcunque conſtantes functiones quantitatum, quarum interpolationes dantur; tum dantur etiam earum interpolationes.

Cor. . Facile deduci poſſunt infinitæ quantitates, quæ ex interpolatione datarum quantitatum interpolari poſſunt.

Inveniatur quæcunque conſtans functio datarum quantitatum, & perficitur coroll.

Et vice versâ ex animadvertendo ſeriem datam eſſe prædictam functionem quantitatum, quarum interpolationes innoteſcunt, conſequetur ejus interpolatio. E.g. Ex conceſsâ interpolatione terminorum 1, 1×2, $1 \times 2 \times 3$, $1 \times 2 \times 3 \times 4$, &c. termini binomialis theorematis $\frac{n}{m}$, $\frac{\frac{n}{m}+1}{2}A$, $\frac{\frac{n}{m}+2}{3}B$, $\frac{\frac{n}{m}+3}{4}C$, &c. ubi A, B, C, &c. præcedentes terminos reſpective denotant, interpolari poſſunt: terminus enim in datâ ſerie 1, 1.2, $1.2.3$, $1.2.3.4$, &c. ad $\left(\frac{n}{m}-2\right)$ diſtantiam a primo ſit π, & quoniam terminus ad diſtantiam z a primo in ſerie 1, 1.2, $1.2.3$, &c. ductus in $z+2$ ſubſequentem præbet terminum; ergo terminus π ad diſtantiam $(z=\frac{n}{m}-2)$ a primo in $z+2=\frac{n}{m}$ ductus præbet terminum ad diſtantiam $z+1=\frac{n}{m}-1$ a primo in eadem ſerie; & ſimiliter termini ad diſtantias $\frac{n}{m}-1$, $\frac{n}{m}$, $\frac{n}{m}+1$, $\frac{n}{m}+2$, &c., etiamque ad diſtantias $\frac{n}{m}-2$, $\frac{n}{m}-3$, $\frac{n}{m}-4$, &c. poſiti evadent $\frac{n}{m}\times\pi$,

$\frac{n}{m} \times \left(\frac{n}{m}+1\right)\pi$, $\frac{n}{m}\left(\frac{n}{m}+1\right)\left(\frac{n}{m}+2\right)\pi$, $\frac{n}{m}\left(\frac{n}{m}+1\right)\left(\frac{n}{m}+2\right)\left(\frac{n}{m}+3\right)\pi$, &c., etiamque π, $\left(\frac{1}{\frac{n}{m}-1}\right)\pi$, $\left(\frac{1}{\frac{n}{m}-1}\right)\left(\frac{1}{\frac{n}{m}-2}\right)\pi$, &c. hinc terminus ad diſtantiam r a primo in ſerie 1, 1 . 2, 1 . 2 . 3, &c. æqualis erit termino ad diſtantiam $r-\frac{n}{m}+2$ a primo in ſerie 1, $\frac{n}{m}$, $\frac{n}{m} \times \left(\frac{n}{m}+1\right)$, &c. in π ducto.

THEOR. XXXIV.

1. Sint termini π, ρ & σ; quorum diſtantiæ a primo in ſerie 1, 1 × 2, 1 × 2 × 3, 1 × 2 × 3 × 4, &c. ſint reſpective $\frac{n}{m}-2$, $\frac{n}{m}+r-1$ & r; tum terminus in ſerie $\frac{n}{m}$, $\frac{n}{m} \times \frac{\frac{n}{m}+1}{2}$, $\frac{n}{m} \times \frac{\frac{n}{m}+1}{2} \times \frac{\frac{n}{m}+2}{3}$, &c. cujus diſtantia e primo ſit r erit $\frac{\rho}{\pi\sigma}$.

2. Datis terminis ſeriei (P) 1, 1 × 2, 1 × 2 × 3, 1 × 2 × 3 × 4, &c. ad quaſlibet diſtantias a primo poſitis, invenire terminum ſeriei (Q) $\frac{m}{r}$, $\frac{m}{r} \cdot \frac{m+n}{r+s}$, …, $\frac{m}{r} \cdot \frac{m+n}{r+s} \cdot \frac{m+2n}{r+2s} \ldots \frac{m+zn}{r+zs}$ ad α diſtantiam a primo conſtitutum. Sint termini ad $\frac{m}{n}-2$ & $\frac{r}{s}-2$ diſtantias a primo ſeriei (P) termino reſpective π & ρ; termini vero ad diſtantias $\frac{n}{m}+\alpha-1$ & $\frac{r}{s}+\alpha-1$ a primo in eâdem ſerie reſpective σ & τ; tum erit terminus quæſitus $\frac{\sigma}{\pi} \times \frac{\rho}{\tau} \times \frac{n^{\alpha+1}}{s^{\alpha+1}}$.

Hæ poſteriores ſeries in priori P continentur.

Cor. . Si detur ſeries (R) terminorum, qui ſint quæcunque algebraica finita functio prædictæ ſeriei $\mathfrak{Q}$; facile ex iiſdem datis inveniri poteſt terminus in prædictâ ſerie R ad quamcunque diſtantiam a primo poſitus.

Eâdem methodo ratiocinari liceat ex interpolationibus ſeriei (P) vel plurium ſerierum (P), ($\mathfrak{Q}$), &c. conceſſis ad interpolationes ſerierum, quæ ſint quæcunque finitæ functiones quantitatum e prædictis ſeriebus deductarum.

Hic animadvertendum eſt, quodſi detur ſeries hujuſmodi alternatim affirmativa & negativa, i. e. ſeries ſit $-a+ab-abc+abcd$ — &c. quæ conficitur e factoribus $-a$, $-b$, $-c$, &c. in ſeſe ductis, minime ejus interpolatio eadem erit ac interpolatio factorum affirmativorum a, $a\times b$, $a\times b\times c$, &c. in priori caſu enim inter ſingulos duos ſucceſſivos terminos nihilo evadet terminus interpolandus, &c.

Series duas vel plures variabiles quantitates involventes vel ſummationem vel interpolationem recipiant ex eâdem methodo, ac ſeries unam ſolummodo variabilem quantitatem habentes.

THEOR. XXXV.

1. Erit $e^{\pm x}=1\pm x+\frac{x^2}{1.2}\pm\frac{x^3}{1.2.3}+\frac{x^4}{1.2.3.4}\pm$ &c. $=\left(1\pm\frac{x}{i}\right)^i$, ubi i eſt infinita quantitas; erit etiam $\frac{e^{+x}+e^{-x}}{2}=1+\frac{x^2}{1.2}+\frac{x^4}{1.2.3.4}+$ &c. $=\frac{\left(1+\frac{x}{i}\right)^i+\left(1-\frac{x}{i}\right)^i}{2}$; & ſic erunt $\frac{e^{+x}-e^{-x}}{2}=x+\frac{x^3}{1.2.3}+$ &c. & $\frac{e^{x\surd(-1)}-e^{-x\surd(-1)}}{2}\times-\surd(-1)=x-\frac{x^3}{1.2.3}+$ &c. $=$ ſin. arcus x, & $\frac{e^{x\surd(-1)}+e^{-x\surd(-1)}}{2}=1-\frac{x^2}{1.2}+\frac{x^4}{1.2.3.4}-$ &c. $=$ coſin. arcus x. Erit etiam $e^{x\surd(-1)}=1-$

$\frac{x^2}{2} + \frac{x^4}{2 \cdot 3 \cdot 4} - \frac{x^6}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} +$ &c. $+ \left(x - \frac{x^3}{1 \cdot 2 \cdot 3} + \text{\&c.}\right) \times \sqrt{(-1)}$ $=$ coſ. $x + \sqrt{(-1)}$ ſin. x; & ſimiliter $e^{-x\sqrt{(-1)}} = 1 - \frac{x^2}{1 \cdot 2} + \frac{x^4}{1 \cdot 2 \cdot 3 \cdot 4} -$ &c. $- \sqrt{(-1)} \left(x - \frac{x^3}{1 \cdot 2 \cdot 3} + \frac{x^5}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} - \text{\&c.}\right) =$ coſin. $x - \sqrt{(-1)}$ ſin. x, hic radius eſt 1.

Forſan haud indignum eſt obſervatu, quod $1^{\pm\sqrt{(-n)}} = 1$.

2. Sint $\alpha, \beta, \gamma, \delta, \varepsilon$, &c. (n) radices æquationis $x^n - 1 = 0$, & erit $\frac{e^{\alpha x} + e^{\beta x} + e^{\gamma x} + \text{\&c.}}{n} = 1 + \frac{x^n}{1 \cdot 2 \cdot 3 \, .. \, n} + \frac{x^{2n}}{1 \cdot 2 \cdot 3 \, .. \, 2n} +$ &c. & $\frac{\alpha^{n-m} e^{\alpha x} + \beta^{n-m} e^{\beta x} + \gamma^{n-m} e^{\gamma x} + \text{\&c.}}{n} = \frac{x^m}{1 \cdot 2 \cdot 3 \, .. \, m} + \frac{x^{n+m}}{1 \cdot 2 \cdot 3 \, .. \, n+m} + \frac{x^{n+2m}}{1 \cdot 2 \cdot 3 \, .. \, n+2m} +$ &c. ubi m minor eſt quam n; & ſic ſit quæcunque exponentialis quantitas, quæ reducatur in ſeriem (P) ſecundum dimenſiones quantitatis x progredientem, viz. $Ax^a + Bx^{a+b} + Cx^{a+2b}$ $+$ &c. & ſi in datâ exponentiali ſcribantur αx, βx, γx, &c. reſpective pro x, & ſunt quantitates reſultantes reſpective A, B, C, D, &c. tum erit $\frac{A + B + C + D + \text{\&c.}}{n} =$ ſummæ ſeriei, cujus termini ſint $Ax^a + \sigma x^{a+nb} +$ &c. ſi vero requiratur ut primus terminus ad $m - 1$ diſtantiam a primo ponatur, viz. ſit ſeries $\pi x^{a+mb} + \rho x^{a+(m+n)b} +$ &c. tum erit ejus ſumma $\frac{\alpha^{n-m} A + \beta^{n-m} B + \gamma^{n-m} C + \text{\&c.}}{n}$.

Idem etiam de ſeriebus ab exponentialibus fluentibus exortis affirmari poteſt.

Cor. 1. Hinc inveniri poſſunt ſummæ ſerierum prædictarum $a(t + t^n + t^{2n} + \text{\&c.}) + b(t' + t^{n+1} + t^{2n+1} + \text{\&c.}) + c(t^2 + t^{n+2} + t^{2n+2} + \text{\&c.})$, ubi t^n, &c. denotat terminum prædictæ ſeriei, cujus diſtantia a primo ſit n, &c. & a, b, c, &c. deſignant quaſcunque invariabiles quantitates.

THEOR.

THEOR. XXXVI.

1. Erit $\left(1 \pm \frac{x}{i}\right)^i = 1 \pm x + \frac{x^2}{1.2} \pm \frac{x^3}{1.2.3} + \&c.$ & exinde $\left(1 \pm \frac{x}{i}\right)^i - 1 = \pm x + \frac{x^2}{1.2} \pm \frac{x^3}{1.2.3} + \&c.$ quantitatis $\left(1 \pm \frac{x}{i}\right)^i - 1 = v^i - 1$ inveniantur quadratici diviſores, qui erunt $\left(1 \pm \frac{x}{i}\right)^2 - 2\alpha\left(1 \pm \frac{x}{i}\right) + 1$, ubi α denotat reſpective coſinus arcuum $0°$, $\frac{4 \times 90°}{i}$, $\frac{8 \times 90°}{i}$, $\frac{12 \times 90°}{i}$, &c. i. e. coſ. arcuum (0π, $\frac{2\pi}{i}$, $\frac{4\pi}{i}$, $\frac{6\pi}{i}$, $\frac{8\pi}{i}$, &c.); vel ſub formulâ $\left(1 \pm \frac{x}{i}\right)^2 - 2\left(1 \pm \frac{x}{i}\right)$ coſin. $\frac{2k\pi}{i} + 1$, ubi k ſucceſſive denotat integros numeros 0, 1, 2, 3, 4, &c. contineantur. In hâc formulâ pro coſ. $\frac{2k\pi}{i}$ ſcribatur ejus valor $1 - \frac{2k^2\pi^2}{i^2}$ prope, & reſultat $\frac{x^2}{i^2} + \frac{4k^2\pi^2}{i^2} \pm \frac{4k^2\pi^2}{i^3} x = \left(1 \pm \frac{x}{i}\right)^2 - 2\left(1 \pm \frac{x}{i}\right)$ coſ. $\frac{2k.\pi}{i} + 1$, ducatur hæc quantitas in $\frac{i^2}{4k^2\pi^2}$ & reſultat $\frac{x^2}{4k^2\pi^2} \pm \frac{x}{i} + 1$; quare expreſſio $e^x - 1 = x\left(1 + \frac{x^2}{1.2} + \frac{x^3}{1.2.3} + \&c.\right)$ habebit diviſores x, $\left(1 + \frac{x}{i} + \frac{x^2}{4\pi^2}\right)$, $\left(1 + \frac{x}{i} + \frac{x^2}{16\pi^2}\right)$, $\left(1 + \frac{x}{i} + \frac{x^2}{36\pi^2}\right)$, &c.

2. Erit $e^x - e^{-x} = \left(1 + \frac{x}{i}\right)^i - \left(1 - \frac{x}{i}\right)^i = 2\left(x + \frac{x^3}{1.2.3} + \frac{x^5}{1.2.3.4.5} + \&c.\right) = v^i - z^i$, ſi modo pro $1 + \frac{x}{i}$ & $1 - \frac{x}{i}$ ſcri-

bantur reſpective v & z; ſed inveniri poſſunt ejus diviſores $v^2 - 2vz$ coſ. $\frac{2k\pi}{i} + z^2 = \frac{4x^2}{i^2} + \frac{4k^2\pi^2}{i^2} - \frac{4k^2\pi^2x^2}{i^4}$ (ob coſin. $\frac{2k\pi}{i} = 1 - \frac{2k^2\pi^2}{i^2}$); ducatur hæc quantitas in $\frac{i^2}{4k^2\pi^2}$, & reſultat formula ejus diviſoris $\frac{x^2}{k^2\pi^2} - \frac{x^2}{i^2} + 1 = \frac{x^2}{k^2\pi^2} + 1$, unde prædicta æquatio habebit diviſores $1 + \frac{x^2}{\pi^2}$, $1 + \frac{x^2}{4\pi^2}$, $1 + \frac{x^2}{9\pi^2}$, &c.

3. Erit $\frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2} + \frac{x^4}{1.2.3.4} + \&c. = \frac{\left(1 + \frac{x}{i}\right)^i + \left(1 - \frac{x}{i}\right)^i}{2} = \frac{v^i + z^i}{2}$, ſi modo pro $1 + \frac{x}{i}$ & $1 - \frac{x}{i}$ ſcribantur reſpective v & z; tum inveniri poſſunt ejus diviſores formulæ $1 + \frac{4x^2}{(2k+1)^2\pi^2}$ & conſequenter erunt $1 + \frac{4x^2}{\pi^2}$, $1 + \frac{4x^2}{9\pi^2}$, $1 + \frac{4x^2}{25\pi^2}$, &c.

Et ſic inveuiri poſſint x, $1 - \frac{x^2}{\pi^2}$, $1 - \frac{x^2}{4\pi^2}$, $1 - \frac{x^2}{9\pi^2}$, &c. diviſores ſeriei $x - \frac{x^3}{1.2.3} + \frac{x^5}{1.2.3.4.5} - \&c.$ etiamque $1 - \frac{4x^2}{\pi^2}$, $1 - \frac{4x^2}{9\pi^2}$, $1 - \frac{4x^2}{25\pi^2}$, &c. diviſores ſeriei $1 - \frac{x^2}{1.2} + \frac{x^4}{1.2.3.4} - \&c.$

4. Erit $e^x - 2$ coſ. $g + e^{-x} = 2\left(1 - \text{coſ.}\, g + \frac{x^2}{1.2} + \frac{x^4}{1.2.3.4} + \&c.\right) = \left(1 + \frac{x}{i}\right)^i - 2$ coſ. $g + \left(1 - \frac{x}{i}\right)^i = v^i - 2$ coſ. $g + z^i$, unde conſtabit ejus diviſorum formula, viz. $v^2 - 2vz$ coſ. $\frac{2k\pi \pm g}{i} + z^2 = 2 + \frac{2x^2}{i^2} - 2\left(1 - \frac{x^2}{i^2}\right)^i$ coſ. $\frac{2(2k\pi \pm g)}{i} =$

$4x^2 +$

$\frac{4x^2}{i^2} + \frac{4(2k\pi \pm g)^2}{i^2}$; ob i infinitum, coſin. $\frac{2(2k\pi \pm g)}{i} = 1 - \frac{2(2k\pi \pm g)^2}{i^2}$.

5. Erit $e^{b+x} \pm e^{c-x} = \left(1 + \frac{b+x}{i}\right)^i \pm \left(1 + \frac{c-x}{i}\right)^i = v^i \pm z^i$, cujus factores habent formulam $v^2 - 2vz \text{ coſ. } \frac{m\pi}{i} + z^2$. ubi m denotat imparem numerum ſi ſignum affixum ſit $+$, ſin vero ſit $-$, tum parem denotat numerum; ſed ob i numerum infinite magnum erit coſ. $\frac{m\pi}{i} = 1 - \frac{m^2\pi^2}{2i^2}$, unde formula factorum prædicta evadet $(v-z)^2 + \frac{m^2\pi^2}{i^2}vz$, in hâc formulâ pro v & z ſcribantur earum valores $1 + \frac{b+x}{i}$ & $1 + \frac{c-x}{i}$, & ducatur quantitas exinde deducta in i^2, & reſultabit $(b-c)^2 + m^2\pi^2 + 4(b-c)x + 4x^2$, dividatur ea per $(b-c)^2 + m^2\pi^2$ & reſultat formula quæſita $1 + \frac{4(b-c)x + 4x^2}{(b-c)^2 + m^2\pi^2}$.

Cor. 2. Ut unitas evadat primus terminus ſeriei; dividatur quantitas $e^{b+x} \pm e^{c-x}$ per $e^b \pm e^c$, & reſultat $\frac{e^{b+x} \pm e^{c-x}}{e^b \pm e^c} = \left(1 + \frac{4(b-c)x + 4x^2}{\pi^2 + (b-c)^2}\right)\left(1 + \frac{4(b-c)x + 4x^2}{9\pi^2 + (b-c)^2}\right)$ &c. ſi m ſit impar, vel $= \left(1 + \frac{2x}{b-c}\right)\left(1 + \frac{4(b-c)x + 4x^2}{4\pi^2 + (b-c)^2}\right)\left(1 + \frac{4(b-c)x + 4x^2}{16\pi^2 + (b-c)^2}\right)$, &c. ſi m ſit impar.

Et ſimiliter inveniri poteſt formula diviſorum quantitatis $e^{a+bx} \pm e^{c+dx} = \left(1 + \frac{a+bx}{i}\right)^i \pm \left(1 + \frac{c+dx}{i}\right)^i = v^i \pm z^i$, &c.

Plures hujuſmodi quantitates in ſeſe ducantur, vel per ſe dividantur; & exinde oriri poſſunt ſeries, quæ recipiant formulas diverſorum generum pro earum diviſoribus.

Si vero a vel b vel c vel d ſint impoſſibiles quantitates, viz. $p\sqrt{(-1)}$ & exponentialium ſumma vel differentia $e^{p\sqrt{(-1)}} \pm e^{-p\sqrt{(-1)}}$ denotat ſinum arcus circuli, qui pro ejus valore in æquationibus prius traditis ſubſtitutus, æquationes ſpecie diverſas reddent.

Cor. 4. Hinc inveniri poſſunt ſummæ plurium ſerierum; e factoribus quidem datis innoteſcunt radices ipſæ, & e terminis datæ æquationis innoteſcunt ſummæ prædictarum radicum, earum quadratorum, cuborum, &c. & denique aggregatum ex quacunque algebraicâ functione ſingularum radicum, cujus functionis ſinguli termini conſtituant terminos ſeriei datæ.

Cor. 4. Sit $z =$ datæ ſeriei $= \alpha \times \beta \times \gamma \times \delta \times$ &c. tum erit $\frac{\dot{z}}{z\dot{x}} = \frac{\ddot{\alpha}}{\alpha\dot{x}} + \frac{\dot{\beta}}{\beta\dot{x}} + \frac{\ddot{\gamma}}{\gamma\dot{x}} +$ &c. ſi modo x ſit variabilis quantitas. E. g. Sit $z = 1 + \frac{-x}{1.2} + \frac{x^2}{1.2.3} +$ &c. $= \left(1 + \frac{x}{i} + \frac{x^2}{4\pi^2}\right)\left(1 + \frac{x}{i} + \frac{x^2}{16\pi^2}\right)$, &c. ubi i eſt infinita quantitas, tum erit $\frac{\dot{z}}{z\dot{x}} = \frac{1}{1.2} + \frac{x}{1\,2} +$ &c. $= \frac{\frac{1}{i} + \frac{2x}{4\pi^2}}{1 + \frac{x}{i} + \frac{x^2}{4\pi^2}} + \frac{\frac{1}{i} + \frac{2x}{16\pi^2}}{1 + \frac{x}{i} + \frac{x^2}{16\pi^2}} +$ &c. & ſic de reliquis.

THEOR. XXXVII.

1. Sit $x = A - \frac{A^3}{1.2.3} + \frac{A^5}{1.2.3.4.5} - \frac{A^7}{1.2.3.4.5.6.7} +$ &c. tum erit x ſinus arcus (A) circuli, cujus radius eſt unitas, & cujus peripheria eſt 2π.

Si vero detur ſinus (x) arcus circuli, tum ejus correſpondentes arcus circuli, quorum ſinus eſt x, erunt reſpective $\frac{m\pi}{n}$, $\frac{n-m}{n}\pi$, $\frac{2n+m}{n}\pi$, $\frac{3n-m}{n}\pi$, &c. $-\frac{n+m}{n}\pi$, $-\frac{2n-m}{n}\pi$, $-\frac{3n+m}{n}\pi$, &c. quæ erunt radices (A) æquationis $x-A+\frac{A^3}{1.2.3}-\frac{A^5}{1.2.3.4.5}+\&c.=0$, & exinde reciprocæ radices erunt reſpective $\frac{n}{m\pi}$, $\frac{n}{(n-m).\pi}$, $\frac{n}{(2n+m).\pi}$, &c. $-\frac{n}{(n+m).\pi}$, $-\frac{n}{(2n-m).\pi}$, &c. & conſequenter per vulgarem algebram erit $\left(1-\frac{nA}{m\pi}\right)\left(1+\frac{nA}{(n+m)\pi}\right)\left(1-\frac{nA}{(n-m)\pi}\right)\left(1+\frac{nA}{(2n-m)\pi}\right)\&c.=1-\frac{A}{x}+\frac{A^3}{1.2.3x}-\frac{A^5}{1.2.3.4.5x}+\&c.$ unde ſumma e ſingulis quantitatibus $\frac{1}{m}$, $-\frac{1}{n+m}$, $\frac{1}{n-m}$, $\frac{-1}{2n-m}$, $\frac{1}{2n+m}$, &c. &c. erit $\frac{\pi}{nx}$; ſumma productorum e ſingulis duabus erit 0; ſummæ contentorum e ſingulis tribus, quatuor, quinque, &c. erunt reſpective $\frac{-\pi^3}{1.2.3xn^3}$, 0, $\frac{\pi^5}{1.2.3.4.5xn^5}$, 0, $\frac{-\pi^7}{1.2.3.4.5.6.7xn^7}$, 0, &c.

Pro $\frac{n}{m\pi}$ ſcribatur a, & erunt reciprocæ radices prædictæ $\frac{1}{a}$, $\frac{1}{\pi-a}$, $\frac{1}{2\pi+a}$, $\frac{1}{3\pi-a}$, $\frac{1}{4\pi+a}$, &c., $-\frac{1}{\pi+a}$, $-\frac{1}{2\pi-a}$, $-\frac{1}{3\pi+a}$, $-\frac{1}{4\pi-a}$, &c. quarum ſumma erit $\frac{1}{a}+\frac{2a}{\pi^2-a^2}-\frac{2a}{4\pi^2-a^2}+\frac{2a}{9\pi^2-a^2}-\frac{2a}{16\pi^2-a^2}+\&c.$

2. Sit æquatio $y = 1 - \frac{A^2}{1 \cdot 2} + \frac{A^4}{1 \cdot 2 \cdot 3 \cdot 4} - \frac{A^6}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6}$ + &c. & erit y coſinus arcus (A) circuli, cujus radius eſt 1, & peripheria dicatur 2π.

Si vero detur y coſinus arcus $A = \frac{r\pi}{n}$, tum erunt correſpondentes arcus circuli, qui ſint radices æquationis $y = 1 - \frac{A^2}{1 \cdot 2} + \frac{A^4}{1 \cdot 2 \cdot 3 \cdot 4} -$ &c. reſpective $\frac{r\pi}{n}, -\frac{r\pi}{n}, \frac{(2n-r) \cdot \pi}{n}, -\frac{(2n-r) \cdot \pi}{n}, \frac{(4n-r)\pi}{n}, -\frac{(4n-r) \cdot \pi}{n}, \frac{(6n-r) \cdot \pi}{n}$, &c. cujus reciprocæ radices erunt $\frac{n}{r\pi}, -\frac{n}{r\pi}, \frac{n}{(2n-r) \cdot \pi}, -\frac{n}{(2n-r)\pi}, \frac{n}{(4n-r)\pi}, -\frac{n}{(4n-r)\pi}$, &c. per vulgarem algebram erit $1 - \frac{A^2}{1 \cdot 2(1-y)} + \frac{A^4}{1 \cdot 2 \cdot 3 \cdot 4(1-y)} - \frac{A^6}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6(1-y)} + \&c. = \left(1 - \frac{nA}{r\pi}\right)\left(1 + \frac{nA}{r\pi}\right)\left(1 - \frac{nA}{(2n-r)\pi}\right)\left(1 + \frac{nA}{(2n-r)\pi}\right)\left(1 - \frac{nA}{(4n-r)\pi}\right)\left(1 + \frac{nA}{(4n-r)\pi}\right)$ &c. & conſequenter ſumma e ſingulis quantitatibus $\frac{n}{r\pi}, -\frac{n}{r\pi}, \frac{n}{(2n-r)\pi}, \frac{-n}{(2n-r)\pi}, \frac{n}{(2n+r)\pi}, \frac{-n}{(2n+r)\pi}, \frac{-n}{(4n+r)\pi}$, &c. nihilo æqualis erit; ſumma vero productorum e ſingulis duabus prædictis quantitatibus erit $\frac{-1}{1 \cdot 2(1-y)}$; aggregata vero contentorum e ſingulis tribus, quatuor, quinque, ſex, &c. erunt reſpective 0, $\frac{1}{1 \cdot 2 \cdot 3 \cdot 4(1-y)}$, 0, $\frac{-1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6(1-y)}$, 0, &c.

Sit $\frac{r\pi}{n} = a$, tum erunt reciprocæ radices prædictæ $\frac{1}{a}$, $\frac{1}{2\pi - a}$, $\frac{1}{2\pi + a}$, $\frac{1}{4\pi - a}$, $\frac{1}{4\pi + a}$, &c., $-\frac{1}{a}$, $-\frac{1}{2\pi - a}$, $+\frac{1}{2\pi + a}$, $-\frac{1}{4\pi - a}$, $-\frac{1}{4\pi + a}$, &c.

Cor. . Hinc e capite 1^{mo}. medit. algeb. facile colligi poſſunt aggregata e quicuſcunque rationalibus, & haud reciprocis functionibus radicum vel æquationis infinitæ (*A*) $1 - \frac{A^2}{1 \cdot 2(1-y)} + \frac{A^4}{1 \cdot 2 \cdot 3 \cdot 4(1-y)} - \&c. = 0$, vel etiam æquationis (*B*) $1 - \frac{A}{x} + \frac{A^3}{1 \cdot 2 \cdot 3x} - \frac{A^5}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5x} + \&c. = 0$, vel earum in ſeſe ductarum.

Ex. . In æquatione (*A*) detur y coſinus arcus nihilo æqualis, tum erit $n : r :: 2 : 1$; ſcribantur igitur 2 & 1 reſpective pro n & r in ejus radicibus prius traditis, & reſultant radices $\frac{2}{\pi}$, $-\frac{2}{\pi}$, $\frac{2}{3\pi}$, $-\frac{2}{3\pi}$, $\frac{2}{5\pi}$, $-\frac{2}{5\pi}$, $\frac{2}{7\pi}$, &c. quarum quadrata erunt $\frac{4}{\pi^2}$, $\frac{4}{\pi^2}$, $\frac{4}{9\pi^2}$, $\frac{4}{9\pi^2}$, $\frac{4}{25\pi^2}$, $\frac{4}{25\pi^2}$, &c. ſed per algebram erit ſumma quadratorum $1 = 8\left(\frac{1}{\pi^2} + \frac{1}{9\pi^2} + \frac{1}{25\pi^2} + \&c.\right)$ & exinde $\frac{\pi^2}{8} = 1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \&c. = S$; ſi vero $s = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \frac{1}{36} + \&c.$ tum erit $S + \frac{1}{4}s = s$, unde $s = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \&c. = \frac{4S}{3} = \frac{\pi^2}{2 \cdot 3}$.

Et ſic inveniatur ſumma quadrato-quadratorum e ſingulis prædictis quantitatibus, quæ evadet $32 \times \left(\frac{1}{\pi^4} + \frac{1}{81\pi^4} + \frac{1}{625\pi^4} + \&c.\right) = \frac{1}{3}$, & exinde $S = 1 + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + \&c. = \frac{\pi^4}{32 \cdot 3}$; ſed aſſumatur

$s = 1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} +$ &c. dividatur hæc æquatio per 2^4, & reſultat $\frac{s}{2^4} = \frac{1}{2^4} + \frac{1}{4^4} + \frac{1}{6^4} + \frac{1}{8^4} +$ &c. unde $S = s - \frac{s}{2^4}$, & conſequenter $\frac{16S}{15} = \frac{\pi^4}{3 \cdot 2 \cdot 3 \cdot 5} = s = 1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \frac{1}{5^4} +$ &c.

Et in genere ſit R ſumma 2λ poteſtatum e ſingulis radicibus prædictis, per prob. 1. medit. algebr. invenienda; tum erit $1 + \frac{1}{3^{2\lambda}} + \frac{1}{5^{2\lambda}} + \frac{1}{7^{2\lambda}} +$ &c. $= \frac{R\pi^2}{2^{2\lambda+1}} = S$, & $s = \frac{2^{2\lambda}}{2^{2\lambda} - 1} S = 1 + \frac{1}{2^{2\lambda}} + \frac{1}{3^{2\lambda}} + \frac{1}{5^{2\lambda}} + \frac{1}{7^{2\lambda}} +$ &c.

Ex. 2. Reciprocæ radices æquationis (B) $1 - \frac{A}{x} + \frac{A^3}{1 \cdot 2 \cdot 3 \cdot x} -$ &c. $= 0$ inventæ fuerunt $\frac{n}{m\pi}$; $\frac{n}{(n-m)\pi}$, $-\frac{n}{(n+m)\pi}$; $\frac{n}{(2n+m)\pi}$, $-\frac{n}{(2n-m)\pi}$; &c. quarum ſumma eſt $\frac{1}{x}$; ergo erit $\frac{1}{2m^2} + \frac{1}{n^2 - m^2} + \frac{1}{4n^2 - m^2} + \frac{1}{9n^2 - m^2} + \frac{1}{16n^2 - m^2} +$ &c. $= \frac{\pi}{2m.n.x}$.

Sit $n = 2m$, unde ſinus $x = 1$, & conſequenter radices prædictæ fient $\frac{2}{\pi}$, $\frac{2}{\pi}$, $-\frac{2}{3\pi}$, $-\frac{2}{3\pi}$, $\frac{2}{5\pi}$, $\frac{2}{5\pi}$, $-\frac{2}{7\pi}$, $-\frac{2}{7\pi}$, &c.

Et conſequenter erit $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} -$ &c. $= \frac{\pi}{4}$; & ſic $1 - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} +$ &c. $= \frac{1}{32}\pi^3$, & generaliter erit $1 - \frac{1}{3^{2\lambda+1}} + \frac{1}{5^{2\lambda+1}} - \frac{1}{7^{2\lambda+1}} +$ &c. $= \frac{N\pi^{2\lambda+1}}{2^{2\lambda+2}}$, ubi N æqualis erit aggregato $2\lambda + 1$ poteſtatis e ſingulis prædictis radicibus, quod facile e prob. 1. medit. algebr. deduci poteſt.

Sit $x = 0$, & conſequenter $m = 0$, & æquatio reſultat $A\left(1 - \frac{A^2}{2 \cdot 3} + \frac{A^4}{2 \cdot 3 \cdot 4 \cdot 5} - \&c.\right) = 0$, & exinde $1 - \frac{A^2}{2 \cdot 3} + \frac{A^4}{2 \cdot 3 \cdot 4 \cdot 5} - \&c.$ $= \left(1 - \frac{A}{\pi}\right)\left(1 + \frac{A}{\pi}\right)\left(1 + \frac{A}{2\pi}\right)\left(1 - \frac{A}{2\pi}\right)\left(1 + \frac{A}{3\pi}\right)\left(1 - \frac{A}{3\pi}\right)$ $(\&c.) = \left(1 - \frac{A^2}{\pi^2}\right)\left(1 - \frac{A^2}{4\pi^2}\right)\left(1 - \frac{A^2}{9\pi^2}\right)$ &c.

Sit $y = 0$ & ſimiliter colligi poteſt $\left(1 - \frac{A^2}{1 \cdot 2} + \frac{A^4}{1 \cdot 2 \cdot 3 \cdot 4} - \&c.\right)$ $= \left(1 - \frac{A^2}{\pi^2}\right)\left(1 - \frac{A^2}{9\pi^2}\right)\left(1 - \frac{A^2}{25\pi^2}\right)$ &c.

3. Sit æquatio $z = A - \frac{A^3}{1 \cdot 2} + \frac{A^5}{1 \cdot 2 \cdot 3 \cdot 4} - \&c.$ In hâc æquatione pro A ſcribatur $\sqrt{(-1)}\,a$, & reſultat æquatio $\frac{z}{\sqrt{(-1)}} = a + \frac{a^3}{2 \cdot 3} + \frac{a^5}{2 \cdot 3 \cdot 4 \cdot 5} + \&c.$ Aſſumatur $z = 0$, & dividatur æquatio reſultans per a, & reſultat æquatio $0 = 1 + \frac{a^2}{2 \cdot 3} + \frac{a^4}{2 \cdot 3 \cdot 4 \cdot 5} + \&c.$ $= \left(1 + \frac{a^2}{\pi^2}\right)\left(1 + \frac{a^2}{4\pi^2}\right)\left(1 + \frac{a^2}{9\pi^2}\right)$ &c.

4. Ob ſin. $x = x\left(1 - \frac{x^2}{\pi^2}\right)\left(1 - \frac{x^2}{4\pi^2}\right)\left(1 - \frac{x^2}{9\pi^2}\right)$ &c. & coſ. x $= \left(1 - \frac{4x^2}{\pi^2}\right)\left(1 - \frac{4x^2}{9\pi^2}\right)$ &c. & pro x ſcribatur $\frac{m\pi}{n}$ & duæ reſultant æquationes, viz. ſin. $\frac{m\pi}{n} = \frac{m\pi}{n}\left(\frac{n^2 - m^2}{n^2}\right)\left(\frac{4n^2 - m^2}{4n^2}\right)\left(\frac{9n^2 - m^2}{9n^2}\right)$ &c. & coſ. $\frac{m\pi}{n} = \left(\frac{n^2 - m^2}{n^2}\right)\left(\frac{9n^2 - m^2}{9n^2}\right)\left(\frac{25n^2 - m^2}{25n^2}\right)$ &c. e priori æquatione ſequitur $\pi = \frac{n}{m}$ ſin. $\frac{m\pi}{n} \times \left(\frac{n^2}{n^2 - m^2}\right)\left(\frac{4n^2}{4n^2 - m^2}\right)$

$(9n^2)$ &c.

$\left(\frac{9n^2}{9n^2-m^2}\right)$ &c. Sit $n = 2m$, & reſultat $\frac{\pi}{2} = \frac{2 \cdot 2}{1 \cdot 3} \cdot \frac{4 \cdot 4}{3 \cdot 5} \cdot \frac{6 \cdot 6}{5 \cdot 7} \cdot \frac{8 \cdot 8}{7 \cdot 9}$. &c. ſit $n = 4m$ & erit ſin. $\frac{m\pi}{n}$ = ſin. $45^{0} = \surd(2)$, & exinde $\frac{\pi}{4\surd(2)} = \frac{4 \cdot 4}{3 \cdot 5} \cdot \frac{8 \cdot 8}{7 \cdot 9} \cdot \frac{12 \cdot 12}{11 \cdot 13}$. &c. dividatur prior reſultans æquatio per poſteriorem, & habebitur $2\surd(2) = \frac{2 \cdot 2}{1 \cdot 3} \cdot \frac{6 \cdot 6}{5 \cdot 7} \cdot \frac{10 \cdot 10}{9 \cdot 11}$. &c.

Et ſic de pluribus hujuſcemodi æquationibus deducendis.

Cor. . Ob ſinum & coſinum per ſeries datos facile erui poſſunt tangens, cotangens, ſecans, coſecans, &c. prædicti arcus $\left(\frac{m\pi}{n}\right)$.

Cor. . Hinc etiam multiplicari, dividi, &c. poſſunt ſinus, coſinus, &c. duorum vel plurium diverſorum arcuum. E. g. $\frac{\text{ſin. ar. } \frac{m\pi}{n}}{\text{ſin. ar. } \frac{k\pi}{n}} = \frac{m}{k}\left(\frac{n^2-m^2}{n^2-k^2}\right)\left(\frac{4n^2-m^2}{4n^2-k^2}\right)\left(\frac{9n^2-m^2}{9n^2-k^2}\right)$ &c.

5. Datâ præcedente æquatione $1 - \frac{A}{x} + \frac{A^3}{2 \cdot 3x} - \frac{A^5}{2 \cdot 3 \cdot 4 \cdot 5x} + \text{\&c.} = 0$, cujus reciprocæ radices ſint $\alpha, \beta, \gamma, \delta$, &c. Fingatur $Z = 1 - \frac{A}{x} + \frac{A^3}{2 \cdot 3x} - \text{\&c.}$ unde conſtat $\frac{\dot{Z}}{Z\dot{A}} = (\alpha + \beta + \gamma + \delta + \text{\&c.}) + (\alpha^2 + \beta^2 + \gamma^2 + \delta^3 + \text{\&c.})A + (\alpha^3 + \beta^3 + \gamma^3 + \delta^3 + \text{\&c.})A^2 + \text{\&c.} = a + bA + cA^2 + \text{\&c.}$ ſed $Z = 1 - \frac{A}{x} + \frac{A^3}{2 \cdot 3x} - \text{\&c.} = 1 - \frac{1}{x}$ ſin. arc. A, & conſequenter $\frac{\dot{Z}}{Z\dot{A}} = \frac{\text{coſ. arc. } A}{x - \text{ſin. arc. } A}$.

Scribatur $\frac{p\pi}{n}P$ pro A in datâ æquatione, & quoniam $\frac{m\pi}{n}, \frac{n-m}{n}\pi$, &c. ſunt radices datæ æquationis, erunt $\frac{p}{m}, \frac{p}{n-m}, -\frac{p}{n+m}, -$

$2n - m,$

$\frac{p}{2n-m}$, $\frac{p}{2n+m}$, &c. reciprocæ radices æquationis reſultantis $1 - \frac{p\pi P}{nx} + \frac{p^3\pi^3 P^3}{2 \cdot 3 n^3 x} - \&c. = 0$: ſtatuantur

$$K = \frac{p}{m} + \frac{p}{n-m} - \frac{p}{n+m} - \frac{p}{2n-m} + \&c.$$

$$L = \frac{p^2}{m^2} + \frac{p^2}{(n-m)^2} + \frac{p^2}{(n+m)^2} + \&c.$$

$$M = \frac{p^3}{m^3} + \frac{p^3}{(n-m)^3} - \frac{p^3}{(n+m)^3} - \&c.$$

&c. &c. &c. & exinde conſtat $K + L + M + \&c. = \frac{p}{m-p} + \frac{p}{n-m-p} - \frac{p}{n+m+p} - \frac{p}{2n-m+p} + \&c. = \frac{p\pi \text{ coſ. arc. } \frac{p\pi}{n}}{nx - n \text{ ſin. arc. } \frac{p\pi}{n}}$, ubi $x = \text{ſin. arc. } \frac{m\pi}{n}$: in hâc ſerie ſcribatur $m = \frac{k+l}{2}$ & $p = \frac{l-k}{2}$, & reſultat ſeries $\frac{1}{k} + \frac{1}{n-l} - \frac{1}{n+l} - \frac{1}{2n-k} + \frac{1}{2n+k} + \&c.$ ſeu $\frac{1}{k} + \frac{2l}{n^2-l^2} - \frac{2k}{4n^2-k^2} + \&c. = \frac{\pi \text{ coſ. arc. } \frac{l-k}{2n}}{n \text{ ſin. arc. } \frac{k+l}{2n}\pi - n \text{ ſin. arc. } \frac{(l-k)\pi}{2n}}$.

Inveniri poſſunt etiam e præcedentibus principiis $\frac{1}{1+b} - \frac{1}{4+b} + \frac{1}{9+b} - \frac{1}{16+b} + \&c. = \frac{1}{2b} - \frac{\pi\surd(b)}{(e^{\pi\surd(b)} - e^{-\pi\surd(b)})b}$ & $\frac{1}{1+b} + \frac{1}{4+b} + \frac{1}{9+b} + \&c. = \frac{(e^{\pi\surd(b)} + e^{-\pi\surd(b)})\pi\surd(b)}{2b(e^{\pi\surd(b)} - e^{-\pi\surd(b)})} - \frac{1}{2b}$.

Erit $\frac{\pi \log. 2}{2} = 1 + \frac{1}{2.3^2} + \frac{1.3}{2.4.5^2} + \frac{1.3.5}{2.4.6.7^2} +$ &c.

Et ſic e meditat. algebraicis colligi poſſunt plurimæ conſimiles ſeries, i. e. ſumma cujuſcunque functionis, in quâ haud reciproca vel nulla dimenſio prædictarum radicum continetur: progredi etiam liceat ad radices æquationis quæ eſt fluxio datæ æquationis; & ſic deinceps. Forſan vero quærat aliquis, quare ratiocinatio, quæ prius tradita fuit de ſeriebus ſinum circuli ex ejus arcu exprimentibus haud applicari poteſt ad ſeries quæ detegunt ſinum, i. e. ordinatam vel ellipſeos vel plurimarum etiam ovalium conſimilium ex earum arcubus: cui reſpondendum eſt, ut ſeries prædictæ haud omnes forſan habent radices poſſibiles, vel omnes radices datæ infinitæ æquationis non in prædictâ ſerie continentur.

Si modo igitur dentur termini infinitæ ſeriei, cujus omnes radices innoteſcunt; tum facile erui poſſunt e præcedente methodo infinitæ ſeries, quarum ſummæ innoteſcunt, i. e. e ſummis ſingulorum valorum quarumcunque algebraicarum functionum prædictarum radicum medit. algeb. capit. primo inveſtigatis.

Hæ ſeries, quarum ſummæ innoteſcunt, haud nunquam uſui inſervire poſſunt in datis ſeriebus ſummandis; reducantur enim datæ ſeries ad plures, ſi ita reduci poſſint, quarum reſultantes termini qui lentiſſime convergant, ſint ſeries, quarum ſummæ innoteſcunt; pro his terminis ſcribantur prædictæ ſummæ, & reſultant ſeries magis convergentes.

THEOR. XXXVIII.

Ex. Datis fluentibus fluxionum $\frac{\dot{x}}{1+ax^n}$, $\frac{\dot{x}}{x}\int.\frac{\dot{x}}{1+ax^n}$, $\frac{\dot{x}}{x}\int.\frac{\dot{x}}{x}\int.\frac{\dot{x}}{1+ax^n}$, $\frac{\dot{x}}{x}\int.\frac{\dot{x}}{x}\int.\frac{\dot{x}}{x}\int.\frac{1}{1+ax^n}$, &c. quæ erunt $x - \frac{ax^{n+1}}{n+1} + \frac{a^2x^{2n+1}}{2n+1} - \frac{a^3x^{3n+1}}{3n+1} + \frac{a^4x^{4n+1}}{4n+1} -$ &c., $x - \frac{ax^{n+1}}{(n+1)^2} + \frac{a^2x^{2n+1}}{(2n+1)^2} - \frac{a^3x^{3n+1}}{(3n+1)^2} +$ &c.,

$x -$

$x - \frac{ax^{n+1}}{(n+1)^3} + \frac{a^2x^{2n+1}}{(2n+1)^3} - \frac{a^3x^{3n+1}}{(3n+1)^3} + \&c., \ldots, x - \frac{ax^{n+1}}{(n+1)^4} + \frac{a^2x^{2n+1}}{(2n+1)^4} - \frac{a^3x^{3n+1}}{(3n+1)^4} + \&c. \&c.$ & fluentibus fluxionum $\frac{x^z\dot{x}}{1+ax^n}$, erui poſſunt ſummæ quarumcunque ſerierum, quarum generalis terminus eſt $\pm \frac{nz+m \,.\, nz+m' \,.\, nz+m'' \,.\, nz+m''' \,.\, \&c. \times a^z \times x^{nz+1}}{nz+l \,.\, nz+l' \,.\, nz+l'' \,.\, \&c. \times (zn+1)^{b-1}}$; ubi l, l', l'', &c., ſint inter ſe diverſæ quantitates, & z eſt diſtantia a primo ſeriei termino.

Pro $\int. \frac{\dot{x}}{1+ax^n}$, $\int. \frac{\dot{x}}{x} \times \int. \frac{\dot{x}}{1+ax^n}$, $\int. \frac{\dot{x}}{x} \int. \frac{\dot{x}}{x} \int. \frac{\dot{x}}{1+ax^n}$, $\int. \frac{\dot{x}}{x} \int. \frac{\dot{x}}{x} \int. \frac{\dot{x}}{x} \int. \frac{\dot{x}}{1+ax^n}$, &c. ſcribantur reſpective $A, B, C, D, \ldots P, \mathcal{Q}, R$ & S & T; ducatur ultima fluens in $x^{l-2}\dot{x}$, & inveniatur fluens fluxionis reſultantis, quæ erit $V = \frac{1}{l-1} x^{l-1} \times (T - \frac{1}{(l-1)} S + \frac{1}{(l-1)^2} R - \frac{1}{(l-1)^3} \mathcal{Q} + \ldots \pm \frac{1}{(l-1)^{b-4}} C \mp \frac{1}{(l-1)^{b-3}} B \pm \frac{1}{(l-1)^{b-2}} A) \mp \frac{1}{(l-1)^{b-1}} \times \int. \frac{x^{l-1}\dot{x}}{1+ax^n}$, ubi b deſignat numerum literarum $A, B, C, \ldots R, S$ & T per unitatem auctum: hæc autem fluens proprie correcta æquat ſummam ſeriei, cujus generalis terminus eſt $\frac{\pm 1}{nz+l \,.\, (nz+1)^{b-1}} \times a^z \times x^{nz+l}$; ducatur prædicta fluens V in $x^{l'-l-1}\dot{x}$ & inveniatur per præcedentem methodum fluens fluxionis $x^{l'-l-1}\dot{x} \times V$, & ea deſignat ſummam ſeriei, cujus generalis terminus eſt $\frac{\pm a^z}{nz+l \,.\, nz+l' \times (zn+1)^{b-1}} \times x^{nz+l'}$, & ſimiliter introducantur reliqui factores in denominatorem: ſit $nz+X$ ultimus factor qui in denominatorem introducitur; tum ducatur fluens prius reſultans W in x^{m-X}, & inveniatur fluxio $(m-X) x^{m-X-1}\dot{x}W + x^{m-X}\dot{W}$; & conſequenter quantitas $Y = (m-X) x^{m-X-1}W + x^{m-X} \times \frac{\dot{W}}{\dot{x}}$, quæ æquat ſummam ſeriei, cujus generalis terminus eſt $\frac{\pm(nz+m).q^{z}\times x^{nz+l+m-1}}{\overline{nz+l}.\overline{nz+l'}..\overline{nz+X}\times(nz+1)^{b-1}}$; tum ducatur quantitas Y in $x^{m'-m+1}$ & reſultat quantitas cujus fluxio per $\dot{x}$ diviſa præbet ſummam ſeriei, cujus generalis terminus eſt $\pm\frac{\overline{nz+m}.\overline{nz+m'}.a^{z}.x^{nz+m'-1}}{\overline{nz+l}.\overline{nz+l'}.\&c.\times(nz+1)^{b-1}}$; & ſic introduci poſſunt quicunque factores in numeratorem; ergo conſtat theorema.

Hæc methodus fallit vel cum $l-1=0$ vel $l'-1=0$ vel $l''-1=0$, &c.

Si unquam occurrat in denominatore vel denominatoribus terminorum ſeriei, cujus ſumma quæritur, factor $nz+l=0$ vel $nz+l'=0$ vel $nz+l''=0$, &c.; tum, ut prius aſſeritur ſemper terminus erui poteſt e fluente fluxionis $x^{n}\dot{x}\int.\left(\frac{\dot{x}}{x}\right)^{n}$, ubi n eſt integer numerus, cujus fluens prius traditur.

2. Sit ſeries, cujus generalis terminus eſt $\frac{\pm a^{z}x^{nz}}{(nz+l')^{b'}\times(nz+l)^{b}}$, & dentur fluentes $A, B, C, \ldots Q, R, S$ & T fluxionum $\frac{x^{l-1}\dot{x}}{1+ax^{n}}$, $\frac{\dot{x}}{x}\int.\frac{x^{l-1}\dot{x}}{1+ax^{n}}$, $\frac{\dot{x}}{x}\int.\frac{\dot{x}}{x}\int.\frac{x^{l-1}\dot{x}}{1+ax^{n}}$, $\frac{\dot{x}}{x}\int.\frac{\dot{x}}{x}\int.\frac{\dot{x}}{x}\int.\frac{x^{l-1}\dot{x}}{1+ax^{n}}$, &c. quarum numerus eſt b; etiamque dentur fluentes $A', B', C', \ldots Q', R', S'$ & T' fluxionum $\frac{x^{l'-1}\dot{x}}{1+ax^{n}}$, $\frac{\dot{x}}{x}\int.\frac{x^{l'-1}\dot{x}}{1+ax^{n}}$, $\frac{\dot{x}}{x}\int.\frac{\dot{x}}{x}\int.\frac{x^{l'-1}\dot{x}}{1+ax^{n}}$, &c. quarum numerus eſt b'; tum erit quantitas $\frac{1}{(l'-l)^{b'}}x^{-l}\Big(T-\frac{b'}{l'-l}S+\frac{b'.\frac{b'+1}{2}}{(l'-l)^{2}}\times R-\frac{b'\times\frac{b'+1}{2}\times\frac{b'+2}{3}}{(l'-l)^{3}}Q\ldots\pm\&c.\times A\Big)+\frac{1}{(l-l')^{b}}\times x^{-l'}\Big(T'-\frac{1}{l-l'}S'+\frac{b.\frac{b+1}{2}}{(l-l')^{2}}R'-\frac{b.\frac{b+1}{2}.\frac{b+2}{3}}{(l-l')^{3}}Q'+\ldots\pm\&c.\ A'\Big)$ ſumma ſeriei quæſita.

Ad eundem modum ſumma ſeriei, cujus generalis terminus eſt $\frac{(az^m + bz^{m-1} + cz^{m-2} + \&c.)\, a^z x^{nz}}{(nz+l)^b \times (nz+l')^{b'} \times (nz+l'')^{b''} \times \&c.}$ ex (b) fluentibus fluxionum $\frac{x^{l-1}\dot{x}}{1+ax^n}$, $\frac{\dot{x}}{x}\int.\frac{x^{l-1}\dot{x}}{1+ax^n}$, $\frac{\dot{x}}{x}\int.\frac{\dot{x}}{x}\int.\frac{x^{l-1}\dot{x}}{1+ax^n}$, &c.; & (b') fluentibus fluxionum $\frac{x^{l'-1}\dot{x}}{1+ax^n}$, $\frac{\dot{x}}{x}\int.\frac{x^{l'-1}\dot{x}}{1+ax^n}$, $\frac{\dot{x}}{x}\int.\frac{\dot{x}}{x}\int.\frac{x^{l'-1}\dot{x}}{1+ax^n}$, &c. & (b'') fluentibus fluxionum $\frac{x^{l''-1}\dot{x}}{1+ax^n}$, $\frac{\dot{x}}{x}\int.\frac{x^{l''-1}\dot{x}}{1+ax^n}$, $\frac{\dot{x}}{x}\int.\frac{\dot{x}}{x}\int.\frac{x^{l''-1}\dot{x}}{1+ax^n}$, &c.; &c.; ſi modo b, b', b'', &c. & m ſint integri numeri, acquiri poteſt.

Hìc obſervandum eſt omnes fluentes prædictas inter valores o & x quantitatis x contentas eſſe, ſi ſeries $Ax^r + Bx^s + \&c.$ aſcendat ſecundum dimenſiones quantitatis x; vel inter infinitum & x ſi ſeries deſcendat ſecundum dimenſiones quantitatis x: ſi vero ſeries $Ax^r + Bx^s + \&c.$ nonnullos affirmativos & nonnullos negativos indices quantitatis x contineat; tum in ſerie aſcendente pro x ſcribatur o, vel in ſerie deſcendente pro x ſcribatur infinita quantitas, & reſultet quantitas A'; & pro fluente prædictæ fluxionis aſſumatur fluens inter valores o & x quantitatis $x + A'$.

THEOR. XXXIX.

1. Erit $\left(1 \pm \frac{a}{x}\right)\left(1 \pm \frac{b}{x}\right)\left(1 \pm \frac{c}{x}\right)\left(1 \pm \frac{d}{x}\right)\left(1 \pm \frac{e}{x}\right) \&c. = 1 \pm (a+b+c+d+e+\&c.) \times \frac{1}{x} + (ab+ac+bc+ad+bd+\&c.) \times \frac{1}{x^2} \pm (abc+abd+acd+bcd+\&c.) \times \frac{1}{x^3} + (abcd+\&c.) \times \frac{1}{x^4} + \&c.$

Cor. 1. Sint $a = \frac{1}{2^n}$, $b = \frac{1}{3^n}$, $c = \frac{1}{5^n}$, $d = \frac{1}{7^n}$, &c. & $x = 1$; tum erit $\frac{1}{M} = \left(1 - \frac{1}{2^n}\right)\left(1 - \frac{1}{3^n}\right)\left(1 - \frac{1}{5^n}\right)\left(1 - \frac{1}{7^n}\right)$ &c. $= 1 - \frac{1}{2^n} - \frac{1}{3^n} - \frac{1}{5^n} + \frac{1}{6^n} - \frac{1}{7^n} +$ &c.

Cor. 2. Sit $a = \frac{1}{2^{2n}}$, $b = \frac{1}{3^{2n}}$, $c = \frac{1}{5^{2n}}$, $d = \frac{1}{7^{2n}}$, &c., & $x = 1$; tum erit $\frac{1}{N} = \left(1 - \frac{1}{2^{2n}}\right)\left(1 - \frac{1}{3^{2n}}\right)\left(1 - \frac{1}{5^{2n}}\right)\left(1 - \frac{1}{7^{2n}}\right)$ &c. $= 1 - \frac{1}{2^{2n}} - \frac{1}{3^{2n}} - \frac{1}{5^{2n}} + \frac{1}{6^{2n}} - \frac{1}{7^{2n}} +$ &c.

In his duobus caſibus ſigna affixa erunt + vel —, prout par vel impar ſit numerus diviſorum primorum numerorum 2, 3, 5, 7, &c. pro denominatore in ſeſe ductorum.

Cor. 3. Erit $\left(1 + \frac{1}{2^n}\right)\left(1 + \frac{1}{3^n}\right)\left(1 + \frac{1}{5^n}\right)$ &c. $= 1 + \frac{1}{2^n} + \frac{1}{3^n} + \frac{1}{5^n} + \frac{1}{6^n} + \frac{1}{7^n} + \frac{1}{10^n} +$ &c.; in quâ continentur omnes naturales numeri, qui non habent quadratum numerum pro diviſore. Et ſimiliter $\left(1 + \frac{1}{2^{2n}}\right)\left(1 + \frac{1}{3^{2n}}\right)\left(1 + \frac{1}{5^{2n}}\right)$ &c. $= 1 + \frac{1}{2^{2n}} + \frac{1}{3^{2n}} + \frac{1}{5^{2n}} + \frac{1}{6^{2n}} +$ &c.

Cor. 4. Sint 2, 3, 5, 7, 11, &c. primi numeri, & erit contentum $(1+2)(1+3)(1+5)(1+7)$ &c. $= 1 + (2 + 3 + 5 + 7 +$ &c.$) + 2(3 + 5 + 7 +$ &c.$) + 3(5 + 7 +$ &c.$) +$ &c. quantitas, in quâ occurrunt omnes naturales numeri præter eos, qui plures diviſores æquales accipiant.

2. Erit $\frac{1}{\left(1 \pm \frac{a}{x}\right)\left(1 \pm \frac{b}{x}\right)\left(1 \pm \frac{c}{x}\right)\left(1 \pm \frac{d}{x}\right) \text{ \&c.}} = 1 \mp (a + b$

+ c

$+ c + d + \&c.) \times \frac{1}{x} + (a^2 + b^2 + c^2 + \&c. + ab + ac + bc +$
$ad + \&c.) \times \frac{1}{x^2} + (a^3 + b^3 + c^3 + \&c. + a^2 b + a^2 c + b^2 c + \&c.$
$+ abc + abd + acd + \&c.) \times \frac{1}{x^3} + (a^4 + b^4 + \&c. + a^3 b + \&c.$
$+ a^2 b^2 + \&c. + a^2 bc + \&c. + abcd + \&c.) \times \frac{1}{x^4} + \&c.$

Cor. 1. Sint $a = \frac{1}{2^n}$, $b = \frac{1}{3^n}$, $c = \frac{1}{5^n}$, &c., & $x = 1$; tum erit $M =$
$\frac{1}{\left(1 - \frac{1}{2^n}\right)\left(1 - \frac{1}{3^n}\right)\left(1 - \frac{1}{5^n}\right)\left(1 - \frac{1}{7^n}\right) \&c.} = 1 + \frac{1}{2^n} + \frac{1}{3^n} + \frac{1}{4^n}$
$+ \frac{1}{5^n} + \frac{1}{6^n} + \frac{1}{7^n} + \frac{1}{8^n} + \&c.$

Cor. 2. Sint $a = \frac{1}{2^{2n}}$, $b = \frac{1}{3^{2n}}$, $c = \frac{1}{5^{2n}}$, &c.; tum erit $N =$
$\frac{1}{\left(1 - \frac{1}{2^{2n}}\right)\left(1 - \frac{1}{3^{2n}}\right)\left(1 - \frac{1}{5^{2n}}\right) \&c.} = 1 + \frac{1}{2^{2n}} + \frac{1}{3^{2n}} + \frac{1}{4^{2n}} + \frac{1}{5^{2n}}$
$+ \frac{1}{6^{2n}} + \&c.$

Cor. 3. Erit $\frac{1}{\left(1 + \frac{1}{2^n}\right)\left(1 + \frac{1}{3^n}\right)\left(1 + \frac{1}{5^n}\right) \&c.} = 1 - \frac{1}{2^n} - \frac{1}{3^n}$
$+ \frac{1}{4^n} - \frac{1}{5^n} + \frac{1}{6^n} - \frac{1}{7^n} - \frac{1}{8^n} + \frac{1}{9^n} - \&c.$, &c.; ſigna affixa erunt affirmativa vel negativa, prout numerus diviſorum, qui ſunt primi numeri, ſit par vel impar.

Cor. 4. Sit $n = 1$, & erit $N = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \&c. = \frac{\pi^2}{6}$,

& $\frac{1}{M} = 1 - \frac{1}{2} - \frac{1}{3} - \frac{1}{5} + \frac{1}{6} - \frac{1}{7} +$ &c. $= 0$, & $\frac{1}{N} = 1 - \frac{1}{2^2} - \frac{1}{3^2} - \frac{1}{5^2} + \frac{1}{6^2}$ &c. $= \frac{6}{\pi^2}$.

3. Erit $L = \left(1 \pm \frac{a}{x}\right)^{\pm m} \times \left(1 \pm \frac{b}{x}\right)^{\pm m} \times \left(1 \pm \frac{c}{x}\right)^{\pm m} \times \left(1 \pm \frac{d}{x}\right)^{\pm m} \times$ &c. $= 1 \pm m (a + b + c + d + \&c.) \frac{1}{x} \pm m \times \frac{\pm m - 1}{2} (a^2 + b^2 + c^2 + \&c.) \frac{1}{x^2} + m^2 (ab + ac + bc + ad + \&c.) \frac{1}{x^2} \pm m \times \frac{\pm m - 1}{2} \times \frac{\pm m - 2}{3} (a^3 + b^3 + \&c.) \frac{1}{x^3} \pm m^3 (abc + abd + acd + bcd + \&c.) \frac{1}{x^3} \pm$ &c.

Cor. 1. Sint $a = \frac{1}{2^n}$, $b = \frac{1}{3^n}$, $c = \frac{1}{5^n}$, $d = \frac{1}{7^n}$, &c. & $x = 1$, tum erit $\left(1 \pm \frac{1}{2^n}\right)^{\pm m} \left(1 \pm \frac{1}{3^n}\right)^{\pm m} \left(1 \pm \frac{1}{5^n}\right)^{\pm m} \left(1 \pm \frac{1}{7^n}\right)^{\pm m}$ &c. $= 1 \pm m \left(\frac{1}{2^n} + \frac{1}{3^n} + \frac{1}{5^n} + \frac{1}{7^n} + \&c.\right) \pm m . \frac{m - 1}{2} \left(\frac{1}{2^{2n}} + \frac{1}{3^{2n}} + \&c.\right) + m^2 \left(\frac{1}{2^n . 3^n} + \frac{1}{2^n . 5^n} + \frac{1}{3^n . 5^n} + \frac{1}{2^n . 7^n} + \frac{1}{3^n . 7^n} + \&c.\right) \pm m . \frac{m - 1}{2} . \frac{m - 2}{3} \left(\frac{1}{2^{3n}} + \frac{1}{3^{3n}} + \frac{1}{5^{3n}} + \&c.\right) \pm m^3 \left(\frac{1}{2^n . 3^n . 5^n} + \frac{1}{2^n . 3^n . 7^n} + \frac{1}{3^n . 5^n . 7^n} + \&c.\right) \pm$ &c.; etiamque erit $\left(1 \pm \frac{1}{2^n}\right)^{-m} \times \left(1 \pm \frac{1}{3^n}\right)^{-m} \times \left(1 \pm \frac{1}{5^n}\right)^{-m} \times \left(1 \pm \frac{1}{7^n}\right)^{-m}$ &c. $= 1 \mp m \left(\frac{1}{2^n} + \frac{1}{3^n} + \frac{1}{5^n} + \frac{1}{7^n} + \&c.\right) - m \times \frac{-m - 1}{2} \times \left(\frac{1}{2^{2n}} + \frac{1}{3^{2n}} + \&c.\right) + m^2 \times \left(\frac{1}{2^n . 3^n} + \frac{1}{2^n . 5^n} + \frac{1}{3^n . 5^n} + \frac{1}{2^n . 7^n} + \frac{1}{3^n . 7^n} + \frac{1}{5^n . 7^n} + \&c.\right) \mp m^3 . \left(\frac{1}{2^n . 3^n . 5^n} + \frac{1}{2^n . 3^n . 7^n} +\right.$

$\frac{1}{2^n.5^n.7^n} + \&c. + \frac{1}{3^n.5^n.7^n} + \frac{1}{2^n.3^n.11^n} + \&c.) - \&c.$; in hoc caſu omnes naturales numeri integri, qui primi ſunt, reciproce in primâ ſummâ continentur, & in n ducuntur, & in ſubſequentibus aggregatis e ſingulis contentis hujuſce generis $\pi^{-rn} \times \rho^{-sn} \times \sigma^{-tn}$ &c., ubi π, ρ, σ, &c. ſunt quicunque diverſi primi, & r, s, t, &c. integri numeri, in ſerie continentur; & hæc aggregata ducuntur in coefficientes $m.\frac{m-1}{2}\ldots\frac{m-r+1}{r} \times m.\frac{m-1}{2}\ldots\frac{m-s+1}{s} \times m.\frac{m-1}{2}\ldots\frac{m-t+1}{t} \times \&c.$

Et ſic deduci poſſunt infinitæ conſimiles propoſitiones ex aſſumptis quibuſcunque factoribus, & reductis aſſumptis quantitatibus per methodos diviſionis, radicum extractionis, &c. ad infinitas ſeries, quæ evadunt convergentes.

5. Erit $M = \frac{1}{\left(1-\frac{1}{2^n}\right)\left(1-\frac{1}{3^n}\right)\left(1-\frac{1}{5^n}\right)\left(1-\frac{1}{7^n}\right)\&c.} = \frac{2^n}{2^n-1} \times \frac{3^n}{3^n-1} \times \frac{5^n}{5^n-1} . \&c. = 1 + \frac{1}{2^n} + \frac{1}{3^n} + \frac{1}{4^n} + \&c.$; & ſimiliter $N = \frac{1}{\left(1-\frac{1}{2^{2n}}\right)\left(1-\frac{1}{3^{2n}}\right)\left(1-\frac{1}{5^{2n}}\right)\left(1-\frac{1}{7^{2n}}\right)\&c.} = \frac{2^{2n}}{2^{2n}-1} \times \frac{3^{2n}}{3^{2n}-1} \times \frac{5^{2n}}{5^{2n}-1} \times \&c. = 1 + \frac{1}{2^{2n}} + \frac{1}{3^{2n}} + \frac{1}{4^{2n}} + \frac{1}{5^{2n}} + \&c.$; unde $\frac{M}{N} = \left(1+\frac{1}{2^n}\right)\left(1+\frac{1}{3^n}\right)\left(1+\frac{1}{5^n}\right)\left(1+\frac{1}{7^n}\right)\&c.$, & $\frac{MM}{N} = \frac{2^n+1}{2^n-1} \times \frac{3^n+1}{3^n-1} \times \frac{5^n+1}{5^n-1} \times \frac{7^n+1}{7^n-1} \times \&c.$

6. Erit $\log. M = \log.\left(1 + \frac{1}{2^n} + \frac{1}{3^n} + \&c.\right) = -\log.\left(1-\frac{1}{2^n}\right) - \log.\left(1-\frac{1}{3^n}\right) - \log.\left(1-\frac{1}{5^n}\right) - \&c. = 1\left(\frac{1}{2^n} + \frac{1}{3^n} + \frac{1}{5^n} + \&c.\right) +$

$\frac{1}{2}\left(\frac{1}{2^{2n}} + \frac{1}{3^{2n}} + \frac{1}{5^{2n}} + \text{\&c.}\right) + \frac{1}{3}\left(\frac{1}{2^{3n}} + \frac{1}{3^{3n}} + \frac{1}{5^{3n}} + \text{\&c.}\right) + \frac{1}{4}\left(\frac{1}{2^{4n}}\right.$ $\left. + \text{\&c.}\right)$

Cor. . Quoniam prius inveſtigatæ fuerunt ſummæ ſerierum $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \text{\&c.} = \frac{\pi^2}{6}$, & $1 + \frac{1}{2^{2n}} + \frac{1}{3^{2n}} + \frac{1}{4^{2n}} + \text{\&c.} = \frac{1}{a}\pi^{2n}$; ubi n eſt integer numerus; & conſequenter invenitur valor fractionis $\frac{1}{\left(1 - \frac{1}{2^{2n}}\right)\left(1 - \frac{1}{3^{2n}}\right)\left(1 - \frac{1}{5^{2n}}\right)\text{\&c.}}$ huic ſeriei æqualis; unde conſtat $2n \log. \pi - \log. a = -\log.\left(1 - \frac{1}{2^{2n}}\right) - \log.\left(1 - \frac{1}{3^{2n}}\right) - \text{\&c.}$ $= 1\left(\frac{1}{2^{2n}} + \frac{1}{3^{2n}} + \frac{1}{4^{2n}} + \text{\&c.}\right) + \frac{1}{2}\left(\frac{1}{2^{4n}} + \frac{1}{3^{4n}} + \text{\&c.}\right) + \frac{1}{3}\left(\frac{1}{2^{6n}} + \frac{1}{3^{6n}}\right.$ $\left. + \text{\&c.}\right) + \text{\&c.}$

Inveniantur valores e pluribus hujuſmodi ſeriebus, i. e. cum $2n$ æquat reſpective α, β, γ, &c. qui ſint $\frac{\pi^\alpha}{b}$, $\frac{\pi^\beta}{c}$, $\frac{\pi^\gamma}{d}$, &c. unde eorum logarithmi erunt reſpective $\alpha \log. \pi - \log. b$; $\beta \log. \pi - \log. c$; $\gamma \log. \pi - \log. d$; &c. inveniantur ſeries, quæ his logarithmis reſpective ſunt æquales. Ducantur hæ ſeries reſpective in quantitates k, l, m, &c. & erit ſerierum reſultantium aggregatum $= (k\alpha + l\beta + m\gamma + \text{\&c.}) \log. \pi - \log. b^k c^l d^m$, &c.

Fiat $k\alpha + l\beta + m\gamma + \text{\&c.} = 0$, & reſultant ſeries, quæ cognitæ quantitatis logarithmo æquales ſunt.

THEOR. XL.

1. Sit $A = 1 + \frac{1}{2^n} + \frac{1}{3^n} + \frac{1}{4^n} + \text{\&c.}$ ducatur hæc æquatio in $1 - \frac{1}{2^n}$ & reſultat $\left(1 - \frac{1}{2^n}\right)A = B = 1 + \frac{1}{3^n} + \frac{1}{5^n} + \frac{1}{7^n} + \text{\&c.}$

ducatur hæc æquatio in $\left(1-\frac{1}{3^n}\right)$ & reſultat $\left(1-\frac{1}{3^n}\right)B=C=1+\frac{1}{5^n}+\frac{1}{7^n}+\frac{1}{11^n}+$ &c. ducatur hæc æquatio in $1-\frac{1}{5^n}$, & reſultat $\left(1-\frac{1}{5^n}\right)C=D=1+\frac{1}{7^n}+\frac{1}{11^n}+$ &c. &c. unde $A=\left(1-\frac{1}{2^n}\right)\left(1-\frac{1}{3^n}\right)\left(1-\frac{1}{5^n}\right)\left(1-\frac{1}{7^n}\right)\left(1-\frac{1}{11^n}\right)$ &c. $=\frac{2^n-1}{2^n}\cdot\frac{3^n-1}{3^n}\cdot\frac{5^n-1}{5^n}\cdot$ &c.

2. Sit $A'=1-\frac{1}{3^n}+\frac{1}{5^n}-\frac{1}{7^n}+\frac{1}{9^n}-\frac{1}{11^n}+$ &c. ducatur hæc æquatio in $1+\frac{1}{3^n}$, & reſultat $\left(1+\frac{1}{3^n}\right)A'=B'=1+\frac{1}{5^n}-\frac{1}{7^n}+$ &c. ducatur reſultans æquatio in $1-\frac{1}{5^n}$, & reſultat $\left(1-\frac{1}{5^n}\right)B'=C'=1-\frac{1}{7^n}-\frac{1}{11^n}+$ &c. unde $A'^{-1}=\left(1+\frac{1}{3^n}\right)\left(1-\frac{1}{5^n}\right)\left(1+\frac{1}{7^n}\right)\left(1+\frac{1}{11^n}\right)$ &c. $=\frac{3^n+1}{3^n}\cdot\frac{5^n-1}{5^n}\cdot\frac{7^n+1}{7^n}\cdot$ &c.

Hìc 1, 3, 5, 7, &c. omnes primos numeros denotant, & ſignum affixum erit + vel —, prout primus numerus ſit $4m-1$ vel $4m-3$, ubi m eſt integer numerus.

3. Ex principiis prius traditis inveniri poteſt valor quantitatis A, cum n ſit quicunque integer numerus; & exinde e Walliſii formulâ $\frac{\pi}{2}=\frac{2.2.4.4.6.6.8.8.\&c.}{1.3.3.5.5.7.7.\&c.}$, vel formulis prius datis deduci poſſunt per multiplicationem vel diviſionem vel radicum extractionem harum & prædictarum formularum plurimæ quantitates, quarum contenta innoteſcunt vel per poteſtates quantitatis π, vel per datas algebraicas quantitates.

Ex. g. Sit $n = 1$, & erit $A = \frac{\pi}{4} = \frac{3}{4} \cdot \frac{5}{4} \cdot \frac{7}{8} \cdot \frac{11}{12} \cdot$ &c. at primo inventum fuit $\frac{\pi^2}{6} = \frac{4}{3} \cdot \frac{3^2}{2 \cdot 4} \cdot \frac{5^2}{4 \cdot 6} \cdot \frac{7^2}{6 \cdot 8} \cdot$ &c. dividatur ſecunda per primam, & exorietur $\frac{2\pi}{4} = \frac{3}{2} \cdot \frac{5}{6} \cdot \frac{7}{6} \cdot \frac{11}{10} \cdot \frac{13}{14} \cdot \frac{17}{18} \cdot$ &c. denuo dividatur hæc æquatio per primam, & reſultat $\frac{4}{2} \cdot \frac{4}{6} \cdot \frac{8}{6} \cdot \frac{12}{10} \cdot \frac{12}{14} \cdot$ &c. $= 2 = \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{6}{5} \cdot \frac{6}{7} \cdot$ &c. dividatur Wallisii æquatio $\frac{\pi}{2} = \frac{2.2.4.4.6.6.8}{1 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdot 7}$ &c. ſeu $\frac{4}{\pi} = \frac{3 \cdot 3}{2 \cdot 4} \cdot \frac{5 \cdot 5}{4 \cdot 6} \cdot \frac{7 \cdot 7}{6 \cdot 8} \cdot$ &c. per $\frac{\pi^2}{8} = \frac{3 \cdot 3}{2 \cdot 4} \cdot \frac{5 \cdot 5}{4 \cdot 6} \cdot \frac{7 \cdot 7}{6 \cdot 8} \cdot$ &c. & reſultat $\frac{32}{\pi^3} = \frac{9 \cdot 9}{8 . 10} \cdot \frac{15 \cdot 15}{14 \cdot 16} \cdot \frac{21 \cdot 21}{20 \cdot 22} \cdot$ &c. ubi in numeratoribus occurrunt omnes numeri impares non primi.

Et ſic ſit $n = 3$ & inveniri poteſt $A = \frac{\pi^3}{32} = \frac{3^3}{3^3 + 1} \cdot \frac{5^3}{5^3 - 1} \cdot \frac{7^3}{7^3 + 1} \cdot \frac{11^3}{11^3 + 1} \cdot \frac{13^3}{13^3 - 1} \cdot$ &c. & quoniam $\frac{\pi^6}{945} = 1 + \frac{1}{2^6} + \frac{1}{3^6} + \frac{1}{4^6} +$ &c. $= \frac{2^6}{2^6 - 1} \cdot \frac{3^6}{3^6 - 1} \cdot \frac{5^6}{5^6 - 1} \cdot$ &c. conſequitur $\frac{\pi^3}{30} = \frac{3^3}{3^3 - 1} \cdot \frac{5^3}{5^3 - 1} \cdot \frac{7^3}{7^3 - 1} \cdot$ &c. ex diviſione poſterioris æquationis per primam habebitur $\frac{16}{15} = \frac{3^3 + 1}{3^3 - 1} \cdot \frac{5^3 - 1}{5^3 + 1} \cdot$ &c.

3. Sit $A = 1 - \frac{1}{2^n} + \frac{1}{4^n} - \frac{1}{5^n} + \frac{1}{7^n} - \frac{1}{8^n} + \frac{1}{10^n} - \frac{1}{11^n} +$ &c. & n impar numerus; & erit $\left(1 + \frac{1}{2^n}\right) A = B = 1 - \frac{1}{5^n} + \frac{1}{7^n} - \frac{1}{11^n} +$ &c. & ſimiliter $\left(1 + \frac{1}{5^n}\right) B = C = 1 + \frac{1}{7^n} - \frac{1}{11^n} + \frac{1}{13^n}$ &c. & ſic deinceps: ex his tandem fiet $A \left(1 + \frac{1}{2^n}\right) \left(1 + \frac{1}{5^n}\right) \left(1 - \frac{1}{7^n}\right)$ &c. $= 1$,

ubi numeri primi unitate excedentes multipla ſenarii habent ſignum $-$; deficientes $+$: ſit $n = 1$ & erit $A = \frac{\pi}{3\sqrt{(3)}} = \frac{2}{3} \cdot \frac{5}{6} \cdot \frac{7}{6} \cdot$ &c. ubi denominatores ſunt omnes præter primum per 6 diviſibiles, per hanc expreſſionem dividatur æquatio $\frac{\pi^2}{6} = \frac{4}{3} \cdot \frac{3 \cdot 3}{2 \cdot 4} \cdot \frac{5 \cdot 5}{4 \cdot 6} \cdot \frac{7 \cdot 7}{6 \cdot 8} \cdot \frac{11 \cdot 11}{10 \cdot 12} \cdot$ &c. & reſultat $\frac{\pi\sqrt{(3)}}{2} = \frac{9}{4} \cdot \frac{5}{4} \cdot \frac{7}{8} \cdot \frac{11}{10} \cdot$ &c. hæc vero per illam diviſa, dabit $\frac{4}{3} = \frac{6}{4} \cdot \frac{6}{8} \cdot \frac{12}{14} \cdot \frac{18}{16} \cdot \frac{18}{20} \cdot$ &c. Et ſic ex iiſdem principiis deduci poſſunt aliæ conſimiles æquationes.

Sit $A = 1 + \frac{1}{3} - \frac{1}{5} - \frac{1}{7} + \frac{1}{11} - \frac{1}{13} -$ &c. $= \frac{\pi}{2\sqrt{(2)}}$, & exinde (A), i. e. $\frac{\pi}{2\sqrt{(2)}}(1 - \frac{1}{3})(1 + \frac{1}{5})(1 + \frac{1}{7})(1 - \frac{1}{11})(1 + \frac{1}{13})$ &c. $= 1$, ubi primorum numerorum formularum $8m + 1$ vel $8m + 3$ ſigna erunt $-$, formularum $8m + 5$ vel $8m + 7$ ſigna erunt $+$; unde $\frac{\pi}{2\sqrt{(2)}} = \frac{3}{2} \cdot \frac{5}{6} \cdot \frac{7}{8} \cdot \frac{11}{10} \cdot \frac{13}{14} \cdot \frac{17}{16} \cdot \frac{19}{18}$ &c. Et ex hâc & prius traditis æquationibus per hanc methodum erui poſſunt plurimæ conſimiles æquationes.

E diviſione factorum in numeratore per eorum correſpondentes in denominatore erui poſſunt diverſæ ſeries, quæ in ſeſe ductæ præbent ſeriem æqualem contento factorum in ſeſe continuo ductorum: in reſolutione hujuſce problematis plerumque uſui inſervire poſſint ea, quæ in tradita fuere.

THEOR. XLI.

Cum detur valor contenti, quod erit fractio, cujus denominator conſtat e pluribus factoribus; reducatur hæc fractio ad infinitam ſeriem terminorum, & conſtat valor infinitæ ſeriei; vel ducatur prædicta fractio in quaſcunque cognitas quantitates, quæ vel diruat quoſdam factores datæ fractionis vel novos adjiciat, & reducatur reſultans fractio ad infinitam ſeriem terminorum, cujus ſumma etiam innoteſcit.

Ex. 1. Detur $\frac{\pi}{4} = \frac{3}{3+1} \cdot \frac{5}{5-1} \cdot \frac{7}{7+1} \cdot$ &c. $= \frac{3 \cdot 5 \cdot 7}{4 \cdot 4 \cdot 8}$ &c. $= \frac{1}{(1+\frac{1}{3})\,(1-\frac{1}{5})\,(1+\frac{1}{7})\text{ \&c.}}$; unde $\frac{\pi}{4} \times \frac{1}{1+\frac{1}{2}} = \frac{\pi}{6} = \frac{1}{(1+\frac{1}{2})\,(1+\frac{1}{3})\,(1-\frac{1}{5})\,(1+\frac{1}{7})\text{ \&c.}}$; reducatur hæc fractio ad infinitam ſeriem, & reſultat $\frac{\pi}{6} = 1 - \frac{1}{2} - \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} - \frac{1}{7}$ &c. ubi primi numeri $4m - 1$ habent ſignum $-$, & $4m + 1$ ſignum $+$; ſi m ſit integer numerus.

Fuit $\frac{\pi}{2} = \frac{1}{(1-\frac{1}{3})\,(1+\frac{1}{5})\,(1-\frac{1}{7})\,(1-\frac{1}{11})\text{ \&c.}} = 1 + \frac{1}{3} - \frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \frac{1}{11} -$ &c. ubi impares numeri ſolummodo occurrunt, & primi numeri formulæ $4m + 1$ habent ſignum $-$, formulæ vero $4m - 1$ habent ſignum $+$; & ſigna compoſitorum numerorum e primis eorum diviſoribus deduci poſſunt: ducatur hæc æquatio in $\frac{1}{1-\frac{1}{2}}$ & reſultat $\pi = \frac{1}{(1-\frac{1}{2})\,(1-\frac{1}{3})\,(1+\frac{1}{5})\text{ \&c.}} = 1 + \frac{1}{2} + \frac{1}{3} +$ &c. ducatur eadem æquatio in $\frac{1}{1+\frac{1}{2}}$ & reſultat $\frac{\pi}{3} = \frac{1}{(1+\frac{1}{2})\,(1-\frac{1}{3})\text{ \&c.}} = 1 - \frac{1}{2} + \frac{1}{3} + \frac{1}{4} -$ &c.

Ducatur æquatio $\frac{\pi}{2} = \frac{1}{(1-\frac{1}{2})\,(1+\frac{1}{3})\,(1-\frac{1}{5})\,(1-\frac{1}{7})\text{ \&c.}}$ in $\frac{1+\frac{1}{3}}{1-\frac{1}{3}} \times 2$, & reſultat $\pi = \frac{1}{(1-\frac{1}{2})\,(1-\frac{1}{3})\,(1-\frac{1}{5})\text{ \&c.}}$ & ſic deinceps; & e lege quam obſervat ſeries ex evolutione fractionis prioris æquationis exorta conſequitur lex quam obſervat ſeries ex poſteriore fractione in ſimplices terminos per præcedentem methodum reductâ.

Sit $o = \frac{1}{(1+\frac{1}{2})\,(1+\frac{1}{3})\,(1+\frac{1}{5})\,(1+\frac{1}{7})\,(1+\frac{1}{11})\text{ \&c.}} = 1 - \frac{1}{2} - \frac{1}{3} - \frac{1}{5} + \frac{1}{6} - \frac{1}{7}$ &c. ducatur hæc æquatio in finitum numerum factorum $1 + \frac{1}{2}$, $1 + \frac{1}{3}$, $1 + \frac{1}{3}$, &c. & æquatio reſultans etiam erit nihilo æqualis;

lis; ducatur etiam æquatio exinde reſultans in finitum numerum factorum hujuſce generis $1 - \frac{1}{2}$, $1 - \frac{1}{3}$, $1 - \frac{1}{5}$, &c. & contentum etiam erit nihilo æquale; in quo caſu omnes primi numeri niſi finitus eorum numerus in ſerie præcedente methodo deductâ ſignum habent —: & ſic de infinitis hujuſcemodi ſeriebus, fractionibus & terminis detegendis, vel rationalibus vel irrationalibus.

1. In genere aſſumantur quæcunque algebraicæ quantitates vel rationales fractiones vel irrationales quantitates; inveniantur quæcunque functiones harum quantitatum, reducantur hæ functiones ad ſeries, ſi modo ſint infinitæ, convergentes; tum inveniuntur ſeries, quarum ſummæ innoteſcunt.

2. Inveniantur quæcunque functiones prædictarum ſerierum & datarum infinitarum, quarum ſummæ innoteſcunt; & ſi modo ſeries exinde reſultantes ſint convergentes, tum inveniuntur ſeries, quarum ſummæ innoteſcunt.

THEOR. XLII.

1. Sit $(x^{\alpha} + x^{\beta} + x^{\gamma} + x^{\delta} + \&c.)\,(x^{a} + x^{b} + x^{c} + x^{d} + \&c.) = x^{a+\alpha} \ldots + Nx^{n}$, & N erit numerus modorum, quibus confici poteſt n ex additione cujuſcunque quantitatis vel a vel b vel c, &c. ad unam e quantitatibus α, β, γ, δ, &c. Si vero b ſit negativa quantitas, tum de quantitatibus α, β, γ, δ, &c. ſubtrahenda eſt b.

2. Et ſic de pluribus factoribus in ſeſe ductis: e. g. ſit $(1 + x^{\alpha}z)\,(1 + x^{\beta}z)\,(1 + x^{\gamma}z)\,(1 + x^{\delta}z)\ \&c. = 1 + Pz + Qz^{2} \ldots Nx^{n}z^{m}$, tum coefficiens N erit diverſorum modorum numerus, quibus numerus n poteſt eſſe ſumma m diverſorum terminorum α, β, γ, δ, &c. ſit $z = 1$ & erit $(1 + x)\,(1 + x^{2})\,(1 + x^{3})\ \&c. = 1 + x + x^{2} + 2x^{3} \ldots Px^{n}$, & P numerus modorum, quibus numerus n ex additione diverſorum terminorum 1, 2, 3, 4, &c. produci poteſt.

3. Sit $\frac{1}{(1 + x^{\alpha}z)\,(1 + x^{\beta}z)\,(1 + x^{\gamma}z)\ \&c.} = 1 + Pz + Qz^{2} + Rz^{3} \ldots Nx^{n}z^{m}$, & erit N numerus modorum, quibus numerus n ex

additione m quantitatum $\alpha, \beta, \gamma, \delta$, &c. $2\alpha, 2\beta, 2\gamma$, &c. $3\alpha, 3\beta, 3\gamma$, &c. produci poteſt.

4. Sit $\frac{1}{(1-x)(1-x^2)(1-x^3)\ldots(1-x^m)} = 1+x+2x^2+\ldots Nx^n$ &c. & erit N numerus modorum, quibus produci poteſt n ex additione numerorum 1, 2, 3, 4, &c. ſupponatur etiam $\frac{x}{1-x} \cdot \frac{x^2}{1-x^2} \cdot \frac{x^3}{1-x^3} \ldots \frac{x^m}{1-x^m} = \frac{x^{m \cdot \frac{m+1}{2}}}{(1-x)(1-x^2)(1-x^3)\ldots(1-x^m)} = x^{m \cdot \frac{m+1}{2}} + \ldots\ldots + Nx^{n+m \cdot \frac{m+1}{2}}$, &c. unde N erit numerus diverſorum modorum quibus numerus $n+m \cdot \frac{m+1}{2}$ dividi poteſt in m inæquales partes, & conſequenter quot modis n produci poteſt ex additione numerorum 1, 2, 3, ... m, tot modis numerus $n+m \cdot \frac{m+1}{2}$ dividi poteſt in m inæquales partes.

5. Sint (A) $\frac{1}{(1-x)(1-x^2)(1-x^3)\ldots(1-x^m)} = 1+Px\ldots Nx^n$, &c. & (B) $\frac{x^m}{(1-x)(1-x^2)(1-x^3)\ldots(1-x^m)} = x^m+P'x^{m+1}\ldots Mx^n$, &c. & exinde $(A-B) = \frac{1-x^m}{(1-x)(1-x^2)\ldots(1-x^m)} = \frac{1}{(1-x)(1-x^2)\ldots(1-x^{m-1})} = 1+Px\ldots(N-M)x^n$, &c. unde differentia inter numerum modorum, quibus n & $n-m$ numeri confici poſſunt ex numeris 1, 2, 3, ... m æqualis erit numero modorum, quibus n confici poteſt e numeris 1, 2, 3, ... $m-1$.

Eodem modo ducatur A vel in $(1-x^m)(1-x^{m-1})$ vel B in $(1-x^{m-1})$; &c. & conſimilia deduci poſſunt theoremata; & ſic deinceps.

6. Eſt $\frac{1}{(1-x)(1-x^3)(1-x^5)\,\&c.} = (1+x)(1+x^2)(1+x^3)$ &

&c. in infinitum $= 1 + x + x^2 + 2x^3 + \ldots Nx^n$: & exinde quot diverſis modis per additionem numerus n confici poteſt ex integris inæqualibus numeris, totidem modis idem numerus n confici poteſt per additionem numerorum modo ſint impares, utrum ſint æquales vel inæquales.

7. Contentum $(x^{-1} + 1 + x^1)(x^{-3} + 1 + x^3)(x^{-9} + 1 + x^9)$ $(x^{-27} + 1 + x^{27})$ &c. præbet omnes exponentes quantitatis x vel ex additione vel ex ſubtractione numerorum 1, 3, 9, 27, &c. & ſimilia etiam de reliquis numeris prædicari poſſunt.

8. Sit $(1 - x)^r (1 - x)^s (1 - x)^t = (1 - x)^{r+s+t} = 1 - (r + s + t)$ $x + \ldots Nx^m$ &c.; tum, ſi numerus modorum, quibus α quantitates in r contineri poſſunt, multiplicetur in numerum modorum, quibus β quantitates in s continentur; & productum reſultans multiplicetur in numerum modorum, quibus $m - \alpha - \beta$ quantitates in t continentur; erit aggregatum e ſingulis contentis reſultantibus N; ubi α, β & γ reſpective denotant quoſcunque numeros 1, 2, 3, &c.: & ſimiliter numerus modorum, quibus m quantitates in $r + s + t$ contineri poſſunt, etiam erit N.

9. Sit $(1 + x^r + x^s)^n = 1 + nx^r \ldots + Nx^m$ &c.: & $\alpha r + \beta s = m$; & inveniatur numerus modorum, quibus α quantitates contineri poſſunt in n; qui ducatur in numerum modorum, quibus β quantitates contineri poſſunt in n; tum erit ſumma e ſingulis his productis $= N$.

Hìc etiam adjicere liceat quamplurimas hujuſce generis propoſitiones; in genere ſi modò data quantitas vel ſeries quocunque modo ex aliis conficiatur, animadvertentur etiam quomodo producentur ejus coefficientes, exponentes, &c. & reſultabunt hujuſce generis propoſitiones: ſi modo in diverſam formulam transformetur data quantitas, ita ut eædem coefficientes, exponentes, &c. vel aliæ ad has aſſignabilem relationem habentes e diverſis modis producentur; tum etiam reſultabunt novæ hujuſce generis propoſitiones: hinc vix ulla datur ſeries, e quâ non facile deduci poſſunt propoſitiones hujuſmodi.

10. In multitudine (n) literarum $a\,b\,c\,d\,e\,f\,g\,h$, &c. numeri modorum, quibus detrahi poſſunt quæcunque duæ proxime ſucceſſivæ, viz. vel

a & b

a & b, vel b & c, vel c & d, &c.; ſemel, bis, ter, quater, &c. erunt $n-1$, $(n-2) \cdot \frac{n-3}{2}$, $(n-3) \cdot \frac{n-4}{2} \cdot \frac{n-5}{3}$, &c.

Cor. Sit continua fractio $\cfrac{a}{b+\cfrac{a}{b+\cfrac{a}{b+\text{\&c. in infin.}}}}$; tum erunt fractiones ſucceſſivæ ad datam vergentes $a \times \frac{1}{b}$, $a \times \frac{b}{b^2+a}$, $a \times \frac{b^2+a}{b^3+2ba}$, $a \times \frac{b^3+2ba}{b^4+3b^2a+a^2}$, $a \times \frac{b^4+3b^2a+a^2}{b^5+4b^3a+3ba^2}$, &c. in genere erit fractio, cujus numerator eſt $a \times (b^{n-1} + (n-2) b^{n-3} a + (n-3) \cdot \frac{n-4}{2} b^{n-5} a^2 + (n-4) \cdot \frac{n-5}{2} \cdot \frac{n-6}{3} b^{n-7} a^3 + (n-5) \cdot \frac{n-6}{2} \cdot \frac{n-7}{3} \cdot \frac{n-8}{4} b^{n-9} a^7 + \text{\&c.})$, denominator vero $b^n + (n-1) b^{n-2} a + (n-2) \cdot \frac{n-3}{2} b^{n-4} a^2 + (n-3) \cdot \frac{n-4}{2} \cdot \frac{n-5}{3} b^{n-6} a^3 + \text{\&c.}$ approximatio ad valorem prædictæ continuæ fractionis.

Hæ fractiones etiam ſic exprimi poſſunt, viz. $a \times \frac{1}{b}$, $a \times \frac{A}{bA+aP}$, $a \times \frac{B}{bB+aQ}$, $a \times \frac{C}{bC+aR}$, $a \times \frac{D}{bD+aS}$, &c.; ubi literæ A, B, C, D, &c. reſpective denotant denominatores; & P, Q, R, S, &c. numeratores præcedentium fractionum.

Cor. Hæ fractiones erunt approximationes ad radicem quadraticæ æquationis $x^2+bx-a=0$, nam erit $x = \cfrac{a}{b+\cfrac{a}{b+\text{\&c.}}} = \frac{a}{b+x}$, & exinde $x^2+bx=a$.

Cor. In datâ quadraticâ $x^2+bx-a=0$ pro x ſcribatur $z-b$, &

& reſultat $z^2 - bz = a$; unde per præcedens coroll. $z = \cfrac{a}{-b+\cfrac{a}{-b+\&c.}}$, & exinde $x = b + \cfrac{a}{-b+\cfrac{a}{-b+\cfrac{a}{-b+\&c.}}}$ altera radix prædictæ quadraticæ; continua fractio $\cfrac{a}{b+\cfrac{a}{b+\cfrac{a}{b+\&c.}}}$ eadem erit ac continua fractio $\cfrac{a}{-b+\cfrac{a}{-b+\cfrac{a}{-b+\&c.}}}$, at ejus negativa; & conſequenter earum approximationes eædem.

Cor. Hinc deduci poſſunt multæ continuæ fractiones, quarum ſummæ differunt tantummodo per datas quantitates; transformetur enim quæcunque quadratica æquatio $x^2 + bx = a$ in alteram $z^2 + Bx = A$ e ſcribendo $z + p$ pro x, & erit

$$\cfrac{a}{b+\cfrac{a}{b+\cfrac{a}{b+\&c.}}} = \cfrac{A}{B+\cfrac{A}{B+\cfrac{A}{B+\&c.}}} + p.$$

2. Et ſimiliter in cubicâ æquatione $x^3 + dx^2 - (a+b)x - ad = 0$ pro x ſcribatur $z + e$ & reſultet æquatio $x^3 + Dx^2 - (A+B)x - AD = 0$, tum erit

$$\sqrt{a+\cfrac{b}{1+\cfrac{d}{\sqrt{a+\cfrac{b}{1+\cfrac{d}{\sqrt{\&c.}}}}}}} =$$

$$\sqrt{A}+\cfrac{B}{1+\cfrac{D}{\sqrt{A}+\cfrac{B}{1+\cfrac{D}{\sqrt{\&c.}}}}}+e.$$

Et ſic facile erui poſſunt infinitæ conſimiles propoſitiones ex aſſumendo diverſas approximationes ad quotientes, radices, &c.

THEOR. XLIII.

Fluentes fluxionum $x^{i}\dot{x}\pi'$ & $x^{i+\alpha}\dot{x}\pi'$, ubi i ſit infinitus numerus, α finitus, π' vero quæcunque algebraica finita functio literæ x, inter valores 0 & 1 quantitatis x poſitæ, erunt inter ſe æquales.

Reducatur enim algebraica functio π' ad terminos aſcendentes ſecundum dimenſiones quantitatis x, qui ſint $a x^{\pi-1} + b x^{\pi+\rho-1} + c x^{\pi+2\rho-1} +$ &c. & evadent duæ fluentes prædictæ reſpective $\frac{a}{i+\pi} + \frac{b}{i+\pi+\rho} + \frac{c}{i+\pi+2\rho} +$ &c. & $\frac{a}{i+\alpha+\pi} + \frac{b}{i+\alpha+\pi+\rho} +$ &c. ſed quoniam i eſt infinitus, hæ duæ ſeries aſcendentes erunt inter ſe in ratione æqualitatis.

PROB. XXXVI.

Quaſdam fluentes evolvere per producta infinita.

1. Fluens fluxionis $\frac{\dot{x}}{\sqrt{(1-x^2)}}$ inter valores (0 & 1) quantitatis x per ex. 1. prob. 23. l. 1. erit $\frac{2.4.6.8..(2i)}{1.3.5.7..(2i-1)} \int. \frac{x^{2i}\dot{x}}{\sqrt{(1-x^2)}}$; & ſic per prædict. exem. erit $\int. \frac{x\dot{x}}{\sqrt{(1-x^2)}} = \frac{3.5.7..(2i+1)}{2.4.6..(2i)} \int. \frac{x^{2i+1}\dot{x}}{\sqrt{(1-x^2)}}$. Sit i infinitus & erit $\int. \frac{x^{2i}\dot{x}}{\sqrt{(1-x^2)}} = \int. \frac{x^{2i+1}\dot{x}}{\sqrt{(1-x^2)}} = \int. \frac{x^{2i+\alpha}\dot{x}}{\sqrt{(1-x^2)}}$, & conſequenter $\int. \frac{x\dot{x}}{\sqrt{(1-x^2)}} (1) : \int. \frac{\dot{x}}{\sqrt{(1-x^2)}} \left(\frac{\pi}{2}\right) :: \frac{1.3.5.7.\&c.}{2.4.6.\&c.}$: 2.

$: \frac{2 . 4 . 6 . \&c.}{1 . 3 . 5 . 7 . \&c.}$, unde $\frac{\pi}{2} = \frac{2 . 2}{1 . 3} . \frac{4 . 4}{3 . 5} . \frac{6 . 6}{5 . 7} . \&c.$

Ex. 2. Per ex. 1. prob. 23. $\int . x^{m-1}\dot{x}(1 - x^n)^{\frac{k-n}{n}}$ (P) & $\int . x^{\mu-1}\dot{x}(1 - x^n)^{\frac{k-n}{n}}$ $(\mathcal{Q})$ inter valores o & 1 quantitatis x poſitæ, erunt reſpective $\frac{(m+k)(m+k+n)(m+k+2n)\ldots(m+k+in)}{m(m+n)(m+2n)\ldots(m+in)} \int . x^{m+in+n-1}\dot{x}$ $(1 - x^n)^{\frac{k-n}{n}}$, & $\frac{(\mu+k)(\mu+k+n)\ldots(\mu+k+in)}{\mu(\mu+n)\ldots(\mu+in)} \int . x^{\mu+in+n-1}\dot{x}$ $(1 - x^n)^{\frac{k-n}{n}}$; cum vero i ſit infinitus, tum evadunt hæ duæ fluentes inter ſe æquales, & conſequenter erit

$$\frac{P}{\mathcal{Q}} = \frac{\mu(m+k)}{m(\mu+k)} . \frac{(\mu+n)(m+k+n)}{(m+n)(\mu+k+n)} . \frac{(\mu+2n)(m+k+2n)}{(m+2n)(\mu+k+2n)} . \&c.$$

Aſſumatur $\mu = n$, vel $2n$ vel $3n$ vel $4n$, &c., ita quidem ut ejus fluens deduci poteſt; e. g. ſit $\mu = n$, & erit $\int . x^{n-1}\dot{x}(1 - x^n)^{\frac{k-n}{n}} =$ conſt. $(C) - \frac{1}{k}(1 - x^n)^{\frac{k}{n}}$ (R); poſito vero $x = o$, evaneſcere oportet fluentem, unde $C = \frac{1}{k}$; ergo erit $\mathcal{Q} = \frac{1}{k}$, & exinde conſtat

$$P = \frac{1}{k} . \frac{\mu(m+k)}{m(\mu+k)} . \frac{(\mu+n)(m+k+n)}{(m+n)(\mu+k+n)} . \&c. = \frac{1}{k} . \frac{n(m+k)}{m(n+k)} . \frac{2n(m+k+n)}{(m+n)(2n+k)} . \frac{3n(m+k+2n)}{(m+2n)(3n+k)} . \&c.$$

Sit $m = \frac{n+\alpha}{2}$ & $k = \frac{n-\alpha}{2}$, & reſultat

$$P = \frac{4 . n}{(n-\alpha)^2} . \frac{2 . 4n^2}{9n^2 - \alpha^2} . \frac{4 . 6n^2}{25n^2 - \alpha^2} . \&c. = \frac{2}{n-\alpha} . \frac{2n . 2n}{(n+\alpha)(3n-\alpha)} . \frac{4n . 4n}{(3n+\alpha)(5n-\alpha)} . \&c. = \frac{\pi}{n \text{ ſin. } \frac{m\pi}{n}} = \frac{\pi}{n \text{ coſ. } \frac{\alpha\pi}{2n}}.$$

Sit $k = \frac{\mu n}{\nu}$, & evadet $\int . x^{m-1}\dot{x}$

$$(1-x^n)^{\frac{\mu}{\nu}-1} = \frac{\nu}{m\mu} \cdot \frac{2(\mu\nu+n\mu)}{(m+n)(\mu+\nu)} \cdot \frac{3(\mu\nu+n\mu+n\nu)}{(m+2n)(\mu+2\nu)} \cdot \frac{4(m\nu+n\mu+2n\nu)}{(m+3n)(\mu+3\nu)} \text{. \&c.}$$

Cum $\int. \frac{x^{m-1}\dot{x}}{\sqrt[n]{(1-x^n)^{n-k}}} = \int. \frac{x^{k-1}}{\sqrt[n]{(1-x^n)^{n-m}}} = \frac{1}{k} \cdot \frac{n(m+k)}{m(k+n)} \cdot \frac{2n(m+k+n)}{(m+n)(k+2n)}$. &c. inter duos valores 0 & 1 quantitatis x poſitæ; & ſimiliter erit $\int. \frac{x^{p-1}\dot{x}}{\sqrt[n]{(1-x^n)^{n-q}}} = \int. \frac{x^{q-1}\dot{x}}{\sqrt[n]{(1-x^n)^{n-p}}} = \frac{p+q}{pq} \cdot \frac{n(p+q+n)}{(p+n)(q+n)} \cdot \frac{2n(p+q+2n)}{(p+2n)(q+2n)}$. &c. & conſequenter $\frac{\int. x^{m-1}\dot{x}(1-x^n)^{\frac{k-n}{n}}}{\int. x^{p-1}\dot{x}(1-x^n)^{\frac{q-n}{n}}}$

$$= \frac{pq(m+k)}{mk(p+q)} \cdot \frac{(p+n)(q+n)(m+k+n)}{(m+n)(k+n)(p+q+n)} \cdot \frac{(p+2n)(q+2n)}{(m+2n)(k+2n)}, \frac{(m+k+2n)}{(p+q+2n)} \text{. \&c.}$$

PROB. XXXVI.

Innvenire diverſa producta ex binis hujuſmodi formulis, quæ inter ſe ſunt æqualia.

Sint producta reſpective $\frac{a+b}{ab} \cdot \frac{n(a+b+n)}{(n+a)(n+b)} \cdot \frac{2n(a+b+2n)}{(a+2n)(b+2n)}$. &c. $\frac{c+d}{cd} \cdot \frac{n(c+d+n)}{(n+c)(n+d)}$. &c. & ſic $\frac{p+q}{pq} \cdot \frac{n(p+q+n)}{(n+p)(n+q)}$. &c., & $\frac{r+s}{rs} \cdot \frac{n(r+s+n)}{(n+r)(n+s)}$. &c. ducantur duo priora producta in ſeſe, & evadit productum $\frac{a+b}{ab} \times \frac{c+d}{cd} \times \frac{n(a+b+n)}{(a+n)(b+n)} \times \frac{n(c+d+n)}{(c+n)(d+n)}$. &c. & ſic ducantur in ſeſe duo poſteriora, & reſultat $\frac{p+q}{pq} \times \frac{r+s}{rs}$. &c. nunc neceſſe eſt ut fiat $(a+b)(c+d)pqrs = abcd(p+q)(r+s)$, & conſequenter ſinguli ex his ſex factoribus utrinque ſint æquales: & exinde vel ſint $s=d$, $r=b$, $q=c$, $p=r+s=b+d$, $c+d=a$, & $a+b=p+q$; vel ſint $s=d$, $r=b$, $q=r+s=b+d$, $p=a$, $c+d=p+q$ & $a+b=c$; tertio vero ſint $s=d$, $r=p+q$, $q=b$, $p=c$, $c+d=a$ & $a+b=r+s=p+q+d$. Hinc oriuntur tres formulæ infinitorum productorum, quæ erunt inter ſe æqualia; etiamque $\int. \frac{x^{p-1}\dot{x}}{\sqrt[n]{(1-x^n)^{n-q}}} \cdot \int. \frac{x^{p+q-1}\dot{x}}{\sqrt[n]{(1-x^n)^{n-r}}} = \int. \frac{x^{q-1}\dot{x}}{\sqrt[n]{(1-x^n)^{n-r}}} \cdot \int. \frac{x^{q+r-1}\dot{x}}{\sqrt[n]{(1-x^n)^{n-p}}} = \int. \frac{x^{p-1}\dot{x}}{\sqrt[n]{(1-x^n)^{n-r}}} \cdot \int. \frac{x^{p+r-1}\dot{x}}{\sqrt[n]{(1-x^n)^{n-q}}}$; quæ fluentes inter valores o & 1 quantitatis x continentur.

Cum vero $p+q=n$, erunt tria prædicta producta $= \frac{\pi}{nr\,\text{ſin.}\,\frac{p\pi}{n}}$.

Et ſic progredi liceat ad deducenda diverſa producta e ternis quatuor, &c. hujuſmodi formulis, quæ inter ſe ſunt æqualia.

Per prob.   prius tradita e fluentibus datarum fluxionum acquiri poſſunt infinita producta, quorum contenta e prædictis fluentibus acquiri poſſunt.

Erit $\int. \frac{x^{m+\alpha n-1}\dot{x}}{(1-x^n)^{\frac{n-k}{n}}} \times \int. \frac{x^{m+k+\alpha n-1}\dot{x}}{(1-x^n)^{\frac{k}{n}}} = \frac{m}{m+\alpha n} \int. \frac{x^{m-1}\dot{x}}{(1-x^n)^{\frac{n-k}{n}}} \cdot \int. \frac{x^{m+k-1}\dot{x}}{(1-x^n)^{\frac{k}{n}}}$, ubi valores ſingularum prædictarum fluentium inter valores o & 1 quantitatis x aſſumendi ſunt: conſtat e prob. 23. l. 1.

Et ſic e datis infinitis productis haud difficilis erit inveſtigatio, utrum prædicta producta ſint prædicti generis, necne; ſi vero ſint producta prædicti generis, tum facile acquiri poſſunt fluentes, quibus æqualia ſunt producta jam prædicta.

Singuli factores facile transformari poſſunt in infinitas alias irrationales, & exinde contenta in infinitis irrationalibus factoribus deduci poſſunt.

PROB. XXVII.

Datâ ſerie ſecundum continua contenta in ſeſe ducta progrediente, invenire ſeriem relationem inter ſucceſſivos terminos exprimentem: ſit data ſeries $p(1+\alpha)(1+\beta)(1+\gamma)$ &c. . . . $(P)(1+\pi)(1+\rho)(1+\sigma)(1+\tau)$ *&c. tum erunt ſucceſſivi termini* $T^m = P\pi$ *&* $T^{m+1} = P(1+\pi)\rho$, *ergo erit relatio quæſita* $(1+\pi)\rho T^m = \pi T^{m+1}$.

Cor. 1. Ex datâ relatione inter ſucceſſivos terminos per prob. 2. lib. præc. erui poteſt, annon ſumma ſeriei ſit infinita; ſed ſeries ſecundum continua contenta progrediens facile transformari poteſt in æquationem relationem inter ſucceſſivos terminos deſignantem; ergo facile deduci poteſt, annon ſeries ſecundum prædicta contenta progrediens ſit finita.

Cor. 2. Facile ex principiis prius traditis innoteſcunt caſus, qui ex datis relationibus inter ſucceſſivos terminos ſeriei præbent fluxionales æquationes, in quibus variabiles quantitates ſunt ſummæ ſerierum & quantitas ſecundum cujus dimenſiones termini aſcendunt vel deſcendunt.

Et vice versâ ſit data prædicta fluxionalis æquatio; vel relatio inter ſucceſſivos terminos, quæ exinde deduci poteſt; deinde ita reducantur æquationes inter ſucceſſivos terminos & æquationem $(1+\pi)\rho T^m = \pi T^{m+1}$ vel quæcunque aliæ datæ conſimiles ex continuâ ſerie deductæ, ita ut exterminentur termini; & reſultat æquatio relationem inter π, ρ, σ, &c. deſignans, ubi π, ρ, σ, &c. ſunt ſucceſſivi valores ejuſdem quantitatis: ex hâc æquatione inveniantur valores π, ρ, σ, &c.; & reſultat ſeries continua quæſita.

Aliter: Aſſumatur functio quantitatis z diſtantiæ a primo ſeriei termino pro ſummâ; deinde inveniantur ſucceſſivi termini T & T', & exinde ex aſſumpto primo termino contentorum, ita ut correſpondeat primo termino ſeriei, cujus termini ſunt T & T', &c.; & tum ex terminis T^m & T^{m+1} & præcedente factore $1+\pi$ datis per æquationem $(1+\pi)\rho T^m = \pi T^{m+1}$ facile acquiri poteſt factor $1+\rho$.

PROB. XXXVIII.

E prob. 23. l. 1. per factores hujuſmodi exprimere fluentem cujuſcunque fluxionis hujuſce formulæ $(a+bx^n)^{m-1}x^{rn-1}\dot{x}$ inter valores o & $\frac{-a}{b}$ quantitatis x^n contentam: fluens fluxionis $(a+bx^n)^{m-1}x^{n-1}\dot{x}$, quæ in genere erit $\frac{1}{mnb}\times(a+bx^n)^m$, inter prædictos valores quantitatis x contenta erit $\frac{a^m}{mnb}$; & conſequenter erit (P) $\int.(a+bx^n)^{m-1}x^{(1+i)n-1}\dot{x}$ $= \frac{1 \cdot 2 \cdot 3 \ldots i}{m+1 \cdot m+2 \ldots m+i} \times \frac{a^i}{b^i} \times \frac{a^m}{mnb}$, & ſic inveniri poteſt (Q) $\int.(a+bx^n)^{m-1}x^{(r+i)n-1}\dot{x} = \frac{r \cdot r+1 \cdot r+2 \ldots (r+i-1)}{m+r \cdot m+r+1 \cdot m+r+2 \ldots (m+r+i-1)}$ $\times \frac{a^i}{b^i} \times \int.(a+bx^n)^{m-1}x^{rn-1}\dot{x}$; nunc fingatur i infinita quantitas, & reſultat $P = Q$, & conſequenter $\frac{r \cdot r+1 \cdot r+2 \ldots r+i-1}{m+r \cdot m+r+1 \cdot m+r+2 \ldots (m+r+i-1)} \times \frac{a^{i-r}}{b^{i-r}} \int.(a+bx^n)^{m-1}x^{rn-1}\dot{x} = \frac{1 \cdot 2 \cdot 3 \ldots i}{m \cdot m+1 \cdot m+2 \ldots m+i} \times \frac{a^{i-1}}{b^{i-1}} \times \frac{a^m}{mnb}$; unde $\int.(a+bx^n)^{m-1}x^{rn-1}\dot{x} = \frac{1 \cdot 2 \cdot 3 \ldots (i) \times m+r \cdot m+r+1 \cdot m+r+2 \ldots m+r+i-1}{r \cdot r+1 \cdot r+2 \ldots (r+i-1) \times m \cdot m+1 \cdot m+2 \ldots m+i} \times \frac{a^{r-1}}{b^{r-1}}$ $\times \frac{a^m}{mnb}$.

Fallet hæc methodus, cum quidam factores in prædictâ fractione contenti evadant nihilo æquales; in his caſibus e transformatione datæ fluxionis in alteram ſæpe deduci poteſt reſolutio, & ex iiſdem principiis,

cipiis; viz. transformationibus facile erui poſſunt infinitæ fluxiones, quarum particulares fluentes innoteſcunt.

Sæpe vero fluentes prædictæ detegi poſſunt ex earum diviſione in alias, ſi enim fluentes prædictæ e ſingulis his innoteſcant, tum fluentes e prædictis exinde inveſtigari poſſunt.

THEOR. XLIV.

Si data ſeries A, cujus ſumma innoteſcit, quæ per datam quantitatem (α) diviſa præbeat ſeriem magis convergentem; & ſimiliter quotiens per aliam datam quantitatem (β) diviſa præbeat ſeriem adhuc magis convergentem; & ſic deinceps in infinitum; & ultima quotiens ſit unitas; tum erit $A = \alpha \times \beta \times \gamma \times \delta \times$ &c.

Ex. 1. Series $1 \pm b + b^2 \pm b^3 + b^4 \pm$ &c. in infinitum per $1 \pm b$ diviſa præbet ſeriem $1 + b^2 + b^4 + b^6 +$ &c. hæc autem ſeries per $1 + b^2$ diviſa dat $1 + b^4 + b^8 + b^{12} +$ &c.; quæ per $1 + b^4$ diviſa præbet ſeriem $1 + b^8 + b^{16} +$ &c., &c.; quarum prima magis convergit quam ſecunda, ſecunda quam tertia, &c.; ſi modo b minor ſit quam 1.

Et conſequenter erit $(1 \pm b)(1 + b^2)(1 + b^4)(1 + b^8)(1 + b^{16})(1 + b^{32})$ &c. $= \dfrac{1}{1 \mp b}$.

Cor. Sit $b = \dfrac{m}{n}$, ubi m minor eſt quam n; tum erit $\left(1 \pm \dfrac{m}{n}\right)\left(1 + \dfrac{m^2}{n^2}\right)\left(1 + \dfrac{m^4}{n^4}\right)\left(1 + \dfrac{m^8}{n^8}\right)$ &c. $= \left(\dfrac{n \pm m}{n}\right)\left(\dfrac{n^2 + m^2}{n^2}\right)\left(\dfrac{n^4 + m^4}{n^4}\right)\left(\dfrac{n^8 + m^8}{n^8}\right)\left(\dfrac{n^{16} + m^{16}}{n^{16}}\right)$ &c. in infinitum $= \dfrac{1}{1 \mp \dfrac{m}{n}} = \dfrac{n}{n \mp m}$.

Ex. 2. Series $1 \pm b + b^2 \pm b^3 + b^4 \pm b^5 + b^6 \pm$ &c. per $1 \pm b + b^2 \pm b^3 + b^4 \ldots \pm b^b$ diviſa præbet ſeriem magis convergentem $1 \pm b^{b+1} + b^{2b+2} \pm b^{3b+3} + b^{4b+4} \pm b^{5b+5} +$ &c.; hæc autem ſeries per ſeriem $1 \pm b^{b+1} + b^{2b+2} \pm b^{3b+3} + b^{4b+4} \ldots\ldots b^{rm+r}$ diviſa præbet ſeriem $1 \pm b^{(r+1)\cdot(b+1)} + b^{2(r+1)(b+1)} \pm b^{3(r+1)(b+1)} + b^{4(r+1)(b+1)} \pm \ldots$ &c.; hæc autem ſeries per ſeriem $1 \pm b^{(r+1)(b+1)} + b^{2(r+1)(b+1)} \pm b^{3(r+1)(b+1)} \ldots b^{s(r+1)(b+1)}$ diviſa dat quotientem $1 \pm b^{(s+1)(r+1)(b+1)} + b^{2(s+1)(r+1)(b+1)} \pm b^{3(s+1)(r+1)(b+1} + \&c.$; & ſic deinceps. Si omnes numeri $b+1$, $r+1$, $s+1$, &c. ſint impares; tum ſigna $\pm$ vel poſſunt eſſe $+$ vel $-$; ſi autem unus e prædictis numeris $b+1$, $r+1$, &c. ſit par, tum omnia ſigna in ſubſequentibus quotientibus erunt $+$.

Hinc, ſi b minor ſit quam unitas, continuum productum $(1 \pm b + b^2 \pm \ldots b^b)(1 \pm b^{b+1} + b^{2b+2} \pm b^{3b+3} \ldots b^{rb+1})(1 \pm b^{(r+1)(b+1)} + b^{2(r+1)(b+1)} \pm b^{3(r+1)(b+1)} + \ldots b^{s(r+1)(b+1)})(1 \pm b^{(s+1)(r+1)(b+1)} + b^{2(s+1)(r+1)(b+1)} \pm \ldots b^{t(s+1)(r+1)(b+1)})(1 \pm b^{(t+1)(s+1)(r+1)(b+1)} + \&c.)$ &c. in infinitum $= \dfrac{1}{1 \mp b}$.

Pro b in hoc contento ſcribatur $\frac{m}{n}$, & reſultat prædictum continuum contentum $\left(1 \pm \frac{m}{n} + \frac{m^2}{n^2} \pm \ldots \frac{m^b}{n^b}\right)\left(1 \pm \frac{m^{b+1}}{n^{b+1}} + \frac{m^{2b+2}}{n^{2b+2}} \pm \ldots \frac{m^{r(b+1)}}{n^{r(b+1)}}\right)\left(1 \pm \frac{m^{(r+1)(b+1)}}{n^{(r+1)(b+1)}} \pm \ldots \frac{m^{s(r+1)(b+1)}}{n^{s(r+1)(b+1)}}\right)$ &c. $= \left(\frac{n^{b+1} \pm m^{b+1}}{(n \mp m)\, n^b}\right)$ $\left(\frac{n^{(r+1)(b+1)} \pm m^{(r+1)(b+1)}}{(n^{b+1} \mp m^{b+1}) \times n^{r \times (b+1)}}\right)\left(\frac{n^{(s+1)(r+1)(b+1)} \pm m^{(s+1)(r+1)(b+1)}}{(n^{(r+1)(b+1)} \mp m^{(r+1)(b+1)})\, n^{s(r+1)(b+1)}}\right)$ &c. in infinitum $= \dfrac{n}{n \mp m}$.

2. Sit ſeries $1 \pm b^{-1} + b^{-2} \pm b^{-3} + b^{-4} \pm \&c.$; tum in præcedentibus continuis factoribus pro b ſcribatur b^{-1}, & reſultat contentum e novis continuis factoribus datæ quantitati æquale.

Ex. 3. Aſſumatur quæcunque algebraica functio quantitatis x, quæ ſit π; in eâ pro x ſcribatur $-x$ & reſultet quantitas A; deinde in π pro x ſcribantur $\sqrt{(-1)}x$ & $-\sqrt{(-1)}x$, & reſultent quantitates B & B', quarum productum $B \times B' = A'$: tum in quantitate π pro x ſcribantur αx, $\alpha' x$, $\alpha'' x$ & $\alpha''' x$; ubi α, α', α'' & α''' ſunt radices æquationis $z^4 + 1 = 0$; & reſultent quantitates C, C', C'' & C''' reſpective: ducantur hæ quantitates in ſeſe, & dicatur contentum reſultans $C \times C' \times C'' \times C''' = A''$; tum pro x in datâ quantitate π ſcribantur βx, $\beta' x$, $\beta'' x$, $\beta''' x$, &c.;

&c.; ubi $\beta, \beta', \beta'', \beta'''$, &c. ſunt radices æquationis $z^8 + 1 = 0$, & reſultent quantitates $D, D', D'', D''', D'''', D''''', D'''''', D'''''''$; ducantur hæ quantitates in ſeſe, & dicatur contentum reſultans $D \times D' \times D'' \times D''' \times$ &c. . . $D^7 = A'''$; & in genere ſint $\varepsilon, \varepsilon', \varepsilon'', \varepsilon'''$, &c. radices æquationis $z^{2a} + 1 = 0$; pro x in datâ quantitate π ſcribantur $\varepsilon x, \varepsilon x, \varepsilon'' x, \varepsilon''' x$, &c., & reſultent quantitates P, P', P'', P''', &c.; ducantur hæ quantitates in ſeſe & ſit contentum reſultans $P \times P' \times P'' \times P''' \times$ &c. $= A'^{a}$; tum erit continuum contentum $A \times A' \times A'' \times A''' \times$ &c. in infinitum $= \frac{1}{\pi}$; ſi modo $\pi = 1 + ax^{\sigma} + bx^{2\sigma} + cx^{3\sigma} +$ &c. in infinitum, & x minor ſit quam ulla radix æquationis $\pi = 0$, vel ejus denominatoris vel cujuſcunque irrationalis quantitatis in prædictâ (π) contentæ nihilo æqualis redditæ; & σ ſit affirmativa quantitas.

Si vero $\pi = \Gamma + ax^{\sigma} + bx^{2\sigma} + cx^{3\sigma} +$ &c., tum $\frac{\Gamma}{\pi} = \frac{A}{\Gamma} \times \frac{A'}{\Gamma^2} \times \frac{A''}{\Gamma^4} \times \frac{A'''}{\Gamma^8} \times \frac{A''''}{\Gamma^{16}} \times$ &c. in infinitum.

Sit σ negativa quantitas, & x major quam ulla radix prædictarum æquationum, & erit etiam $\frac{\Gamma}{\pi} = \frac{A}{\Gamma} \times \frac{A'}{\Gamma^2} \times \frac{A''}{\Gamma^4} \times$ &c.

Si modo quantitas π transformetur in alteram ρ, & incognita quantitas x ſit data algebraica functio incognitæ quantitatis (z) in quantitate ρ contentæ; tum ex methodo hìc traditâ erui poſſunt continua contenta quantitatibus $\frac{1}{\pi}$ & $\frac{1}{\rho}$ reſpective & conſequenter inter ſe æqualia.

4. Si π ſit quæcunque fluxio $\lambda \dot{x}$, ubi λ eſt algebraica functio quantitatis x; in fluente $\int . \lambda \dot{x} = V$ pro x ſcribatur $-x$ & reſultet quantitas A; deinde in V pro x ſcribantur $\surd(-1)\, x$ & $-\surd(-1)\, x$ & reſultent quantitates B & B', quarum productum $B \times B' = A'$; & ſic progrediendum eſt, ut in priori caſu; & reſultent quantitates A'', A''',

&

& tum erit $\frac{\Gamma}{V} = \frac{A}{\Gamma} \times \frac{A'}{\Gamma^2} \times \frac{A''}{\Gamma^4} \times \frac{A'''}{\Gamma^8} \times$ &c., ſi modo $V = \Gamma + ax^\sigma + bx^{2\sigma} +$ &c., & x minor ſit quam quæcunque radix æquationis $\lambda = q$, vel ejus denominatoris vel cujuſcunque irrationalis quantitatis in quantitate λ contentæ & nihilo æqualis redditæ; & σ ſit affirmativa quantitas; at x major ſit quam ulla prædicta radix, ſi σ ſit negativa quantitas.

In hoc caſu æqualia continua contenta deduci poſſunt ex conſimilibus transformationibus iis in prioribus caſibus traditis.

PROB. XXXIX.

1. *Invenire fluentem fluxionis* $\frac{z^{m-1}\dot{z}}{1+z^n}$ *inter valores* 0 & *infinitum quantitatis* z.

1$^{\text{mo}}$. Inveniatur fluens prædicta inter valores 0 & 1 quantitatis z contenta per ſeriem aſcendentem $\frac{1}{m}z^m - \frac{1}{m+n}z^{m+n} + \frac{1}{m+2n}z^{m+2n} - \frac{1}{m+3n}z^{m+3n} +$ &c. deinde inveniatur fluens fluxionis prædictæ inter valores 1 & infinitum quantitatis z per ſeriem deſcendentem $\frac{1}{m-n}z^{m-n} - \frac{1}{m-2n}z^{m-2n} + \frac{1}{m-3n}z^{m-3n} +$ &c. Nunc animadvertendum eſt m minorem eſſe quam n, aliter fluentem quæſitam eſſe infinitam, & conſequenter fluentem quæſitam æqualem eſſe ſummæ duarum ſerierum $\frac{1}{m} - \frac{1}{m+n} + \frac{1}{m+2n} -$ &c. & $-\left(\frac{1}{m-n} - \frac{1}{m-2n} + \frac{1}{m-3n} - \text{\&c.}\right)$ quæ ſumma $\frac{1}{m} + \frac{1}{n^2-m^2} - \frac{1}{4n^2-m^2} + \frac{1}{9n^2-m^2} - \frac{1}{16n^2-m^2} + \text{\&c.} = \frac{\pi}{n \text{ ſin. } \frac{m}{n}\pi}$, quod e præcedentibus facile colligi poteſt.

Conſtat e præcedente capite ſeriem aſcendentem eſſe convergentem, cum z minor ſit quam 1; deſcendentem ſeriem eſſe convergentem, cum z major ſit quam 1.

Erit $\int.\frac{z^{m-1}\dot{z}}{1+z^{n}}=\frac{1}{m}\times\frac{z^{m}}{1+z^{n}}+\frac{1}{m}\times\frac{n}{n+m}\times\frac{z^{n+m}}{(1+z^{n})^{2}}+\frac{1}{m}\times\frac{n}{n+m}\times\frac{2n}{2n+m}\times\frac{z^{2n+m}}{(1+z^{n})^{3}}+\&c.$; hæc ſeries convergit, cum $\frac{z^{n}}{1+z^{n}}$ minor ſit quam ± 1; divergit autem, cum major ſit.

2. Sit fluxio $\frac{(az^{m-1}+bz^{m-2}+\&c.)\dot{z}}{z^{tn}-pz^{tn-n}+q^{tn-2n}-\&c.}$, ubi tn major eſt quam m; ſint radices æquationis $z^{t}-pz^{t-1}+qz^{t-2}-\&c.=0$, reſpective α, β, γ, δ, &c. & erit $\frac{(az^{m-1}+\&c.)\dot{z}}{z^{tn}-pz^{tn-n}+\&c.}=\frac{(Az^{s-1}+\&c.)\dot{z}}{z^{n}-\alpha}+\frac{(Bz^{s-1}+\&c.)\dot{z}}{z^{n}-\beta}+\&c.$ unde ex hoc problemate inveniri poſſunt fluentes inter valores 0 & infinitum quantitatis z contentæ.

3. Sit fluxio $\frac{z^{s-1}\dot{z}}{z^{n}+1}$, in eâ pro z ſcribatur $\frac{x}{a}$, & reſultabit $\frac{a^{n-s}x^{s-1}\dot{x}}{x^{n}+a^{n}}$, unde fluens fluxionis $\frac{x^{s-1}\dot{x}}{x^{n}+a^{n}}$ inter valores 0 & infinitum quantitatis x erit $a^{s-n}\times\frac{\pi}{n\times\sin.\frac{s}{n}\pi}$. Sit fluxio $\frac{\alpha x^{s-1}\dot{x}}{x^{n}+a^{n}}+\frac{\beta x^{s-1}\dot{x}}{x^{n}+b^{n}}=\frac{x^{n+s-1}\dot{x}}{x^{n}+a^{n}.x^{n}+b^{n}}$; hinc $\alpha b^{n}+\beta a^{n}=0$ & $\alpha+\beta=1$; & fluens inter valores 0 & infinitum quantitatis x erit $(\alpha\times a^{s-n}+\beta\times b^{s-n})\times\frac{\pi}{n\times\sin.\frac{s}{n}\pi}$.

Sit $a^{n}=(l+\surd(l^{2}-1))k^{n}$, & $b^{n}=(l-\surd(l^{2}-1))k^{n}$; i. e. ſit fluxio $\frac{x^{n+s-1}\dot{x}}{x^{2n}+2lk^{n}x^{n}+k^{2n}}$, & facile inveniri poſſunt quantitates α & β, & exinde fluens quæſita $(\alpha a^{s-n}+\beta b^{s-n})\times\frac{\pi}{n\times\sin.\frac{s}{n}\pi}=$ (cum l minor ſit quam ± 1) $\dfrac{k^{s-n}\times\pi}{\surd(1-l^2)\,n\,\text{ſin.}\,\frac{s\pi}{n}}\times$ ſin. arcus $\frac{s}{n}\rho$, &c., ubi ρ eſt arcus cujus coſinus eſt l.

Hujuſce generis plura problemata deduci poſſunt; transformetur enim data fluxio in alteram, cujus variabilis quantitas y datam habeat relationem ad variabilem datæ fluxionis quantitatem x; & ſi modo cum $x=\alpha$ tum $y=\pi$, & cum $x=\beta$ tum $y=\rho$; duæ fluentes inter valores α & β, π & ρ duarum variabilium x & y contentæ erunt reſpective inter ſe æquales. Et ſic de pluribus hujuſce generis transformationibus; & ex his ſeriebus deduci poſſunt aliæ inter ſe æquales, &c.

THEOR. XLV.

1. Sint x & X ſinus arcuum, qui ſunt inter ſe ut $n:1$, & radius (r); tum erit $X=nx+\dfrac{1-n^2}{2\cdot3r^2}\times Ax^3+\dfrac{9-n^2}{4\cdot5r^2}\times Bx^5+\dfrac{25-n^2}{6\cdot7r^2}\times Cx^7+\dfrac{49-n^2}{8\cdot9r^2}\times Dx^9+$ &c.

2. Sint x & X ſinus arcuum, qui ſunt inter ſe ut $n:m$; tum erit $X=\dfrac{n}{m}x+\dfrac{1-\frac{n^2}{m^2}}{2\cdot3r^2}Ax^3+\dfrac{9-\frac{n^2}{m^2}}{4\cdot5r^2}Bx^5+\dfrac{25-\frac{n^2}{m^2}}{6\cdot7r^2}+$ &c. $=\dfrac{n}{m}x+\dfrac{m^2-n^2}{2\cdot3m^2r^2}Ax^3+\dfrac{9m^2-n^2}{4\cdot5m^2r^2}Bx^5+\dfrac{25m^2-n^2}{6\cdot7m^2r^2}Cx^7+$ &c. $=\surd\left(\dfrac{r^2-x^2}{r^2}\right)\times\left(\dfrac{n}{m}x+\dfrac{4-\frac{n^2}{m^2}}{2\cdot3r^2}\times Ax^3+\dfrac{16-\frac{n^2}{m^2}}{4\cdot5r^2}\times Bx^5+\dfrac{36-\frac{n^2}{m^2}}{6\cdot7r^2}Cx^7+\text{&c.}\right)$ $=\surd\left(\dfrac{r^2-x^2}{r^2}\right)\left(\dfrac{n}{m}x+\dfrac{4m^2-n^2}{2\cdot3m^2r^2}Ax^3+\dfrac{16m^2-n^2}{4\cdot5m^2r^2}Bx^5+\dfrac{36m^2-n^2}{6\cdot7m^2r^2}Cx^7+\text{&c.}\right)$: cum $\frac{n}{m}$ ſit impar numerus, tum abrumpitur ſeries $\frac{n}{m}x$ +

$+ \frac{m^2 - n^2}{2 \cdot 3 m^2 r^2} A x^3 + \frac{9 m^2 - n^2}{4 \cdot 5 m^2 r^2} B x^5 + \&c.$; cum autem $\frac{n}{m}$ ſit par numerus, tum abrumpitur ſeries $\surd \left(\frac{r^2 - x^2}{r^2}\right) \left(\frac{n}{m} x + \frac{4 m^2 - n^2}{2 \cdot 3 m^2 r^2} A x^3 + \frac{16 m^2 - n^2}{4 \cdot 5 m^2 r^2} B x^5 + \&c.\right)$.

3. Sit x ſinus arcus A, tum coſinus arcus $\frac{n}{m} A$ erit $r - \frac{n^2 x^2}{1 \cdot 2 m^2 r} - \frac{4 - \frac{n^2}{m^2}}{3 \cdot 4 r^2} \times A x^4 - \frac{16 - \frac{n^2}{m^2}}{5 \cdot 6 r^2} \times B x^6 - \frac{36 - \frac{n^2}{m^2}}{7 \cdot 8 r^2} C x^8 + \&c. = r - \frac{n^2}{1 \cdot 2 m^2 r} x^2 - \frac{4 m^2 - n^2}{3 \cdot 4 m^2 r^2} \times A x^4 - \frac{16 m^2 - n^2}{5 \cdot 6 m^2 r^2} \times B x^6 - \frac{36 m^2 - n^2}{7 \cdot 8 m^2 r^2} C x^8 - \&c. = \surd(r^2 - x^2) \times \left(1 + \frac{1 - \frac{n^2}{m^2}}{2 r} x^2 + \frac{9 - \frac{n^2}{m^2}}{3 \cdot 4 r^2} \times A x^4 + \frac{25 - \frac{n^2}{m^2}}{5 \cdot 6 r^2} \times B x^6 + \frac{49 - \frac{n^2}{m^2}}{7 \cdot 8 r^2} \times C x^8 + \&c.\right) = \surd(r^2 - x^2) \left(1 + \frac{m^2 - n^2}{2 m^2 r^2} x^2 + \frac{9 m^2 - n^2}{3 \cdot 4 m^2 r^2} \times A x^2 + \frac{25 m^2 - n^2}{5 \cdot 6 m^2 r^2} B x^4 + \&c.\right)$. Cum $\frac{n}{m}$ ſit par numerus, tum ſeries $r - \frac{n^2}{1 \cdot 2 m^2 r} x^2 - \frac{4 m^2 - n^2}{3 \cdot 4 m^2 r^2} \times A x^4$ &c. abrumpitur; cum autem $\frac{n}{m}$ ſit impar numerus, tum poſterior ſeries $\surd(r^2 - x^2) \left(1 + \frac{m^2 - n^2}{2 m^2 r^2} x^2 + \frac{9 m^2 - n^2}{3 \cdot 4 m^2 r^2} \times A x^4 + \frac{25 m^2 - n^2}{5 \cdot 6 m^2 r^2} \times B x^6 + \&c.\right)$ terminat.

In omnibus his ſeriebus literæ A, B, C, D, &c. præcedentium terminorum coefficientes reſpective denotant.

Omnes hæ ſeries ſemper convergent; nam ſinus x non poteſt eſſe major quam $+ r$ vel $- r$.

Cor. Sit x ſinus quantitas valde parva & conſequenter ejus arcui prope æqualis, & $n\frac{x}{r}$ non major quam 1, tum hæ ſeries ex arcu nx dato inveniunt ejus ſinum vel coſinum prope: ſi vero propior requiratur approximatio, ea facile deduci poteſt.

Facile conſimiles ſeries per quaſlibet duas ſubſequentes radium, ſinum, coſinum, tangentem, cotangentem, coſecantem, &c. vel plures deduci poſſunt.

4. Si prædictæ reducantur ad ſeries ſecundum dimenſiones quantitatis x deſcendentes; tum erit $X = \frac{n}{m}x + \frac{m^2 - n^2}{2 \cdot 3 m^2 r^2} A x^3 + \frac{9 m^2 - n^2}{4 \cdot 5 m^2 r^2}$

$B x^5 + \text{\&c.} = \frac{(-4)^{\frac{n-m}{2m}} x^{\frac{n}{m}}}{r^{\frac{n-m}{m}}} - \frac{n \cdot n - m \cdot r^2}{4 \cdot n - m \cdot m} A x^{\frac{n-2m}{m}} - \frac{n - 2m \cdot n - 3m \cdot r^2}{8 \cdot n - 2m \cdot m}$

$B x^{\frac{n-4m}{m}} - \frac{n - 4m \cdot n - 5m \cdot r^2}{12 \cdot n - 3m \cdot m} C x^{\frac{n-6m}{m}} - \frac{n - 6m \cdot n - 7m \cdot r^2}{16 \cdot n - 4m \cdot m}$

$D x^{\frac{n-8m}{m}} - \text{\&c.} = \sqrt{(r^2 - x^2)} \left(\frac{n}{m}\frac{x}{r} + \frac{4m^2 - n^2}{2 \cdot 3 m^2 r^2} A x^3 + \frac{16 m^2 - n^2}{4 \cdot 5 m^2 r^2}\right.$

$\left. B x^5 \text{ \&c.}\right) = \sqrt{(r^2 + x^2)} \left(2(-4)^{\frac{n-2m}{2m}} \times \frac{x^{\frac{n-m}{m}}}{r^{\frac{n-m}{m}}} + \frac{(n-m) \times (n-2m)}{4 \cdot (m-n) : m}\right.$

$\times A \left(\frac{x}{r}\right)^{\frac{n-3m}{m}} + \frac{(n-3m) \cdot (n-4m)}{8 \times (2m-n) \times m} \times B \left(\frac{x}{r}\right)^{\frac{n-5m}{m}} + \frac{(n-5m) \cdot (n-6m)}{12 \times (3m-n) \cdot m}$

$\left. \times C \left(\frac{x}{r}\right)^{\frac{n-7m}{m}} + \text{\&c.}\right).$

5. Coſinus $y = r - \frac{n^2}{1 \cdot 2 m^2 r} x^2 - \frac{4m^2 - n^2}{3 \cdot 4 m^2 r^2} A x^4 - \frac{16 m^2 - n^2}{5 \cdot 6 m^2 r^2} B x^6$

$- \text{\&c.} = -2 \times (-4)^{\frac{n-2m}{2m}} \frac{x^{\frac{n}{m}}}{r^{\frac{n-m}{m}}} + \frac{n \times (n-m)}{4m \times (m-n)} A \frac{x^{\frac{n-2m}{m}}}{r^{\frac{n-3m}{m}}} + \frac{(n-2m)}{8m} \cdot$

$\frac{(n-3m)}{(2m-n)} B \frac{x^{\frac{n-4m}{m}}}{r^{\frac{n-5m}{m}}} + \frac{(n-4m) \times (n-5m)}{12m \times (3m-n)} C \frac{x^{\frac{n-6m}{m}}}{r^{\frac{n-7m}{m}}} + \text{\&c.} = \sqrt{(r^2 - x^2)}$

$(1 +$

$(1 + \frac{m^2 - n^2}{2m^2r^2}x^2 + \frac{9m^2 - n^2}{3 \cdot 4m^2r^2}Ax^4 + \frac{25m^2 - n^2}{5 \cdot 6m^2r^2}Bx^6 + \&c.) =$
$\sqrt{(r^2 - x^2)}\,((-4)^{\frac{n-m}{2m}} \times \left(\frac{x}{r}\right)^{\frac{n-m}{m}} + \frac{n-m \cdot n-2m}{4m \times (m-n)}A\left(\frac{x}{r}\right)^{\frac{n-3m}{m}} +$
$\frac{n-3m \cdot n-4m}{8m \cdot 2m-m}B\left(\frac{x}{r}\right)^{\frac{n-5m}{m}} + \frac{n-5m \cdot n-6m}{12m \cdot 3m-n} \times C\left(\frac{x}{r}\right)^{\frac{n-7m}{m}}$
$+ \&c.)$.

Conſimiles ſeries facile deduci poſſunt pro ſinubus vel coſinubus, tangentibus vel cotangentibus, ſecantibus vel coſecantibus, verſis ſinubus, &c. in terminis ſecundum dimenſiones coſinus, tangentis, &c. progredientibus.

6. Exhinc datâ ſerie ſecundum dimenſiones quantitatis x progrediente a ſcribendo in eâ pro x, x^2, x^3, &c. vel $x^{\frac{n}{m}}$, $x^{\frac{n}{m}-1}$, $x^{\frac{n}{m}-2}$, &c. earum valores hìc traditos, & ducendo hos valores in incognitas coefficientes, & æquando coefficientes correſpondentium terminorum reſultantis æquâtionis, ita ut evaneſcant quantitas x & ejus poteſtates vel radices; ex æquationibus hinc reſultantibus erui poſſunt prædictæ coefficientes quæſitæ.

7. Sint s & c ſinus & coſinus arcus A, cujus radius ſit 1; & per sn, cn; $s(n-2)$, $c(n-2)$; $s(n-4)$, $c(n-4)$; &c. deſignentur ſinus & coſinus arcuum nA, $(n-2)A$, $(n-4) \times A$, &c. tum ex principiis prius traditis erui poſſunt $s^n = \pm \frac{1}{2^{n-1}}(sn - ns(n-2) + n \cdot \frac{n-1}{2}s(n-4) - n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3}s(n-6) + n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3} \cdot \frac{n-3}{4}s(n-8) - \&c.)$ cum n ſit impar numerus; ſignum erit $+$, ſi $n = 4m + 1$, ubi m eſt integer numerus; ſin aliter $-$: ſi vero n ſit par numerus, &c., tum erit $s^n = \pm \frac{1}{2^{n-1}}(cn - nc(n-2) + n \cdot \frac{n-1}{2}c(n-4) - n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3}c(n-6) + \&c.)$. Signum erit $+$, ſi $n =$

$n = 4m$; ſin aliter $-$; ultimus autem terminus erit $\frac{1}{2}\times n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3} \cdots \frac{\frac{1}{2}n+1}{\frac{1}{2}n}$. Et ſimiliter erit $c^n = \frac{1}{2^{n-1}}(cn + nc(n-1) + n \cdot \frac{n-1}{2}c(n-2) + n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3}c(n-3) + \&c.)$. Cum n ſit par numerus, ultimus terminus erit $\frac{1}{2}\times n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3} \cdots \frac{\frac{1}{2}n+1}{\frac{1}{2}n}$.

Ex hinc datâ ſerie ſecundum dimenſiones quantitatis s vel x progrediente deduci poteſt ſeries ei æqualis ſecundum quantitates sn, $s(n-2)$, $s(n-4)$, &c. & cn, $c(n-2)$, $c(n-4)$, &c. progrediens, cum n ſit integer numerus.

Cor. Hinc data algebraica æquatio $x^n - px^{n-1} + qx^{n-2} - \&c. = 0$ transformari poteſt in æquationem $as(n) + bc(n-1) + a's(n-2) + b'c(n-3) + a''s(n-4) + b''c(n-5) + \&c. = 0$, vel $acn + bs(n-1) + a'c(n-2) + b's(n-3) + a''s(n-4) + b''s(n-5) + \&c. = 0$, prout n ſit impar vel par numerus: tum datâ approximatione α ad valorem s, & conſequenter ad c & $s2$ & $c2$, $s3$ & $c3$, $s4$ & $c4 \ldots sn$ & cn; quoniam fluxiones quantitatum sn & cn ſunt $n \times c(n) \times \dot{A}$ & $n \times sn \times \dot{A}$; ubi A eſt arcus, cujus ſinus eſt s; & ſi e & f ſint approximationes ad sn & cn, e' & f' ad $s(n-1)$ & $c(n-1)$, e'' & f'' ad $s(n-2)$ & $c(n-2)$, &c.; & ſcribantur hæ approximationes pro ſuis valoribus in datis æquationibus, & ſit quantitas reſultans B, erit $\frac{-B}{nacn + (n-1)bs(n-1) + (n-2)a'c(n-2) + (n-3)b's(n-3) + \&c.}$, vel $\frac{-B}{nasn + (n-1)bc(n-1) + (n-2)a's(n-2) + \&c.}$ propior approximatio: & ſic deinceps.

PROB. XL.

Sit poteſtas $(1+x+x^2)^n = (1+x(1+x))^n = x^n(1+x)^n + n x^{n-1}(1+x)^{n-1} + n \cdot \frac{n-1}{2} x^{n-2} \cdot (1+x)^{n-2} + n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3} x^{n-3}(1+x)^{n-3} +$ &c. *coefficientem medii ejus termini* (x^n) *definire.*

Coefficiens erit $1 + \frac{n . n-1}{1 . 1} + \frac{n(n-1)(n-2)(n-3)}{1 . 2 . 1 . 2} + \frac{n(n-1)(n-2)(n-3)(n-4)(n-5)}{1 . 2 . 3 . 1 . 2 . 3} +$ &c.

Conſtat ex binomiali theoremate.

Cor. . Sint P, Q & R reſpective coefficientes mediorum terminorum ad poteſtates n, $n+1$ & $n+2$; tum e ſubſtitutione conſtabit $\frac{n+2}{n+1}(R-Q)-(Q-P)=4P$.

Cor. . Quoniam $\frac{n+2}{n+1}(R-Q)-(Q-P)=4P$, erit etiam $\frac{n+3}{n+1}(S-R)-(R-Q)=4Q$, ubi P, Q, R & S ſunt coefficientes mediorum terminorum reſpective; ita reducantur hæ duæ æquationes ad unam, ut exterminetur n, & reſultat æquatio relationem inter S, R, Q & P deſignans.

Et ſic progredi liceat ad inveniendos terminos medios, &c. (n) poteſtatis aliarum. E. g. Quantitatis (A) $(a+bx+cx^2)^n$ medius terminus erit $b^n + \frac{n . n-1}{1 . 1} b^{n-2}ac + \frac{n . (n-1)(n-2)(n-3)}{1 . 2 . 1 . 2} b^{n-4}a^2c^2 + \frac{n . (n-1)(n-2)(n-3)(n-4)(n-5)}{1 . 2 . 3 . 1 . 2 . 3} b^{n-6}a^3c^3 +$ &c.; & exinde $\frac{n+2}{n+1}(R-bQ)-b(Q-bP)=4acP$, ſi modo P, Q & R denotent coefficientes mediorum terminorum quantitatis A ad poteſtates n, $n+1$ & $n+2$ elevatæ.

PROB.

PROB. XLI.

Ex datis fluentibus vel integralibus, quæ a ſe invicem deduci poſſint; invenire infinitas ſeries, quæ etiam a ſe invicem poſſunt inveſtigari.

Reducantur ſingulæ datæ fluentes vel integrales ad infinitas ſeries ſecundum dimenſiones variabilis x progredientes, & exorientur ſeries, quæ a ſe invicem deduci poſſunt.

Ex. 1. Ex datâ fluente $\int.(a+bx^n)^m \times x^{pn-1}\dot{x}$ erui poteſt fluens fluxionis $(a+bx^n)^{m+r} \times x^{pn+vn-1}\dot{x}$, ubi r & v integri ſunt numeri vel affirmativi vel negativi, m vero & p haud integri affirmativi numeri; reducantur hæ fluxiones ad infinitas ſeries, & reſultant $a^m x^{pn-1}\dot{x} + m a^{m-1} b x^{(p+1)n-1}\dot{x} + m.\frac{m-1}{2} a^{m-2} b^2 x^{(p+2)n-1}\dot{x} +$ &c. & $a^{m+r} x^{pn+vn-1}\dot{x} + (m+r).a^{m+r-1} b x^{(p+v+1)n-1}\dot{x} + (m+r).\frac{m+r-1}{2} a^{m+r-2} b^2 x^{(p+v+2)n-1}\dot{x}$ + &c. quarum fluentes erunt reſpective $\frac{a^m x^{pn}}{pn} + \frac{m a^{m-1} b x^{(p+1)n}}{(p+1)n}$ + &c. & $\frac{a^{m+r} x^{pn+vn}}{(p+v)n} + \frac{(m+r) a^{m+r-1} b x^{(p+v+1)n}}{(p+v+1)n}$ + &c. unde conſtat ſummam ſeriei $\frac{1}{p+v} + \frac{m+r}{p+v+1} \times x^n + \frac{(m+r).(m+r-1)}{2.(p+v+2)} x^{2n} + \frac{(m+r).(m+r-1).(m+r-2)}{2.3.(p+v+3)} x^{3n}$ + &c. (ſi modo v & r ſint integri numeri vel affirmativi vel negativi, & p & m haud integri affirmativi numeri) deduci poſſe e ſummâ ſeriei $\frac{1}{p} + \frac{m}{p+1} x^n + \frac{m.(m-1)}{2.(p+2)} x^{2n} + \frac{m.(m-1).(m-2)}{2.3.(p+3)} x^{3n}$ + &c.

Cor. Datâ ſummâ unius ſeriei prædicti generis, ex eâ deduci poſſunt ſummæ omnium ſerierum ejuſdem generis, quicunque integri numeri ſint valores quantitatum r & v: ex fluente enim fluxionis

$(a+bx^n)^{m+r}\,x^{pn+vn-1}\,\dot{x}$ ad unum integrum valorem quantitatum r & v reſpective erui poſſunt fluentes ejuſdem fluxionis ad omnes integros valores quantitatum r & v.

Cor. 2. Ex datâ fluente $\int.\,(a+bx^n)^{m-1}\,x^{pn-1}\,\dot{x}$ inveniri poteſt fluens fluxionis $(a+bx^n)^{m-1}\,x^{pn+vn-1}\,\dot{x} = (a+bx^n)^m \times \left(\frac{1}{rnb}\,x^{(q-1)n} - \frac{(q-1)a}{(r-1)b}\,A\,x^{(q-2)n} + \text{\&c. ad } v \text{ terminos}\right) \pm \frac{p}{p+m}\cdot\frac{p+1}{p+m+1} \ldots \frac{p+v-1}{p+m+v-1}\cdot\frac{a^v}{b^v}\int.\,(a+bx^n)^{m-1}\,x^{pn-1}\,\dot{x}$, ubi $p+v=q$, $q+m-1=r$; & A, &c. ſint coefficientes præcedentium terminorum.

Reducantur ſinguli termini ad infinitas ſeries ſecundum reciprocas dimenſiones quantitatis x^n progredientes, & ex æquatis correſpondentibus terminis in quibus eædem inveniuntur dimenſiones $(p+v+m-w)$ quantitatis x^n, reſultabit æquatio

$$\frac{(m-1).(m-2).(m-3)\ldots(m-w+1)}{1\;.\;2\;.\;3\ldots w-1} \times \frac{1}{p+v+m-w} = m.\frac{m-1}{2}.\frac{m-2}{3}\ldots\frac{m-w+2}{w-1} \times \frac{1}{p+m+v-1} - m.\frac{m-1}{2}.\frac{m-2}{3}\ldots\frac{m-w+3}{w-2} \times \frac{1}{p+m+v-1} \times \frac{p+v-1}{p+v+m-2} + m.\frac{m-1}{2}.\frac{m-2}{3}\ldots\frac{m-w+4}{w-3} \times \frac{1}{p+m+v-1} \times \frac{p+v-1}{p+m+v-2} \times \frac{p+v-2}{p+m+v-3} - \ldots \pm m.\frac{m-1}{2}.\frac{m-2}{3}\ldots\frac{m-w+v+1}{w-v} \times \frac{1}{p+m+v-1} \times \frac{p+v-2}{p+m+v-2} \times \frac{p+v-3}{p+m+v-3}\ldots\frac{p+1}{p+m} \pm \text{\&c.} \pm \frac{p}{p+m}.\frac{p+1}{p+m+1}.\frac{p+2}{p+m+2}\ldots\frac{p+v-1}{p+m+v-1} \times (m-1).\frac{m-2}{2}.\frac{m-3}{3}\ldots\frac{m-w+v}{w-v} \times \frac{1}{p+m+v-w};$$

ſi modo v & w ſint integri, p vero & m haud integri numeri: hic animadvertendum eſt quod omnes termini poſt w primos rejiciendi ſunt; etiamque adjiciendum eſt quod coefficiens termini, cujus diſtantia a primo ſit $w-1$, invenietur $\frac{1}{p+m+v-1}$. $\frac{p+v-1}{p+m+v-2} \cdot \frac{p+v-2}{p+m+v-3} \cdot \, . \, . \frac{p+v-w+1}{p+m+v-w}$.

2. Transformari poſſunt diverſis modis datæ fluxiones in alias, quarum fluentes e fluentibus datarum fluxionum deduci poſſunt; reducantur hæ fluxiones ad ſeries ſecundum dimenſiones quarundam quantitatum progredientes, & inveniantur fluentes; & reſultant ſeries, quæ a ſe invicem deduci poſſunt: e. g. ſit fluxio data $(a+bx^n)^{m+r} \times x^{(p+v)n-1}\dot{x} = (b+ax^{-n})^{m+r} \times x^{(m+r+p+v)n-1}\dot{x}$; reducantur hæ fluxiones ad terminos ſecundum dimenſiones quantitatis x^n vel x^{-n} vel $(a+bx^n)^m$ vel $b+ax^{-n}$, &c. progredientes, & inveniantur fluentes ſerierum reſultantium; & facile conſtat ſeries reſultantes, modo convergant & proprie corrigantur, a ſe invicem erui poſſe.

Ex. 2. Sint ſeries $a^{m+r} x^{(p+v)n} \Big(\Big(\frac{1}{(p+v)n} + (m+r)\frac{bx^n}{(p+v+1)na}$ $+ (m+r).\Big(\frac{c}{(p+v+2)na} + \frac{m+r-1}{2} \times \frac{b^2}{a^2} \times \frac{1}{p+v+2}\Big)x^{2n}$ $+ \&c.\Big) = \int. (a+bx^n+cx^{2n})^{m+r} \times x^{(p+v)n-1}\dot{x}$. Ex ſummis duarum ſerierum independentium hujuſce generis erui poſſunt ſummæ omnium ſerierum ejuſdem generis, quicunque ſint integri valores quantitatum r & v; & exinde inveniri poſſunt ſummæ omnium ſerierum exprimentium fluentes fluxionum $(c+bx^{-n}+ax^{-2n})^{m+r} \times x^{(2(m+r)+p+v)n-1}\dot{x}$ $= (a+bx^n+cx^{2n})^{m+r} \times x^{(p+v)n-1}\dot{x}$, quarum termini ſecundum dimenſiones quantitatis x^n vel ejus reciprocas progrediuntur; vel ſerierum, quæ exprimunt fluentes fluxionum ex prædictis per transformationes, &c. reſultantium.

Et ſic in multis aliis caſibus.

Eadem etiam affirmari poſſunt, ſi modo hæ ſeries incipiant a terminis, quorum diſtantiæ a primis ſint reſpective r, s, t, &c.; vel ſeries conſtent ex quibuſque terminis prædictarum ſerierum, quorum diſtantiæ a ſe invicem ſint reſpective $\rho, \sigma, \tau,$ &c. vel ſucceſſivi termini ducantur in integram functionem $az^m + bz^{m-1} + cz^{m-2} +$ &c. quantitatis z diſtantiæ a primo ſeriei termino reſpective, ubi m eſt integer affirmativus numerus.

PROB. XLII.

Invenire fluentem fluxionis $\frac{x^\alpha \dot{x}}{1 \pm x^n}$; *ubi* α *eſt irrationalis quantitas.*

1^mo^. Aſſumatur π fractio prope $= \alpha$, & inveniatur $\int . \frac{x^\pi \dot{x}}{1 \pm x^n}$, quæ ſit V; tum erit fluens quæſita $= V \times x^{\alpha - \pi}$ prope.

2^do^. Ex eâdem methodo inveniatur $W = \int . (\alpha - \pi) \times \int . V x^{\alpha - \pi - 1} \dot{x}$ prope; tum erit $V - W$ quantitas ad fluentem magis approximans; & ſic deinceps.

Eadem principia etiam applicari poſſunt ad inveniendas approximationes ad fluentes omnium fluxionum.

THEOR. XLVI.

Summæ omnium ſerierum $\frac{1}{\alpha . \beta . \gamma . \delta . \&c.} + \frac{1}{\alpha + 1 . \beta + 1 . \gamma + 1 . \delta + 1 . \&c.} \times x^b + \frac{1}{\alpha + 2 . \beta + 2 . \gamma + 2 . \delta + 2 . \&c.} x^{2b} + \&c. = S$ deduci poſſunt e ſummis ſerierum $\frac{1}{\alpha} + \frac{1}{\alpha + 1} x^b + \frac{1}{\alpha + 2} x^{2b} + \&c. = A$, $\frac{1}{\beta} + \frac{1}{\beta + 1} x^b + \frac{1}{\beta + 2} x^{2b} + \&c. = B$, $\frac{1}{\gamma} + \frac{1}{\gamma + 1} x^b + \frac{1}{\gamma + 2} x^{2b} + \&c. = C$, $\frac{1}{\delta} + \frac{1}{\delta + 1} x^b + \frac{1}{\delta + 2} x^{2b} + \&c. = D$, &c.: erunt $\frac{1}{\alpha - \beta . \alpha - \gamma . \alpha - \delta . \&c.} \times A + \frac{1}{\beta - \alpha . \beta - \gamma . \beta - \delta . \&c.} B +$

$\gamma - \alpha$

$\frac{1}{\gamma-\alpha\,.\,\gamma-\beta\,.\,\gamma-\delta\,.\,\&c.}\times C+\frac{1}{\delta-\alpha\,.\,\delta-\beta\,.\,\delta-\gamma\,.\,\&c.}D+\&c.$ $=\pm S$; erit +, cum numerus factorum α, β, γ, &c. ſit par, ſin aliter —.

Ex his ſeriebus igitur facile erui poſſunt ſummæ omnium ſerierum, quarum generalis terminus eſt $\frac{az^m+bz^{m-1}+\&c.}{pz^n+qz^{n-1}+\&c.}\times e^{bz}$, ubi z eſt diſtantia a primo datæ ſeriei termino, ni duæ quantitates α, β, γ, &c. ſint inter ſe æquales.

Cor. Hinc $\frac{1}{\alpha+z}\times\frac{1}{\alpha-\beta\,.\,\alpha-\gamma\,.\,\alpha-\delta\,.\,\&c.}+\frac{1}{\beta+z}\times\frac{1}{\beta-\alpha\,.\,\beta-\gamma\,.\,\beta-\delta\,.\,\&c.}+\frac{1}{\gamma+z}\times\frac{1}{\gamma-\alpha\,.\,\gamma-\beta\,.\,\gamma-\delta\,.\,\&c.}+\frac{1}{\delta+z}\times\frac{1}{\delta-\alpha\,.\,\delta-\beta\,.\,\delta-\gamma\,.\,\&c.}+\&c.=\frac{\pm 1}{\alpha+z\,.\,\beta+z\,.\,\gamma+z\,.\,\delta+z\,.\,\&c.}$: ſignum erit +, ſi numerus factorum $\alpha-\beta$, $\alpha-\gamma$, $\alpha-\delta$, &c. ſit par; ſin aliter —.

PROB. XLIII.

Invenire ſummam quarumque fractionum, cum earum denominatores nihilo æquales, & conſequenter fractiones evadant infinitæ.

Supponantur quantitates in factoribus contentæ, quæ evadant nihilo æquales, variabiles; & augeantur per incrementa quam minima; reducantur fractiones reſultantes ad communem denominatorem, & ſimul addantur, &c.; tum fractio reſultans reducatur ad minimos terminos per diviſionem ejus numeratoris & denominatoris per maximum communem diviſorem, & fractio reſultans erit ſumma quæſita.

Ex. 1. Invenire ſummam fractionum $\frac{1}{\alpha-\beta\,.\,\alpha-\gamma}+\frac{1}{\beta-\alpha\,.\,\beta-\gamma}$ cum α fiat $=\beta$: augeantur quantitates α & β per e & o; tum evadent fractiones $\frac{1}{\alpha-\beta+e-o\,.\,\alpha-\gamma+e} + \frac{1}{\beta-\alpha+o-e\,.\,\beta-\gamma+o}$ $= \frac{(\beta-\gamma+o)-(\alpha-\gamma+e)}{\alpha-\beta+e-o\,.\,\alpha-\gamma+e\,.\,\beta-\gamma+o}$ (ob $\beta-\alpha=o$) $=$ $\frac{o-e}{e-o\,.\,\alpha-\gamma+e\,.\,\beta-\gamma+o}$ (ob e & o quam minima) $= -$ $\frac{1}{\alpha-\gamma\,.\,\beta-\gamma} = -\frac{1}{(\alpha-\gamma)^2}$.

Et ſic ſumma fractionum $\frac{1}{\alpha-\beta\,.\,\alpha-\gamma\,.\,\alpha-\delta} + \frac{1}{\beta-\alpha\,.\,\beta-\gamma\,.\,\beta-\delta}$ $+ \frac{1}{\gamma-\alpha\,.\,\gamma-\beta\,.\,\gamma-\delta} = \frac{1}{(\alpha-\delta)^3}$, cum $\alpha=\beta=\gamma$; &c. & ſimiliter ſint (n) quantitates $\alpha, \beta, \gamma, \delta$, &c. inter ſe æquales & θ non $=\alpha$; tum erit $\frac{1}{\alpha-\beta\,.\,\alpha-\gamma\,.\,\alpha-\delta \times \&c. \times \alpha-\theta} + \frac{1}{\beta-\alpha\,.\,\beta-\gamma\,.\,\beta-\delta\,.\,\&c.\,.\,\beta-\theta}$ $+ \frac{1}{\gamma-\alpha\,.\,\gamma-\beta\,.\,\gamma-\delta \times \&c.\,\gamma-\theta} + \&c. = \frac{\pm 1}{(\alpha-\theta)^n}$.

THEOR. XLVII.

1. Summa ſeriei $\frac{1}{\alpha\beta\gamma\delta\,\&c.} \pm \frac{1}{\alpha+1\,.\,\beta+1\,.\,\gamma+1\,.\,\delta+1\,\&c.}\,x^b$ $+ \frac{1}{\alpha+2\,.\,\beta+2\,.\,\gamma+2\,.\,\delta+2\,\&c.}\,x^{2b} \pm$ &c. ſemper deduci poteſt e fluentibus fluxionum $\frac{x^{b\alpha-1}\dot{x}}{1\pm x^b}, \frac{x^{b\beta-1}\dot{x}}{1\pm x^b}, \frac{x^{b\gamma-1}\dot{x}}{1\pm x^b}, \frac{x^{b\delta-1}\dot{x}}{1\pm x^b}$, &c.: 1^{mo}. ſit b affirmativa quantitas, & hæ ſeries ſemper convergent, cum x minor ſit quam 1; nunquam autem, cum major ſit.

2. Cum vero $x=1$ & denominator ſit $1+x^b$; vel $x=-1$, cum b ſit impar numerus, vel fractio cujus denominator ſit impar numerus, & denominator ſit $1-x^b$; tum ſeries ſemper convergent: ſi autem $x=1$, & denominator ſit $1-x^b$; vel $x=-1$, cum b ſit impar nu-

merus vel fractio ut antea & denominator $1+x^{b}$; tum fluentes ſingularum fluxionum $\frac{x^{b\alpha-1}\dot{x}}{1\pm x^{b}}$, $\frac{x^{b\beta-1}\dot{x}}{1\pm x^{b}}$, $\frac{x^{b\gamma-1}\dot{x}}{1\pm x^{b}}$, &c. erunt infinitæ: at hæ fluentes ducuntur reſpective in fractiones $\frac{1}{\alpha-\beta\,.\,\alpha-\gamma\,.\,\alpha-\delta\,.\,\&c.}$ $=\alpha'$, $\frac{1}{\beta-\alpha\,.\,\beta-\gamma\,.\,\beta-\delta\,.\,\&c.}=\beta'$, $\frac{1}{\gamma-\alpha\,.\,\gamma-\beta\,.\,\gamma-\delta\,.\,\&c.}=\gamma'$, $\frac{1}{\delta-\alpha\,.\,\delta-\beta\,.\,\delta-\gamma\,.\,\&c.}=\delta'$, &c.; quarum ſumma nihilo æqualis eſt; & ex fluentium in has fractiones reſpective ductarum ſummâ evaneſcat logarithmus infinitus vel quantitas infinita.

Si quantitates α, β, γ, δ, &c. ſint affirmativæ, tum inveniantur fluentes prædictarum fluxionum inter valores 0 & 1 quantitatis x.

3. Si quæcunque quantitates α, β, &c. ſint negativæ, i. e. $-\alpha$, $-\beta$, &c.; tum pro $-\alpha$, $-\beta$, &c. ſcribantur $l-\alpha$, $l'-\beta$, &c., ubi l, l', &c. ſunt integri numeri proxime majores quam α, β, &c. reſpective; & inveniantur fluentes omnium fluxionum $\frac{x^{(l-\alpha)b-1}\dot{x}}{1\pm x^{b}}$, $\frac{x^{(l'-\beta)b-1}\dot{x}}{1\pm x^{b}}$, &c. inter valores 0 & x quantitatis x contentæ, quæ ſint A, B, C, &c. & quæ ductæ reſpective in α', β', &c. conſtituant ſummam $\alpha'A+\beta'B+$ &c. finitam; deinde inveniatur ſumma $-x^{-\alpha b}\left(\frac{1}{\alpha b}\pm\frac{x^{b}}{(\alpha-1)b}+\frac{x^{2b}}{(\alpha-2)b}\pm\frac{x^{3b}}{(\alpha-3)b}\cdot\cdot\frac{x^{(l-1)b}}{(\alpha-l+1)b}\right)\times\alpha'=P$, $-x^{-\beta b}\left(\frac{1}{\beta b}\pm\frac{x^{b}}{(\beta-1)b}+\frac{x^{2b}}{(\beta-2)b}\cdot\cdot\frac{x^{(l'-1)b}}{(\beta-l'+1)b}\right)\times\beta'=\mathcal{Q}$, &c.; i. e. inveniantur fluentes inter valores infinitum & x quantitatis x fluxionum $\frac{(x^{-\alpha b-1}\smile x^{(l-\alpha)b-1})\dot{x}}{1\pm x^{b}}=x^{-\alpha b-1}\dot{x}\pm x^{-(\alpha-1)b-1}\dot{x}+x^{-(\alpha-2)b-1}\dot{x}\pm$ &c. $=P$, $\frac{(x^{-\beta b-1}\smile x^{(l'-\beta)b-1})\dot{x}}{1\pm x^{b}}=x^{-\beta b-1}\dot{x}\pm$ &c. $=\mathcal{Q}$, &c.; ducantur hæ quantitates reſpective in α', β', &c.; tum erit ſumma ſeriei quæſita $=\alpha'(A\mp P)+\beta'(B\mp\mathcal{Q})+$ &c. $=\alpha'A+\beta'B+\gamma'C+$ &c. $\mp\alpha'P\mp\beta'\mathcal{Q}\mp$ &c.

4. Si vero requiratur ſumma prædictæ ſeriei inter duos valores m & m' quantitatis z contenta: inveniantur fluentes fluxionum $\frac{x^{(m+\alpha)b-1} \backsim x^{(m'+\alpha)b-1}}{1 \pm x^b}\dot{x} = A$, $\frac{x^{(m+\beta)b-1} \backsim x^{(m'+\beta)b-1}}{1 \pm x^b}\dot{x} = B$, $\frac{x^{(m+\gamma)b-1} \backsim x^{(m'+\gamma)b-1}}{1 \pm x^b}\dot{x} = C$, &c. inter valores o & x quantitatis x, cum $m+\alpha$, $m'+\alpha$; $m+\beta$, $m'+\beta$; $m+\gamma$, $m'+\gamma$; &c. ſint affirmativæ quantitates: ſin vero quædam e prædictis ſint negativæ quantitates; e. g. ſint $m+\alpha$ & $m'+\alpha$ negativæ quantitates, tum inveniatur fluens (P) fluxionis $\frac{x^{(m+\alpha)b-1} \backsim x^{(m'+\alpha)b-1}}{1 \pm x^b}\dot{x}$ inter valores x & infinitum quantitatis x: ſin vero $m+\alpha$ ſit negativa & $m'+\alpha$ affirmativa quantitas; & $\alpha+l$ affirmativa quantitas, ubi l eſt minimus integer numerus, qui poteſt ſummam $\alpha+l$ reddere affirmativam; inveniatur fluens inter valores x & infinitum quantitatis x fluxionis $\frac{x^{(m+\alpha)b-1} \backsim x^{(\alpha+l)b-1}}{1 \pm x^b} \times \dot{x}$, quæ ſit P; deinde inveniatur fluens inter valores o & x quantitatis x contenta fluxionis $\frac{x^{(\alpha+l)b-1} \backsim x^{(\alpha+m')b-1}}{1 \pm x^b}\dot{x}$, quæ ſit P'; tum fluens fluxionis $\frac{x^{(m+\alpha)b-1} \backsim x^{(m'+\alpha)b-1}}{1 \pm x^b}\dot{x}$ quæſita erit $P' + P = A$; & ſic de ſingulis reliquis fluentibus B, C, &c. corrigendis; & exinde ſumma quæſita erit $\alpha' A + \beta' B + \gamma' C +$ &c.

5. Si α vel β vel γ, &c. ſit negativus numerus & inter m & m' contentus; tum unus terminus prædictæ ſeriei evadit infinitus, i. e. $\frac{1}{0}$; ſin autem plures termini evadant infiniti; & coefficientes, in quas ducitur idem infinitus valor, ſimul ſumptæ nihilo ſint æquales; tum ſumma quæſita poteſt eſſe finita; ſin aliter vero non.

6. Si vero fluens ſit $\int. \frac{z^{b\alpha-1}\dot{z}}{a \pm b z^b} = \frac{1}{a} \times \frac{1}{b\alpha} z^{b\alpha} \pm \frac{b}{a^2} \times \frac{1}{b \times (\alpha+1)} z^{b(\alpha+1)} +$ &c., tum facile reduci poteſt ad prædictam formulam $\frac{1}{a}$

$a^{\alpha} \times$

$\frac{a^{\alpha}}{b^{\alpha}} \times f.\,\frac{x^{b\alpha-1}\dot{x}}{1 \pm x^{b}}$ e ſcribendo $\frac{a^{\frac{1}{b}}}{b^{\frac{1}{b}}}\,x$ pro x. Et ſimiliter fluxio $\frac{x^{b\alpha-1}\dot{x}}{1 \pm x^{b}}$ reduci poteſt ad fluxionem $-\frac{z^{-(\alpha-1)b}\dot{z}}{z^{b} \pm 1}$ e ſcribendo $\frac{1}{z}$ pro x, &c.

7. Sit data ſeries $\frac{1}{\alpha.\alpha'.\alpha''.\alpha'''.\&c.\times\beta.\beta'.\beta''.\beta'''.\&c.\times\gamma.\gamma'.\gamma''.\gamma'''.\&c.\times\&c.}$
$+\frac{1}{\alpha+1.\alpha'+1.\alpha''+1.\alpha'''+1.\&c.\times\beta+1.\beta'+1.\beta''+1.\&c.}$
$\overline{\times\gamma.\gamma'+1.\gamma''+1.\&c..\&c.} + \frac{1}{\alpha+2.\alpha'+2.\&c.\times\beta+2.\beta'}$
$\overline{+2.\&c..\gamma+2.\gamma'+2.\&c..\&c.} \dots \frac{1}{\alpha+z.\alpha'+z.\alpha''+z.\&c.}$
$\overline{\times\beta.\beta'+z.\beta''+z.\&c.\times\gamma+z.\gamma'+z.\&c.\times\&c.}$ in infinitum; ubi z eſt diſtantia a primo datæ ſeriei termino; & $\alpha-\alpha'$, $\alpha-\alpha''$, $\alpha-\alpha''$, &c.; $\beta-\beta'$, $\beta-\beta''$, $\beta-\beta'''$, &c.; $\gamma-\gamma'$, $\gamma-\gamma''$, $\gamma-\gamma'''$, &c., &c., ſunt integri numeri; at $\alpha-\beta$, $\alpha-\gamma$, $\beta-\gamma$, &c. non ſunt integri numeri: tum, ſi ſumma fractionum $\frac{1}{\alpha-\alpha'.\alpha-\alpha''.\alpha-\alpha'''.}$
$\overline{\&c.\times\alpha-\beta.\alpha-\beta'.\alpha-\beta''.\alpha-\beta'''.\&c.\times\alpha-\gamma.\alpha-\gamma'.\alpha-\gamma''.\&c..\&c.}$
$+\frac{1}{\alpha'-\alpha.\alpha'-\alpha''.\alpha'-\alpha'''.\&c..\alpha'-\beta.\alpha'-\beta'.\alpha'-\beta''.\alpha'-\beta'''.\&c..\alpha'-\gamma.}$
$\overline{\alpha'-\gamma'.\alpha'-\gamma''.\&c.} + \frac{1}{\alpha''-\alpha.\alpha''-\alpha'.\alpha''-\alpha'''.\&c..\alpha''-\beta.\alpha''-\beta'.}$
$\overline{\alpha''-\beta''.\alpha''-\beta'''.\&c.\times\alpha''-\gamma.\alpha''-\gamma'.\alpha''-\gamma''.\alpha''-\gamma'''.\&c..\&c.} +$
$\frac{1}{\alpha'''-\alpha.\alpha'''-\alpha'.\alpha'''-\alpha''.\&c.\times\alpha'''-\beta.\alpha'''-\beta'.\alpha'''-\beta''.\alpha'''-\beta'''.\&c.\times}$
$\overline{\alpha'''-\gamma.\alpha'''-\gamma'.\alpha'''-\gamma''.\alpha'''-\gamma'''.\&c..\&c.} + \&c. = 0$, etiamque ſumma fractionum

fractionum

$$\frac{1}{\beta-\alpha.\beta-\alpha'.\beta-\alpha''.\beta-\alpha'''.\&c.\times\beta-\beta'.\beta-\beta''.\beta-\beta'''.\&c.\times\beta-\gamma.\beta-\gamma'.\beta-\gamma''.\beta-\gamma'''.\&c..\&c.}+\frac{1}{\beta'-\alpha.\beta'-\alpha'.\beta'-\alpha''.\beta'-\alpha'''.\&c.\times\beta'-\beta.\beta'-\beta''.\beta'-\beta'''.\&c.\times\beta'-\gamma.\beta'-\gamma'.\beta'-\gamma''.\beta'-\gamma'''.\&c..\&c.}+\frac{1}{\beta''-\alpha.\beta''-\alpha'.\beta''-\alpha''.\beta''-\alpha'''.\&c.\times\beta''-\beta.\beta''-\beta'.\beta''-\beta'''.\&c.\times\beta''-\gamma.\beta''-\gamma'.\beta''-\gamma''.\beta''-\gamma'''.\&c.\times\&c.}+\frac{1}{\beta'''-\alpha.\beta'''-\alpha'.\beta'''-\alpha''.\beta'''-\alpha'''.\&c.\times\beta'''-\beta.\beta'''-\beta'.\beta'''-\beta''.\&c.\times\beta'''-\gamma.\beta'''-\gamma'.\beta'''-\gamma''.\beta'''-\gamma'''.\&c.\times\&c.}+\&c.=0;$$

&

$$\frac{1}{\gamma-\alpha.\gamma-\alpha'.\gamma-\alpha''.\gamma-\alpha'''.\&c.\times\gamma-\beta.\gamma-\beta'.\gamma-\beta''.\gamma-\beta'''.\&c.\times\gamma-\gamma'.\gamma-\gamma''.\gamma-\gamma'''.\&c.\times\&c.}+\frac{1}{\gamma'-\alpha.\gamma'-\alpha'.\gamma'-\alpha''.\gamma'-\alpha'''.\&c.\times\gamma'-\beta.\gamma'-\beta'.\gamma'-\beta''.\gamma'-\beta'''.\&c.\times\gamma'-\gamma.\gamma'-\gamma''.\gamma'-\gamma'''.\&c.\times\&c.}+\frac{1}{\gamma''-\alpha.\gamma''-\alpha'.\gamma''-\alpha''.\gamma''-\alpha'''.\&c.\times\gamma''-\beta.\gamma''-\beta'.\gamma''-\beta''.\gamma''-\beta'''.\&c.\times\gamma''-\gamma.\gamma''-\gamma'.\gamma''-\gamma'''.\&c.\times\&c.}+\frac{1}{\gamma'''-\alpha.\gamma'''-\alpha'.\gamma'''-\alpha''.\gamma'''-\alpha'''.\&c.\times\gamma'''-\beta.\gamma'''-\beta'.\gamma'''-\beta''.\gamma'''-\beta'''.\&c.\times\gamma'''-\gamma.\gamma'''-\gamma'.\gamma'''-\gamma''.\&c.\times\&c.}+\&c.=0,$$

&c.; ſumma datæ ſeriei in finitis terminis exprimi poteſt; ſin aliter vero non: hoc facile conſtat per additionem, &c. prius traditam.

8. Sit b negativa quantitas, & ſi requirantur fluentes fluxionum $\frac{x^{b\alpha-1}\dot{x}}{1\pm x^{b}}$, $\frac{x^{b\beta-1}\dot{x}}{1\pm x^{b}}$, $\frac{x^{b\gamma-1}\dot{x}}{1\pm x^{b}}$, &c. ad inveniendam ſummam ſeriei $\frac{1}{\alpha.\beta.\gamma.\delta\&c.}+\frac{1}{\alpha+1.\beta+1.\gamma+1.\&c.}x^{b}+\frac{1}{\alpha+2.\beta+2.\gamma+2.\&c.}\times x^{2b}+\&c.$

&c. deſcendentis ſecundum dimenſiones quantitatis x^b; quæ ſeries ſemper converget, cum x major ſit quam 1; ſin minor ſit, diverget.

Si $x = 1$; haud multum diverſa erit ſeries, cum b ſit negativa, quam cum b ſit affirmativa quantitas; & de eâ fere eadem prædicari poſſunt.

In hoc caſu pro inveſtigandis fluentibus fluxionum $\frac{x^{b\alpha-1}\dot{x}}{1 \pm x^b}$, $\frac{x^{b\beta-1}\dot{x}}{1 \pm x^b}$, $\frac{x^{b\gamma-1}\dot{x}}{1 \pm x^b}$, &c. inter valores m & m' quantitatis z diſtantiæ a primo ſeriei termino, ſi α, β, γ, &c. ſint affirmativæ quantitates, inveniatur fluens fluxionis $\frac{x^{(m+b)\alpha-1} \backsim x^{(m'+b)\alpha-1}}{1 \pm x^b}\dot{x}$ (inter valores infinitum & x quantitatis x) $= A$: ſi autem quædam α, β, γ, &c. ſint negativæ; e. g. ſit α negativa quantitas, & $l + \alpha$ minima affirmativa, ubi l eſt integer affirmativus numerus inter numeros m & m' poſitus; inveniatur fluens fluxionis $\frac{x^{(m+\alpha)b-1} \backsim x^{(l+\alpha)b-1}}{1 \pm x^b}\dot{x}$ inter valores o & x quantitatis x, quæ ſit P; deinde inveniatur fluens (A') fluxionis $\frac{x^{(l+\alpha)b-1} \backsim x^{(m'+\alpha)b-1}}{1 \pm x^b}\dot{x}$ inter valores infinitum & x quantitatis x, tum erit fluens quæſita $P \pm A' = A$; & ſic de reliquis; ergo ſumma ſeriei quæſita erit $\alpha' A + \beta' B +$ &c.

Omnia de hoc caſu, cum b ſit negativa quantitas, affirmari poſſunt; quæ prius affirmantur de caſu, cum b ſit affirmativa quantitas.

Conſimilia etiam applicari poſſunt ad inveniendas ſummas ſerierum, quarum generalis terminus habeat formulam $\frac{az^m + bz^{m-1} + \&c.}{pz^n + qz^{n-1} + \&c.}$; ubi z eſt diſtantia a primo ſeriei termino.

9. Sint datæ ſeries $\frac{1}{\alpha^2} + \frac{1}{(\alpha+1)^2}x^b + \frac{1}{(\alpha+2)^2}x^{2b} + \frac{1}{(\alpha+3)^2}x^{3b} + \&c. = A$, $\frac{1}{\alpha} + \frac{1}{\alpha+1}x^b + \frac{1}{\alpha+2}x^{2b} + \&c. = A'$, $\frac{1}{\beta} + \frac{1}{\beta+1}x^b + \frac{1}{\beta+2}x^{2b} + \&c. = B$, $\frac{1}{\gamma} + \frac{1}{\gamma+1}x^b + \frac{1}{\gamma+2}x^{2b} + \&c. = C$, &c.;

generales termini harum ſerierum ſunt reſpective $\frac{1}{(\alpha+z)^2}x^{z}$, $\frac{1}{\alpha+z}x^{z}$, $\frac{1}{\beta+z}x^{z}$, $\frac{1}{\gamma+z}$, &c.; ducantur hæ fractiones in coefficientes a, b, c, d, e, &c. inveſtigandas, & reſultant $\frac{a}{(\alpha+z)^2}$, $\frac{b}{\alpha+z}$, $\frac{c}{\beta+z}$, $\frac{d}{\gamma+z}$, &c.; reducantur hæ fractiones ad communem denominatorem & ſimul addantur, & reſultat ſumma

$$\frac{a\times(\beta+z)(\gamma+z)\&c.+b(\alpha+z)(\beta+z)(\gamma+z)\&c.+c\times(\alpha+z)^2(\gamma+z)\&c.+d(\alpha+z)^2(\beta+z)\&c.+\&c.}{(\alpha+z)^2\times\beta+z\,.\,\gamma+z\,.\,\&c.};$$

fiat numerator $((b+c+d)+\&c.)z^{l-1}+(a+b(\alpha+\beta+\gamma+\&c.)+c(2\alpha+\gamma+\&c.)+d(2\alpha+\beta+\&c.)+\&c.)z^{l-2}+\&c.=1$, quicunque ſit valor quantitatis z; & conſequenter coefficientes e ſingulis poteſtatibus quantitatis z nihilo reſpective æquales, & coefficiens quæ non ducitur in poteſtatem quantitatis z erit 1, i. e. $b+c+d+\&c.=0$, $a+b(\alpha+\beta+\&c.)+c(2\alpha+\gamma+\&c.)+\&c.=0$; &c.; & $a\beta\gamma\&c.+b\alpha\beta\gamma\&c.+c\alpha^2\gamma\delta\&c.+d\alpha^2\beta\delta\&c.+\&c.=P=1$; & reſultant $l-1$ ſimplices æquationes involventes l incognitas quantitates a, b, c, d, &c.; e quibus deduci poſſunt rationes, quas habent inter ſe incognitæ a, b, c, d, &c. & ex ultimâ æquatione $P=1$ erui poſſunt quantitates ipſæ a, b, c, d, &c. & conſequenter

$$\frac{a}{(\alpha+z)^2}+\frac{b}{\alpha+z}+\frac{c}{\beta+z}+\frac{d}{\gamma+z}+\&c.=\frac{1}{(\alpha+z)^2.\,\beta+z\,.\,\gamma+z\,.\,\delta+z\,.\&c.}:$$

e. g. ſint $\gamma=0$, $\delta=0$, &c., tum erit $a=\frac{1}{\beta-\alpha}$, $b=-\frac{1}{(\beta-\alpha)^2}$ & $c=-b$; & conſequenter $\frac{1}{\beta-\alpha}\times\frac{1}{(\alpha+z)^2}-\frac{1}{(\beta-\alpha)^2}\times\frac{1}{\alpha+z}+\frac{1}{(\beta-\alpha)^2}\times\frac{1}{\beta+z}=\frac{1}{(\alpha+z)^2.\,\beta+z}$; ergo erit ſumma $\frac{1}{\beta-\alpha}A-$

$\frac{1}{(\beta-\alpha)^2}A' + \frac{1}{(\beta-\alpha)^2}B$ æqualis ſummæ ſeriei $\frac{1}{\alpha^2\beta} + \frac{1}{(\alpha+1)^2.\beta+1}$ $x^b + \frac{1}{(\alpha+2)^2.\beta+2}x^{2b} + \text{\&c.}$

10. Sit ſeries $\frac{1}{\alpha^m.\beta^n.\gamma^r.\delta^s.\text{\&c.}} \pm \frac{1}{(\alpha+1)^m.(\beta+1)^n.(\gamma+1)^r.(\delta+1)^s.\text{\&c.}}$ $x^b + \frac{1}{(\alpha+2)^m \times (\beta+2)^n \times (\gamma+2)^r \times (\delta+2)^s}x^{2b} \pm \text{\&c.}$; tum ejus ſumma deduci poteſt ex datis ſummis ſerierum $\frac{1}{\alpha} \pm \frac{1}{\alpha+1}x^b + \frac{1}{\alpha+2}$ $x^{2b} \pm \text{\&c.} = A$, $\frac{1}{\alpha^2} \pm \frac{1}{(1+\alpha)^2}x^b + \frac{1}{(\alpha+2)^2}x^{2b} \pm \text{\&c.} = A'$, $\frac{1}{\alpha^3} \pm$ $\frac{1}{(\alpha+1)^3}x^b + \frac{1}{(\alpha+2)^3}x^{2b} \pm \frac{1}{(\alpha+3)^3}x^{3b} + \text{\&c.} = A''$, ... $\frac{1}{\alpha^m} \pm \frac{1}{(\alpha+1)^m}$ $+ \frac{x^{2b}}{(\alpha+2)^m} \pm \frac{x^{3b}}{(\alpha+3)^m} + \text{\&c.} = \overset{m-1}{A'}$; etiamque $\frac{1}{\beta} \pm \frac{x^b}{\beta+1} +$ $\frac{x^{2b}}{\beta+2} \pm \frac{x^{3b}}{\beta+3} + \text{\&c.} = B$, $\frac{1}{\beta^2} \pm \frac{x^b}{(\beta+1)^2} + \frac{x^{2b}}{(\beta+2)^2} \pm \text{\&c.} = B'$, $\frac{1}{\beta^3} \pm \frac{x^b}{(\beta+1)^3} + \frac{x^{2b}}{(\beta+2)^3} \pm \text{\&c.} = B''$, ... $\frac{1}{\beta^n} \pm \frac{x^b}{(\beta+1)^n} + \frac{x^{2b}}{(\beta+2)^n}$ $\pm \frac{x^{3b}}{(\beta+3)^n} \pm \text{\&c.} = \overset{m-1}{B'}$; & $\frac{1}{\gamma} \pm \frac{x^b}{\gamma+1} + \frac{x^{2b}}{\gamma+2} \pm \text{\&c.} = C$, $\frac{1}{\gamma^2}$ $\pm \frac{x^b}{(\gamma+1)^2} + \frac{1}{(\gamma+2)^2} \pm \text{\&c.} = C'$, ... $\frac{1}{\gamma^r} \pm \frac{x^b}{(\gamma+1)^r} + \frac{x^{2b}}{(\gamma+2)^r}$ $\pm \frac{x^{3b}}{(\gamma+3)^r} + \text{\&c.} = \overset{r-1}{C'}$, & $\frac{1}{\delta} \pm \frac{x^b}{\delta+1} + \frac{x^{2b}}{\delta+2} \pm \frac{x^{3b}}{\delta+3} + \text{\&c.} =$ D, ... $\frac{1}{\delta^s} \pm \frac{x^b}{(\delta+1)^s} + \frac{x^{2b}}{(\delta+2)^s} \pm \frac{x^{3b}}{(\delta+3)^s} + \text{\&c.} = \overset{s-1}{D'}$, &c.; i.e. dentur ſummæ ſerierum, quarum generales termini ſunt $\frac{\pm 1}{\alpha+z}$, $\frac{\pm 1}{(\alpha+z)^2}$, $\frac{\pm 1}{(\alpha+z)^3}$, $\frac{\pm 1}{(\alpha+z)^4}$, ... $\frac{\pm 1}{(\alpha+z)^m}$; $\frac{\pm 1}{\beta+z}$, $\frac{\pm 1}{(\beta+z)^2}$, $\frac{\pm 1}{(\beta+z)^3}$,

...

$\ldots \frac{\pm 1}{(\beta + z)^n}$; $\frac{\pm 1}{\gamma + z}$, $\frac{\pm 1}{(\gamma + z)^2}$, $\frac{\pm 1}{(\gamma + z)^3}$, $\ldots \frac{\pm 1}{(\gamma + z)^r}$; $\frac{\pm 1}{\delta + z}$, $\frac{\pm 1}{(\delta + z)^2}$, $\ldots \frac{\pm 1}{(\delta + z)^s}$; &c.; ducantur hi generales termini in coefficientes deducendas $a, a', a'', \ldots \overset{m-1}{a'}$; $b, b', b'', \ldots \overset{n-1}{b'}$; $c, c', c'', \ldots \overset{r-1}{c'}$; $d, d', d'', \ldots \overset{s-1}{d'}$, &c.; reſpective; & deinde reducantur omnes fractiones reſultantes ad communem denominatorem, qui erit $(\alpha + z)^m (\beta + z)^n (\gamma + z)^r (\delta + z)^s$ &c.; & ſimul addantur; tum fingantur coefficientes ſingularum poteſtatum quantitatis z in numeratore contentarum nihilo reſpective æquales, & reſultant $m + n + r + s$ &c. $- 1$ æquationes habentes $(m + n + r + s +$ &c.$)$ plures incognitas quantitates $a, a', \ldots \overset{m-1}{a'}$; $b, b', b'', \ldots \overset{n-1}{b'}$; $c, c', c'', \ldots \overset{r-1}{c'}$; $d, d', d'', \ldots \overset{s-1}{d'}$; &c. per unitatem quam numerus æquationum; ex his $(m + n + r + s +$ &c. $- 1)$ ſimplicibus æquationibus erui poſſunt rationes, quas habent inter ſe ſingulæ incognitæ quantitates $a, a', \ldots \overset{m-1}{a'}$; $b, b', \ldots \overset{n-1}{b'}$; $c, c', \ldots \overset{r-1}{c'}$; $d, d', \ldots \overset{s-1}{d'}$, &c.: etiamque reſultat ſimplex æquatio, ex fingendo terminum, in quo haud continetur z, unitati æqualem; & exinde erui poſſunt quantitates $a, a', a'', \ldots \overset{m-1}{a'}$; b, b', b'', &c., $\overset{n-1}{b'}$; $c, \ldots \overset{r-1}{c'}$, $d, \ldots \overset{s-1}{d'}$, &c.

Summa ſeriei quæſitæ erit $= aA + a'A' + a''A'' + \ldots \overset{m-1}{a'} \overset{m-1}{A'} + bB + b'B' + b''B'' + \ldots + \overset{m-1}{b'} \overset{m-1}{B'} + cC + c'C' + c''C'' + \ldots \overset{r-1}{c'} \overset{r-1}{C'} + dD + d'D' + d''D'' + \ldots \overset{s-1}{d'} \overset{s-1}{D'} +$ &c.

Cor. Si $m = 1$, tum coefficiens a in ſummam ſeriei $\left(\frac{1}{\alpha} \pm \frac{x^b}{\alpha + 1} +\right.$ &c. $\left.= A\right)$ ducenda $= \frac{1}{(\alpha - \beta)^n \times (\alpha - \gamma)^r \times (\alpha - \delta)^s \times \text{\&c.}}$: ſi vero m ſit numerus major quam 1, tum vel e prædictis ſimplicibus æquationibus, vel e prob. 43. deduci poſſunt coefficientes a, a', a'', &c.

Cor. Summa ſeriei $\frac{1}{\alpha} \pm \frac{x^b}{\alpha + 1} + \frac{x^{2b}}{\alpha + 2} \pm$ &c. inter quoſcunque duos valores, quorum diſtantiæ a primo dantur, deduci poteſt e fluente fluxionis $\int.\frac{x^{\alpha b-1}\dot{x}}{1\pm x^{b}}$ proprie correctâ; etiamque ſumma ſeriei $\frac{1}{\alpha^{2}}\pm\frac{x^{b}}{(\alpha+1)^{2}}+\frac{x^{2b}}{(\alpha+2)^{2}}\pm\frac{x^{3b}}{(\alpha+3)^{2}}+$ &c. deduci poteſt generaliter e fluente fluxionis $\int.\frac{\dot{x}}{x}\int.\frac{x^{\alpha b-1}\dot{x}}{1\pm x^{b}}$ vere correctâ; quæ dici poteſt fluens ſecundi ordinis: & ſimiliter ſummæ ſerierum $\frac{1}{\alpha^{3}}\pm\frac{x^{b}}{(\alpha+1)^{3}}+\frac{x^{2b}}{(\alpha+2)^{3}}\pm\frac{x^{3b}}{(\alpha+3)^{4}}+$ &c., $\frac{1}{\alpha^{4}}\pm\frac{x^{b}}{(\alpha+1)^{4}}+\frac{x^{2b}}{(\alpha+2)^{4}}\pm\frac{x^{3b}}{(\alpha+3)^{4}}+$ &c., &c., deduci poſſunt e fluentibus fluxionum $\int.\frac{\dot{x}}{x}\int.\frac{\dot{x}}{x}\int.\frac{x^{\alpha b-1}\dot{x}}{1\pm x^{b}}$, $\int.\frac{\dot{x}}{x}\int.\frac{\dot{x}}{x}\int.\frac{\dot{x}}{x}\int.\frac{x^{\alpha b-1}\dot{x}}{1\pm x^{b}}$, &c. vere correctis; quæ dici poſſunt fluentes tertii, quarti, &c. ordinum: fluens ſuperiori ordinis nunquam generaliter exprimi poteſt per fluentem inferioris ordinis.

Et ſic de ſummis reliquarum ſerierum prædictarum inveſtigandis.

Cor. Hinc generaliter detegi poteſt ſumma ſeriei inter quoſcunque terminos contenta; cujus generalis terminus ſit quæcunque rationalis functio quantitatis z diſtantiæ a primo ſeriei termino in x^{bz} ducta, viz. $\frac{az^{m}+bz^{m-1}+cz^{m-2}+\text{\&c.}}{fz^{n}+gz^{n-1}+bz^{n-2}+\text{\&c.}}\times x^{bz}$, ubi m & n ſunt integri numeri; ſi modo generaliter dentur fluentes omnium fluxionum ſubſequentium formularum $\frac{x^{b\pi-1}\dot{x}}{1\pm x^{b}}$, $\frac{\dot{x}}{x}\int.\frac{x^{b\pi-1}\dot{x}}{1\pm x^{b}}$, $\frac{\dot{x}}{x}\int.\frac{\dot{x}}{x}\int.\frac{x^{b\pi-1}\dot{x}}{1\pm x^{b}}$, $\frac{\dot{x}}{x}\int.\frac{\dot{x}}{x}\int.\frac{\dot{x}}{x}\int.\frac{x^{b\pi-1}\dot{x}}{1\pm x^{b}}$, $\frac{\dot{x}}{x}\int.\frac{\dot{x}}{x}\int.\frac{\dot{x}}{x}\int.\frac{\dot{x}}{x}\int.\frac{x^{b\pi-1}\dot{x}}{1\pm x^{b}}$, &c. ubi b & π ſint quæcunque quantitates poſſibiles vel impoſſibiles: i. e. ſi detur generaliter fluens fluxionis $\frac{x^{b\pi-1}\dot{x}}{1\pm x^{b}}$, tum dabitur ſumma omnis ſeriei, cujus generalis terminus habeat prædictam formulam, & in cujus denominatore

$fz^{n}+gz^{n-1}+$ &c. haud contineantur duo vel plures diviſores vel factores inter ſe æquales; i. e. denominator $fz^{n}+gz^{n-1}+$ &c. non dividi poteſt per quantitatem formulæ $(\sigma z+\rho)^2$; & ſimiliter ſumma omnis ſeriei, cujus generalis terminus prædictam habeat formulam, & in cujus denominatore $fz^{n}+gz^{n-1}+$ &c. haud contineatur cubicus diviſor vel factor $(\sigma z+\rho)^3$, deduci poteſt e fluentibus fluxionum formularum $\frac{x^{bn-1}\dot{x}}{1\pm x^{b}}$ & $\frac{\dot{x}}{x}\int.\frac{x^{bn-1}\dot{x}}{1\pm x^{b}}$; & ſic deinceps.

Cor. Sit data ſeries, cujus generalis terminus eſt $\frac{\pm 1}{(\alpha+z)^{m}(\beta+z)^{n}(\gamma+z)^{r}(\delta+z)^{s}\,\&c.}x^{bz}$, & dentur ſummæ ſerierum inter valores H & K quantitatis z contentarum $\frac{1}{\alpha+1}\pm\frac{1}{\alpha+2}x^{b}+\frac{1}{\alpha+3}x^{2b}\pm$ &c., $\frac{1}{\alpha^{2}}\pm\frac{1}{(\alpha+1)^{2}}x^{b}+\frac{1}{(\alpha+2)^{2}}x^{2b}\pm\frac{1}{(\alpha+3)^{2}}x^{3b}+$ &c., $\frac{1}{\alpha^{3}}\pm\frac{x^{b}}{(\alpha+1)^{3}}+\frac{x^{2b}}{(\alpha+2)^{3}}\pm$ &c., … $\frac{1}{\alpha^{m}}\pm\frac{x^{b}}{(\alpha+1)^{m}}+\frac{x^{2b}}{(\alpha+2)^{m}}\pm$ &c.; & $\frac{1}{\beta}\pm\frac{x^{b}}{\beta+1}+\frac{x^{2b}}{\beta+2}\pm$ &c., $\frac{1}{\beta^{2}}\pm\frac{x^{b}}{(\beta+1)^{2}}+\frac{x^{2b}}{(\beta+2)^{2}}\pm$ &c., … $\frac{1}{\beta^{n}}\pm\frac{x^{b}}{(\beta+1)^{n}}+\frac{x^{2b}}{(\beta+2)^{n}}\pm$ &c.; & $\frac{1}{\gamma}\pm\frac{x^{b}}{\gamma+1}+\frac{x^{2b}}{\gamma+2}\pm$ &c., $\frac{1}{\gamma^{2}}\pm\frac{x^{b}}{(\gamma+1)^{2}}+\frac{x^{2b}}{(\gamma+2)^{b}}\pm$ &c., … $\frac{1}{\gamma^{r}}\pm\frac{x^{b}}{(\gamma+1)^{r}}+\frac{x^{2b}}{(\gamma+2)^{r}}\pm$ &c.; & $\frac{1}{\delta}\pm\frac{x^{b}}{\delta+1}+\frac{x^{2b}}{\delta+2}\pm$ &c., $\frac{1}{\delta^{2}}\pm\frac{x^{b}}{(\delta+1)^{2}}+\frac{x^{2b}}{(\delta+2)^{2}}+$ &c., … $\frac{1}{\delta^{s}}\pm\frac{x^{b}}{(\delta+1)^{s}}+\frac{x^{2b}}{(\delta+2)^{s}}\pm$ &c., &c.; tum ex iis deduci poteſt ſumma datæ ſeriei inter prædictos valores H & K contenta: ſummæ autem prædictarum ſerierum inter valores H & K quantitatis z contentarum ex fluentibus fluxionum $\frac{x^{b\alpha-1}\dot{x}}{1\pm x^{b}}$, $\frac{\dot{x}}{x}\int.\frac{x^{b\alpha-1}\dot{x}}{1\pm x^{b}}$, $\frac{\dot{x}}{x}\int.\frac{\dot{x}}{x}\int.\frac{x^{b\alpha-1}\dot{x}}{1\pm x^{b}}$, $\frac{\dot{x}}{x}\int.\frac{\dot{x}}{x}\int.\frac{\dot{x}}{x}$

$\int.x$

$\int . \frac{x^{b\alpha-1}\dot{x}}{1 \pm x^{b}}$ uſque ad m terminos; $\frac{x^{b\beta-1}\dot{x}}{1 \pm x^{b}}$, $\frac{\dot{x}}{x}\int . \frac{x^{\beta b-1}\dot{x}}{1 \pm x^{b}}$, $\frac{\dot{x}}{x}\int . \frac{\dot{x}}{x}\int . \frac{x^{\beta b-1}\dot{x}}{1 \pm x^{b}}$, uſque ad n terminos; $\frac{x^{\gamma b-1}\dot{x}}{1 \pm x^{b}}$, $\frac{\dot{x}}{x}\int . \frac{x^{\gamma b-1}\dot{x}}{1 \pm x^{b}}$, &c. uſque ad r terminos; & ſic deinceps; inter valores o & x quantitatis x contentis exprimi poſſunt.

Sit b affirmativa quantitas, hæ ſeries convergent, cum x inter 1 & -1 contineatur; & ſi x major ſit quam 1, tum ſemper divergent: ſi vero $x = 1$, tum ſemper converget, necne; prout numerus dimenſionum quantitatis x in numeratore ſeriei ad affirmativam reductæ minor ſit per quantitatem majorem quam unitatem quam ejus dimenſiones in denominatore, necne.

11. Erit ſeries $\frac{1}{r} \pm \frac{m}{(r+1)} . \frac{bx^{n}}{a} + m . \frac{m-1}{2} \times \frac{1}{(r+2)} . \frac{b^{2}x^{2n}}{a^{2}} \pm m . \frac{m-1}{2} . \frac{m-2}{3} \times \frac{1}{(r+3)} . \frac{b^{3}x^{3n}}{a^{3}} + m . \frac{m-1}{2} . \frac{m-2}{3} . \frac{m-3}{4} \times \frac{1}{(r+4)} \times \frac{b^{4}x^{4n}}{a^{4}} + \&c. = \frac{n}{a^{m}x^{rn}} \int . (a \pm bx^{n})^{m} x^{rn-1} \dot{x}$ vere correctæ; ducatur hæc æquatio in $x^{bn-1}\dot{x}$ & inveniatur fluens utriuſque partis æquationis reſultantis, & evadet $\frac{1}{rb} \pm \frac{m}{\overline{r+1} . \overline{b+1}} \times \frac{bx^{n}}{a} + \frac{m . (m-1)}{2 . \overline{r+2} . \overline{b+2}} \times \frac{b^{2}x^{2n}}{a^{2}} \pm \frac{m . (m-1) . (m-2)}{2 . 3 . (r+3) . (b+3)} \times \frac{b^{3}x^{3n}}{a^{3}} + \frac{m . (m-1) . \overline{m-2} . \overline{m-3}}{2 . 3 . 4 . (r+4) . (b+4)} \times \frac{b^{4}x^{4n}}{a^{4}} \pm \&c. = \frac{n^{2}x^{-bn}}{a^{m}} \int . x^{(b-r)n-1}\dot{x} \int . x^{rn-1}\dot{x} (a \pm bx^{n})^{m}$; ducatur hæc æquatio in $x^{kn-1}\dot{x}$, & inveniatur fluens æquationis reſultantis, quæ erit $\frac{1}{rbk} \pm \frac{m}{\overline{r+1} . \overline{b+1} . \overline{k+1}} \times \frac{bx^{n}}{a} + \frac{m . \overline{m-1}}{2 . \overline{r+2} . \overline{b+2} . \overline{k+2}} \times \frac{b^{2}x^{2n}}{a^{2}} \pm \&c. = \frac{n^{3}}{a^{m}} x^{-kn} \int . x^{(k-b)n-1}\dot{x} \int . x^{(b-r)n-1}\dot{x} \int . x^{rn-1}\dot{x} (a \pm bx^{n})^{m}$; & ſic deinceps; & conſtat, ſi modo m, r, b, k, &c., ſint quantitates, quæ habent formulam $\pm l + \frac{1}{2}$ vel $\pm l$, ubi l eſt integer numerus, prædictas fluentes inveniri poſſe finitorum terminorum, circularium arcuum & logarithmorum ope.

Cor. Hinc ſit generalis terminus $m \cdot \frac{m-1}{2} \cdot \frac{m-2}{3} \cdot \cdot \frac{m-z+1}{z} \times \frac{1}{r+z} \cdot \frac{az^{M}+bz^{M-1}+cz^{M-2}+\&c.}{fz^{N}+gz^{N-1}+\&c.} \times e^{z}$, ubi $e=\frac{bx^{n}}{a}$ & M & N ſunt integri numeri; & omnes diviſores denominatoris $fz^{N}+gz^{N-1}+\&c.$ habeant formulam $z \pm l \pm \frac{1}{2}\lambda$, in quâ l & λ ſunt integri numeri vel o, & z eſt diſtantia a primo ſeriei termino, & m & r quantitates formulæ prius traditæ $\pm l \pm \frac{1}{2}\lambda$; & haud duo vel plures diviſores ſunt inter ſe æquales; tum ejus ſumma detegi poteſt ope finitorum terminorum, circularium arcuum & logarithmorum.

Cor. Conſtat e ſummis ſerierum, quarum generales termini reſpective ſunt $V \times \frac{x^{z}}{(z \pm l \pm \frac{1}{2}\lambda)^{b}} = L$, $V \times \frac{x^{z}}{(z \pm l \pm \frac{1}{2}\lambda)^{b-1}} = L'$, $V \times \frac{x^{z}}{(z \pm l \pm \frac{1}{2}\lambda)^{b-2}} = L''$, . . . $V \times \frac{x^{z}}{(z \pm l \pm \frac{1}{2}\lambda)^{2}} = L^{\overset{b-2}{\prime}}$, $V \times \frac{x^{z}}{(z \pm l \pm \frac{1}{2}\lambda)} = L^{\overset{b-1}{\prime}}$; & $V \times \frac{x^{z}}{(z \pm l' \pm \frac{1}{2}\lambda')^{b'}} = M$, $V \times \frac{x^{z}}{(z \pm l' \pm \frac{1}{2}\lambda')^{b'-1}} = M'$, $\frac{x^{z}}{(z \pm l' \pm \frac{1}{2}\lambda')^{b'-2}} = M''$, . . . $V \times \frac{x^{z}}{(z \pm l' \pm \frac{1}{2}\lambda')^{2}} = M^{\overset{b-2}{\prime}}$, $V \times \frac{x^{z}}{(z \pm l' \pm \frac{1}{2}\lambda)} = M^{\overset{b-1}{\prime}}$, etiamque $V \times \frac{x^{z}}{(z \pm l'' \pm \frac{1}{2}\lambda'')^{b''}} = N$, $V \times \frac{x^{z}}{(z \pm l'' \pm \frac{1}{2}\lambda'')^{b''-1}} = N'$, . . . $V \times \frac{x^{z}}{(z \pm l'' \pm \frac{1}{2}\lambda'')^{2}} = N^{\overset{b-2}{\prime}}$, $V \times \frac{x^{z}}{(z \pm l'' \pm \frac{1}{2}\lambda)} = N^{\overset{b-1}{\prime}}$; &c. per prob. acquiri poſſe ſummam ſeriei, cujus generalis terminus eſt $V \times \frac{az^{M}+bz^{M-1}+\&c.}{(z \pm l \pm \frac{1}{2}\lambda)^{b} \times (z \pm l' \pm \frac{1}{2}\lambda')^{b'} \times (z \pm l'' \pm \frac{1}{2}\lambda'')^{b''} \times \&c.}$, ſi modo M, b, b', b'', &c. ſint integri numeri; conſtat etiam ex additione prius uſitatâ generalem terminum eſſe $aL + a'L' + a''L'' + \&c. + bM + b'M' + \&c.$; & coefficientes a, a', &c., b, b', &c. exinde inveſtigari poſſe.

Sint $2l, 2l', 2l''$,&c.; $2\lambda, 2\lambda', 2\lambda''$,&c. integri numeri & $V = m \cdot \frac{m-1}{2} \cdot \frac{m-2}{3} \ldots \frac{m-z+1}{z} \times \frac{1}{\mu+z}$; tum ſummæ prædictarum ſerierum detegi poſſunt e fluentibus fluxionum formularum $\int . x^{\mu n-1}\dot{x}(a+bx^n)^m$, $\int . \frac{\dot{x}}{x} \int . (a+bx^n)^m x^{\mu n-1}\dot{x}$, $\int . \frac{\dot{x}}{x} \int . \frac{\dot{x}}{x} \int . (a+bx^n)^m x^{\mu n-1}\dot{x}$, $\int . \frac{\dot{x}}{x} \int . \frac{\dot{x}}{x} \int . \frac{\dot{x}}{x} \int . (a+bx^n)^m x^{\mu n-1}\dot{x}$, &c.; ubi quantitas μ habet prædictam formulam $\mp l \pm \frac{1}{2}\lambda$, l & λ exiſtentibus integris numeris.

Ex his fluentibus generaliter deductis detegi poſſunt ſummæ ſerierum inter quoſcunque duos valores quantitatis z poſitæ; etiamque, ſi modo dentur fluentes harum fluxionum inter quoſcunque duos valores quantitatis x, erui poſſunt correſpondentes prædictarum ſerierum ſummæ.

E principiis prius traditis dijudicari poſſunt harum ſerierum convergentiæ.

Si modo fluentes harum fluxionum per finitos terminos, circulares arcus, & logarithmos exprimi poſſint; tum per finitos terminos, circulares arcus & logarithmos etiam exprimi poſſunt fluentes, quæ exprimunt ſummam e ſingulis terminis prædictæ ſeriei, quorum diſtantiæ a ſe invicem ſit H. Si enim ſeries ſecundum dimenſiones quantitatis x^n progrediantur; tum in fluente inventâ pro x^n ſcribantur αx^n, βx^n, γx^n, &c., ubi α, β, γ, &c. ſunt radices æquationis $w^H - 1 = 0$; & deinde per methodum in medit. algebr. datam progrediendum eſt.

Conſimiles etiam propoſitiones erui poſſunt de ſeriebus, quarum ſummæ exprimi poſſunt per ellipticos arcus & alias fluentes.

12. Sit $\int . \frac{\dot{x}}{1+x} = \log.(1+x) = a$, tum erit $x = a + \frac{a^2}{1.2} + \frac{a^3}{1.2.3} +$ &c., & $e^a = 1 + x$; hinc e medit. algebr. $\frac{\alpha^{n-m}e^{\alpha a} + \beta^{n-m}e^{\beta a} + \gamma^{n-m}e^{\beta a} + \delta^{n-m}e^{\delta a} + \&c.}{n}(P) = \frac{a^m}{1.2.3..m} + \frac{a^{n+m}}{1.2.3..n+m} + \frac{a^{2n+m}}{} +$

$\frac{a^{2n+m}}{1.2.3..2n+m} + \frac{a^{3n+m}}{1.2.3..3n+m} +$ &c.; ubi $\alpha, \beta, \gamma, \delta,$ &c. ſunt (n) radices æquationis $x^n - 1 = 0$. 2. Erit $e^{-a} = 1 - a + \frac{a^2}{1.2} - \frac{a^3}{1.2.3} +$ &c. & exinde $\frac{\alpha^{n-m} e^{-\alpha a} + \beta^{n-m} e^{-\beta a} + \gamma^{n-m} e^{-\gamma a} + \&c.}{n} (P) = \pm \frac{a^m}{1.2.3..m} \pm \frac{a^{n+m}}{1.2..n+m} \pm \frac{a^{2n+m}}{1.2..2n+m} \pm$ &c.: ſigna affixa erunt $+$, ſi n & m ſint pares numeri; erunt $-$, ſi n ſit par & m impar; erunt alternatim $+$ & $-$, ſi n ſit impar; ſignum affixum primo termino erit $+$, ſi m ſit par, ſin aliter $-$; & exinde uterque caſus evadit idem, cum n & m ſint pares numeri.

Sint α & δ correſpondentes impoſſibiles radices, quæ ſint $e^{\alpha} + e^{\delta} = e^{a+b\sqrt{(-1)}} + e^{a-b\sqrt{(-1)}} = e^{a} (e^{b\sqrt{(-1)}} + e^{-b\sqrt{(-1)}})$; ergo ex ſummis omnium quantitatum hujuſce formulæ $e^{b\sqrt{(-1)}} + e^{-b\sqrt{(-1)}} = 2 (1 - \frac{b^2}{1.2} + \frac{b^4}{1.2.3.4} - \frac{b^6}{1.2.3.4.5.6} + \&c.) = 2$ coſ. arcus b circuli, cujus radius eſt 1, erui poſſunt ſummæ omnium quantitatum formulæ (P), vel ſerierum $1 \pm \frac{1}{m+1.m+2..n+m} a^n + \frac{1}{m+1.m+2..2n+m} a^{2n} \pm$ &c. erit $\frac{e^{a\sqrt{(-1)}} - e^{-a\sqrt{(-1)}}}{2\sqrt{(-1)}} = a - \frac{a^3}{1.2.3} + \frac{a^5}{1.2..5} - \frac{a^7}{1.2..7} +$ &c. $= x$ ſin. prædicti arcus (a).

2. Sint fluentes $\int. \frac{\dot{x}}{\sqrt{(1 - \alpha x^2)}} = a$, $\int. \frac{\dot{x}'}{\sqrt{(1 - \beta x'^2)}} = a$, $\int. \frac{\dot{x}''}{\sqrt{(1 - \gamma x''^2)}} = a$, &c.; tum erunt $x = a - \frac{\alpha a^3}{1.2.3} + \frac{\alpha^2 a^5}{1.2..5} -$ &c., $x' = a - \frac{\beta a^3}{1.2.3} +$ &c., $x'' = a - \frac{\gamma a^3}{1.2.3} +$ &c.; ſi modo $\int. \frac{\dot{x}}{\sqrt{(1 - \alpha x^2)}} = a$, $\int. \frac{\dot{x}'}{\sqrt{(1 - \beta x'^2)}} = a$, &c. inter valores o & x, o & x', &c.

x'', &c. quantitatum x, x', &c. contineantur; etiamque $\frac{\alpha^{n-m}\dot{x} + \beta^{n-m}\dot{x}' + \gamma^{n-m}\dot{x}'' + \&c.}{n} \pm \frac{\pm a^m}{1.2.3..2m} \pm \frac{a^{2n+m}}{1.2.3..2n+2m} \pm \frac{a^{4n+m}}{1.2.3..4n+m} \pm$ &c. ſigna affixa leges prius traditas obſervant.

Hæ ſeries evadunt eædem ac priores, & conſequenter ex quantitatibus $e^{\alpha a}$, $e^{\beta a}$, $e^{\gamma a}$, &c. deduci poſſunt quantitates x, x', x'', &c.: $\int. \frac{\dot{x}}{\sqrt{(1-\alpha x^2)}} = a$, $\int. \frac{\dot{x}'}{\sqrt{(1-\beta x'^2)}} = a$, $\int. \frac{\dot{x}''}{\sqrt{(1-\gamma x''^2)}} = a$, &c. inter valores 0 & x, 0 & x', 0 & x'', &c. quantitatum x, x', x'', &c. contentæ erunt arcus circuli, cujus radius eſt 1 & ſinus poſſibiles vel impoſſibiles quantitates reſpective $x\sqrt{(\alpha)}$, $x'\sqrt{(\beta)}$, &c. in $\frac{1}{\sqrt{(\alpha)}}$, $\frac{1}{\sqrt{(\beta)}}$, $\frac{1}{\sqrt{(\gamma)}}$, &c. reſpective ductæ; i. e. erunt $x = a - \frac{\alpha a^3}{1.2.3} + \&c.$, $x' = a - \frac{\beta a^3}{1.2.3} + \frac{\beta^2 a^5}{1.2..5} - \&c.$, &c.: & $\int. \frac{-\dot{x}}{\sqrt{(1-\alpha x^2)}} = a$, $\int. \frac{-\dot{x}'}{\sqrt{(1-\beta x'^2)}} = a$, $\int. \frac{-\dot{x}''}{\sqrt{(1-\gamma x''^2)}} = a$, &c. inter valores 1 & x, 1 & x', 1 & x'', &c. quantitatum x, x', x'', &c. contentæ erunt arcus circuli, cujus radius eſt 1, & coſinus poſſibiles vel impoſſibiles quantitates $x\sqrt{(\alpha)}$, $x\sqrt{(\beta)}$, $x\sqrt{(\gamma)}$, &c. in $\frac{1}{\sqrt{(\alpha)}}$, $\frac{1}{\sqrt{(\beta)}}$, $\frac{1}{\sqrt{(\gamma)}}$, &c. reſpective ductæ; i. e. erunt $x = 1 - \frac{\alpha a^2}{1.2} + \frac{\alpha^2 a^4}{1.2.3.4} - \&c.$, $x' = 1 - \frac{\beta a^2}{1.2} + \&c.$, &c.

3. Sit $a + \frac{a^2}{1.2} + \frac{a^3}{1.2.3} + \frac{a^4}{1.2.3.4} + \&c. = x$, ubi $\dot{a} = \frac{\dot{x}}{1+x}$; ducatur hæc æquatio in $a^{m-1}\dot{a}$, & inveniatur fluens æquationis reſultantis, quæ erit $\frac{1}{m+1}a^{m+1} + \frac{1}{m+2} \times \frac{1}{1.2}a^{m+2} + \frac{1}{m+3} \times \frac{1}{1.2.3}a^{m+3} +$

$a^{m+3} + \&c. = \int . a^{m-1} x \dot{a} = \int . a^{m-1} \frac{x\dot{x}}{1+x} = a^{m-1} (x - \frac{1}{m} a) - (m-1)$ $a^{m-2} (x - \frac{1}{m-1} a) + (m-1) . (m-2) (x - \frac{1}{m-2} a) - (m-1) .$ $(m-2) . (m-3) (x - \frac{1}{m-3} a) + \&c.$

4. Ad eundem modum ſit $a - \frac{a^2}{1 . 2} + \frac{a^3}{1 . 2 . 3} - \&c. = x$; ubi $\dot{a} = \frac{\dot{x}}{1-x}$; ducatur hæc æquatio in $a^{m-1} \dot{a}$ & reſultat æquatio $a^m \dot{a}$ $- \frac{a^{m+1}}{1 . 2} \dot{a} + \&c. = a^{m-1} \dot{a} x$; cujus fluens erit $\frac{a^{m+1}}{m+1} - \frac{a^{m+2}}{m+2} + \&c. =$ $a^{m-1} (-x + \frac{1}{m} a) - (m-1) a^{m-2} (-x + \frac{1}{m-1} a) + (m-1) . (m$ $- 2) a^{m-3} (-x + \frac{1}{m-2} a) - \&c.$

5. Et ſimiliter ſit æquatio $a - \frac{a^3}{2 . 3} + \frac{a^5}{2 . 3 . 4 . 5} - \&c. = x$, ubi $\dot{a} = \frac{\dot{x}}{\surd(1-x^2)}$; ducatur prior æquatio in $a^{m-1} \dot{a}$ & inveniatur fluens reſultantis fluxionis, quæ erit $\frac{a^m}{m+1} - \frac{a^{m+3}}{2 . 3 . (m+3)} + \frac{a^{m+5}}{2 . 3 . 4 . 5 . (m+5)}$ $- \&c. = - a^{m-1} \times \surd(1-x^2) + (m-1) a^{m-2} x + (m-1) . (m-2)$ $a^{m-3} \times \surd(1-x^2) - (m-1) . (m-2) . (m-3) a^{m-4} x - \&c. = -$ $(a^{m-1} - (m-1) . (m-2) a^{m-3} + (m-1) . (m-2) . (m-3) . (m-4)$ $a^{m-5} - \&c.) \surd(1-x^2) + ((m-1) . a^{m-2} - (m-1) . (m-2) .$ $(m-3) a^{m-4} + \&c.) x$.

6. Sit æquatio $1 - \frac{a^2}{1 . 2} + \frac{a^4}{1 . 2 . 3 . 4} - \&c. = x$, ubi $\dot{a} = \frac{-\dot{x}}{\surd(1-x^2)}$; ducatur hæc æquatio in $a^{m-1} \dot{a}$ & inveniatur fluens fluxionis reſultantis, quæ erit $\frac{a^m}{m} - \frac{a^{m+2}}{1 . 2 . (m+2)} + \frac{a^{m+4}}{1 . 2 . 3 . 4 . (m+4)} - \&c. =$ $(a^{m-1} + (m-1) . (m-2) a^{m-3} + \&c.) \surd(1-x^2) + ((m-1) a^{m-2}$ $+ (m-1) . (m-2) . (m-3) a^{m-4} + \&c.) x$.

Et conſimilia applicari poſſunt ad omnes caſus prius traditos; in omnibus hiſce caſibus ſeries (quæ denotat ſummam) terminat cum m ſit integer affirmativus numerus: deinde ducantur æquationes reſultantes in $a^{m'-1}\dot{a}$, & inveniantur fluentes æquationum reſultantium, &c. introducitur novus factor in ſinguli termini datæ ſeriei denominatorem; & ſerierum ſummæ in finitis terminis exprimi poteſt, ſi modo m & m' ſint integri numeri.

7. Hinc conſtat ex hâc methodo deduci poſſe in finitis terminis ſummam omnis ſeriei, cujus generalis terminus eſt $\frac{\pm a^{lz+b}}{1\,.\,2\,.\,3\,.\,4\,.\,.\,nz \times (z+m)\,.\,(z+m')\,.\,(z+m'')\,.\,\&c.}$, ubi z eſt diſtantia a primo ſeriei termino; & m, m', m'', &c. ſunt quicunque diverſi integri numeri.

8. Sit æquatio $a+\frac{a^2}{1\,.\,2}+\frac{a^3}{1\,.\,2\,.\,3}+\&c.=x$, ducatur hæc æquatio in a^r & reſultat $a^{r+1}+\frac{a^{r+2}}{1\,.\,2}+\frac{a^{r+3}}{1\,.\,2\,.\,3}+\&c.=a^r x$; inveniatur ejus fluxio, & reſultat $(r+1)a^r+\frac{(r+2)a^{r+1}}{1\,.\,2}+\frac{(r+3)a^{r+2}}{1\,.\,2\,.\,3}+\&c.=ra^{r-1}x+a^r\frac{\dot{x}}{\dot{a}}=ra^{r-1}x+a^r(1+x)$ ob $\dot{a}=\frac{\dot{x}}{(1+x)}$; ducatur hæc æquatio in $a^{r'-r+1}$ & inveniatur fluxio æquationis reſultantis, quæ erit $((r+1).(r'+1)a^{r'}+\frac{(r+2).(r'+2)}{1\,.\,2}a^{r'+1}+\&c.)\dot{a}=(ra^{r'}x+a^{r'+1}(1+x))=rr'a^{r'-1}x\dot{a}+ra^{r'}\dot{x}+(r'+1)a^{r'}(1+x)\dot{a}+a^{r'+1}\dot{x}=(rr'a^{r'-1}x+(r+r'+1+a')a^{r'}(1+x)\dot{a}$; dividatur hæc æquatio per $\dot{a}$ & reſultat æquatio quæſita.

Ducatur æquatio reſultans in $a^{r''-r'+1}$ & inveniatur æquationis reſultantis fluxio, & introducitur novus factor in numeratorem; & ſic deinceps. Per eandem methodum introduci poſſunt conſimiles factores in numeratores omnium ſerierum hìc traditarum. Literæ r, r', r'', &c. denotent quaſcunque quantitates poſſibiles vel impoſſibiles; correſpondentes impoſſibiles radices ſint $r = e + f\sqrt{(-1)}$ & $r' = e - f\sqrt{(-1)}$; ducantur eædem functiones harum radicum in ſeriem poſſibiles quantitates ſolummodo involventem, & poſt ſingulam multiplicationem per methodum hìc traditam inveniantur fluxiones vel fluentes, & ſeries reſultans erit poſſibilis.

9. Hinc conſtat ex hâc methodo deduci poſſe in finitis terminis ſummam omnis ſeriei, cujus generalis terminus eſt $\frac{\pm a^{lz+b} \times (Az^{m} + Bz^{m-1} + Cz^{m-2} + \&c.)}{1.2.3..nz.(z+k).(z+k').(z+k'').(z+k''').\&c.}$; ubi z ut antea denotat diſtantiam a primo datæ ſeriei termino, & literæ k, k', k'', &c. deſignant integros diverſos numeros.

Eadem etiam perfici poſſunt ex additione ſerierum hìc traditarum in invariabiles quantitates deducendas ductarum.

Si requirantur ſeries, e quarum ſummis evaneſcunt termini, qui circulares arcus & logarithmos continent; inveniantur ſummæ per methodos prius traditas; & fiant termini, qui involvunt independentes circulares arcus & logarithmos nihilo reſpective æquales, & ex æquationibus reſultantibus erui poſſunt ſeries quæſitæ.

Hæ ſeries etiam inveniri poſſunt ex prædictâ additione terminorum.

10. Et ſimiliter ex datis ſummis ſerierum, quarum generales termini ſunt $\frac{a^{lz+b}}{1.2.3..nz.(\alpha+z)}$, $\frac{a^{lz+b}}{1.2.3..nz.\overline{\beta+z}}$, $\frac{a^{lz+b}}{1.2.3..nz.\overline{\gamma+z}}$, $\frac{a^{lz+b}}{1.2.3..nz.\overline{\delta+z}}$, &c. & principiis hìc traditis erui poſſunt ſummæ omnium ſerierum formulæ, cujus generalis terminus exprimitur per fractionem, cujus numerator eſt $a^{lz+b} \times (Az^{m} + Bz^{m-1} + Cz^{m-2} + \&c.)$, & denominator $1.2.3.nz \times (\alpha+z).(\alpha+z+1).(\alpha+z+2)..(\alpha+z+e) \times (\beta+z)(\beta+z+1)(\beta+z+2)..(\beta+z+e') \times (\gamma+z)(\gamma+z+1)(\gamma+z+2)..(\gamma+z+e'') \times (\delta+z)(\delta+z+1)(\delta+z+2)..(\delta+z+e''') \times \&c.$; ni quilibet factores in denominatore contenti ſint inter ſe æquales.

Ex ſummâ ſeriei, cujus generalis terminus eſt $\frac{\pm a^{lz+b}}{1\,.\,2\,.\,3\,.\,.\,z\,.\,(\alpha+z)}$, datâ per principia hic tradita erui poteſt ſumma ſeriei, cujus generalis terminus eſt $\frac{\pm a^{lz+b}}{1\,.\,2\,.\,3\,.\,.\,nz\,.\,(\alpha+z)}$.

11. Sit ſeries $Ax^n + Bx^{n+m} + Cx^{n+2m} + Dx^{n+3m} + \&c. = y$, cujus indices variabilis quantitatis x ſint in arithmeticâ progreſſione; ducantur ſucceſſivi termini ſeriei reſpective in terminos ſucceſſivos trium arithmeticarum ſerierum $rz + s$, $r'z + s'$ & $r''z + s''$, ubi z eſt diſtantia a primo datæ ſeriei termino, & reſultant tres ſeries $sAx^n + (r+s)Bx^{n+m} + (2r+s)Cx^{n+2m} + \&c. = S$, $s'Ax^n + (r'+s')Bx^{n+m} + (2r'+s')Cx^{n+2m} + \&c. = S'$, $s''Ax^n + (r''+s'')Bx^{n+m} + (2r''+s'')Cx^{n+2m} + \&c. = S''$: ex ſummis duarum priorum ſerierum inter ſe independentium datis acquiri poteſt tertiæ ſumma: aſſumantur enim duæ æquationes $er + e'r' = r''$ & $es + e's' = s''$, e quibus erui poſſunt e & e', & exinde $eS + e'S' = S''$ ſummæ quæſitæ.

Et ſimiliter ducantur ſucceſſivi termini ſeriei reſultantis in ſucceſſivos ſecundæ arithmeticæ ſeriei terminos, & reſultat ſeries, cujus ſumma deduci poteſt ex ſummis trium ejuſdem generis ſerierum: & ſimiliter ducatur data ſeries in (b) arithmeticas ſeries ſucceſſive; tum ſumma ſeriei reſultantis erui poteſt e ſummis $(b+1)$ independentium ſerierum ejuſdem generis datis; vel quod idem eſt, ducantur ſucceſſivi termini prædictæ ſeriei in ſucceſſivos terminos quantitatis ${}'Az^m + {}'Bz^{m-1} + {}'Cz^{m-2} + \&c. = (rz+s)(tz+l)(t'z+l')(t''z+l'')\ \&c.$, ubi z diſtantia a primo datæ ſeriei termino ſucceſſive denotat 0, 1, 2, 3, &c. tum ex ſummis $(m+1)$ ſerierum reſultantium inter ſe independentium datis erui poſſunt ſummæ omnium ejuſdem generis ſerierum: vel magis generaliter ducantur ſucceſſivi termini ſeriei $Ax^n + Bx^{n+m} + Cx^{n+2m} + \&c.$ in $a\varphi : z + b\varphi' : z + c : \varphi'' : z + d : \varphi''' : z + \&c. = P$, & ſimiliter in $a'\varphi : z + b'\varphi' : z + c'\varphi'' : z + \&c. = \mathcal{Q}$; etiamque in $a''\varphi : z + b''\varphi' : z + c''\varphi'' : z + \&c.,\ \&c. = R$, ubi $\varphi : z$, $\varphi' : z$, $\varphi'' : z$, &c. ſunt diverſæ datæ functiones quantitatis (z) diſtantiæ a primo datæ ſeriei termino; & P, $\mathcal{Q}$, R, &c. ſunt (n) quantitates inter ſe independentes, & ſint ſummæ ſerierum reſultantium reſpective A', A'', A''', A'''',

&c.; tum ex ſummis (n) diverſarum reſultantium hujuſce generis ſerierum erui poſſunt ſummæ omnium ejuſdem generis ſerierum; e. g. ſit ſumma ſeriei reſultantis ex ducendo ſucceſſivos terminos datæ ſeriei $Ax^n + Bx^{n+m} +$ &c. in ſucceſſivos & correſpondentes terminos quantitatis $(\alpha\phi : z + \beta\phi' : z + \gamma\phi'' : z + \delta\phi''' : z +$ &c. $= T) = B$.

Fiant $pa + qa' + ra'' +$ &c. $= \alpha$, $pb + qb' + rb'' +$ &c. $= \beta$, $pc + qc'' + rc' +$ &c. $= \gamma$, $pd + qd' + rd'' +$ &c. $= \delta$, &c.; & reſultant n æquationes totidem (n) incognitates $p, q, r, s,$ &c. involventes, e quibus inveniantur quantitates $p, q, r, s,$ &c.; tum erit $pA' + qA'' + rA''' + sA'''' +$ &c. $= B$.

Cor. Quoniam $y = Ax^n + Bx^{n+m} + Cx^{n+2m} +$ &c., & exinde $\alpha x^{\alpha-1} y + x^\alpha \frac{\dot{y}}{\dot{x}} = (\alpha + n) Ax^{\alpha+n-1} + (\alpha + n + m) Bx^{n+m-1} + (\alpha + n + 2m) Cx^{n+2m-1} +$ &c.; & ſimiliter multiplicetur hæc æquatio in x^β, & inveniatur fluxio æquationis reſultantis, & ſequitur ſumma ſeriei reſultantis ex multiplicatione datæ ſeriei in duas arithmeticas ſeries: & ſic ex datâ ſeriei ſummâ acquiri poteſt ſumma ſeriei reſultantis ex multiplicatione terminorum datæ ſeriei in quotlibet arithmeticas ſeries.

12. Sit ſeries $A + Bx^n + Cx^{2n} + Dx^{3n} +$ &c. $= P$; tum erit $\frac{1}{\alpha} A + \frac{1}{\alpha + n} Bx^n + \frac{1}{\alpha + 2n} Cx^{2n} + \ldots \frac{1}{\alpha + mz} x^{mz} +$ &c. $= \frac{1}{\alpha} (P - \frac{1}{x^\alpha} \int . x^\alpha \dot{P})$; ducatur hæc æquatio in $x^{\beta-1}\dot{x}$, & inveniatur ejus fluens, quæ erit $\frac{1}{\alpha . \beta} A + \frac{1}{\alpha + n . \beta + n} Bx^n + \frac{1}{\alpha + 2n . \beta + 2n} Cx^{2n} +$ &c. $= \frac{1}{\alpha\beta} P - \frac{1}{\alpha} \times \frac{1}{\beta - \alpha} x^{-\alpha} \int . x^\alpha \dot{P} - \frac{1}{\beta} \times \frac{1}{\alpha - \beta} x^{-\beta} \int . x^\beta \dot{P}$; & in genere erit ſumma ſeriei $\frac{A}{\alpha\beta\gamma\delta \text{ \&c.}} + \frac{Bx^n}{\alpha + n . \beta + n . \gamma + n . \delta + n . \text{\&c.}} + \frac{Cx^{2n}}{\alpha + 2n . \beta + 2n . \gamma + 2n . \delta + 2n} + \frac{Dx^{3n}}{\alpha + 3n . \beta + 3n . \gamma + 3n . \delta + 3n} +$ &c. $= \frac{1}{\alpha\beta\gamma\delta \text{ \&c.}} \times P - \frac{1}{\alpha} . \frac{1}{\beta - \alpha} . \frac{1}{\gamma - \alpha} . \frac{1}{\delta - \alpha} .$ &c. $\times x^{-\alpha} \int . x^\alpha \dot{P}$

$$-\frac{1}{\beta}\cdot\frac{1}{\alpha-\beta}\cdot\frac{1}{\gamma-\beta}\cdot\frac{1}{\delta-\beta}\cdot\&c.\times x^{-\beta}\int.x^{\beta}\dot{P}-\frac{1}{\gamma}\cdot\frac{1}{\alpha-\gamma}\cdot\frac{1}{\beta-\gamma}\cdot\frac{1}{\delta-\gamma}\cdot\&c.\times x^{-\gamma}\int.x^{\gamma}\dot{P}-\frac{1}{\delta}\cdot\frac{1}{\alpha-\delta}\cdot\frac{1}{\beta-\delta}\cdot\frac{1}{\gamma-\delta}\cdot\&c.\times x^{-\delta}\int.x^{\delta}\dot{P}-\&c.$$

Hoc facile deduci poteſt ex hoc theoremate, viz. $\frac{1}{\alpha}\cdot\frac{1}{\beta-\alpha}\cdot\frac{1}{\gamma-\alpha}\cdot\&c.+\frac{1}{\beta}\cdot\frac{1}{\alpha-\beta}\cdot\frac{1}{\gamma-\beta}\cdot\frac{1}{\delta-\beta}\cdot\&c.+\frac{1}{\gamma}\cdot\frac{1}{\alpha-\gamma}\cdot\frac{1}{\beta-\gamma}\cdot\frac{1}{\delta-\gamma}\cdot\&c.+\&c.+\frac{1}{\delta}\cdot\frac{1}{\alpha-\delta}\cdot\frac{1}{\beta-\delta}\cdot\frac{1}{\gamma-\delta}\cdot\&c.+\&c.=\frac{1}{\alpha\beta\gamma\delta\&c.}$.

Cor. Hinc, ſi ſumma (P) ſeriei, cujus generalis terminus ſit ϕ, detur; ſumma ſeriei, cujus generalis terminus eſt $\frac{\phi}{\alpha+z.\beta+z.\gamma+z.\delta+z.\&c.}$; vel magis generaliter $\frac{\phi\times(az^{m}+bz^{m-1}+cz^{m-2}+\&c.)}{\alpha+z.\beta+z.\gamma+z.\delta+z.\&c.}$ ubi m eſt integer numerus; pendet ex fluentibus fluxionum $x^{\alpha}\dot{P}$, $x^{\beta}\dot{P}$, $x^{\gamma}\dot{P}$, $x^{\delta}\dot{P}$, &c.; vel quod idem eſt ex ſummis ſerierum $nx^{\alpha+n}\left(\frac{B}{\alpha+n}+\frac{2C}{\alpha+2n}x^{n}+\frac{3D}{\alpha+3n}x^{2n}+\frac{4E}{\alpha+4n}x^{3n}+\&c.\right)$, $nx^{\beta+n}\left(\frac{1}{\beta+n}+\frac{2C}{\beta+2n}x^{n}+\frac{3D}{\beta+3n}x^{2n}+\&c.\right)$, $nx^{\gamma+n}\left(\frac{1}{\gamma+n}+\frac{2C}{\gamma+2n}x^{n}+\frac{3D}{\gamma+3n}x^{2n}+\&c.\right)$, $nx^{\delta+n}\left(\frac{B}{\delta+n}+\frac{2C}{\delta+2n}x^{n}+\frac{3D}{\gamma+3n}x^{2n}+\&c.\right)$, &c.; ni duæ vel plures e quantitatibus α, β, γ, δ, &c. ſint inter ſe æquales; in quo caſu e præcedentibus petenda eſt ſumma.

13. Sit $(a+bx^{n})^{m}=A+Bx^{n}+Cx^{2n}+\&c.=P$; ducatur hæc æquatio in $x^{rn-1}\dot{x}$, & inveniatur fluens æquationis reſultantis $\phi\times x^{rn}=A'+B'x^{n}+C'x^{2n}+\&c.$; ducatur hæc æquatio in $x^{sn-1}\dot{x}$ & inveniatur fluens æquationis reſultantis; & ſic deinceps; & reſultant

ſeries $\frac{A}{r.s.t.\&c.} + \frac{Bx^{n}}{r+n.s+n.t+n.\&c.} + \frac{Cx^{2n}}{r+2n.s+2n.t+2n.\&c.} + \&c.$; cujus ſumma exprimi poteſt in finitis terminis, ſi literæ r, s, t, &c. integros denotent numeros; per prædictos & circulares arcus & logarithmos, ſi denotent affirmativas fractiones, quarum denominatores non majores ſunt quam 2; per prædictos & ellipticos & hyperbolicos arcus, ſi denominatores non majores ſint quam 3; &c.

14. Sit $(a+bx^{n}+cx^{2n}\ldots x^{ln})^{m} = A+Bx^{n}+Cx^{2n} = P$, ducatur hæc æquatio in $x^{rn-1}\dot{x}$, $x^{r'n-1}\dot{x}$, $x^{r''n-1}\dot{x}$, &c. & inveniantur fluentes æquationum reſultantium $\phi = A'+B'x^{n}+C'x^{2n}+\&c.$; ducantur æquationes reſultantes in $x^{sn-1}\dot{x}$, $x^{s'n-1}\dot{x}$, $x^{s''n-1}\dot{x}$, &c., & inveniantur fluentes fluxionum reſultantium, & ſic deinceps; tum, ſi r, r', r'', &c., s, s', s'', &c., &c. ſint integri numeri, ex $(l-1)$ fluentibus independentibus prædictis erui poſſunt reliquæ: ſi vero differentiæ inter prædictas quantitates r, r', r'', &c.; s, s', s'', &c.; &c.; ſint integri numeri, tum ex l independentibus erui poſſunt reliquæ; & ſic deinceps.

Cor. Erunt $\int.x^{\beta-\alpha-1}\dot{x}\int.x^{\alpha-1}\dot{x}P = \int.x^{\alpha-\beta-1}\dot{x}\int.x^{\beta-1}\dot{x}P$; $\int.x^{\gamma-\beta-1}\dot{x}\int.x^{\beta-\alpha-1}\dot{x}\int.x^{\alpha-1}\dot{x}P = \int.x^{\gamma-\alpha-1}\dot{x}\int.x^{\alpha-\beta-1}\dot{x}\int.x^{\beta-1}\dot{x}P = \int.x^{\beta-\gamma-1}\dot{x}\int.x^{\gamma-\alpha-1}\dot{x}\int.x^{\alpha-1}\dot{x} = \int.x^{\beta-\alpha-1}\dot{x}\int.x^{\alpha-\gamma-1}\dot{x}\int.x^{\gamma-1}\dot{x} = \int.x^{\alpha-\beta-1}\dot{x}\int.x^{\beta-\gamma-1}\dot{x}\int.x^{\gamma-1}\dot{x} = \int.x^{\alpha-\gamma-1}\dot{x}\int.x^{\gamma-\beta-1}\dot{x}\int.x^{\beta-1}\dot{x}$, & ſic deinceps: ſi modo omnes fluentes inter eoſdem valores quantitatis x contineantur; ſi vero quartus adjiciatur index δ, tum reſultant $1.2.3.4 = 24$ fluentes inter ſe æquales; & ſic deinceps.

THEOR. XLVIII.

Sint T^{m}, T^{n}, T^{r}, T^{s}, &c. termini ſeriei A, quorum diſtantiæ a primo vel quocunque alio ſeriei termino ſint reſpective m, n, r, s, &c. i. e. in generali termino A', pro z diſtantiâ a primo ſeriei termino ſcribantur $z+m$, $z+n$, $z+r$, $z+s$, &c., & reſultent termini T^{m}, T^{n}, T^{r}, T^{s}, &c. ſint $T'^{m'}$, $T'^{n'}$, $T'^{r'}$, $T'^{s'}$, &c. termini ſeriei B, quorum diſtantiæ a primo ſint reſpective m', n', r', s', &c.; i. e. in generali termino B' pro z diſtantiâ a primo ſeriei termino ſcribantur $z + m'$, $z + n'$, $z + r'$, $z + s'$, &c., & reſultent ſeriei termini $T'^{m'}$, $T'^{n'}$, $T'^{r'}$, $T'^{s'}$, &c. ſint etiam $T''^{m''}$, $T''^{n''}$, $T''^{r''}$, $T''^{s''}$, &c. termini ſeriei C, quorum diſtantiæ a primo ſint reſpectivè m'', n'', r'', s'', &c.; i. e. in generali termino C' pro z diſtantiâ a primo ſeriei termino ſcribantur $z + m''$, $z + n''$, $z + r''$, $z + s''$, &c., & reſultent termini $T''^{m''}$, $T''^{n''}$, $T''^{r''}$, $T''^{s''}$, &c.; & ſic deinceps.

Aſſumatur quantitas $aT^{m} + bT^{n} + cT^{r} + dT^{s} +$ &c. $+ a'T'^{m'} + b'T'^{n'} + c'T'^{r'} + d'T'^{s'} +$ &c. $+ a''T''^{m''} + b''T''^{n''} + c''T''^{r''} + d''T''^{s''} +$ &c. pro termino ſeriei quæſitæ, tum ex ſummis (S, S', S'', &c.) datarum ſerierum A, B, C, &c. acquiri poteſt ſumma ſeriei reſultantis quæſitæ, erit enim $(a + b + c + d +$ &c.$) \times S + (a' + b' + c' + d' +$ &c.$) \times S' + (a'' + b'' + c'' + d'' +$ &c.$) \times S'' +$ &c. $- a(T^{0} + T^{1} + T^{2} + \,.\,.\, T^{m-1}) - b(T^{0} + T^{1} + T^{2} + \,.\,.\, T^{n-1}) - c(T^{0} + T^{1} + T^{2} + \,.\,.\, T^{r-1}) - d(T^{0} + T^{1} + T^{2} + \,.\,.\, T^{s-1}) -$ &c. $- d'(T'^{0} + T'^{1} + T'^{2} \,.\,.+ T'^{m'-1}) - b'(T'^{0} + T'^{1} + T'^{2} + T'^{3} + \,.\,.+ T'^{n'-1}) - c'(T'^{0} + T'^{1} + T'^{2} + \,.\,.+ T'^{r'-1}) - d'(T'^{0} + T'^{1} + \,.\,.\, T'^{s'-1}) +$ &c. $- a''(T''^{0} + T''^{1} + T''^{2} + \,.\,.+ T''^{m''-1}) - b''(T''^{0} + T''^{1} + T''^{2} + T''^{3} + \,.\,.+ T''^{n''-1}) - c''(T''^{0} + T''^{1} + T''^{2} + \,.\,.+ T''^{r''-1}) -$ &c.

Sit $a + b + c + d +$ &c. $= o$, & $a' + b' + c' + d' +$ &c. $= o$, & $a'' + b'' + c'' + d'' +$ &c. $= o$, &c. tum in finitis prædictis terminis exprimi poteſt ſumma reſultantis ſeriei.

PROB. XLVI.

Invenire infinitas ſeries, quarum ſummæ innoteſcant.

1. Aſſumatur quæcunque quantitas pro ſummâ, reducatur ea per diviſionem, extractionem radicum, &c. ad ſeriem convergentem, & reſultat ſeries, cujus ſumma innoteſcit.

2. Reducatur aſſumpta quantitas ad ſeriem ſecundum dimenſiones quarumcunque perparvarum quantitatum (x) in eâ contentarum progrediens, & exoriuntur ſeries, quarum ſummæ innoteſcunt: annon prædictæ ſeries convergent, deduci poteſt e principiis prius traditis;

ditis; viz. ex ſupponendo denominatorem & ſingulas irrationales quantitates nihilo æquales, & deducendo minimam radicem (α) quantitatis x in æquationibus reſultantibus; & ſi x minor ſit quam α, tum ſeries aſcendens ſemper converget; ſi autem x major ſit quam maxima prædictarum radicum, tum ſemper converget, ſi modo ſit ſeries ſecundum dimenſiones quantitatis x deſcendens.

3. Aſſumantur quantitates pro ſummis, deinde reducantur quantitates aſſumptæ ad ſeries ſecundum dimenſiones unius, duarum, trium, &c. quantitatum, vel ſecundum quaſdam datas functiones quarundam quantitatum progredientes; ita quidem ut ſeries evadant convergentes; & deducuntur ſeries, quarum ſummæ innoteſcunt.

4. Ducantur æquationes deductæ in fluxiones formularum, quæ reddent fluentes reſultantium æquationum facile integrabiles; & exinde erui poſſunt ſeries ſummabiles; & ſic deinceps.

5. Inveniantur diverſi valores quantitatum aſſumptarum per quaſdam functiones ſecundum dimenſiones quarundam perparvarum quantitatum aſcendentes vel permagnarum deſcendentes; & exinde ſæpe facilius erui poſſunt fluentes earum ſummarum, differentiarum, &c.; ſed animadvertendum eſt, ſi aſcendens ſeries convergat, tum deſcendens diverget, ni omnes radices quantitatum aſſumptarum poſſibiles vel impoſſibiles inter ſe quodammodo habeantur æquales; in quo caſu nonnunquam convergent & nonnunquam divergent & aſcendens & deſcendens ſeries, & in algebraicis quantitatibus utræque plerumque evadent eædem vel una affirmativa & altera negativa & æqualis: in fluentibus proprie correctis eadem affirmari poſſunt.

6. Ex theor. $P + \left(\frac{\dot{P}}{\dot{x}}\right) e + \frac{1}{2}\left(\frac{\ddot{P}}{\dot{x}^2}\right) e^2 +$ &c. & $P + \left(\left(\frac{\dot{P}}{x}\right) e + \left(\frac{\dot{P}}{\dot{y}}\right) o + \text{\&c.}\right) +$ &c. erui poſſunt ſummæ multarum ſerierum.

Ex. 1. Erit $a^{b+n} = a^b\left(1 + nA + \frac{n^2A^2}{1.2} + \frac{n^3A^3}{1.2.3} + \frac{n^4A^4}{1.2.3.4} + \text{\&c.}\right)$, ubi A eſt logarithmus quantitatis a.

Ex. 2. Erit $\surd(b+ax+an) = A^{\frac{1}{2}} + \frac{an}{2A^{\frac{1}{2}}} - \frac{1}{4} \times \frac{a^2n^2}{2A^{\frac{3}{2}}} + \frac{1}{4} \times \frac{3}{2} \times \frac{a^3n^3}{2.3A^{\frac{5}{2}}} - \frac{1}{4} \times \frac{3}{2} \times \frac{5}{2} \times \frac{a^5n^5}{2.3.4A^{\frac{7}{2}}} + \frac{1}{4} \times \frac{3}{2} . \frac{5}{2} . \frac{7}{2} \times \frac{a^7n^7}{2.3.4.5A^{\frac{9}{2}}} -$ &c., ubi $A = \surd(b+ax)$.

Ex. 3. Erit $(x+b+n)^{x+n} = (x+b)^x \left(1 + \left(\frac{x}{x+b} + \log.(x+b)\right) n\right) + \frac{1}{2}n^2(x.(x-1).(x+b)^{x-2} + 2(x+b)^{x-1} + 2x \times (x+b)^{x-1} \times \log.(x+b) + (x+b)^x \times \log.(x+b)^2) +$ &c.

Cum plures variabiles in aſſumptâ quantitate contineantur, tum ex theor. 22. l. 3. erui poſſunt plures ſeries, quarum ſummæ innoteſcunt.

Convergentiæ harum ſerierum ex principiis prius traditis dijudicari poſſunt.

7. Et ſic ex aſſumptis quantitatibus per quamcunque methodum continuo inveniantur quantitates, quæ ad eas ſemper propius accedunt, & ultimo propius accedunt quam pro datâ quâvis differentiâ, tum inveniuntur ſeries, quarum ſummæ innoteſcunt: hinc ex omnibus methodis approximationes ad quaſcunque radices vel quantitates continuo deducendi prius traditis erui poſſunt ſeries, quarum ſummæ ſint radices, &c. ipſæ. E. g. Sint quæcunque quantitates perparvæ, & reducantur quæcunque aſſumptæ quantitates, ita ut progrediantur ſecundum dimenſiones perparvarum quantitatum, & reſultant ſeries convergentes, quarum ſummæ ſunt quantitates aſſumptæ.

8. Sit P quæcunque functio quantitatis $(x) = 0$, i. e. $P = 0$; pro valore quantitatis x aſſumatur a, & ſcribatur a pro x in functione P, & reſultet quantitas A; ſcribatur etiam in quantitatibus $\frac{\dot{P}}{\dot{x}}$, $\left(\frac{\ddot{P}}{\dot{x}^2}\right)$, $\left(\frac{\dddot{P}}{\dot{x}^3}\right)$, $\left(\frac{\ddddot{P}}{\dot{x}^4}\right)$, &c.; a pro x, & reſultent π, π', π'', π''', &c.; tum, ſi $\pi((a+z)-a) + \frac{1}{2}\pi'((a+z)^2-a^2) + \frac{1}{2.3}\pi''((a+z)^3-a^3) + \frac{1}{2.3.4}((a+z)^4-a^4) +$ &c. minor ſit quam A, & inter a & $a+z$

nulla contineatur poſſibilis vel impoſſibilis radix æquationis $\left(\frac{\dot{P}}{\dot{x}}\right) = 0$, erit radix datæ æquationis major quam $a + z$.

Eadem principia etiam ad duas vel plures æquationes duas vel plures incognitas quantitates habentes, etiamque ad ſucceſſivas approximationes detegendas applicari poſſunt.

9. Aſſumantur quæcunque quantitates vel algebraicæ vel fluxionales vel incrementiales, reducantur eæ ad diverſas ſeries ſecundum dimenſiones diverſarum perparvarum vel permagnarum quantitatum progredientes, & reſultant diverſæ ſeries inter ſe æquales; vel reducantur prædictæ quantitates in diverſas ſeries, & inveniantur fluentes vel integrales inter eoſdem valores earundem variabilium contentæ, & reſultant diverſæ ſeries inter ſe æquales.

10. Sint P & Q infinitæ ſeries, quæ convergunt, cum x ſit quæcunque quantitas minor quam α, tum etiam ſeries progrediens ſecundum dimenſiones quantitatis $(x) = \text{ſ}.\,Px^{m}\dot{x}\;\text{ſ}.\,Qx^{n}\dot{x}$ vel $= \text{ſ}.\,x^{n'}\dot{x}\;\text{ſ}.\,x^{m'}\dot{x}$ &c. $\times\,\text{ſ}.\,Px^{m}\dot{x}\;\text{ſ}.\,Qx^{n}\dot{x}$ converget, cum x minor ſit quam α.

2. Si prædictæ quantitates ſemper convergant, cum x major ſit quam α, tum etiam ſeries prædictæ $= \text{ſ}.\,Px^{n}\dot{x} \times \text{ſ}.\,Qx^{n}\dot{x}$ &c. convergent, cum x major ſit quam α.

Conſimilia etiam affirmari poſſunt de pluribus quantitatibus ejuſdem generis P, Q, R, S, &c.

THEOR. XLIX.

Sit data ſeries ſecundum dimenſiones quantitatis x progrediens, & v functio quantitatis x perparva, vel permagna; tum inveniatur $x = \phi : v$, ſcribatur $\phi : v$ in datâ ſerie pro x, & reducatur reſultans ad ſeriem ſecundum dimenſiones quantitatis v progredientem, ita ut exoriatur ſeries convergens, & invenitur ſeries cujus ſumma innoteſcit.

Sunt caſus, in quibus ſeries reſultans nunquam converget; e. g. ſit ſeries $x - \frac{1}{2}x^{2} + \frac{1}{3}x^{3} - \frac{1}{4}x^{4} +$ &c., & $\sqrt{(1 - x^{2})} = v$ perparva quantitas, & exinde $x = \sqrt{(1 - v^{2})}$, ſcribatur hæc quantitas pro x in datâ ſerie, & reſultat ſeries $(1 - v^{2})^{\frac{1}{2}} - \frac{1}{2}(1 - v^{2})^{\frac{2}{2}} + \frac{1}{3}(1 - v^{2})^{\frac{3}{2}}$ — &

$- \&c. = (1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \&c.) - \frac{1}{2}(1 - 1 + 1 - 1 + 1 - \&c.)$ $v^2 + \frac{1}{8}(1 \times (1-2) - 1 \times (2-2) + 1 \times 1 - 1 \times 2 + 1 \times 3 - \&c.)$ $v^4 + \&c.$; quæ ſeries nunquam converget, nam $x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \&c. = \int . \frac{\dot{x}}{1+x} = \frac{-v\dot{v}}{\sqrt{(1-v^2)} \times (1 + \sqrt{(1-v^2)})}$; ſed radix æquationis $1 + \sqrt{(1-v^2)} = 0$ erit $v = 0$, ergo ſeries ſecundum dimenſiones quantitatis v progrediens nunquam converget, cum v ſit finita quantitas.

THEOR. L.

Erit $\frac{u}{u+1} = \frac{\dot{u}}{u} - \frac{\dot{u}}{u^2} + \frac{\dot{u}}{u^3} - \frac{\dot{u}}{u^4} + \&c.$, unde log. $(u+1)$ — log. $u = \frac{1}{u} - \frac{1}{2u^2} + \frac{1}{3u^3} - \&c.$; at log. $(1+u) = \int . \frac{\dot{u}}{1+u} = u - \frac{1}{2}u^2 + \frac{1}{3}u^3 - \&c.$, unde log. $u = u - u^{-1} - \frac{u^2 - u^{-2}}{2} + \frac{u^3 - u^{-3}}{3} - \&c.$: hæc ſeries nunquam converget, cum u ſit poſſibilis quantitas: ſi autem u ſit impoſſibilis $a + b\sqrt{(-1)}$, & $a^2 + b^2 = 1$, tum ſeries ſemper converget.

THEOR. LI.

Sit data æquatio relationem inter P, Q, R, &c. exprimens, quarum haud duæ in ſeſe ducuntur, vel ſimul involvuntur; ducatur data æquatio in ρ'; ubi ρ' eſt fluxio functionis e ſingulis quantitatibus P, Q, R, &c., ſi modo fluens requiratur; vel incrementum functionis e prædictis quantitatibus, ſi modo integralis requiratur; tum inveniri poteſt fluens vel integralis æquationis reſultantis.

Ex. Sint quantitates, quæ ſint quæcunque functiones quantitatis $x \pm x^{-1}$; vel quantitatis, in quâ x & x^{-1} ſimiliter involvuntur; tum ea facile reduci poteſt ad functionem ſinuum & coſinuum arcuum circuli.

Erunt $\frac{(x + \sqrt{(x^2-1)})^{\frac{n}{m}} - (x - \sqrt{(x^2-1)})^{\frac{n}{m}}}{2\sqrt{(-1)}}$ & $\frac{(x + \sqrt{(x^2-1)})^{\frac{n}{m}}}{2}$

$\underline{+(x-\sqrt{(x^2-1)})^{\frac{n}{m}}}$ ſinus & coſinus arcus circuli, qui erit ad arcum cujus ſinus eſt $\sqrt{(1-x^2)}$ & coſinus $x :: n : m$. Si quantitates in prædictâ functione ſint $z^{\frac{n}{m}} - z^{\frac{-n}{m}}$ vel $z^{\frac{n}{m}} + z^{\frac{-n}{m}}$, tum pro z ſcribatur $x + \sqrt{(x^2-1)}$, & pro $z^{\frac{n}{m}} - z^{\frac{-n}{m}}$ & $z^{\frac{n}{m}} + z^{\frac{-n}{m}}$ ſcribantur $2\sqrt{(-1)}\, s^{\frac{n}{m}}$ & $2\, c^{\frac{n}{m}}$, ubi $s^{\frac{n}{m}}$ eſt ſinus arcus $\frac{n}{m}A$ & $c^{\frac{n}{m}}$ coſinus ejuſdem arcus, & coſinus arcus A eſt x & radius 1.

Cor. Hinc, ſi in æquatione $u - u^{-1} - \frac{u^2 - u^{-2}}{2} + \frac{u^3 - u^{-3}}{3} - \&c. = \int . \frac{\dot{u}}{u}$ pro u ſcribatur $x + \sqrt{(x^2-1)}$, reſultat $s - \frac{s'}{2} + \frac{s''}{3} - \&c. = \int . \frac{\dot{x}}{\sqrt{(-1)} . \sqrt{(x^2-1)}} = \int . \frac{\dot{x}}{\sqrt{(1-x^2)}} = z$ arcui, cujus radius eſt 1 & coſin. $= x$.

2. Denotentur ſinus & coſinus arcus $q\,x$ per $\overset{(q)}{s}$ & $\overset{(q)}{c}$ reſpective, tum erunt $\int . \frac{\overset{(q)}{s}\,\dot{x}}{\sqrt{(1-x^2)}}$ & $\int . \frac{\overset{(q)}{c}\,\dot{x}}{\sqrt{(1-x^2)}}$ reſpective $\frac{\overset{(q)}{c}}{q}$ & $\frac{\overset{(q)}{s}}{q}$; nam ſit ſinus $\overset{(q)}{s} = s'$, tum erit $\frac{\dot{s}'}{\sqrt{(1-s'^2)}} = \frac{\dot{s}'}{\overset{(q)}{c}} = \frac{q\,\dot{x}}{\sqrt{(1-x^2)}}$ & exinde $\frac{s'}{q} = \int . \frac{\overset{(q)}{c}\,\dot{x}}{\sqrt{(1-x^2)}}$; & ad eundem modum inveniri poteſt $\frac{\overset{(q)}{c}}{q} = \int . \frac{\overset{(q)}{s}\,\dot{x}}{\sqrt{(1-x^2)}}$.
Hoc etiam verum eſt, cum q ſit fractio.

3. Fluens fluxionis $\left(\overset{(q)}{s}{}^{m} \times \overset{(q)}{c}{}^{n} \times \frac{q\,\dot{x}}{\sqrt{(1-x^2)}}\right) = \frac{1}{m+1}\overset{(q)}{s}{}^{m+1}\,\overset{(q)}{c}{}^{n-1} - \frac{1}{m+1} \times \frac{n-1}{m+3}\overset{(q)}{s}{}^{m+3} \times \overset{(q)}{c}{}^{n-3} + \frac{1}{m+1} \times \frac{n-1}{m+3} \times \frac{n-3}{m+5}\overset{(q)}{s}{}^{m+5}\,\overset{(q)}{c}{}^{n-5} - \frac{1}{m+1} \times \frac{n-1}{m+3} \times \frac{n-3}{m+5} \times \frac{n-5}{m+7}\overset{(q)}{s}{}^{m+7}\,\overset{(q)}{c}{}^{n-7} + \frac{1}{m+1} \times \frac{n-1}{m+3} \times \frac{n-3}{m+5} \times \frac{n-5}{m+7} \times$

$\times \frac{n-7}{m+9} \times \overset{(q)}{s}{}^{m+9} \times \overset{(q)}{c}{}^{n-9} -$ &c.; hæc ſeries ſemper terminat, cum n ſit impar numerus: vel fluens prædicta erit $\frac{1}{n+1} \overset{(q)}{c}{}^{n+1} \overset{(q)}{s}{}^{m-1} - \frac{1}{n+1} \cdot \frac{m-1}{n+3} \overset{(q)}{c}{}^{n+3} \times \overset{(q)}{s}{}^{m-3} + \frac{1}{n+1} \times \frac{m-1}{n+3} \times \frac{m-3}{n+5} \times \overset{(q)}{c}{}^{n+5} s^{m-5} - \frac{1}{n+1} \times \frac{m-1}{n+3} \cdot \frac{m-3}{n+5} \cdot \frac{m-5}{n+7} \times \overset{(q)}{c}{}^{n+7} s^{m-7} +$ &c., quæ ſemper terminat cum m ſit impar numerus.

4. Ducatur prædicta æquatio $s - \frac{s'}{2} + \frac{s''}{3} - \frac{s'''}{4} +$ &c. $= z$ in $\frac{\dot{x}}{\sqrt{(1-x^2)}}$ & inveniatur fluens æquationis reſultantis, deinde ducatur æquatio reſultans in $\frac{\dot{x}}{\sqrt{(1-x^2)}}$ & inveniatur fluens æquationis reſultantis; & ſic deinceps; & reſultant ſeries, quarum ſummæ innoteſcunt.

5. Fluens fluxionis $\left(\frac{u^{m-1}\dot{u}}{(1+u^n)^e}\right) = \frac{u^m}{m} - \frac{e\,u^{m+n}}{m+n} + e \cdot \frac{e+1}{2} \cdot \frac{u^{m+2n}}{m+2n}$ &c. $- M'$ vel $= - \frac{u^{m-en}}{en-m} + \frac{e\,u^{m-en-n}}{en-m+n} - e \times \frac{e+1}{2} \cdot \frac{u^{m-en-2n}}{en+2n-m}$ &c. $+ M''$, ubi $M' = \frac{1}{m} - \frac{e}{m+n} + e \cdot \frac{e+1}{2} \times \frac{1}{m+2n}$ &c., & $M'' = \frac{1}{en-m} - \frac{e}{en+n-m} + e \cdot \frac{e+1}{2} \cdot \frac{1}{en+2n-m}$ &c.; quæ erunt fluentes fluxionum $\frac{u^{m-1}\dot{u}}{(1+u^n)^e}$ & $\frac{u^{en-m-1}\dot{u}}{(1+u^n)^e}$ inter valores o & 1 quantitatis u contentarum: ducatur hæc æquatio in $u^{en-2m-1}\dot{u}$, & invenientur fluentes flux. reſult. $\frac{u^{en-m}}{m\,.\,en-m} - \frac{e\,u^{en+n-m}}{m+n\,.\,en+n-m} + e \cdot \frac{e+1}{2} \frac{u^{en+2n-m}}{m+2n\,.\,en+2n-m}$ &c. $- \frac{M'u^{en-2m}}{en-2m} + \frac{M'}{en-2m} = \frac{u^{-m}}{m\,.\,en-m} - \frac{e\,u^{-m-n}}{m+n\,.\,en+n-m} + e \cdot \frac{e+1}{2} \frac{u^{-m-2n}}{m+2n\,.\,en+2n-m}$ &c. $+$

$M''u$

$\frac{M''u^{en-2m}}{en-2m} - \frac{M''}{en-2m}$; ducatur hæc æquatio in $\frac{u^{m-\frac{1}{2}en}}{2\sqrt{(-1)}}$, & reſultat $\frac{u^{\frac{1}{2}en} - u^{-\frac{1}{2}en}}{2\sqrt{(-1)}\,m\,.\,en-m} - e\frac{u^{\frac{1}{2}en+n} - u^{-\frac{1}{2}en-n}}{2\sqrt{(-1)}\,.\,m+n\,.\,en+n-m} +$ &c. $= 0$; & per principia prius tradita e ſcribendo $x+\sqrt{(x^2-1)}$ pro u facile reduci poteſt data æquatio ad ſeriem ſinuum: ducatur iterum iterumque reducta æquatio in $\frac{\dot{x}}{\sqrt{(1-x^2)}}$; & inveniantur fluentes reſultantium æquationum; & reſultant novæ ſeries, quarum ſummæ innoteſcunt.

6. Erit $(2\overset{(\frac{n}{2})}{c})^{-e} = ((x+\sqrt{(x^2-1)})^{\frac{n}{2}} + (x-\sqrt{(x^2-1)})^{\frac{n}{2}})^{-e} = (x+\sqrt{(x^2-1)})^{\frac{1}{2}en} - e(x+\sqrt{(x^2-1)})^{\frac{1}{2}en+n} +$ &c. $= (x-(\sqrt{x^2-1}))^{\frac{1}{2}en} - e(x-\sqrt{(x^2-1)})^{\frac{1}{2}n+n} +$ &c.; & exinde $((x+\sqrt{(x^2-1)})^{\frac{1}{2}en} - (x-\sqrt{(x^2-1)})^{\frac{1}{2}en}) - e((x+\sqrt{(x^2-1)})^{\frac{1}{2}en+n} - (x-\sqrt{(x^2-1)})^{\frac{1}{2}en+n}) +$ &c. $= s^{\frac{1}{2}en} - e\,s^{\frac{1}{2}en+n} +$ &c. $= 0$; vel ducatur $2(\overset{(\frac{n}{2})}{c})^{-e} = (x+\sqrt{(x^2-1)})^{\frac{1}{2}en} - e(x+\sqrt{(x^2-1)})^{\frac{1}{2}en+n} +$ &c. in $(x+\sqrt{(x^2-1)})^{m-\frac{1}{2}en}$ & $(2\overset{(\frac{n}{2})}{c})^{-e} = (x-\sqrt{(x^2-1)})^{\frac{1}{2}en} - e(x-\sqrt{(x^2-1)})^{\frac{1}{2}en+n} +$ &c. in $(x-\sqrt{(x^2-1)})^{m-\frac{1}{2}en}$, & ſubducatur poſterior reſultans æquatio de priori, & dividatur per $2\sqrt{(-1)}$ & reſultabit $\frac{\overset{(m-\frac{1}{2}en)}{s}}{(2\overset{(\frac{n}{2})}{c})^{e}} = \overset{(m)}{s} - e\overset{(m+n)}{s} + e\,.\,\frac{e+1}{2}\overset{(m+2n)}{s} -$ &c.; ducatur hæc æquatio in $\frac{\dot{x}}{\sqrt{(1-x^2)}}$, & inveniatur fluens æquationis reſultantis; deinde ducatur fluens prædicta in $\frac{\dot{x}}{\sqrt{(1-x^2)}}$ & inveniatur fluens æquationis reſultantis; & ſic deinceps; & reſultabunt ſeries quarum ſummæ innoteſcunt.

7. Erit $\frac{1}{(x-\sqrt{(x^2-1)})^{\frac{1}{2}n} + y(x+\sqrt{(x^2-1)})^{\frac{1}{2}n}} = (x+\sqrt{(x^2-1)})^{\frac{n}{2}}$

$-y$

$-y(x+\sqrt{(x^2-1)})^{\frac{3n}{2}}+$ &c., &c., $\frac{1}{(x+\sqrt{(x^2-1)})^{\frac{1}{2}n}+y(x-\sqrt{(x^2-1)})^{\frac{1}{2}n}}=(x-\sqrt{(x^2-1)})^{\frac{1}{2}n}-y(x-\sqrt{(x^2-1)})^{\frac{3}{2}n}+$ &c.; ſubtrahatur alter æquatio de alterâ, & dividatur per $2\sqrt{(-1)}$, reſultat $(1-y)\times\frac{\overset{(\frac{1}{2}n)}{s}}{1+y^2+2y\overset{(n)}{c}}=\overset{(\frac{n}{2})}{s}-y\overset{(\frac{3n}{2})}{s}+$ &c.: multiplicetur prima æquatio per $(x+\sqrt{(x^2-1)})^{\frac{n}{2}}$ & ſecunda per $(x-\sqrt{(x^2-1)})^{\frac{n}{2}}$ & ſubtrahatur alter de alterâ, & dividatur per $2\sqrt{(-1)}$, & reſultat $\frac{\overset{(n)}{s}}{1+y^2+2y\overset{(n)}{c}}=\overset{(n)}{s}-\overset{(2n)}{s}y+$ &c.: multiplicentur hæ æquationes per $\frac{\dot{x}}{\sqrt{(1-x^2)}}$ & inveniantur fluentes æquationum reſultantium terminorum, & deducuntur ſeries, quarum ſummæ innoteſcunt: vel multiplicetur prima æquatio per $(x+\sqrt{(x^2-1)})^{m-\frac{n}{2}}$; & ſecunda per $(x-\sqrt{(x^2-1)})^{m-\frac{n}{2}}$, & collocentur quantitates in ordine $(x+\sqrt{(x^2-1)})^b\pm(x-\sqrt{(x^2-1)})^b=H$, quæ denotat $\overset{(b)}{s}$ ſinum vel coſinum arcus bz, ubi z eſt arcus, cujus coſinus eſt x; vel ſcribatur y^b pro y, & deinde multiplicentur æquationes per $y^{b-1}\dot{y}$ & inveniantur fluentes, ex hypotheſi quod y ſolummodo ſit variabilis; & conſequuntur ſeries, quarum ſummæ innoteſcunt.

8. In ſummis ſerierum prius traditarum inveſtigandis, haud neceſſario introducendi ſunt ſinus & coſinus arcuum nz, in eorum loco retineantur quantitates algebraicæ $\frac{(x+\sqrt{(x^2-1)})^n+(x-\sqrt{(x^2-1)})^n}{2}=P$ & $\frac{(x+\sqrt{(x^2-1)})^n-(x-\sqrt{(x^2-1)})^n}{2\sqrt{(-1)}}=Q$.

Aſſumatur log. $u=u-u^{-1}-\frac{u^2-u^{-2}}{2}+$ &c.; ducatur hæc æquatio in $\frac{\dot{u}}{u}$, & reſultat fluxio, cujus fluens eſt $u+u^{-1}-\frac{u^2+u^{-2}}{2.2}$

$+ \frac{u^3 + u^{-3}}{3 \cdot 3} + \&c. = \int \text{log. } u \times \frac{\dot{u}}{u} = \frac{1}{2} (\text{log. } u)^2$; hæc ſeries ut prædicitur ſemper divergit, ni u ſit impoſſibilis quantitas $x + \sqrt{(x^2 - 1)}$: log. impoſſibilis quantitatis denotat circularem arcum, &c.: & ſimiliter erui poſſunt omnes ſeries, quæ ex introductione ſinuum & coſinuum per hanc methodum deduci poſſunt.

9. Sint duæ æquationes $A = 0$ & $B = 0$, in quibus α in unâ ſimiliter involvitur ac β in alterâ; tum ſubtrahatur una æquatio de alterâ, & pro α & β ſcribantur $x + \sqrt{(x^2 - 1)}$ & $x - \sqrt{(x^2 - 1)}$; & $\overset{(b)}{s}$ & $\overset{(b)}{c}$ pro $\frac{(x + \sqrt{(x^2 - 1)})^b - (x - \sqrt{(x^2 - 1)})^b}{2\sqrt{(-1)}}$ & $\frac{(x + \sqrt{(x^2 - 1)})^b + (x + \sqrt{(x^2 - 1)})^b}{2}$, & reſultat æquatio, in quâ involvuntur ſinus & coſinus arcuum bz; e. g. ſint duæ æquationes $(a + b)^q = a^q + q a^{q-1} b + q \cdot \frac{q-1}{2} a^{q-2} b^2 + q \cdot \frac{q-1}{2} \cdot \frac{q-2}{3} a^{q-3} b^3 + \&c.$ & $(a + c)^q = a^q + q a^{q-1} c + q \cdot \frac{q-1}{2} a^{q-2} c^2 + \&c.$; ſubtrahatur poſterior æquatio de priori; deinde pro c ponatur eadem functio quantitatis $x + \sqrt{(x^2 - 1)}$ ac b ſit quantitatis $x - \sqrt{(x^2 - 1)}$; & facile transformari poteſt æquatio reſultans in æquationes ſinus & coſinus arcuum bz involventes.

Et ſimiliter transformari poſſunt prædictæ æquationes, &c. in æquationes, quæ ſunt conſimiles functiones quantitatum $x + \sqrt{(x^2 - 1)}$ & $x - \sqrt{(x^2 - 1)}$; vel quarumcunque aliarum datarum quantitatum; ducantur æquationes reſultantes in fluxiones, quarum fluentes ſunt functiones e ſingulis prædictis quantitatibus; reſultant ſeries, quarum ſummæ innoteſcunt e fluentibus quantitatum in æquationibus contentarum; & ſic deinceps.

PROB. XLV.

Invenire ſummam ſeriei $x + \frac{x^2}{2^2} + \frac{x^3}{3^2} +$ *&c. cum* $x = \frac{1}{2}$.

Erit log. $\frac{1}{1-x} = x + \frac{x^2}{2} + \frac{x^3}{3} +$ &c. in hâc æquatione pro x ſcribatur $\frac{v}{v-1}$ & reſultat log. $\frac{1}{1-\frac{v}{v-1}} =$ log. $(1-v) = \frac{v}{v-1} + \frac{v^2}{2(v-1)^2} + \frac{v^3}{3(v-1)^3} +$ &c. $= (1-v^{-1})^{-1} + \frac{(1-v^{-1})^{-2}}{2} + \frac{(1-v^{-1})^{-3}}{3} +$ &c.; in hâc æquatione pro $1-v$ ſcribatur u & reſultat log. $u = 1 - u^{-1} + \frac{(1-u^{-1})^2}{2} + \frac{(1-u^{-1})^3}{3} +$ &c. $= -(1-u) - \frac{(1-u)^2}{2} - \frac{(1-u)^3}{3} -$ &c., quæ prius inventa fuit $= u - u^{-1} - \frac{u^2 - u^{-2}}{2} + \frac{u^3 - u^{-3}}{3} -$ &c.; prima ſeries converget, cum $1 - u^{-1}$ minor ſit quam ± 1; ſecunda cum $1-u$ ſit etiam minor quam ± 1: ſed $\int. \frac{\dot{x}}{x}$ log. $\frac{1}{1-x} = x + \frac{x^2}{2^2} + \frac{x^3}{3^2} +$ &c., & $\int. \frac{\dot{x}}{1-x}$ log. $x = (1-x) + \frac{(1-x)^2}{2^2} + \frac{(1-x)^3}{3^2} +$ &c. $- (1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} +$ &c. $= A)$, & exinde conſtat log. $x \times$ log. $\frac{1}{1-x} = \int. \frac{\dot{x}}{x} \times$ log. $\frac{1}{1-x} + \int. \frac{\dot{x}}{1-x} \times$ log. $x = (x + \frac{x^2}{2^2} + \frac{x^3}{3^2} +$ &c.$) + (1 - x + \frac{(1-x)^2}{2^2} + \frac{(1-x)^3}{3^2} +$ &c. $- A)$; fingatur $x = \frac{1}{2}$, & reſultat log. $\frac{1}{2} \times$ log. $2 = 2 \times (\frac{1}{2} + \frac{\frac{1}{4}}{2^2} + \frac{\frac{1}{8}}{3^2} + \frac{\frac{1}{16}}{4^2} +$ &c.$) - (A = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} +$ &c.$)$ & exinde valor ſeriei $x + \frac{x^2}{4} + \frac{x^3}{9} +$ &c. cum $x = \frac{1}{2}$.

Et ex conſimilibus principiis detegi poteſt ſumma ſeriei $x + \frac{x^2}{2^2} + \frac{x^3}{3^2} +$ &c. $= \frac{2a^2}{3}$, $\frac{a^2}{3} - \frac{1}{2}(\log. 2)^2$, $\frac{4a^2}{15} - \left(\log. \frac{\surd(5)-1}{2}\right)^2$, vel $\frac{2}{15}a^2 - \left(\log. \frac{\surd(5)-1}{2}\right)^2$; prout x fit $= 1$, $\frac{1}{2}$, $\frac{3-\surd(5)}{2}$ vel $\frac{\surd(5)-1}{2}$.

THEOR. LII.

1. Erit $z+1 \,.\, z+2 \,.\, z+3 \ldots\ldots nz = 1 \,.\, 2 \,.\, 3 \,.\,.\, (n-1) \times (n+1) \,.\, n+2 \,.\, n+3 \ldots (2n-1) \times (2n+1) \,.\, 2n+2 \,.\, 2n+3 \,.\,.\, (3n-1) \times (3n+1) \,.\, 3n+2 \,.\, 3n+3 \ldots (4n-1) \times (4n+1) \,.\, 4n+2 \,.\, 4n+3$ &c. ad $(nz-1) \times n^z$; nam $\frac{1 \,.\, 2 \,.\, 3 \,.\,.\, nz}{1 \,.\, 2 \,.\, 3 \,.\,.\, z} = z+1 \,.\, z+2 \,.\, z+3 \,.\,.\, nz = \frac{1 \,.\, 2 \,.\, 3 \,.\,.\, nz}{n \,.\, 2n \,.\, 3n \,.\,.\, nz} \times n^z = 1 \,.\, 2 \,.\, 3 \,.\, n-1 \,.\, n+1 \,.\, n+2 \,.\,.\, 2n-1 \,.\, 2n+1 \,.\, 2n+2 \,.\,.\, 3n-1 \,.\, 3n+1 \ldots (nz-1) \times n^z$; ſit $n = 2$ & evadet $z+1 \,.\, z+2 \,.\, z+3 \ldots nz = 1 \cdot 3 \cdot 5 \cdot 7 \ldots (2z-1) \times 2^z$.

2. Sit z infinita quantitas & erit $z^z : (z-n)^z : 1 : 1 - n + \frac{n^2}{2} - \frac{n^3}{2 \cdot 3} + \frac{n^4}{2 \cdot 3 \cdot 4} -$ &c., & $z^z : (z+n)^z :: 1 : 1 + n + \frac{n^2}{2} + \frac{n^3}{1 \,.\, 2 \,.\, 3} + \frac{n^4}{1 \,.\, 2 \cdot 3 \cdot 4} +$ &c. unde tres quantitates $(z-n)^z$, z^z & $(z+n)^z$ erunt ultimo in ratione quantitatum α, 1 & β; ubi α & β ſunt numeri, qui correſpoudeant logarithmis n & $-n$. Erunt $z^z : (z+n)^z :: 1 : 1 + n + \frac{1}{2}n^2 + \frac{1}{6}n^3 +$ &c. etiamque $(z+n)^z : (z+n)^{z+n} :: 1 : (z+n)^n :: 1 : z^n$ & conſequenter $z^z : (z+n)^{z+n} :: 1 : (1 + n + \frac{1}{2}n^2 + \frac{1}{6}n^3 +$ &c.$)\, z^n$.

Et ſic facile deduci poſſunt innumeræ propoſitiones hujuſmodi, quæ oſtendunt limites, ad quos continuo vergunt datæ algebraicæ vel exponentiales quantitates, uſque donec z evadat infinita.

3. In genere, ſi dentur quæcunque irrationales quantitates, earum rationes inter ſe ad infinitam diſtantiam detegere.

Primo rejiciantur omnes quantitates in iis, quæ ad infinitam diſtantiam infinite minores ſint quam reliquæ, i. e. nihil valent per extractionem radicum, &c. deinde reducantur reliquæ in ſeries convergentes, & ex iis deduci poſſunt relationes ad infinitam diſtantiam quæſitæ.

4. 1. Datâ quantitate A involvente alias z, &c. infinite magnas; invenire annon A finita ſit: ſi in finito numero factorum contineatur A; tum ex ejus reductione ad ſeries ſecundum dimenſiones quantitatis z progredientes conſtat, annon quantitas A ſit finita.

2. Sint datæ æquationes, quæ vere conſtituuntur, cum z ſit quæcunque finita quantitas; tum etiam vere conſtituentur, cum z evadat infinita: & ſi in datis æquationibus pro z ſcribatur $z+n$ vel nz, &c., veræ erunt æquationes reſultantes: e. g. aſſumatur $1^2 . 2^2 . 3^2 .. z^2 \times \frac{e^{2z}}{z^{2z+1}} = 2\pi$, & exinde $1^2 . 2^2 .. z^2 \times (z+1)^2 \times \frac{e^{2z+2}}{(z+1)^{2z+3}} = 2\pi$, unde $2\pi \times (z+1)^2 \times \frac{e^2 \times z^{2z+1}}{(z+1)^{2z+3}} = 2\pi$, & $e^2 \times \frac{z^{2z+1}}{(z+1)^{2z+1}} = 1$, & conſequenter $\frac{(z+1)^{2z+1}}{z^{2z+1}} = 1 + 2 + \frac{4}{1.2} + \frac{8}{2.3} + \frac{16}{2.3.4} + \frac{32}{2.3.4.5} + \frac{64}{2.3.4.5.6} +$ &c. $= e^2$, aliter æquatio aſſumpta non poteſt eſſe vera.

3. Augeatur infinita quantitas (z) per (n) finitas quantitates a, b, c, &c.; & ſint quantitates reſultantes per earum ſubſtitutiones pro z in datâ quantitate A, B, C, D, &c.; tum erit $\alpha A + \beta B + \gamma C + \delta D +$ &c. $= 0$, cum $\alpha + \beta + \gamma + \delta +$ &c. $= 0$, & A ſit finita: e. g. ſit præcedens æquatio $1^2 . 2^2 . 3^2 . 4^2 .. z^2 \frac{e^{2z}}{z^{2z+1}} = 2\pi$, pro z ſcribatur $z+1$ & reſultat $1^2 . 2^2 . 3^2 .. z^2 \times \frac{e^{2z+2}}{(z+1)^{2z+1}} = 2\pi$, & exinde $1^2 . 2^2 . 3^2 ..$

$z^2 \times e^{2z} \times \left(\frac{e^2}{(z+1)^{2z+1}} - \frac{1}{z^{2z+1}}\right) = -1^2 . 2^2 . 3^2 \ldots z^2 \times e^{2z} \left(\left(1 + 2 + \frac{4}{1.2} + \frac{8}{1.2.3} + \&c. - e^2\right) z^{-2z-1} - \&c.\right)$, at ſit $\left(1 + 2 + \frac{4}{2} + \frac{8}{1.2.3} + \&c.\right) - e^2 = 0$, & ſubſequentes quantitates erunt infinite parvæ, cum z evadat infinita: ergo conſtat exemplum.

4. Inveniantur ſucceſſivi termini datæ quantitatis, & per principia prius tradita dijudicari poteſt, utrum ſeries ſit finita necne.

5. Sit fluxio $(a + bx^n)^m x^{rn-1} \dot{x}$, inveniatur fluens ($A$) hujuſce fluxionis inter valores 0 & $-\frac{a}{b}$ quantitatis x^n poſita; deinde per prob. 23. inveniatur fluens (P) fluxionis $(a + bx^n)^{m \pm h} x^{(r \pm i)n-1} \dot{x}$ inter prædictos valores, ubi h & i ſunt infinitæ quantitates; vel altera infinita, altera vero finita vel nulla: per eandem methodum inveniatur fluens (B) fluxionis $(a + bx^n)^s x^{tn-1} \dot{x}$ inter eoſdem valores poſita & exinde per prædictum prob. inveniatur fluens (Q) fluxionis $(a + bx^n)^{s \pm h} x^{(t \pm i)n-1} \dot{x}$; & ex æquando fluentes P & Q conſequitur relatio inter A & B quæ involvit quantitates infinite magnas: ducantur plures hujuſmodi quantitates in ſeſe, vel inveniantur fluentes prædictarum fluxionum ex diverſis modis; & ad eas applicentur conſimilia principia; & reſultant æquationes involventes quantitates infinite magnas, vel potius limites ad quos quantitates appropinquant prius quam quædam quantitates in iis contentæ evadant infinite magnæ.

6. Erit $1^2 \times 2^2 \times 3^2 \ldots z^2 \times \frac{e^{2z}}{z^{2z+1}} = 2\pi$ peripheria circuli, cujus radius eſt 1: & $(1 - m^2) \times (2^2 - m^2) \times (3^2 - m^2) \ldots (z^2 - m^2) \times \frac{e^{2z}}{z^{2z+1}} = \frac{m}{2} \times$ ſin. arc. mS; etiamque $(z + m) \times (z + m + 1) \times (z + m + 2) \ldots (2z + m) \times \frac{e^z}{2^{2z} z^z} = 2^{m-\frac{1}{2}}$.

PROB.

PROB. XLVI.

Datâ ſerie (P) $a + bx^n + cx^{2n} + dx^{3n} +$ &c. . . $Dx^{(m-3)n} + Cx^{(m-2)n} + Bx^{(m-1)n} + Ax^{mn} +$ &c. *in infinitum progrediente, ubi* a, b, c, d, &c. D, C, B, A, &c. *ſint quæcunque datæ functiones quantitatis* z *diſtantiæ a primo ſeriei termino; invenire legem, quam obſervant coefficientes ejus* (P) *quadrati, &c.*

Aſſumatur A coefficiens generalis termini cujuſcunque x^{mn}, tum erit $2(aA + bB + cC +$ &c.) coefficiens termini x^{mn} ſeriei, quæ ſit quadratum ſeriei P: ſint vero l & L coefficientes quorumcunque terminorum x^{rn} & $x^{(m-r)n}$; tum erit Ll generalis terminus ſeriei $aA + bB + cC +$ &c. Ex hoc vero generali termino dato deducatur ſumma ſeriei in infinitum, quæ correcta præbet ſeriei ſummam $aA + bB + cC +$ &c. i. e. generalem terminum ſeriei $(P^2) = a^2 + 2abx^2 +$ &c.

2. Ex iiſdem principiis etiam inveniri poteſt generalis terminus ſeriei, quæ ſit $P^s = a^s + sa^{s-1}bx^2 +$ &c. ſi modo ſummæ ſerierum exinde ortarum in finitis terminis exprimi poſſint: in his autem ſeriebus continentur una, duæ vel plures variabiles quantitates; e. g. ſit $s = 2$, & una ſolummodo in ſerie reſultante continetur variabilis quantitas; ſit $s = 3$, & una vel duæ variabiles quantitates in ſeriebus reſultantibus continentur; ſit $s = 4$, & una vel duæ vel tres variabiles quantitates in ſeriebus reſultantibus continentur; & ſic deinceps. Facile conſtant generales termini ad has ſeries, quæcunque ſit poteſtas s. Et ſic de lege cujuſcunque functionis hujuſce ſeriei vel plurium ſerierum inveſtigandâ.

NOVA METHODUS DIFFERENTIARUM.

THEOR. LIII.

Sit data ſeries in omni parte celeriter convergens $ax + bx^2 + cx^3 + dx^4 + ex^5 +$ &c. in eâ pro x ſcribantur ſucceſſive p, $2p$, $3p$, $4p$, $5p$, &c.;

$5p$, &c.; quantitates reſultantes erunt $ap + bp^2 + cp^3 +$ &c. $= S^1$, $2ap + 4bp^2 + 8cp^3 +$ &c. $= S^2$, $3ap + 9bp^2 + 27cp^3 +$ &c. $= S^3$, $4ap + 16bp^2 + 64cp^3 +$ &c. $= S^4$, &c.; & dentur prædictæ ſummæ S^1, S^2, S^3, S^4 & S^5; invenire ſummam S^6 prope.

Fingatur $\alpha S^1 + \beta S^2 + \gamma S^3 + \delta S^4 + \varepsilon S^5 = \alpha(ap + bp^2 + cp^3 +$ &c.$) + \beta(2ap + 4bp^2 + 8cp^3 +$ &c.$) + \gamma(3ap + 9bp^2 + 27cp^3 +$ &c.$) + \delta(4ap + 16bp^2 + 64cp^3 +$ &c.$) +$ &c. $= (\alpha + 2\beta + 3\gamma + 4\delta +$ &c.$)ap + (\alpha + 4\beta + 9\gamma + 16\delta +$ &c.$)bp^2 + (\alpha + 8\beta + 27\gamma + 64\delta +$ &c.$)cp^3 + (\alpha + 16\beta + 81\gamma + 256\delta +$ &c.$)dp^4 +$ &c. $= S^6 = 6ap + 36bp^2 + 216cp^3 + 1296dp^4 +$ &c.

Fiant correſpondentes termini hujuſce æquationis reſpective inter ſe æquales, & exinde reſultant æquationes

	Differentiæ	
$\alpha+2\beta+3\gamma+4\delta+5\varepsilon=6$ $\alpha+4\beta+9\gamma+16\delta+25\varepsilon=36$ $\alpha+8\beta+27\gamma+64\delta+125\varepsilon=216$ $\alpha+16\beta+81\gamma+256\delta+625\varepsilon=1296$ $\alpha+32\beta+243\gamma+1024\delta+3125\varepsilon=7776$	$2\beta+6\gamma+12\delta+20\varepsilon=30=\lambda$ $4\beta+18\gamma+48\delta+100\varepsilon=180=\mu$ $8\beta+54\gamma+192\delta+500\varepsilon=1080=\nu$ $16\beta+162\gamma+768\delta+2500\varepsilon=6480=\xi$	$6\gamma+24\delta+60\varepsilon=120=\mu-2\lambda=\pi$ $18\gamma+96\delta+300\varepsilon=720=\nu-2\mu=\rho$ $54\gamma+384\delta+1500\varepsilon=4320=\xi-2\nu=\sigma$ $24\delta+120\varepsilon=360=\rho-3\pi=\tau$ $96\delta+600\varepsilon=2160=\sigma-3\rho=\upsilon$ $120\varepsilon=720=\upsilon-4\tau$

unde $\varepsilon = 6$, $\delta = -15$, $\gamma = 20$, $\beta = -15$, $\alpha = 6$; & conſequenter $S^6 = 6S^5 - 15S^4 + 20S^3 - 15S^2 + 6S^1$ prope; & in genere erit $S^n = nS^{n-1} - n.\frac{n-1}{2}S^{n-2} + n.\frac{n-1}{2}.\frac{n-2}{3}S^{n-3} - n.\frac{n-1}{2}.\frac{n-2}{3}.\frac{n-3}{4}S^{n-4} \ldots \pm n.\frac{n-1}{2}.\frac{n-2}{3}S^3 \mp n.\frac{n-1}{2}S^2 \pm nS^1$ prope; ubi coefficientes inveniuntur eædem ac in ſerie $-e^n + ne^{n-1}f - n.\frac{n-1}{2}e^{n-2}f^2 +$ &c. $= -(e-f)^n$, ſi modo primus & ultimus terminus abjiciantur.

Cor. Sit $n = 5$, reſultat $S^5 = 5S^4 - 10S^3 + 10S^2 - 5S^1 = 5(S^4 - S^1) - 10(S^3 - S^2)$; ſint S^1, S^2, S^3 & S^4 logarithmi rationum $r : r + p$, $r : r + 2p$, $r : r + 3p$, $r : r + 4p$ reſpective; tum erit S^5 logar. rationis $r : r + 5p$ prope: in genere erit $S^n = n(S^{n-1} \pm S^1) - n.\frac{n-1}{2}(S^{n-2} \pm S^2) + n.\frac{n-1}{2}.\frac{n-2}{3}(S^{n-3} \pm S^3) -$ &c.

1. 2. Iiſdem literis eaſdem quantitates denotantibus, erit $S^{m+n} = \frac{m+n\,.\,n+m-1\,.\,m+n-2\,\ldots\,m+2}{1\,.\,2\,.\,3\,.\,,\,.\,n-1} \times S^{n-1} - \frac{n-1}{1} A \times \frac{m+1}{m+2} S^{n-2} + \frac{n-2}{2} \times B \times \frac{m+2}{m+3} S^{n-3} - \frac{n-3}{3} \times C \times \frac{m+3}{m+4} S^{n-4} + \frac{n-4}{4} \times D \times \frac{m+4}{m+5} S^{n-5} - \frac{n-5}{5} \times E \times \frac{m+5}{m-6} \times S^{n-6} +$ &c. uſque ad terminum S^1 prope, ubi literæ A, B, C, D, E,&c. præcedentes coefficientes reſpective denotant, & m eſt integer numerus.

THEOR. LIV.

1. Si in præcedente ſerie ſcribantur $+p$, $-p$, $2p$, $-2p$, $3p$, $-3p \ldots np$, $-np$; exorientur quantitates $ap + bp^2 + cp^3 + dp^4 +$ &c. $= S^1$, $-ap + bp^2 - cp^3 + dp^4 -$ &c. $= S^{-1}$, $2ap + 4bp + 8cp^3 +$ &c. $= S^2$, $-2ap + 4bp^2 - 8cp^3 +$ &c. $= S^{-2}$, $3ap + 9bp^2 + 27cp^3 +$ &c. $= S^3$, $-3ap + 9bp^2 - 27cp^3 +$ &c. $= S^{-3}$, &c. tum erit ſumma $S^m = \frac{m\,.\,(m^2-1)\,.\,(m^2-4)\,.\,(m^2-9)\,.\,(m^2-16)\ldots(m^2-(n-1)^2)\,.\,(m-n)}{n\,.\,(n^2-1)\,.\,(n^2-4)\,.\,(n^2-9)\ldots(n^2-(n-1)^2)\times 2n = 1\,.\,2\,.\,3\,.\,4\ldots 2n} S^{-n} + A \times \frac{m+n}{m-n} S^{+n} - \frac{2n}{1} \times B \times \frac{m-n}{m+(n-1)} S^{-n+1} - C \times \frac{m+(n-1)}{m-(n-1)} S^{n-1} + \frac{2n-1}{2} D \times \frac{m-(n-1)}{m+(n-2)} S^{-n+2} + \frac{m+(n-2)}{m-(n-2)} E S^{n-2} - \frac{2n-2}{3} F \times \frac{m-(n-2)}{m+(n-3)} S^{-n+3} - G \times \frac{m+(n-3)}{m-(n-3)} S^{n-3} +$ &c. prope; ubi literæ A, B, C, D, &c. præcedentes coefficientes reſpective denotant: generaliter vero coefficientes terminorum S^{-n+s} & S^{n-s} erunt reſpective $\frac{2n-s+1}{s} \times L \times \frac{m-(n-s+1)}{m+(n-s)}$ & $M \times \frac{m \pm (n-s)}{m - (n-s)}$, ubi literæ L & M earum præcedentes coefficientes reſpective denotant, quæ coefficientes affirmativæ vel negativæ aſſumendæ ſunt, prout s ſit par vel impar numerus.

2. Si pro x in prædictâ ſerie ſcribantur p, $-p$, $2p$, $-2p$, $3p$, $-3p \ldots\ldots (n-1)p$, $-(n-1)p$, np reſpective; tum erit

$$S^m = \frac{m\,.\,(m^2-1)\,.\,(m^2-4)\,.\,(m^2-9)\ldots(m^2-(n-1)^2)}{1\,.\,2\,.\,3\,.\,4\ldots(2n-1)}S^n - A \times \frac{m-n}{m+(n-1)}S^{-n+1} - \frac{2n-1}{1} \times B \times \frac{m+(n-1)}{m-(n-1)}S^{n-1} + C \times \frac{m-(n-1)}{m+(n-2)}S^{-n+2} + \frac{2n-2}{2} \times D \times \frac{m+(n-2)}{m-(n-2)}S^{n-2} - \&c.$$

prope, ubi A, B, C, &c. præcedentes coefficientes ut antea denotant, & generaliter coefficientes terminorum S^{-n+s} & S^{n-s} erunt reſpective $L \times \frac{m-(n-s+1)}{m+n-s}$ & $\frac{2n-s}{s} \times M \times \frac{m+(n-s)}{m-(n=s)}$; L & M earum præcedentes coefficientes reſpective denotantibus, quæ autem coefficientes aſſumendæ ſunt affirmativæ vel negativæ, prout s ſit par vel impar numerus.

Cor. 1. Si m ſit negativa quantitas; tum in hiſce ſeriebus pro m ſcribenda eſt $-m$, & reſultat quantitas quæſita.

Cor. 2. Cum numerus terminorum S^1, S^2, S^3, &c. detur, tum hæc methodus plerumque magis converget quam ea in theor. præcedente tradita; & conſimilis propoſitio etiam vera eſt de ſeriebus in theor. ſubſeq. contentis.

THEOR. LV.

In ſerie prædictâ $ax + bx^2 + cx^3 + dx^4 + \&c.$ pro x ſcribantur reſpective p, q, r, s, t, &c., & m; & dicantur ſummæ ſerierum reſultantium reſpective S^p, S^q, S^r, S^s, &c., & S^m; tum erit

$$S^m = \frac{m \times (m-q) \times (m-r) \times (m-s) \times (m-t) \times \&c.}{p \times (p-q) \times (p-r) \times (p-s) \times (p-t) \times \&c.}S^p + \frac{m\,.\,(m-p) \times (m-r) \times (m-s) \times (m-t) \times \&c.}{q\,.\,(q-p)\,.\,(q-r)\,.\,(q-s)\,.\,(q-t) \times \&c.} \times S^q + \frac{m\,.\,(m-p)\,.\,(m-q)\,.\,(m-s)\,.\,(m-t) \times \&c.}{r\,.\,(r-p)\,.\,(r-q)\,.\,(r-s)\,.\,(r-t) \times \&c.} \times S^r + \&c. = am + bm^2 + cm^3 + dm^4 + \&c.$$

prope; hìc vero animadveradvertendum eſt coefficientem cujuſcunque termini S^{π} eſſe $\frac{m\,.\,(m-p)\,.\,(m-q)\,.\,(m-r)\,.\,(m-s)\,.\,\&c.}{\pi\,.\,(\pi-p)\,.\,(\pi-q)\,.\,(\pi-r)\,.\,(\pi-s)\,.\,\&c.}$, in cujus numeratore haud continetur factor $m-\pi$.

Cor. Hinc facile deduci poſſunt quædam arithmeticæ propoſitiones, e. g. ſint quæcunque quantitates p, q, r, s, t, &c.; & m; tum per theorema $\frac{m\,.\,(m-q)\,.\,(m-r)\,.\,\&c.}{p\,.\,(p-q)\,.\,(p-r)\,.\,\&c.} \times S^{p} + \frac{m\,.\,(m-p)\,.\,(m-r)\,.\,\&c.}{q\,.\,(q-p)\,.\,(q-r)\,.\,\&c.} S^{q} + \frac{m\,.\,(m-p)\,.\,(m-q)\,.\,\&c.}{r\,.\,(r-p)\,.\,(r-q)\,.\,\&c.} S^{r} + \&c. = am + bm^{2} + cm^{3} + \&c.$ prope; unde $\frac{m\,.\,(m-q)\,.\,(m-r)\,.\,\&c.}{p\,.\,(p-q)\,.\,(p-r)\,.\,\&c.} \times (ap + bp^{2} + cp^{3} + \&c.) + \frac{m\,.\,(m-p)\,.\,(m-r)\,.\,\&c.}{q\,.\,(q-p)\,.\,(q-r)\,.\,\&c.} \times (aq + bq^{2} + cq^{3} + \&c.) + \frac{m\,.\,(m-p)\,.\,(m-q)\,.\,\&c.}{r\,.\,(r-p)\,.\,(r-q)\,.\,\&c.} \times (ar + br^{2} + cr^{3} + \&c.) + \&c. = am + bm^{2} + c^{3} + \&c.$, & conſequenter ex æquatis inter ſe correſpondentibus terminis $\frac{m : (m-q)\,.\,(m-r)\,.\,(m-s)\,.\,\&c.}{p(p-q)\,.\,(p-r)\,.\,(p-s)\,.\,\&c.} (A)p + \frac{m\,.\,(m-p)\,.\,(m-r)\,.\,(m-s)\,.\,\&c.}{q(q-p)\,.\,(q-r)\,.\,(q-s)\,.\,\&c.} (B)q + \frac{m\,.\,(m-p)\,.\,(m-q)\,.\,(m-s)\,.\,\&c.}{r(r-p)\,.\,(r-q)\,.\,(r-s)\,.\,\&c.} (C)r + \&c. = pA + qB + rC + \&c. = m$; & in genere $p^{i}A + q^{i}B + r^{i}C + \&c. = m^{i}$; ſi modo i ſit integer & non major quam numerus quantitatum p, q, r, s, t, &c.

Summa fractionum $\frac{P}{\alpha-\beta\,.\,\alpha-\gamma\,.\,\alpha-\delta\,.\,\alpha-\varepsilon\,.\,\&c.} + \frac{Q}{\beta-\alpha\,.\,\beta-\gamma\,.\,\beta-\delta\,.\,\beta-\varepsilon\,.\,\&c.} + \frac{R}{\gamma-\alpha\,.\,\gamma-\beta\,.\,\gamma-\delta\,.\,\gamma-\varepsilon\,.\,\&c.} + \frac{S}{\delta-\alpha\,.\,\delta-\beta\,.\,\delta-\gamma\,.\,\delta-\varepsilon\,.\,\&c.} + \&c. = \pi$; ubi P, Q, R, S, &c. ſunt rationales & integrales functiones quantitatum α, β, γ, δ, &c.;

i. e. functiones, in quibus nullus negativus vel fractionalis index continetur: & in quantitate P ſimiliter involvuntur omnes quantitates β, γ, δ, &c. præter unam α; & ſic in quantitatibus Q, R, S, &c. ſimiliter involvuntur omnes prædictæ quantitates præter unam β, γ, δ, &c. reſpective: & in quantitate P ſimiliter involvuntur quantitates β, γ, δ, ε, &c.; ac quantitates α, γ, δ, ε, &c. in Q; & α, β, δ, ε, &c. in R; α, β, γ, ε, &c. in S, &c.: & ultimo functio quantitatis α in P eadem eſt ac functio quantitatis β in Q, & utraque eadem ac functio quantitatis γ in C, & ſic deinceps: tum, ſi numerus literarum α, β, γ, δ, &c. ſit n; & dimenſiones literarum α, β, γ, &c. in quantitatibus P, Q, R, S, &c. contentarum ſint m, $m-1$, $m-2$, &c.; ubi n & m ſunt integri numeri; erit π functio rationalis & integralis quantitatum α, β, γ, δ, &c., in quâ ſimiliter involvuntur quantitates α, β, γ, δ, &c.; cujus dimenſiones quantitatum in eâ contentarum haud exſuperant $m-n$; ſi m ſit minor quam n, tum erit $\pi = 0$.

Facile ex hiſce vel pluribus ſimplicibus æquationibus erui poſſunt plures hujuſmodi propoſitiones.

THEOR. LVI.

Sit ſeries celeriter convergens $a x^b + b x^{b+k} + c x^{b+2k} + d x^{b+3k} + \&c.$, in eâ pro x ſcribantur reſpective p, q, r, s, t, &c. π, & m; & dicantur ſummæ exinde reſultantes reſpective S^p, S^q, S^r, S^s, &c. & S^m; tum erit

$$S^m = \frac{m^b \times (m^k - q^k) \times (m^k - r^k) \times (m^k - s^k) \times \&c.}{p^b (p^k - q^k) \times (p^k - r^k) \times (p^k - s^k) \times \&c.} \times S^p + \frac{m^b \times (m^k - p^k)}{q^b (q^k - p^k) \times}$$

$$\frac{\times (m^k - r^k) \times (m^k - s^k) \times \&c.}{(q^k - r^k) \times (q^k - s^k) \times \&c.} \times S^q + \frac{m^b \times (m^k - p^k) \times (m^k - q^k) \times}{r^b (r^k - p^k) \times (r^k - q^k) \times}$$

$$\frac{(m^k - s^k) \times \&c.}{(r^k - s^k) \times \&c.} S^r + \&c. = a m^b + b m^{b+k} + c m^{b+2k} + d m^{b+3k} + \&c.$$

prope; coefficiens cujuſcunque termini S^π erit $\frac{m^b \times (m^k - p^k) \times (m^k - q^k)}{\pi^b (\pi^k - p^k) \times (\pi^k - q^k)}$ $\frac{\times (m^k - r^k) \times \&c.}{\times (\pi^k - r^k) \times \&c.}$, in cujus numeratore haud continetur factor $m^k - \pi^k$.

Cor. 1. Hinc conſtat ſummam e ſingulis quantitatibus $\frac{(m^k-q^k)\times(m^k-r^k)\times(m^k-s^k)\times\&c.}{(p^k-q^k)\times(p^k-r^k)\times(p^k-s^k)\times\&c.}(A)+\frac{(m^k-p^k)\times(m^k-r^k)\times(m^k-s^k)\times\&c.}{(q^k-p^k)\times(q^k-r^k)\times(q^k-s^k)\times\&c.}(B)+\frac{(m^k-p^k)\times(m^k-q^k)\times(m^k-s^k)\times\&c.}{(r^k-p^k)\times(r^k-q^k)\times(r^k-s^k)\times\&c.}(C)+\&c.=1$; etiamque $p^{fk}A+q^{fk}B+r^{fk}C+s^{fk}D+\&c.=m^{fk}$, ſi modo f ſit integer & non major quam numerus quantitatum p, q, r, s, &c.

Cor. 2. Ducantur factores e ſingulis numeratoribus in ſeſe; e. g. $(m^k-q^k)\times(m^k-r^k)\times(m^k-s^k)\times(m^k-t^k)\times\&c.=\pm q^k r^k s^k t^k\&c.\mp(q^k r^k s^k\&c.+q^k s^k t^k\&c.+r^k s^k t^k\&c.+\&c.)m^k\pm(q^k r^k\&c.+q^k s^k\&c.+r^k s^k\&c.+q^k t^k\&c.)m^{2k}\mp(q^k\&c.+r^k\&c.+s^k\&c.+t^k\&c.)m^{3k}\pm\&c.$; & ſic de ſingulis reliquis numeratoribus: in his numeratoribus reſultantibus rejiciantur omnes termini, in quibus haud inveniuntur eædem (l) dimenſiones quantitatis m^k, & pro ſingulis numeratoribus aſſumatur coefficiens quantitatis m^{lk}; tum erit ſumma ſingularum fractionum reſultantium nihilo æqualis; ni $l=0$, in quo caſu erit ſumma prædicta $=1$.

Omnia hæc facile conſtant e ſcribendo pro x in aſſumptâ quantitate ejus valores p, q, r, s, &c. ſucceſſive.

Eadem etiam perfici poſſunt pro ſeriebus diverſarum formularum.

E libro tertio conſtat, ut ſeries hoc modo deductæ plerumque cæteris paribus magis celeriter convergent, cum quantitates interpolandæ aſſumantur in quâdam geometricâ quam in quâvis aliâ ratione.

In prob. hujuſce generis reſolvendis primum inveniendi ſunt termini tabulæ a quæſito haud longe diſtantes; inter quos nulli evadunt vel infiniti, vel nihilo æquales.

Hinc facile erui poſſunt quam proxime areæ vel centra gravitatis, &c. curvarum & ſolidorum a datis ordinatis & ſe invicem haud longe diſtantia, &c.

THEOR. LVII.

1. In quantitate y quæſitâ ſint a, b, c, d, e, &c. ſucceſſivi valores quantitatis x; & S^a, S^b, S^c, S^d, S^e, &c. eorum correſpondentes valores quantitatis y; tum poteſt eſſe

$$y = \frac{(x-b)(x-c)(x-d)(x-e)\,\&c.}{(a-b)(a-c)(a-d)(a-e)\,\&c.} \times S^a + \frac{(x-a)\times(x-c)(x-d)(x-e)\,\&c.}{(b-a)(b-c)(b-d)(b-e)\,\&c.} \times S^b + \frac{(x-a)(x-b)\times(x-d)(x-e)\,\&c.}{(c-a)(c-b)(c-d)(c-e)\,\&c.} S^c + \frac{(x-a)(x-b)(x-c)\times(x-e)\,\&c.}{(d-a)(d-b)(d-c)(d-e)\,\&c.} \times S^d + \frac{(x-a)(x-b)(x-c)(x-d)\times\&c.}{(e-a)(e-b)(e-c)(e-d)\,\&c.} \times S^e + \&c.$$

2. Aliter: Eadem quantitas y ſic exprimi poteſt

$$y = S^a + (x-a)\left(\frac{1}{a-b} S^a + \frac{1}{b-a} S^b\right) + (x-a)(x-b)\left(\frac{1}{a-b} \times \frac{1}{a-c} \times S^a + \frac{1}{b-a} \times \frac{1}{b-c} S^b + \frac{1}{c-a} \times \frac{1}{c-b} S^c\right) + (x-a)(x-b)(x-c)\left(\frac{1}{a-b} \times \frac{1}{a-c} \times \frac{1}{a-d} S^a + \frac{1}{b-a} \times \frac{1}{b-c} \times \frac{1}{b-d} S^b + \frac{1}{c-a} \times \frac{1}{c-b} \times \frac{1}{c-d} S^c + \frac{1}{d-a} \times \frac{1}{d-b} \times \frac{1}{d-c} S^d\right) + (x-a)(x-b)(x-c)(x-d)\left(\frac{1}{a-b} \times \frac{1}{a-c} \times \frac{1}{a-d} \times \frac{1}{a-e} S^a + \frac{1}{b-a} \times \frac{1}{b-c} \times \frac{1}{b-d} \times \frac{1}{b-e} \times S^b + \frac{1}{c-a} \times \frac{1}{c-b} \times \frac{1}{c-d} \times \frac{1}{c-e} S^c + \frac{1}{d-a} \times \frac{1}{d-b} \times \frac{1}{d-c} \times \frac{1}{d-e} S^d + \frac{1}{e-a} \times \frac{1}{e-b} \times \frac{1}{e-c} \times \frac{1}{e-d} S^e\right) + \&c.$$

PROB. XLVII.

Aſſumatur data quantitas $A + Bx + Cx^2 + Dx^3 + \ldots + Lx^{n-1} = y$, & dentur (n) valores $\alpha, \beta, \gamma, \delta, \ldots \chi$ quantitatis z, qui vere correſpondent (n) valoribus $\pi, \rho, \sigma, \tau, \ldots \phi$ quantitatis y; & e regulâ prius traditâ deduci poſſunt; i. e. ſint $A + B\alpha + C\alpha^2 + D\alpha^3 + \ldots + L\alpha^{n-1} = \pi$, $A + B\beta + C\beta^2 + D\beta^3 + \ldots + L\beta^{n-1} = \rho$, $A + B\gamma + C\gamma^2 + D\gamma^3 + \ldots L\gamma^{n-1} = \sigma$, &c., $A + B\chi + C\chi^2 + D\chi^3 \ldots + L\chi^{n-1} = \phi$; dentur etiam (m) valores a, b, c, d, &c. quantitatis (x), quibus correſpondent (m) valores P, Q, R, S, &c. quantitatis y; ſed cum regulâ prius traditâ, viz. $A + Bx + Cx^2 + Dx^3 +$ &c. $= y$ haud concordant; eorum errores ſint reſpective p, q, r, s, &c.; i. e. ſint $A + Ba + Ca^2 + Da^3 + \ldots + La^{n-1} = P - p$, $A + Bb + Cb^2 + Db^3 \ldots + Lb^{n-1} = Q - q$, $A + Bc + Cc^2 + Dc^3 \ldots Lc^{n-1} = R - r$, &c.: corrigere aſſumptam quantitatem $A + Bx + Cx^2 + Dx^3 + \ldots + Lx^{n-1} = y$.

Scribantur $\dfrac{p}{(a-\alpha)\times(a-\beta)\times(a-\gamma)\ldots(a-\chi)} = M$, $\dfrac{q}{(b-\alpha)\times(b-\beta)\times(b-\gamma)\times(b-\delta)\ldots\times(b-\chi)} = N$, $\dfrac{r}{(c-\alpha)\times(c-\beta)\times(c-\gamma)\ldots(c-\chi)} = O$, $\dfrac{s}{(d-\alpha)\times(d-\beta)\times(d-\gamma)\ldots(d-\chi)} = P$, $\dfrac{t}{(e-\alpha)\times(e-\beta)\times\text{\&c.}} = Q$, &c.

Quantitas pro correctione aſſumatur

$$(x-\alpha)\times(x-\beta)\times(x-\gamma)\times(x-\delta)\times\ldots\times(x-\chi)\Big(M + (x-a)\Big(\frac{1}{a-b}M + \frac{1}{b-a}N\Big) + (x-a)\times(x-b)\Big(\frac{1}{(a-b)\times(a-c)}M + \frac{1}{(b-a)\times(b-c)}N + \frac{1}{(c-a)\times(c-b)}O\Big) + (x-a)\times(x-b)\times(x-c)\Big(\frac{1}{(a-b)\times(a-c)\times(a-d)}M + \frac{1}{(b-a)\times(b-c)\times(b-d)}N + \frac{1}{(c-a)\times(c-b)\times(c-d)}O + \frac{1}{(d-a)\times(d-b)\times(d-c)}P\Big) + (x-a)\times(x-b)\times(x-c)\times(x-d)\Big(\frac{1}{(a-b)\times(a-c)\times(a-d)\times(a-e)}M + \frac{1}{(b-a)\times(b-c)\times(b-d)}$$

$\frac{}{(b-d)\times(b-e)}N + \frac{1}{(c-a)\times(c-b)\times(c-d)\times(c-e)}O +$ $\frac{1}{(d-a)\times(d-b)\times(d-c)\times(d-e)}P + \frac{1}{(e-a)\times(e-b)\times(e-c)\times(e-d)}$ $Q) +$ &c.) Lex, quam obſervant ſucceſſive coefficientes prædictorum terminorum, erit $\frac{1}{(a-b)\times(a-c)\times(a-d)\times(a-e)\times \&c.}$, $\frac{1}{(b-a)\times(b-c)\times(b-d)\times(b-e)\times \&c.}$, $\frac{1}{(c-a)\times(c-b)\times(c-d)\times(c-e)\times \&c.}$, $\frac{1}{(d-a)\times(d-b)\times(d-c)\times(d-e)\times \&c.}$, $\frac{1}{(e-a)\times(e-b)\times(e-c)\times(e-d)\times \&c.}$, &c.: termini autem ſucceſſivi ſunt M, N, O, P, Q, R, &c.; etiamque ſucceſſive ducuntur reſultantes quantitates in $(x-a)$, $(x-a)\times(x-b)$, $(x-a)(x-b)(x-c)$, $(x-a)(x-b)(x-c)(x-d)$, &c.

In multis caſibus præſtat, ut ex repetitis operationibus per primam correctionem in hâc regulâ traditam, viz. prima correctio $= \frac{(x-\alpha)\times(x-\beta)\times(x-\gamma)\,..\,(x-\chi)}{(a-\alpha)\times(a-\beta)\times(a-\gamma)\,..\,(a-\chi)} \times p$, ſecunda $= \frac{(x-\alpha)\times}{(b-\alpha)\times}$ $\frac{(x-\beta)\times(x-\gamma)\,..\,(x-\chi)\times(x-a)}{(b-\beta)\times(b-\gamma)\,..\,(b-\chi)\times(b-a)} \times q$, &c. detegantur novæ correctiones, uſque donec reſultat correctio quæſita.

2. Sint π, ρ, σ, τ, &c. quantitates, quæ omnes erunt nihilo æquales, cum $x = \alpha$ vel β vel γ vel δ, &c.; etiamque erunt reſpective P, R, S, T, &c., cum x ſit reſpective a, b, c, d, &c.: ſit λ quantitas nihilo æqualis, cum $x = b$ vel c vel d, &c. at $= L$, cum $x = a$: & ſimiliter ſit $\mu = o$, cum $x = a$, vel c vel d, &c., at $= m$, cum $x = b$; & ſic ſit $\nu = o$, cum $x = a$ vel b vel d, &c., at $= n$, cum $x = c$; & ſic deinceps; deinde aſſumi poteſt correctio quæſita $= \frac{\pi}{P} \times \frac{\lambda}{L} \times p$ $+ \frac{\rho}{R} \times \frac{\mu}{m} \times q + \frac{\sigma}{S} \times \frac{\nu}{n} \times r +$ &c.

Ex principiis hìc traditis, &c. & datis n valoribus datæ quantitatis y correſpondentibus, & erroribus (m) valorum prædictæ quantitatis; cum duæ vel plures variabiles $(x, y, z, \&c.)$ quantitates in datâ æquatione contineantur; tum ex aſſumptis $(m+n)$ æquationibus hujuſce generis $a + bx + cz + \&c. + dx^2 + ezx + fz^2 + \&c. + gx^3 + hx^2z + \&c. + \&c. = y$, quæ correſpondent $(m+n)$ diverſis correſpondentibus valoribus quantitatum $(x, y, z, \&c.)$ erui poſſunt $(m+n)$ coefficientes a, b, c, &c. quæſitæ.

METHODUS CORRESPONDENTIUM VALORUM.

PROB. XLVIII.

1. Sint a, b, c, d, &c. dati valores quantitatis x, cujus ſint S^a, S^b, S^c, S^d, &c. correſpondentes valores quantitatis y: invenire functionem quantitatis (x), quæ pro valoribus $(a, b, c, d, \&c.)$ quantitatis (x) habet S^a, S^b, S^c, S^d, &c. correſpondentes valores quantitatis (y).

Aſſumatur $$y = \frac{(x-b)\times(x-c)\times(x-d)\times(x-e)\times\&c.}{(a-b)\times(a-c)\times(a-d)\times(a-e)\times\&c.}\times S^a$$ $$+\frac{(x-a)\times(x-c)\times(x-d)\times(x-e)\times\&c.}{(b-a)\times(b-c)\times(b-d)\times(b-e)\times\&c.}\times S^b + \frac{(x-a)\times(x-b)\times(x-d)\times(x-e)\times\&c.}{(c-a)\times(c-b)\times(c-d)\times(c-e)\times\&c.}\times S^c + \frac{(x-a)\times(x-b)\times(x-c)\times(x-e)\times\&c.}{(d-a)\times(d-b)\times(d-c)\times(d-e)\times\&c.}$$ $$\times S^d + \&c.;$$ tum evadent valores quantitatis (y) reſpective S^a, S^b, S^c, S^d, &c.: cum eorum correſpondentes valores quantitatis (x) fiant a, b, c, d, &c. reſpective.

2. Iiſdem poſitis; cum x evadat a, b, c, d, &c.; tum A, B, C, D, &c. evadat $\alpha, \beta, \gamma, \delta$, &c. reſpective; aſſumatur $$y = \frac{A}{\alpha}\times\frac{(x-b)\times(x-c)\times(x-d)\times\&c.}{(a-b)\times(a-c)\times(a-d)\times\&c.}\times S^a + \frac{B}{\beta}\times\frac{(x-a)\times(x-c)\times(x-d)\times\&c.}{(b-a)\times(b-c)\times(b-d)\times\&c.}\times S^b +$$ $$+\frac{C}{\gamma}\times\frac{(x-a)\times(x-b)\times(x-d)\times\&c.}{(c-a)\times(c-b)\times(c-d)\times\&c.}\times S^c + \frac{D}{\delta}\times\frac{(x-a)\times(x-b)}{(d-a)\times(d-b)}$$

× (x

$\frac{\times (x-c) \times \&c.}{\times (d-c) \times \&c.} \times S^{d} + \&c.$; in quo caſu valores quantitatis (y) erunt etiam reſpective S^{a}, S^{b}, S^{c}, S^{d}, &c., cum valores quantitatis (x) ſint a, b, c, d, &c.: e. g. ſint A, B, C, D, &c. reſpective x, tum erit $\alpha = a$, $\beta = b$, $\gamma = c$, $\delta = d$, &c.; unde $y = \frac{x \times (x-b) \times (x-c) \times (x-d) \times \&c.}{a \times (a-b) \times (a-c) \times (a-d) \times \&c.} \times S^{a} + \frac{x \times (x-a) \times (x-c) \times (x-d) \times \&c.}{b \times (b-a) \times (b-c) \times (b-d) \times \&c.} \times S^{b} + \frac{x \times (x-a) \times}{c \times (c-a) \times}$ $\frac{(x-b) \times (x-d) \times \&c.}{(c-b) \times (c-d) \times \&c.} \times S^{c} + \&c.$; quæ eadem erit ac quantitas e methodo differentiarum deducta.

THEOR. LVIII.

Quantitates aſſumptæ A, B, C, D, &c. fiant $\alpha, \beta, \gamma, \delta$, &c.; cum x evadat a, b, c, d, &c. reſpective: deinde aſſumantur quantitates p, q, r, s, &c.; quæ nihilo ſunt æquales, cum $x = a$; etiamque p', q', r', s', &c. quæ nihilo ſunt æquales, cum $x = b$; & ſimiliter p'', q'', r'', s'', &c. nihilo æquales, cum $x = c$; & ſic de quantitatibus p''', q''', r''', s''', &c.; p'''', q'''', &c.; quæ nihilo evadunt æquales, cum $x = d, e$, &c. reſpective: ſint H, I, K, L, &c. valores quantitatum $p' \times p'' \times p''' \times p'''' \times$ &c., $q \times q'' \times q''' \times q'''' \times$ &c., $r \times r' \times r''' \times r'''' \times$ &c., $s \times s' \times s'' \times s'''' \times$ &c., &c., cum x evadat a, b, c, d, &c., reſpective; tum aſſumatur $y = \frac{A}{\alpha} \times \frac{p' \times p'' \times p''' \times \&c.}{H} \times S^{a} + \frac{B}{\beta} \times \frac{q \times q'' \times q''' \times \&c.}{I} \times S^{b} + \frac{C}{\gamma} \times \frac{r \times r' \times}{K}$ $\frac{r'' \times \&c.}{} \times S^{c} + \&c.$; & S^{a}, S^{b}, S^{c}, S^{d}, &c. erunt valores quantitatis (y), qui correſpondent valoribus a, b, c, d, &c. quantitatis x reſpective.

THEOR. LIX.

Sit æquatio, quæ ſit functio quantitatum $(x, y, z$, &c.$)$, in quâ continentur (n) coefficientes ad libitum aſſumendæ; dentur etiam (n)

correſpondentes valores ſingularum incognitarum quantitatum (x, y, z, &c.); invenire coefficientes prædictas.

Pro incognitis quantitatibus (x, y, z, &c.) ſcribantur earum (n) correſpondentes valores, reſultant (n) æquationes totidem (n) incognitas quantitates, i. e. (n) coefficientes ad libitum aſſumendas, habentes; e quibus detegi poſſunt coefficientes ipſæ.

2. Sint (m) æquationes ($n+m$) incognitas quantitates (x. y, z, &c.) habentes, & in ſingulis hiſce æquationibus contineantur (r) incognitæ coefficientes ad libitum aſſumendæ; tum ex datis (r) correſpondentibus valoribus ſingularum incognitarum quantitatum (x, y, z, &c.) per methodum prius traditam erui poſſunt coefficientes ipſæ.

PROB. XLIX.

Invenire quantitatem, quæ evadat reſpective S, S', S'', S''', &c.; *cum* $x=\alpha$, $y=\beta$, $z=\gamma$, &c.; $x=\alpha'$, $y=\beta'$, $z=\gamma'$, &c.; $x=\alpha''$, $y=\beta''$, $z=\gamma''$, &c.; &c.

Aſſumantur quæcunque functiones (H, K, L, &c.; A, B, C, D, &c.) quantitatum (x, y, z, &c.); in functionibus H, K, L, &c. pro incognitis quantitatibus (x, y, z, &c.) ſcribantur reſpective α, β, γ, &c.; α', β', γ', &c.; α'', β'', γ'', &c.; α''', β''', γ''', &c.; &c.; & reſultent quantitates h, k, l, &c. reſpective: in functione (A) pro incognitis quantitatibus (x, y, z, &c.) ſcribantur prædictæ quantitates α, β, γ, &c.; α', β', γ', &c.; α'', β'', γ'', &c.; &c.; reſultent quantitates a, a', a'', a''', &c.: & ſimiliter in functionibus B, C, D, &c.; ſcribantur pro incognitis (x, y, z, &c.) prædictæ quantitates & reſultent quantitates b, b', b'', &c.; c, c', c'', &c.; d, d', d'', &c. Quantitas quæſita poteſt eſſe

$$\frac{H}{h}\times\frac{(A-a')\times(A-a'')\times(A-a''')\times\&c.}{(a-a')\times(a-a'')\times(a-a''')\times\&c.}\times S+\frac{K}{k}\times\frac{(B-b)\times(B-b'')\times(B-b''')\times\&c.}{(b'-b)\times(b'-b'')\times(b'-b''')\times\&c.}\times S'$$
$$+\frac{L}{l}\times\frac{(C-c)\times(C-c')\times(C-c''')\times\&c.}{(c''-c)\times(c''-c')\times(c''-c''')\times\&c.}\times S''+\&c.$$

Cor. Si requiratur, ut quantitas detegenda ſit maxime ſimplex; tum pro H, K, L, &c. aſſumatur invariabilis quantitas; & pro A, B,

C, &c. aſſumantur functiones quantitatum (x, y, z, &c.) hujuſce formulæ $ex + fy + gz +$ &c. $+ d$; ubi literæ e, f, g, &c. & d invariabiles quantitates reſpective denotant.

PROB. L.

Datâ nonnullorum caſuum reſolutione, & formulâ in quâ generalis continetur; eam detegere.

Primo aſſumatur caſus, ſi modo fieri poſſit, in quo una ſolummodo continetur incognita quantitas, & ex eo caſu deduci poteſt incognita quantitas: & ſic de ſingulis reliquis incognitis quantitatibus, i. e. continuo inveniantur caſus, in quibus una reliqua vel ea cum quibuſdam prius inventis ſolummodo contineatur, & exinde deduci poteſt prædicta reliqua quantitas: ſi vero non innoteſcat methodus inveniendi caſum, in quo una ſolummodo incognita quantitas contineatur; inveniantur duo independentes caſus, in quibus duæ ſolummodo contineantur incognitæ quantitates: & ſi haud dentur duo independentes caſus, in quibus ſolummodo contineantur quæcunque duæ incognitæ quantitates cum prædictis inventis, inveniantur tres caſus, in quibus continentur tres incognitæ quantitates cum prius inventis; & ſic deinceps: & exinde detegi poſſunt incognitæ quantitates in reſolutione contentæ, & conſequenter reſolutio quæſita.

Ex. Sit formula $V = \frac{x^2z + bxy + 1}{fx + gz}$; & detur caſus, cum x & y nihilo ſint reſpective æquales, viz. ſit $z = 3$ & $V = 2$; tum reſultat $\frac{1}{gz} = V$, & exinde $g = \frac{1}{z \times V} = \frac{1}{2 \cdot 3}$: 2$^{\text{do}}$. detur caſus cum $y = 0$ & $x = 5$, viz. ſit $z = 4$ & $V = 3$, tum erit $V = \frac{x^2z + 1}{fx + \frac{1}{6}z} = \frac{25z + 1}{fx + \frac{4}{6}}$ $= 3$, & exinde $f = \frac{101 - 2}{15} = 6\frac{3}{5}$; & ſimiliter ſint x, y & z reſpective 1, 2 & 3 cum $V = 6$, tum erit $6 = \frac{3 + 2b + 1}{6\frac{3}{5} \times 1 + \frac{1}{6} \times 3}$; & exinde $b = 19\frac{3}{10}$.

Cor. 1. Hoc problema ad reſolutionem quam plurimorum problematum applicari poteſt, nam in multis caſibus facile innoteſcet formula prædicta.

Ex. 1. Sit æquatio $x^n - px^{n-1} + qx^{n-2} - rx^{n-3} + sx^{n-4} -$ &c. $= o$, cujus radices ſint $\alpha, \beta, \gamma, \delta$, &c.; invenire æquationem cujus radices (z) ſunt $\alpha + \beta, \alpha + \gamma, \alpha + \delta, \beta + \gamma, \beta + \delta, \gamma + \delta$, &c.: formula æquationis erit $z^{n \cdot \frac{(n-1)}{2} = m} + apz^{m-1} + (a'p^2 + bq)z^{m-2} + (a''p^3 + b'pq + cr)z^{m-3} + (a'''p^4 + b''p^2q + c'pr + dq^2 + es)z^{m-4} +$ &c. $= o$; ſinguli enim termini eidem poteſtati (z^{m-x}) quantitatis (z) annectendi eaſdem (α) habent dimenſiones quantitatum p, q, r, s, &c.; ſi modo dimenſiones quantitatum p, q, r, s, &c. ſint reſpective 1, 2, 3, 4, &c. 1^{mo}. Aſſumatur æquatio $x^n - x^{n-1} = o$, in quâ q, r, s, &c. deſunt & $p = 1$, tum erunt valores quantitatis (x) reſpective 1, 0, 0, 0, &c., & conſequenter $(n-1)$ valores quantitatis (z) erunt 1, cæteræ vero nihilo æquales; unde æquatio erit $z^m - (n-1)z^{m-1} + (n-1) \cdot \frac{n-2}{2} z^{m-2} - (n-1) \cdot \frac{n-2}{2} \cdot \frac{n-3}{3} z^{m-3} +$ &c. $= o$; & exinde $a = -(n-1)$, $a' = (n-1) \cdot \frac{n-2}{2}$, $a'' = -(n-1) \cdot \frac{n-2}{2} \cdot \frac{n-3}{3}$, &c.: 2^{do}. aſſumatur $x^n - x^{n-2} = o$, ubi $q = -1$ & p, q, r, s, &c. deſunt, cujus radices ſunt 1 & -1, & 0, 0, 0, &c.; unde $(n-2)$ valores quantitatis z erunt 1, & $n-2$ etiam -1; cæteræ vero erunt nihilo æquales; & æquatio erit $z^m - (n-2)z^{m-2} + (n-2) \cdot \frac{n-3}{2} z^{m-4} -$ &c. $= o$; unde b, d, &c. coefficientes terminorum q, q^2, q^3, &c. erunt reſpective $-(n-2)$, $(n-2) \cdot \frac{n-3}{2}$, $-(n-2) \cdot \frac{n-3}{2} \cdot \frac{n-4}{3}$, &c.: & ſimiliter ex aſſumptâ æquatione hujuſce formulæ $x^n - px^{n-1} + qx^{n-2} = o$, cujus duæ radices $(\alpha$ & $\beta)$ innoteſcunt & cæteræ nihilo ſunt æquales, detegi poſſunt coefficientes terminorum pq & p^2q^2; & ex duabus æquationibus prædictæ formulæ $x^n - px^{n-1} + qx^{n-2} = o$, quarum $(n-2)$ radices in duabus reſpectivis æquationibus nihilo ſunt æquales, duæ vero reliquæ innoteſcunt & non ſunt eædem in utrâque æquatione, deduci poſſunt coefficientes terminorum p^3q & pq^2; p^4q & p^2q^2: & ſimiliter ex tribus diverſis æquationibus prædictæ formulæ detegi poſſunt coefficientes terminorum p^5q, p^3q^2, pq^3; p^6q, p^4q^2, p^2q^3: & ſic deinceps: eadem methodus etiam applicari poteſt ad terminos, in quibus continentur literæ p, q, r, &c.

THEOR. LX.

Nonnullæ coefficientes unius erui poſſunt e coefficientibus alterius caſus; quod plerumque fieri poteſt, cum unus ſit particularis caſus alterius; vel quendam nexum habent inter ſe duorum caſuum quædam coefficientes.

Deducantur coefficientes terminorum in unâ quantitate, & e datâ relatione deduci poſſunt coefficientes alterius quantitatis.

Ex. Sit quantitas A in quâ continentur literæ a, b, c, d, &c.; ſit etiam quantitas B, in quâ omnes literæ a, b, c, d, &c. non involventes literas p, q, r, &c. ſimiliter involvuntur ac in quantitate A; tum deducatur eadem functio quantitatum A & B; quæ ſit E & F; in his quantitatibus omnes literæ, quæ non involvunt literas p, q, r, s, &c.; ſimiliter involventur.

Ex. 1. Sit æquatio data inferioris ordinis $x^4 + qx^2 - rx + s = 0$, cujus radices ſint α, β, γ, δ; & æquatio, cujus radices ſint $\alpha + \beta$, $\alpha + \gamma$, $\alpha + \delta$, $\beta + \gamma$, $\beta + \delta$, $\gamma + \delta$, erit $v^6 + 2qv^4 + (q^2 - 4s)v^2 - r^2 = 0$.

Ducatur data æquatio $x^4 + qx^2 - rx + s = 0$ in ſimplicem æquationem $(x = 0)$, cujus radix eſt (0); & ſit æquatio $x^5 + qx^3 - rx^2 + sx = 0$, cujus radices erunt α, β, γ, δ, 0; & radices æquationis ſimiliter exinde reſultantis erunt $\alpha + \beta$, $\alpha + \gamma$, $\beta + \gamma$, $\alpha + \delta$, $\beta + \delta$, $\gamma + \delta$, α, β, γ, δ; & æquatio ipſa $(v^4 + qv^2 - rv + s) \times (v^6 + 2qv^4 + (q^2 - 4s) \times v^2 - r^2) = 0$; cujus æquationis omnes termini iidem erunt, ac termini æquationis ſimiliter reſultantis e datâ æquatione $x^5 + qx^3 - rx^2 + sx - t = 0$; in quibus continentur ſolummodo literæ q, r, s.

Per æquationem ſimiliter reſultantem in hoc caſu deſignatur æquatio, cujus radices ſunt $\lambda + \mu$, $\lambda + \nu$, $\lambda + \pi$, $\lambda + \rho$, $\mu + \nu$, $\mu + \pi$, $\mu + \rho$,

$\mu + \rho$, $\nu + \pi$, $\nu + \rho$, $\pi + \rho$; ſi modo λ, μ, ν, π, ρ ſint reſpective radices æquationis $x^5 + qx^3 - rx^2 + sx - t = 0$.

SCHOLIUM.

Finis huic operi non eſt imponendus, priuſquam paucula de methodo reſolvendi hæc cum multis aliis problematibus adjiciantur, quæ dici poteſt methodus deductionis & reductionis; omnia enim ferè mathematica nihil aliud volunt, quam e datâ methodo deductionis quantitates reducere: per deductionis methodum deſigno quamcunque notam operationem; E. g. vel quantitatum additionem, ſubductionem, multiplicationem, diviſionem, extractionem earum radicum, &c. quoniam hæc operatio bene nota eſſe ſupponitur, ſemper perfici poteſt exinde deductio: ſi vero exigat problema, ut iterum iterumque repetatur data operatio, in multis caſibus propter calculi laborem ad reductionis methodum confugere haud inutile videtur; E. g. Sint $ax + bx^2 + cx^3 + \&c. = y$, $az + bz^2 + cz^3 + \&c. = x$, $av + bv^2 + cv^3 + \&c. = z$, $aw + bw^2 + cw^3 + \&c. = v$, &c. facile conſtat ut labor hujus calculi repetitis operationibus valde creſcit; & idem affirmari poteſt de inveniendis primis, ſecundis, tertiis, &c. fluxionibus quantitatis $(a + bx^n + cx^{2n} + \&c.) \times (e + fx^n + gx^{2n} + \&c.)^m$.

2. 1. Ex datâ quantitate deductâ & deductionis methodo ejus reductionem invenire, eſt problema maximè generale; hujuſce problematis particulares erunt ſubſequentes caſus, viz. ſit x quantitas invenienda, & a quantitas deducta; & methodus deductionis talis ſit, ut $Ax^n + Bx^{n+1} + Cx^{n+2} + \&c. = a$; reductio vero hujus quantitatis (a) ad quæſitam x eadem eſt, ac reſolutio algebraicæ æquationis $Ax^n + Bx^{n+1} + \&c. = a$. 2. Sit quantitas deducta quæcunque fluens, & data deductio ſit vulgaris methodus inveniendi fluxiones datarum fluentium, & methodus reductionis erit methodus inveniendi fluentem ex datâ fluxione: hoc vero problema plerumque reſolutionem recipiet ex aſſumptâ quantitate generali (cujus irrationalitas vel formula ferè deduci poteſt e datâ quantitate & datâ etiam deductionis methodo) pro quantitate quæſitâ; ſcribatur hæc quantitas pro ejus valore, & e datâ deductionis methodo inveniatur altera quantitas, quâ datæ quantitati æquali eſſe ſuppoſitâ, exinde erui poteſt problematis reſolutio: e. g. ſit data quantitas $(a^2 + 7x + x^2) \times (a^2 + x^2)^{\frac{5}{2}} = W$, cujus quantitas reducta ſit A, & methodus deductionis $A + \frac{\dot{A}}{\dot{x}}$, i. e. $A + \frac{\dot{A}}{\dot{x}} = W$; ſed ex hâc methodo conſtat quæſitam quantitatem A, ſi modo algebraice exprimi poſſit, eandem habere irrationalitatem ac quantitatem W; aſſumatur igitur $(a^2 + x^2)^{\frac{7}{2}} \times (p + qx + rx^2 + \&c.) = A$, & e prædictâ methodo inveniri poſſunt $p = 1$, $q = 0$, $r = 0$, &c. ergo quantitas quæſita erit $(a^2 + x^2)^{\frac{7}{2}}$; generalis autem reſolutio erit $\frac{\int . e^x (a^2 + 7x + x^2) \times (a^2 + x^2)^{\frac{7}{2}} \dot{x}}{e^x} + be^{-x} = A$, ubi b eſt quantitas ad libitum aſſumenda; unde, ſi generalis requiratur reſolutio quantitatis A, aſſumi debet reſolutio, quæ prædictam generalem continet.

Cor. Hinc conſtat e præcedenti ſubſtitutione deductum eſſe ſolummodo particularem valorem quantitatis A, quoniam ſubſtitutio ſolummodo eſt particularis & non continet in ſe omnes valores quantitatis A.

3. Conſimilia etiam principia applicari poſſunt ad reductionem datæ quantitatis in plures, quarum methodi deductionum dantur; & ſic de æquationibus inter quaſcunque quantitates: hinc ex datâ methodo deductionis erui poſſunt quælibet quantitates, quas reducere liceat; aſſumantur enim quæcunque quantitates & ex iis per datam methodum deductionis inveniantur quantitates quæſitæ.

4. Hìc vero animadvertendum eſt, ut talis ſit deductionis methodus, quæ (ſi modo eadem data quantitas ſub diverſâ ſpecie lateat) diverſam haud ſpecie ſed re præbeat quantitatem deductam: e. g. ſit methodus deductionis talis ut e datâ quantitate $ax^n y^m$ deducatur quantitas $n^2 a x^{n-1} y^m + m^2 a x^n y^{m-1}$; ſit data quantitas x^{n+m}, & quantitas per prædictam methodum inveſtigationis inventa erit $(n + m)^2 x^{n+m-1}$; at $x^{n+m} = x^n \times x^m$, & quantitas per eandem methodum inventa $n^2 x^{n-1} \times x^m + m^2 x^n \times x^{m-1} = (n^2 + m^2) x^{n+m-1}$, ſed $n^2 + m^2$ non eſt $(n + m)^2$, ergo ex eâdem quantitate x^{n+m} ſub diverſâ ſpecie $(x^n \times x^m)$ latente per hanc methodum deductionis diverſæ exorientur quantitates,

tes, & methodus deductionis habet relationem ad qualitatem æque ad quantitatem, ni ex aliis datis ad finitum reſponſionum numerum reſtinguatur quæſtio propoſita: ſi modo talis ſit methodus deductionis, ut eædem quantitates ſub diverſis formulis latentes ſemper eaſdem præbeant quantitates; tum quantitas data utcunque in diverſam formulam vel etiam in infinitas approximationes transformari poteſt, & exinde omnes regulæ de approximationibus in hoc & præcedente libro traditæ mutatis mutandis ad hunc caſum applicari poſſunt.

5. Sæpe ex repetitis n operationibus & animadverſione methodorum, e quibus formantur termini, conſtabit lex, quam obſervat ſeries, quæ exprimit quantitatem deductam per operationem n vicies repetitam, quæ quantitas dicatur terminus ad n diſtantiam; pro n ſcribatur $\frac{r}{m}$, & invenietur ſeries, quæ exprimit quantitatem in eadem ſcalâ ad $\frac{r}{m}$ diſtantiam. E. g. Ex operatione inveniendi fluxiones datarum quantitatum ſæpius repetitâ conſtabit fluxio rectanguli xy ordinis vero $(n) = y\overset{n}{\dot{x}} + n\dot{y}\overset{n-1}{\dot{x}} + n.\frac{(n-1)}{2}\ddot{y}\overset{n-2}{\dot{x}} + n.\frac{n-1}{2}.\frac{n-2}{3}\dddot{y}\overset{n-3}{\dot{x}} + \&c.$ In hâc fluxione pro n ſcribatur $\frac{r}{m}$, & reſultat quantitas in eâdem ſcalâ ad $\frac{r}{m}$ diſtantiam poſita. Ex. 2. Fluxio r ordinis quantitatis x^n invenietur $n.(n-1).(n-2)\ldots(n-r+1)x^{n-r}\dot{x}^r + r.\frac{r-1}{2}\times n.(n-1)\ldots(n-r+2)x^{n-r+1}\dot{x}^{r-2}\ddot{x} + r.\frac{r-1}{2}.\frac{r-2}{3}\times n.(n-1).(n-2)\ldots(n-r+3)x^{n-r+2}\dot{x}^{r-3}\dddot{x} + \&c.$ quæ etiam ita ſcribi poteſt $nx^{n-1}\overset{r}{\dot{x}} + r\times n.(n-1)x^{n-2}\dot{x}\overset{r-1}{\dot{x}} + r.\frac{r-1}{2}\times n$

$.$ $+ r.\frac{n-1}{2}.n$

$(n-1).(n-2)x^{n-3}\overset{r-2}{\dot{x}}\dot{x}^2 + \&c.$ Sit terminus $\overset{a}{\dot{x}}{}^{\alpha}\times\overset{b}{\dot{x}}{}^{\beta}\times\overset{c}{\dot{x}}{}^{\gamma}\times\overset{d}{\dot{x}}{}^{\delta}\times$

$(n-1)x^{n-2}\overset{r-2}{\dot{x}}\ddot{x}$

&c.;

&c.; ubi $a\alpha + b\beta + c\gamma +$ &c. $= r$; tum ejus coefficiens erit $N \times n \times (n-1).(n-2) \times (n-3) \dots (n-t+1)$, ubi $t = \alpha + \beta + \gamma + \delta +$ &c.; & N numerus qui eſt functio quantitatis r & prædictorum numerorum $a, \alpha, b, \beta, c, \gamma,$ &c. facile deducenda: in hâc ſerie pro r ſcribatur $\frac{k}{l}$, & conſtabit lex quam obſervat quantitas ad diſtantiam $\frac{k}{l}$ in ſcalâ fluxionum poteſtatis x^n poſita.

Sit r negativus numerus & ſeries prædicta denotet quantitatis x^n fluentes; ſed hic animadvertendum eſt, quodſi numerus factorum ſit negativus, tum deſignat factores in denominatore contentos, ſi numerus factorum affirmativus denotet factores in numeratore. E. g. Requiratur fluens r ordinis quantitatis x^n, hic r eſt negativus numerus, & numerus factorum $(n . n-1 \dots (n-r+1))$ eſt r; ergo numerus factorum in denominatore contentorum erit r, & ultimus factor $(n-r+1)$ erit in hoc caſu $(n+r)$, & ſeries prædicta evadet

$$\frac{x^{n+r}}{n+1 . n+2 \dots (n+r) . \dot{x}^r} + r . \frac{r+1}{2} \times \frac{x^{n+r+1}\ddot{x}}{(n+1).(n+2)\dots(n+r+1).\dot{x}^{r+2}} - r . \frac{r+1}{2} . \frac{r+2}{3} \times \frac{x^{n+r+2}\dddot{x}}{(n+1).(n+2)\dots(n+r+2) \times \dot{x}^{r+3}} + \&c.$$

hujuſce ſeriei lex e præcedente conſequitur: ſi vero requiratur terminus in ſcalâ fluentium hic traditarum quantitatis x^n ad diſtantiam $-\frac{k}{l}$ a primo poſitus; ſi $\frac{k}{l}$ ſit integer affirmativus numerus, tum ſolummodo requiruntur fluentes datæ quantitatis x^n; ſi vero $\frac{k}{l}$ ſit fractio ad minimos terminos reducta, tum inveniantur quantitates in ſcalâ ſubſequente (ubi diſtantiæ a primo ſint, &c. $-2, -1, 0, 1, 2,$ &c. & earum correſpondentes termini, &c., $\frac{1}{n+1 . n+2}, \frac{1}{n+1}, 1, n, n . (n-1)$, &c.) quarum diſtantiæ a primo termino ſint reſpective $-\frac{k}{l}$, $-\frac{k-1}{l}, -\frac{k-2}{l} \dots -\frac{k-l+1}{l}$; pro his l quantitatibus inventis ſcribantur A, B, C, D, &c. reſpective; tum erunt quantitates in eâdem ſcalâ, quarum diſtantiæ a ſecundo ſit $-\frac{k}{l}$, $-\frac{k-1}{l}$, $-\frac{k-2}{l}$, &c. reſpective $\left(n+\frac{k}{l}\right)A$, $\left(n+\frac{k-1}{l}\right)B$, $\left(n+\frac{k-2}{l}\right)C$, &c. & ſic deinceps; & quantitas quæſita erit $Ax^{n+\frac{k}{l}}\dot{x}^{-\frac{k}{l}}$, ſi $\overset{\frac{k}{l}}{\dot{x}}$ ſit conſtans quantitas.

Facile etiam conſtat, ſi modo ſit v quantitas, cujus fluxio $(\overset{\frac{1}{l}}{\dot{v}})$ ordinis $\frac{1}{l}$ vel quantitas in ſcalâ fluxionum ad diſtantiam $\frac{1}{l}$ a datâ fluente poſita ſit conſtans, ubi l ſit integer numerus; & ſit $\overset{r+\frac{1}{l}}{\dot{z}}$ datâ fluxio; eſſe fluentem $r+\frac{1}{l}$ ordinis $=$ (ubi r eſt integer numerus) $z + Av^{r} + Bv^{r-\frac{1}{l}} + Cv^{r-\frac{2}{l}} + \&c. \ldots Fv^{\frac{2}{l}} + Gv^{\frac{1}{l}} + H$, ubi A, B, $C \ldots F$, G, H ſunt invariabiles quantitates ad libitum aſſumendæ.

Ferè omnia etiam, quæ prius tradita fuere de fluxionibus, fluxionalibus æquationibus & earum multiplicatoribus, &c. ad haſce interpolationes applicari poſſunt: hìc forſan haud indignum eſt obſervatu, quod ſeries, cujus termini hanc habeant formulam Ax^{ez+f}, ubi z eſt diſtantia a primo ſeriei termino, & e & f ſunt invariabiles quantitates, & A algebraica functio vel rationalis vel irrationalis literæ z, ſemper exprimi poteſt vel per fluxionalem æquationem, vel per interpolationem ſeriei, quæ eſt reſolutio fluxionalis æquationis & conſequenter eſt reſolutio fluxionalis æquationis ordinis vero fractionalis: ſi enim A ſit rationalis functio quantitatis z, tum erit reſolutio fluxionalis æquationis; ſi vero radices rationalis functionis contineat quantitas A, tum plerumque recurrendum eſt ad interpolationem ſerierum ex fluxionalibus æquationibus exortarum.

Dimenſiones cujuſcunque incognitæ quantitatis in ſucceſſivis terminis datæ ſeriei per inæquales differentias augeantur vel diminuantur,

tur, tum ſumma ſeriei nec per finitam algebraicam nec fluxionalem æquationem exprimi poteſt.

Sit ſeries, cujui termini per hanc formulam Ax^{α} exprimuntur, ubi α eſt quæcunque functio quantitatis z diſtantiæ a primo ſeriei termino vel algebraica vel fluxionalis, vel incrementialis vel formulæ, cujus differentia inter numerum factorum in numeratore & denominatore contentorum haud eadem manet; e prædictis principiis, i. e. terminis ad infinitam diſtantiam conſtitutis, inveniri poteſt, utrum convergit, necne; etiamque annon exprimi poteſt per algebraicam vel fluxionalem, &c. æquationem.

De hâc deductionum & reductionum doctrinâ, i. e. ſubſtitutionibus mentionem fieri hîc cenſui conſilio, ut via in hâc maxime generali ſcientiâ ad multo majora ſternatur.

MVSEVM
BRITAN
NICVM

CORRIGENDA.

SÆPE pro in lege duct. vel multiplicat. in, & poſt verba reduct. & reſolv. pro in lege ad, & pro incrementialis, lege incrementalis, & pro integralis, lege integrale, & nonnunquam pro fiunt vel fiant, lege evadunt vel evadant, &c.; pag. 18. lin. 11. pro β & α, lege β' & α'; l. 12. pro $\frac{1}{\quad}$, lege $\frac{x}{\quad}$; pag. 19. l. 13. pro poteſt, lege poteſt, necne; pag. 20. l. 17. pro (v), lege (v), necne; p. 26. l. antepenult. pro P, lege $\dot{P}$; pag. 45. l. 27. pro affirmativa, lege negativa; pag. 53. l. 10. pro bx, lege bx^n; pag. 56. l. 8. pro v, lege v, l, t, &c.; l. antepenult. pro fluxione, lege fluxio; pag. 58. l. 13. dele $)^t$, & pro U, lege U'; pag. 60. l. ultim. pro $n+2$, lege $n+1$, & pro $m-1$, lege $m+1$; pag. 61. l. 1. pro $n-2$, lege $n+1$; pag. 64. l. 8. pro par, lege impariter par; l. 15. pro $\pm p$, lege $\mp p$; l. antepenult. & penult. pro αx, βx & γx, lege $2\alpha x$, $2\beta x$ & $2\gamma x$; pag. 68. l. 11. pro ſingularium, lege ſingularum; pag. 71. l. 1. pro $\dot{x}^n$, lege x^n; pag. 72. l. 1. pro , cujus, lege in $\dot{v}$ ducta, cujus; pag. 73. l. 16. pro $Q\times$, lege $Q\times(\quad)$; pag. 74. l. 15. poſt $(\pi, \rho, \sigma, \tau$, &c.), lege etiamque $(\pi^2, \rho^2, \sigma^2$, &c., π^3, ρ^3, σ^3, &c., &c.); pag. 75. l. 8. pro ſuos, lege ejus; pag. 77. l. 3. pro $\ldots +$, lege $\ldots\,.+E$; pag. 80. l. 20. pro fluxione, lege fluxionem; pag. 81. l. 20. pro $\dot{z}$, lege $z^{-1}\dot{z}$; pag. 82. l. 6. pro functio, lege rationalis functio; pag. 83. l. 8. dele V; l. antepenult. pro ſumma, lege earum ſumma; pag. 86. l. 5. pro x ſemel, lege z; pag. 95. l. 27. pro $m\pm r$, lege $m\pm\lambda$; pag. 97. l. 9. pro reducendæ, lege reducantur; pag. 98. l. 26. pro s, lege ſ; pag. 99. l. 19. pro X, lege $\dot{X}$; pag. 100. l. 16. pro σ & υ, lege σ, τ & υ; pag. 106. l. 6. pro $\dot{y}$, lege y; pag. 110. l. 15. pro pro unde, lege unde $\frac{P}{Q}$; pag. 124. l. 18. pro $=\dot{o}$, lege $=o$, &c.; pag. 131. l. 4. pro exigat, lege exigere; pag. 132. l. 13. pro δx, lege δy; pag. 143. l. 7. pro coefficientibus, lege coefficientibus correſpondentium terminorum; l. 17. pro $\dot{x}$, lege x; pag. 149. l. 10. pro ſuo, lege ejus; pag. 150. l. 23. pro utriſque partibus, lege ad utraſque partes; pag. 152. l. 24. & pag. 153. l. 4. pro alteri parti, lege ad alteram partem; l. 22. pro fluenti, lege ad fluentem; pag. 157. l. 23. pro $\dot{V}'$, lege $\dot{V}'=o$; pag. 160. l. ultim. pro ſ. $X\dot{x}$, lege $X\dot{x}$; pag. 165. l. 23. pro $n-m$, lege m; pag. 166. l. 7. pro Q, lege $-Q$; l. 7. pro $-r)=o$, lege $-r))=o$; pag. 171. l. 21. pro fluxionum, lege fluentes fluxionum; pag. 172. l. 17. pro $m-1+1$, lege $m-n+1$; l. 19. pro æquatione, lege quantitate; pag. 173. l. 17. pro prius, lege poſtea; pag. 180. l. 5. pro $)2$, lege $)^2$; l. 6. pro $R\ \dot{R}\ \dot{P}\ \ddot{P}$, lege $\dot{R}\ \dot{P}\ \ddot{P}$; pag. 184. l. 2. pro $\ddot{\pi}$, lege π; pag. 193. l. 24. pro x^n, lege $\dot{x}^n$ & $\dot{y}^n$; pag. 194. l. 19. $\frac{q\dot{x}}{q}$, lege $\frac{q\dot{x}}{p}$; pag. 200. l. 25. pro x, lege $\dot{x}$; pag. 205. l. 1. pro $x^{l+n-b-1}$, lege $x^{l+n-b-1}+$ &c.; l. 20. pro vel, lege aliter; pag. 206. l. antepenult. dele $\dot{x}$; pag. 207. l. penult. pro $X\dot{x}$, lege $X\dot{x}^2$; pag. 210. l. 8. pro notas, lege datas, & pro inveniatur, lege tum invenietur; pag. 212. l. 6. pro $X\dot{y}^2$, lege $\dot{X}\dot{y}^2$; l. 10. pro Ppx, lege $Pp\dot{x}$; l. 15. pro $Pq\dot{x}$, lege $Pq\dot{y}$; pag. 213. l. 6. pro $\rho\dot{y}$, lege $\rho\ddot{y}$, & pro $\dot{\sigma}$, lege σ; l. ultim. pro τM, lege τM; pag. 225. l. 2. pro habet, lege habent; pag. 231. l. 7. pro xz, lege x^2; pag. 237. l. 7. pro transformantur, lege transformare; l. 8. pro reducatur æquatio in, lege reducere æquationem ad; pag. 239. l. 6. pro prædictis, lege e prædictis; pag. 254. l. 2. pro b^2-a, lege b^2-ca; l. 4. pro equat, lege æquat; pag. 260. l. 18. pro $+K$, lege $\times I$; l. 19. pro I, lege H; l. 20. pro H, lege G; pag. 263. l. penult. collocentur n, $n-1$, $n-2$ ſupra $\dot{y}, \dot{y}, \dot{y}$; pag. 266. l. 6. pro e fluxionali æquatione (m) ordinis, lege e fluxionalibus æquationibus $(m-1, m-2, m-3$, &c.) ordinum; & pro deduci poteſt, lege deduci poteſt; in his duobus caſibus A, B, C, &c., N, M, L, &c.; p, q, r, &c. denotant datas functiones quantitatis x; pag. 268. l. 11 & 14. pro x, y, &c., lege p, x, y, &c.; l. 16. pro y, x & $\dot{x}$, lege p, y, x & $\dot{x}$; l. 17. pro x, lege p & x, & pro ſit variabilis, lege ſint variabiles; l. antepenult. pro $\beta\dot{y}$, lege $p\beta\dot{x}$; pag. 272. l. 6. pro $\dot{y}$ ſemel, lege y; pag. 275. l. 3. pro $\overset{n-1}{\dot{y}}$ & $\overset{n-2}{\dot{y}}$ ſemel lege $\overset{n-1}{\dot{z}}$ & $\overset{n-2}{\dot{z}}$; pag. 282. l. 8. pro $n+1$, n & $n-1$, lege n, $n-1$ & $n-2$; l. 13. pro $\dot{x}^{n-1}$,

CORRIGENDA.

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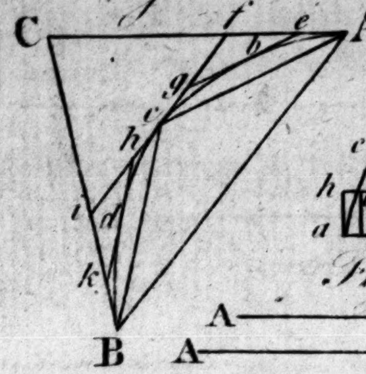
Fig. 1.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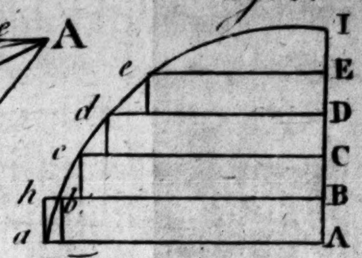
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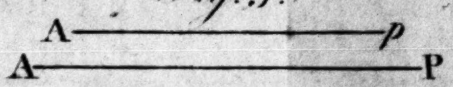

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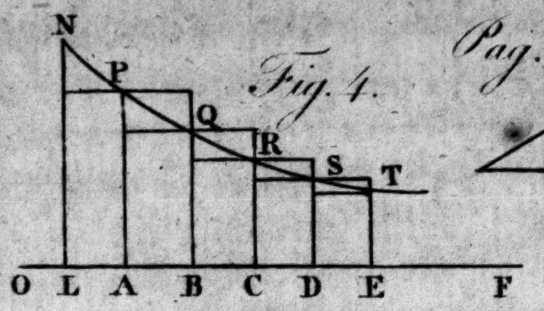
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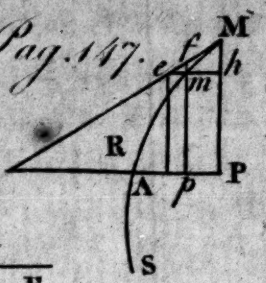
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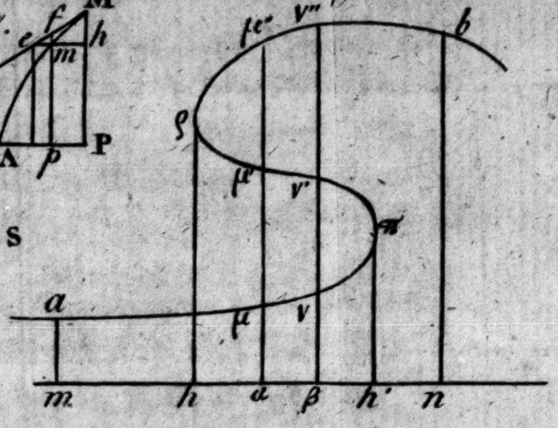
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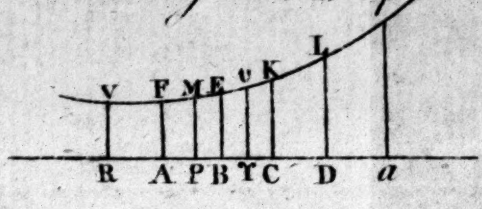
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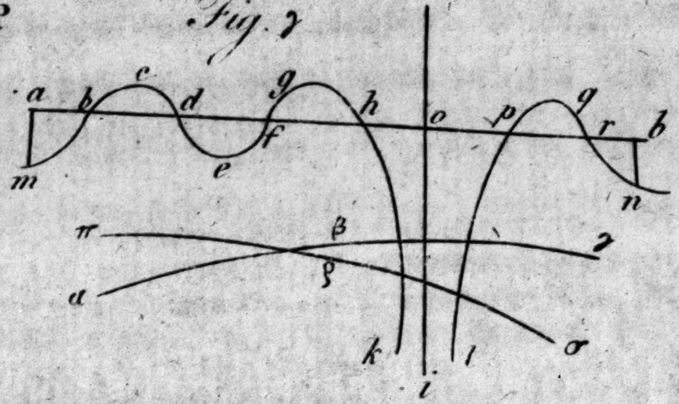
Fig. γ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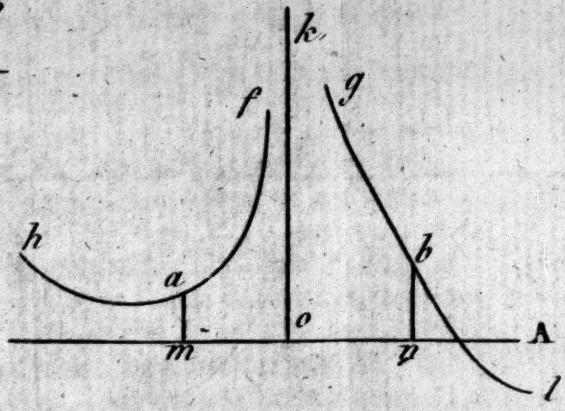
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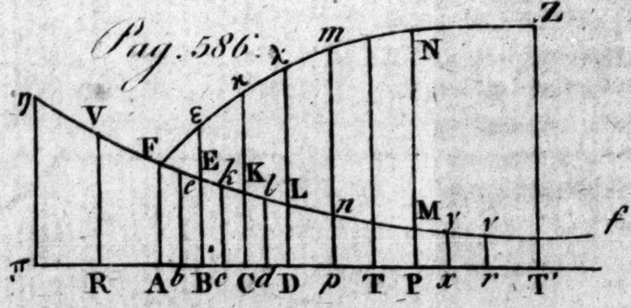
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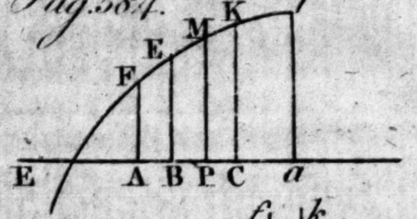
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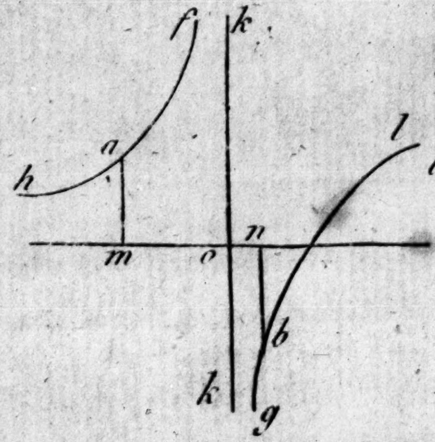
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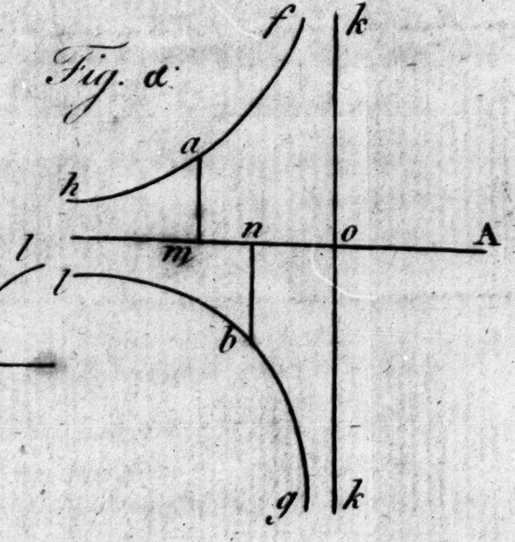
Fig. α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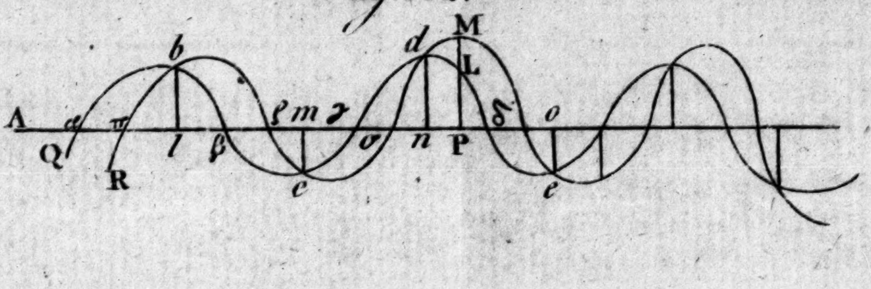
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